Tomorrow is the day after Doomsday

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Lots of people have produced rules for working out the day of the week corresponding to any given date. One need merely add components for the century, year of century, month, and day of month, reduce modulo 7 and then start counting at the right place. But since the month components are essentially random numbers most people soon forget the rule. [In the version known as Zeller's congruence the month numbers are produced by a uniform formula, but this is just complicated enough to be easily forgotten.]

The Doomsday rule is one I worked out last year in an attempt to overcome these difficulties. In it one computes Doomsday for the given year and then computes dates in that year relative to Doomsday. The rule has the additional advantage that when one knows Doomsday for the year one has in effect the complete calendar for that year at one's fingertips — so all the man in the street need do is remember to update Doomsday at about the time he remembers to put the new year on his cheques.

Doomsdays in a given year

Doomsday for a given year is defined to be the day of the week on which the last day of February falls. For 1973, Doomsday is Wednesday: The displayed table shows how to find a Doomsday in any given month. In February we see that a Doomsday is Feb 28 or 29 according as the year is ordinary or leap. In January, a Doomsday is Jan 31 or 32 in the same circumtances, (Of course Jan 32 = Feb 1, but we should think of it as Jan 32.)

Otherwise 2 Doomsday in the nth month is the nth day, if n is even, and the (n±4)th day, if n is odd. The sign is + for long odd months (31 days), and – for short ones (30 days), and it is fairly easy to remember than the only short odd months are September and November.

	DOOMSDAYS
Jan	31/32
Feb	28/29
Mar	3 + 4 = 7
Apr	4
May	5 + 4 = 9
June	6
July	7 + 4 = 11
Aug	8
Sep	9 – 4 = 5
Oct	10
Nov	11 – 4 = 7
Dec	12

Summary: 'Last' in Jan and Feb, otherwise nth in even months, (n±4)th in odd ones.

By adding and subtracting 7s we can find other Doomsdays in these months, and then by the "last-friday-was-the-twentyfirst-so-today-is-the-twentyfifth" technique we can locate any particular date.

Examples

August 19 1973? August is the 8th month, so that August 8th, and therefore August 22nd are Doomsdays (Wednesdays in 1973), so August 19th is a Sunday.

September 24th? September is the 9th month, and is short, so September 9 – 4 and September 26 are Doomsdays, so September 24th 1973 is a Monday.

II you do things exactly this way you will gradually remember more and more Doomsdays throughout the year. Don't say things like Sep 5 = Doomsday, 24 – 5 = 19 –2 (mod 7), so Sep 24 = Doomsday – 2. This kind of calculation

is prone to sign errors and does not help you to accumulate more mental Doomsdays.

Doomsdays for the century years.

One first needs to know Doomsdays for the century years. All the practical man need know is that Doomsday for 1900 was a Wednesday. (We say simply "1900 was a Wednesday".) However, we assert that in the Julian system (which was used in England before September 1752) the years 0, 700, 1400, 2100, ... are Sundays (they were more Godly then!), and that each century after one of these retards Doomsday by 1 day. In the Gregorian system (after September 1752) 0, 400, 800, 1200, 1600, ... are Tuesdays, and each century after the most recent of these retards Doomsday by 2 days. In particular, 1900 = 3 centuries past 1600 = Tuesday – 6 = Wednesday, as we asserted.

Doomsdays for years in a given century.

Each ordinary year has its Doomsday 1 day later than the previous year, and each leap year 2 days later. It follows that within any given century a dozen years advances Doomsday by 12 + 3 = 15 days = 1 day. ("A dozen years is but a day.") So we add to the Doomsday for the century year the number of dozens of years thereafter, the remainder, and the number of fours in the remainder. It is easiest to say these numbers aloud, as in the examples. Remember, 1900 = Wednesday.

1946? We have 46 = 3 dozen and 10, so we say

"1946 = Wednesday, 3 dozen, 10, and 2 = Thursday."
(Thursday being found by adding 3, 10, and 2 to Wednesday.)

1973? Wednesday, 6 dozen and 1 = Wednesday, as we know.
1990? Wednesday, 8 dozen[1], 5, and 1 = Thursday.

1752 September 2 (Julian)?

[1] ?! sic.

1400 = Sunday, so 1700 = Sunday – 3 = Thursday, so
1752 = Thursday, 4 dozen, 4, and 1 = Saturday, so
September 5 would be Saturday, and September 2 = Wednesday.

1752 September 14 (Gregorian)?

1600 = Tuesday, so 1700 = Tuesday – 2 = Sunday, so
1752 = Sunday, 4 dozen, 4, and 1 = Tuesday, so
September 5 and 12 would be Tuesdays, so September 14 = Thursday.

In fact in this country Wednesday September 2 and Thursday September 14 1752 were consecutive days, since this was the year we changed from Julian to Gregorian.

Note on changes in the calendar.

The Julian system in which every fourth year has an extra day was introduced by Julius Caesar on the advice of the astronomer Sosigenes. For some time the calendar had been at the mercy of Roman officials who more or less arranged things to suit themselves, and there had, for instance, even been one year with a month of 45 days. Sosigenes recommended a regular alternation of 30 and 31 day months which would have been adhered to had not both Julius and Augustus Caesar needed the months named after them to have 31 days, which was achieved by breaking the regular alternation and shortening February still more.

The Gregorian system, in which years divisible by 100 but not 400 are not leap years was introduced by Pope Gregory XII. Roman catholic countries changed in 1582, but protestant countries resisted this piece of popery for several hundred years, and then changed at various times. Some eastern European countries changed only this century, Sweden managed the change most elegantly, by simply omitting all leap years between 1800 and 1840 inclusive. So in working out dates for the intervening Period, one must be sure where one's problem originated.

The reason for the change was of course that the astronomical year has 365.2422 days rather than 365 ¼, and the inaccuracy had gradually accumulated until it was 10 or 11 days. There is still a residual inaccuracy which many people

have remarko would be partly cured by making the years divisible by 4000 not leap years, but with any luck the whole ungainly system will be dead by then!)

Another annoyance is that the conventional starting date for the year has not always been January 1 (as we have supposed in the Doomsday rule). A number of different dates have been used at various times, even in this country. Jan 1 and Dec 25 (of what we should call the previous year) were both used at about the end of the first millennium, but March 25 then became more or less universal. So for instance March 24 1583 and March 25 1584 were consecutive days.

This convention for starting the year is the Old Style, the January 1 convention being the New Style. Unfortmately these terms are offen ueed incorrectly to refer to the Julian and Gregorian systems, since in fact the Act of Parliament establishing the Gregorian system in England also finally decreed that the New Style was henceforth to be used for all legal purposes.

In fact the change from Old to New style dating had been accomplished long before. From about 1600 to 1700 opinion had gradually hardened in favour of the New Style. In the changeover period we usually find the double dating convention — thus February

14 166 ⅝ denotes the date which would be Feb 14 1665 (Old Style), and Feb 14 1666 (New Style). The situation is further complicated by the fact that dates in history hooks have often been transposed into New Style even when they refer to periods when Old Style was the only one in use. Also, dates in English history books for the deaths of French kings (say) might be in either the Julian or Gregorian system for the period between 1582 and 1752, according as the original source was English or French.

The fact that for various financial purposes the year starts on April 5 is an interesting consequence of the various changes. Originally it started in the first day (March 25) of the calendar year. When New Style was adopted, this remained the start of the financial year, although no longer the start of the calendar year. When we changed from Julian to Gregorian, this became April 5, since obviously no one was to pay a full year's interest on a year that was eleven days short!

So apply the method for historical dates with some caution. But the rule really comes Into its own for dates within any given year. Throw your calendar away after a quick glance to find the Doomsday, for when you know Doomsday you will know it all! But when you impress your friends with the Doomsday rule, remember to give credit where eredit is due! I do this now by noting that I found the Doomsday rule by simplifying (almost beyond recognition!) a rule given by Lewis Carrol in Nature, 1872.