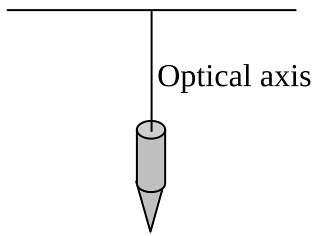
Basic Geometry of Photogrammetry

Topics to be Covered

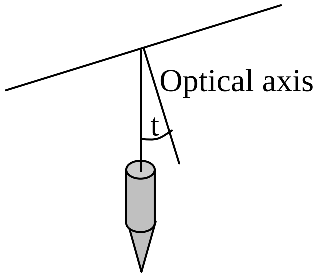
- Geometric types of airphotos
- Taking vertical airphotos
- Geometry of maps vs. airphotos
- Photographic scale
- Ground control
- Area measurement
- Object height determination
    - from relief displacement
    - from parallax measurement
- Mapping from photographs
    - stereoplotters
    - orthophotos
- Digital photogrammetry

Geometric Types of Airphotos

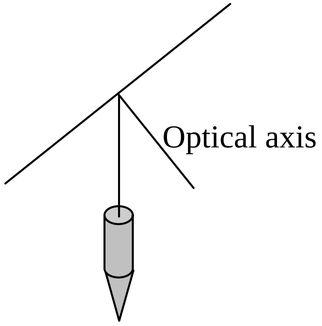
"Vertical" - taken with the optical axis of the camera in a vertical position



"Tilted" - produced by an unintentional inclination of the optical axis from vertical

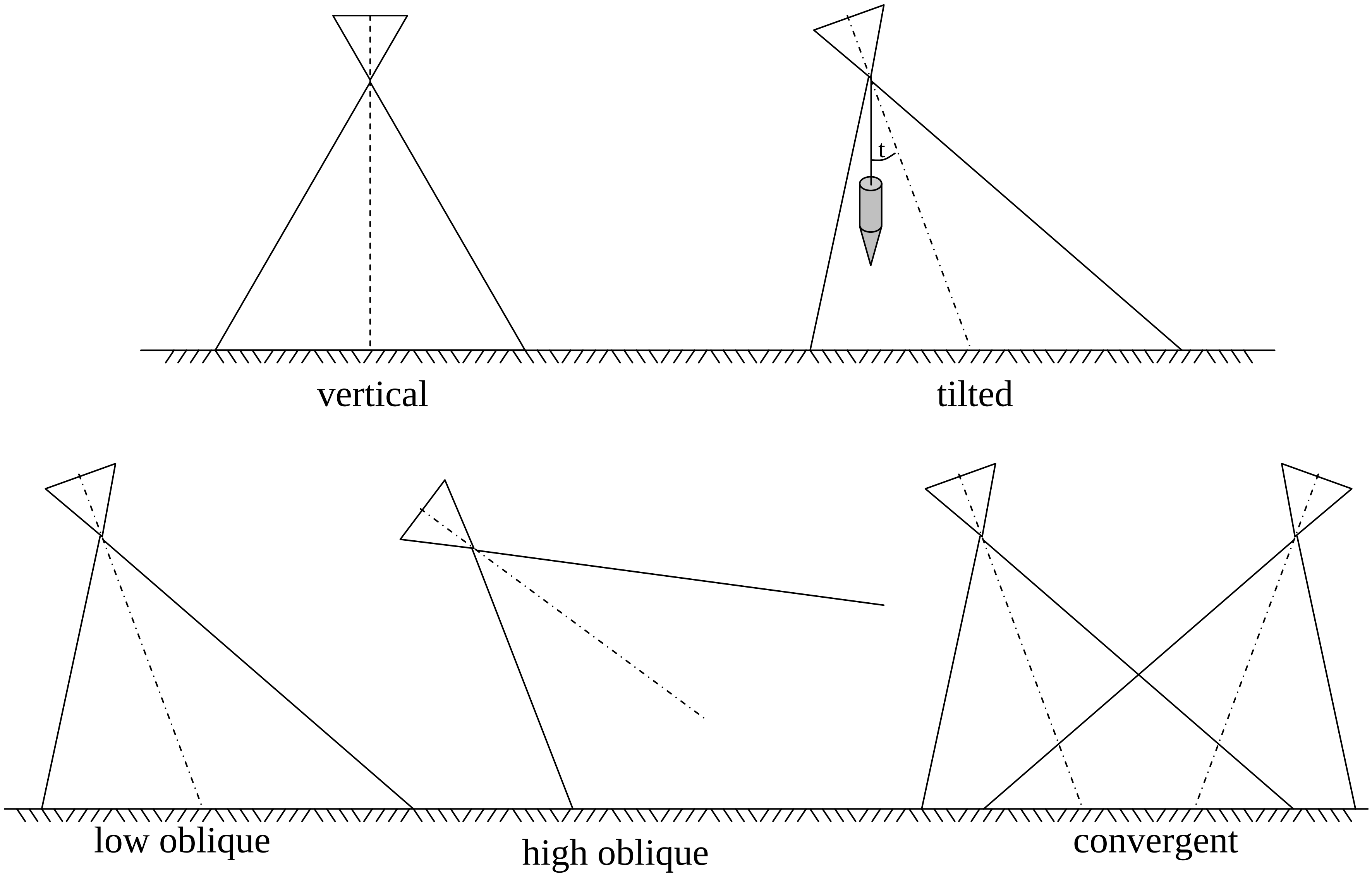


"Oblique" - produced when the optical axis is intentionally inclined from vertical



- high oblique: contains image of horizon
- low oblique: horizon not visible
- convergent obliques: sequential overlapping low obliques

Geometric Types of Airphotos



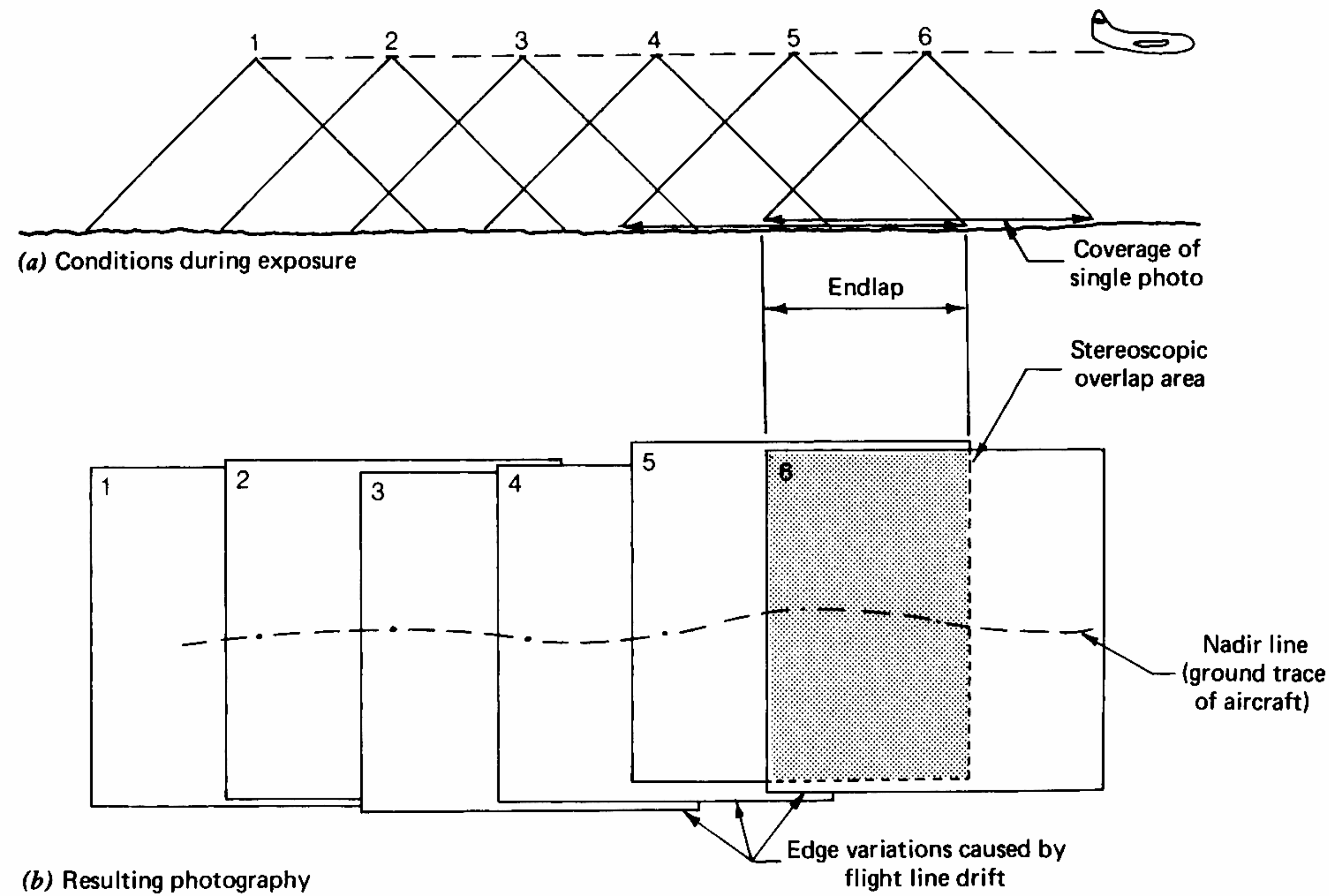


Figure 3.1 Photographic coverage along a flight strip.

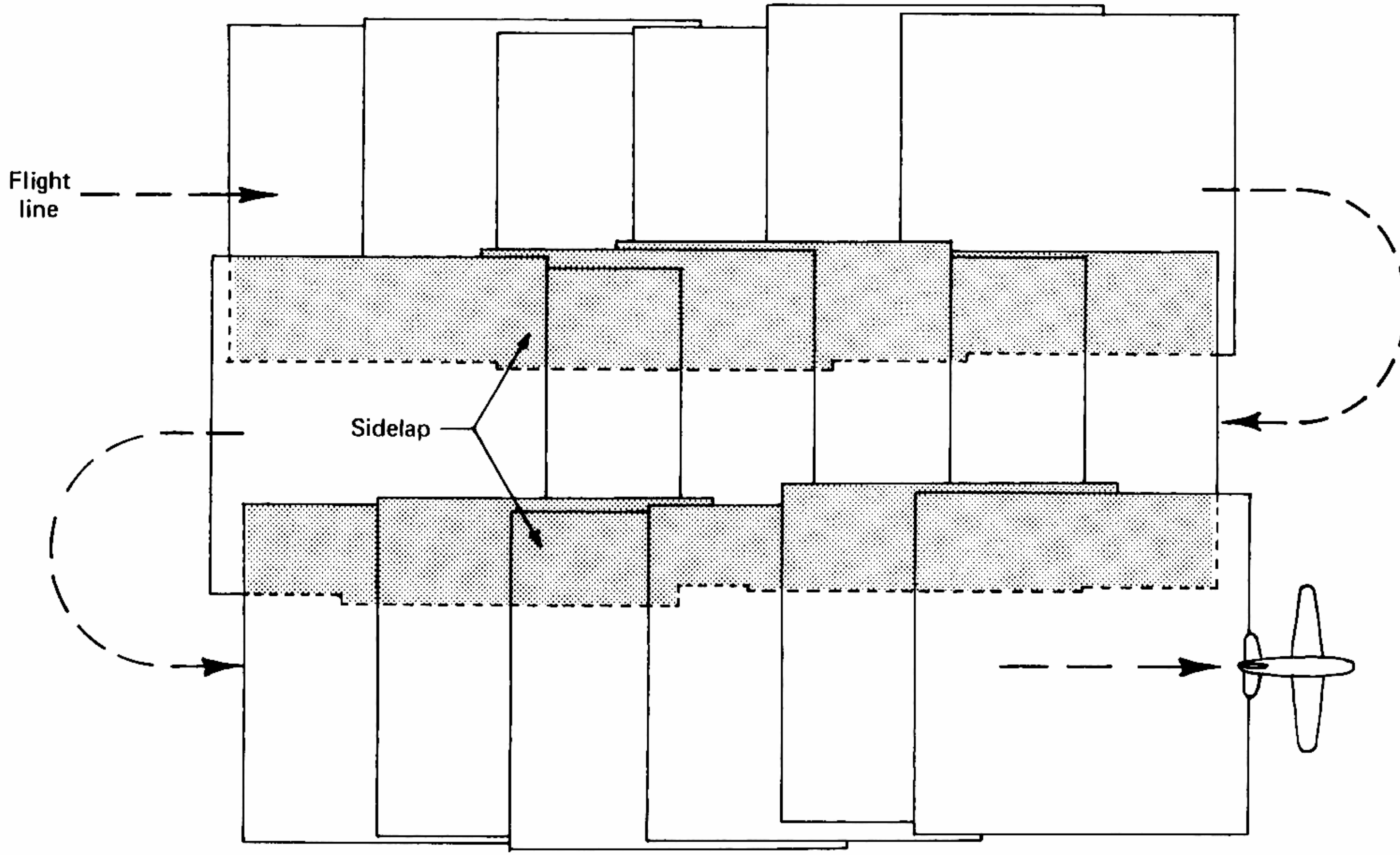


Figure 3.4 Adjacent flight lines over a project area.

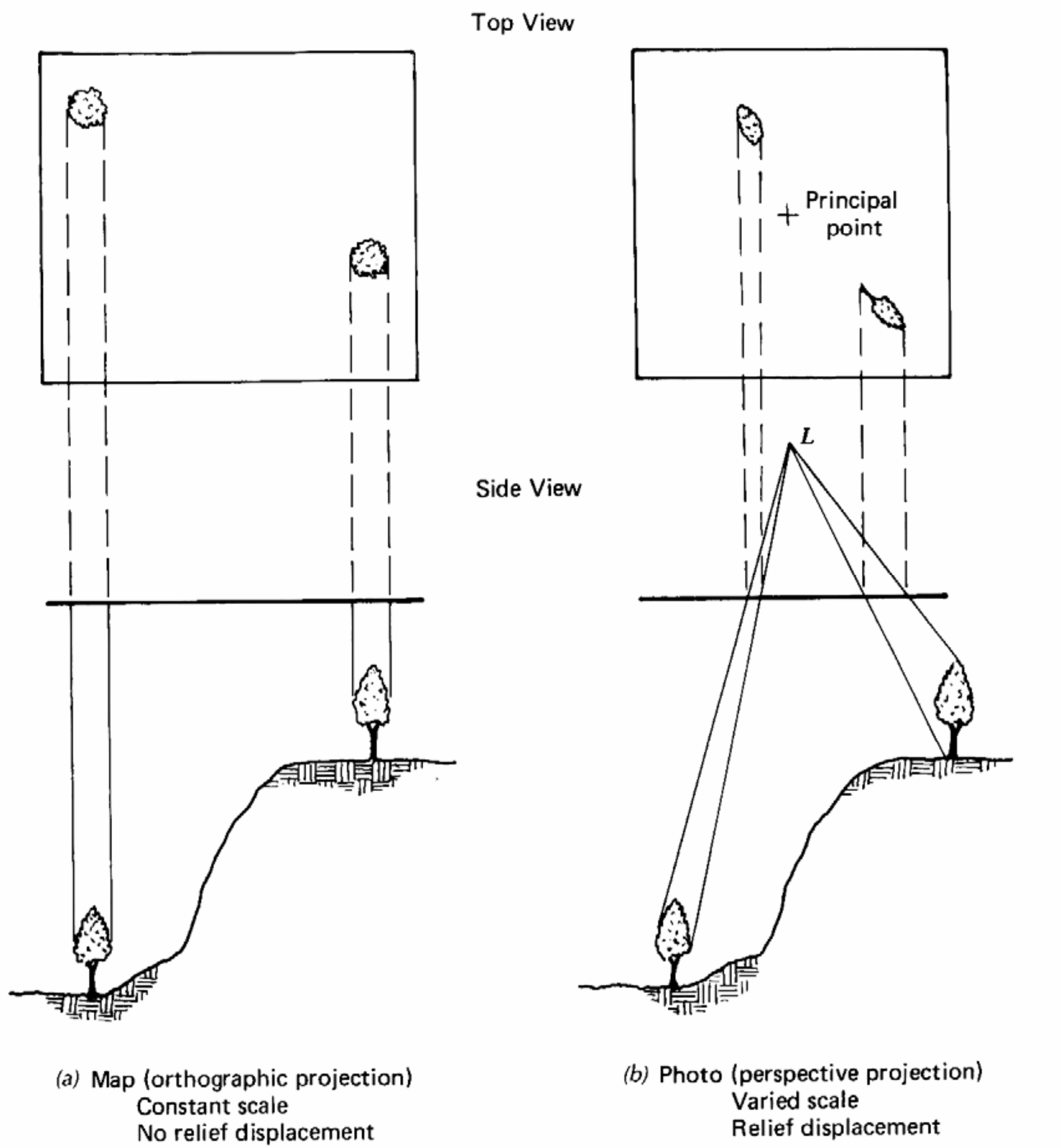
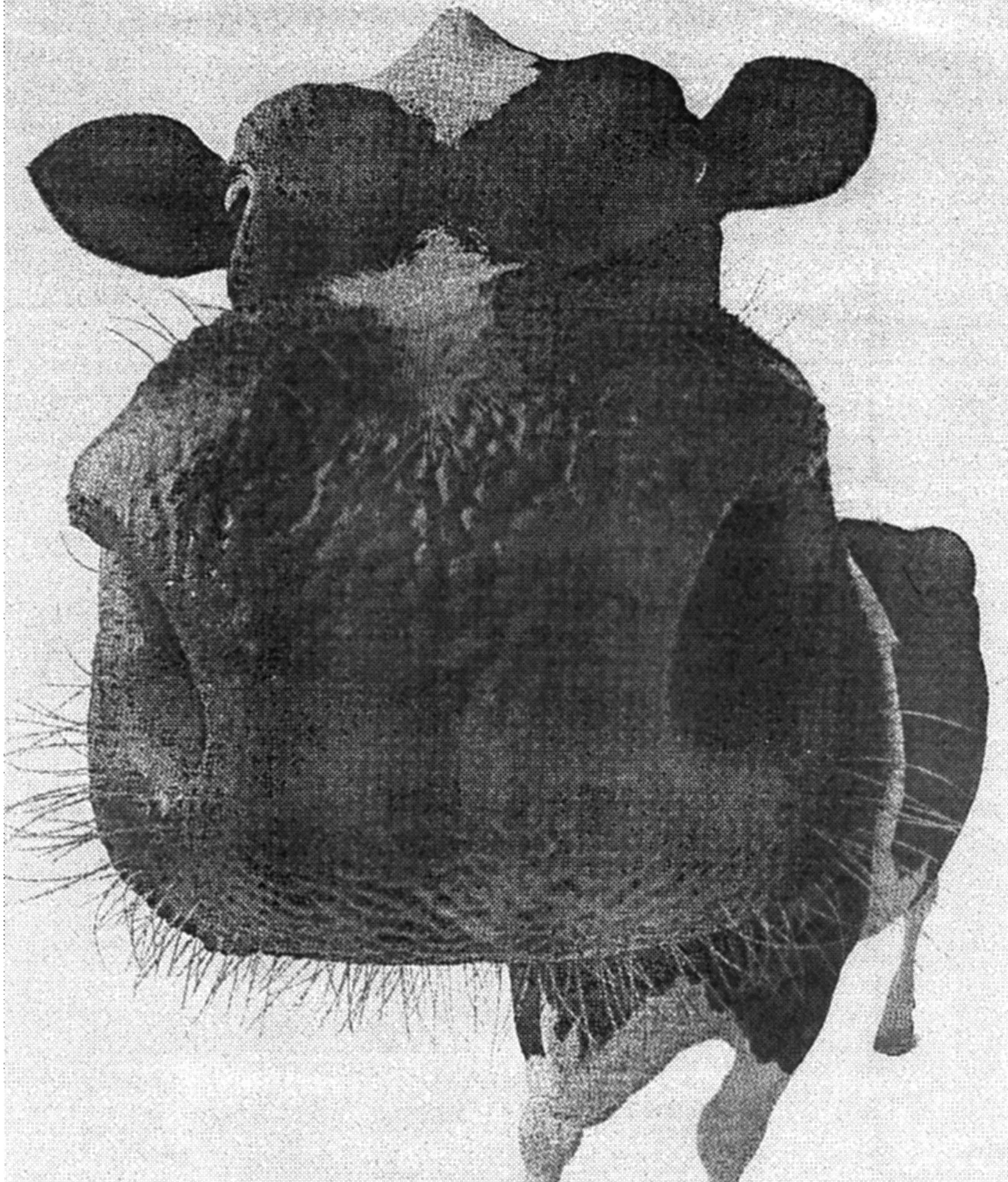


Figure 3.8 Comparative geometry of a map and a vertical aerial photograph.

Illustration of Perspective Distortion



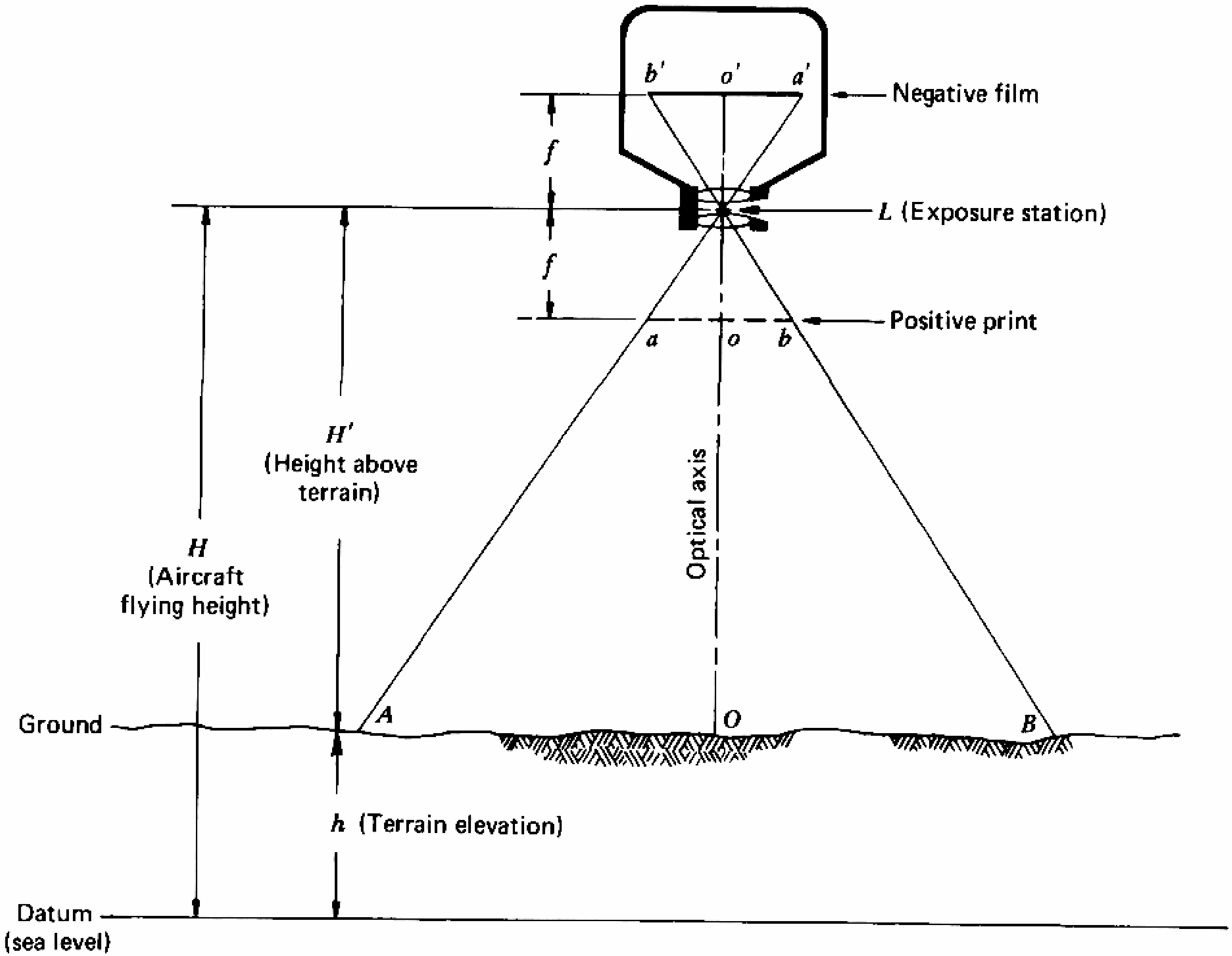


Figure 3.7 Scale of a vertical aerial photograph taken over flat terrain.

Scale

$$\text{Scale} = \frac{\text{Photo distance}}{\text{Ground distance}} = \frac{d}{D}$$

Scale

<u>Expressing Scale</u>

Engineer’s scale (unit equivalents)	1” = 400’
Representative fraction	1/4800
Ratio	1:4800

Scale

Small vs. Large Scale

Which is the smaller scale

1:10,000 or 1:120,000

$\frac{1}{10,000}$ = larger scale

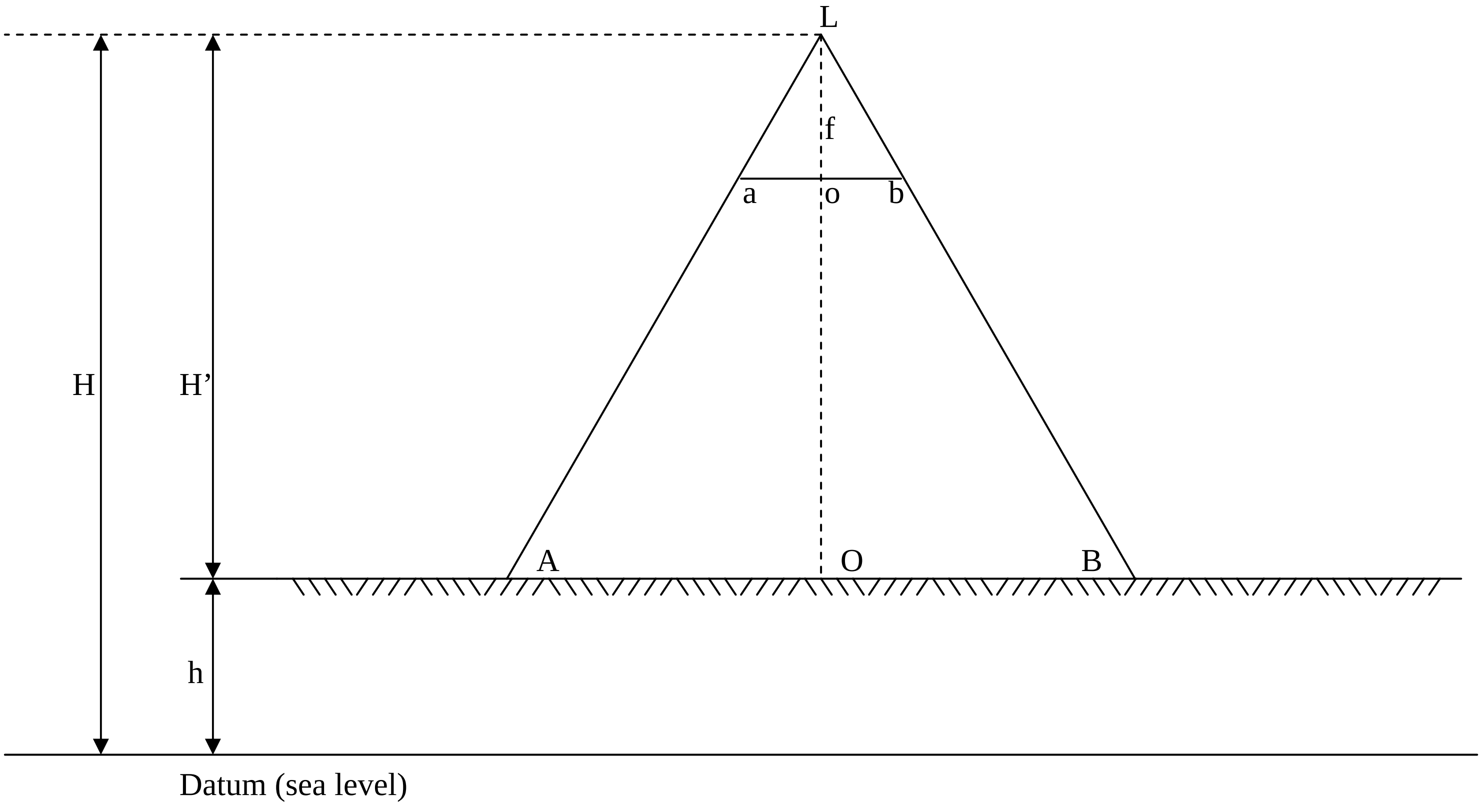
$\frac{1}{120,000}$ = smaller scale

Scale

<u>Photo Scale Ranges</u>

Small scale	1:50,000 or smaller
Medium scale	1:12,000-1:50,000
Large scale	1:12,000 or larger

Scale Over Flat Terrain

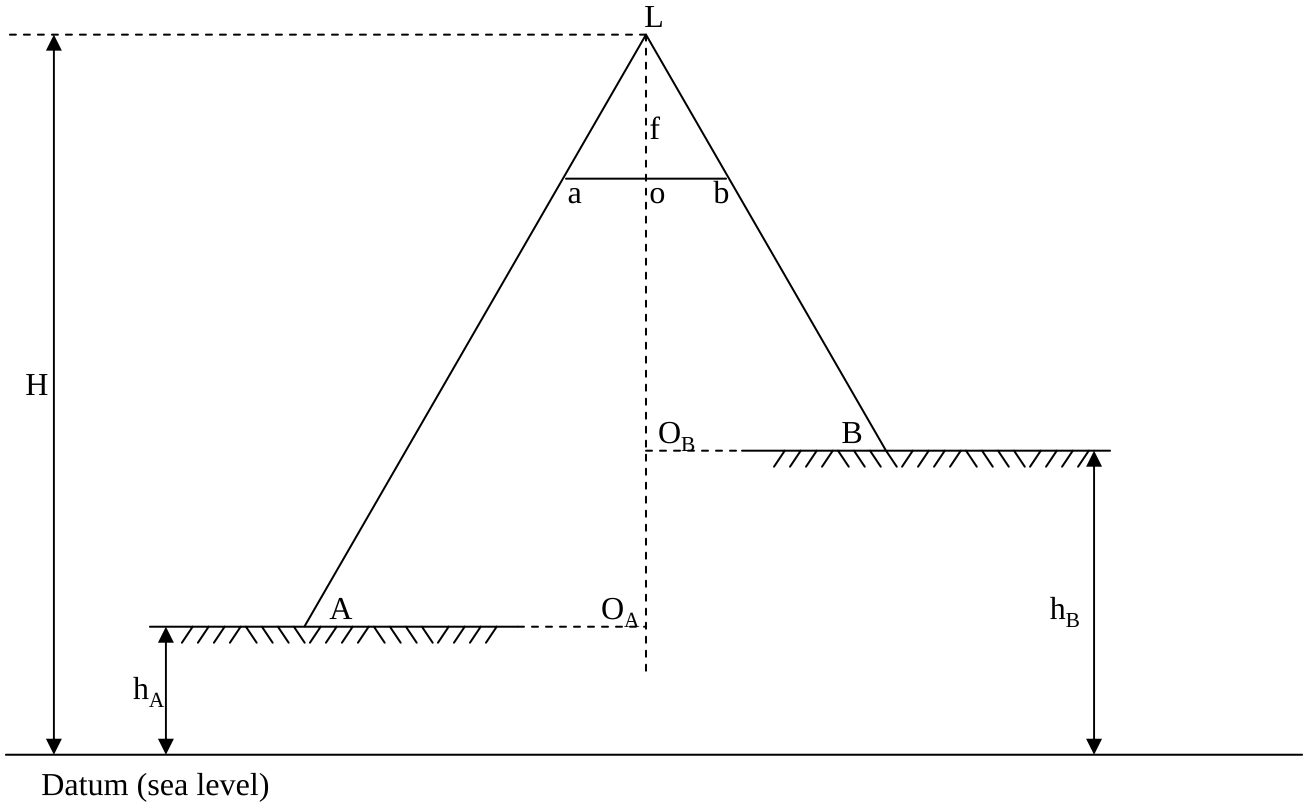


$$\text{Scale} = \frac{\text{Photo distance}}{\text{Ground distance}} = \frac{\overline{ab}}{\overline{AB}} = \frac{f}{H'} = \frac{f}{H - h}$$

Scale

$$\text{Scale} = \frac{\text{Camera focal length}}{\text{Flying height above terrain}} = \frac{f}{H'}$$

Scale Over Variable Terrain



For any point P:

$$\text{Scale P} = \frac{f}{H - h_P}$$

"Average" scale:

$$S_{avg} = \frac{f}{H - h_{avg}}$$

$$\text{Scale A} = \frac{\overline{oa}}{\overline{O_A A}} = \frac{f}{H - h_A}$$

$$\text{Scale B} = \frac{\overline{ob}}{\overline{O_B B}} = \frac{f}{H - h_B}$$

Sample Scale Computations

$$\text{Scale} = \frac{\text{Photo distance}}{\text{Ground distance}} = \frac{\text{Focal length}}{\text{Height above ground}} = \frac{f}{H - h}$$

<u>Case 1 Distance between two ground features known</u>

Example: 1-mile section line measures 1.76” on photo

$$scale = \frac{photo\ dist.}{ground\ dist.} = \frac{1.76''}{5280'} = \frac{1''}{3000'}\ or\ 1:36{,}000$$

Applying scale: Field measures 0.44” long on photo

$$ground\ dist. = \frac{3000'}{1''} \times 0.44'' = 1320'\ or\ 0.25\ mile$$

Sample Scale Computations

$$\text{Scale} = \frac{\text{Photo distance}}{\text{Ground distance}} = \frac{\text{Focal length}}{\text{Height above ground}} = \frac{f}{H - h}$$

<u>Case 2 Flying height and focal length known</u>

Example: Focal length = 6”; ht. above ground = 3000’

$$scale = \frac{focal\ length}{ht.\ above\ ground} = \frac{6''}{3000'} = \frac{1''}{500'}\ or\ 1:6{,}000$$

Applying scale: Field measures 1.43” long on photo

$$ground\ dist. = \frac{500'}{1''} \times 1.43'' = 715'$$

Sample Scale Computations

<u>Case 3 Flying height H and scale at various elevations sought</u>

Example: Assume the section line in Case 1 is at an elevation of 1200’ above datum, and focal length is 6”.

Find: H.

$$scale = \frac{f}{H-h} \qquad \frac{1''}{3000'} = \frac{6''}{H-1200'} \qquad H = \frac{6''(3000')}{1''} + 1200'$$

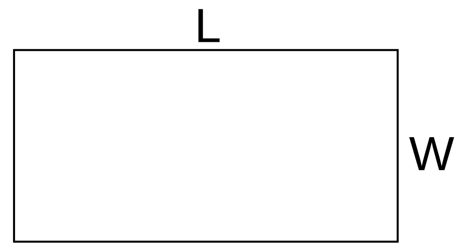
$$\boxed{H = 19{,}200'}$$

Find: The scale applicable to points at an elevation of 2000’ above datum.

$$scale = \frac{f}{H-h} = \frac{6''}{19{,}200' - 2000'} = \frac{1''}{2867'}$$

$$\boxed{scale = 1:34{,}400}$$

Determining Ground Areas From Photo Measurements



Assumptions:

(1) Flat terrain
(2) Rectangular object

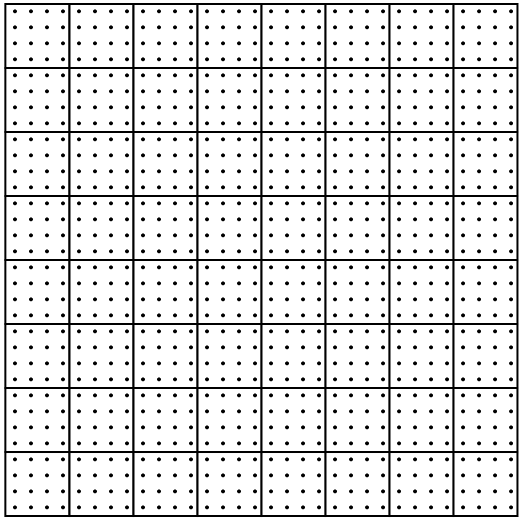
Approach:

ground area = ground length x ground width

$$= \frac{1}{photo\ scale} \times (photo\ length) \times \frac{1}{photo\ scale} \times (photo\ width)$$

$$= photo\ area \times \frac{1}{(photo\ scale)^2}$$

Dot Grid for Area Measurement



Sample Area Computations

$$ground\ area = photo\ area \times \frac{1}{(photo\ scale)^2}$$

<u>Case 1 Measure length and width of field and compute area.</u>

Example: Field measures 1.50” by 3.00” on 1:12,000 photo (1” = 1000’)

$$ground\ area = 4.50\ sq.\ in. \times \frac{(1000)(1000)\ sq.\ ft.}{1\ sq.\ in.} = 4{,}500{,}000\ sq.\ ft. = 103\ acres$$

Sample Area Computations

<u>Case 2 Measure area with a dot grid.</u>

Example: Flooded area covered by 43 dots on a 50 dot/sq. in. grid

$$\text{grid factor} = \frac{1\,\text{sq. in.}}{50\,\text{dots}} \times \frac{(1000)(1000)\ \text{sq. ft.}}{1\,\text{sq. in.}} = \frac{20{,}000\,\text{sq. ft.}}{\text{dot}} = \frac{0.46\,\text{acres}}{\text{dot}}$$

$$\text{ground area} = 43\,\text{dots} \times \frac{0.46\ \text{acres}}{\text{dot}} = 19.78\ \text{or}\ 20\ \text{acres}$$

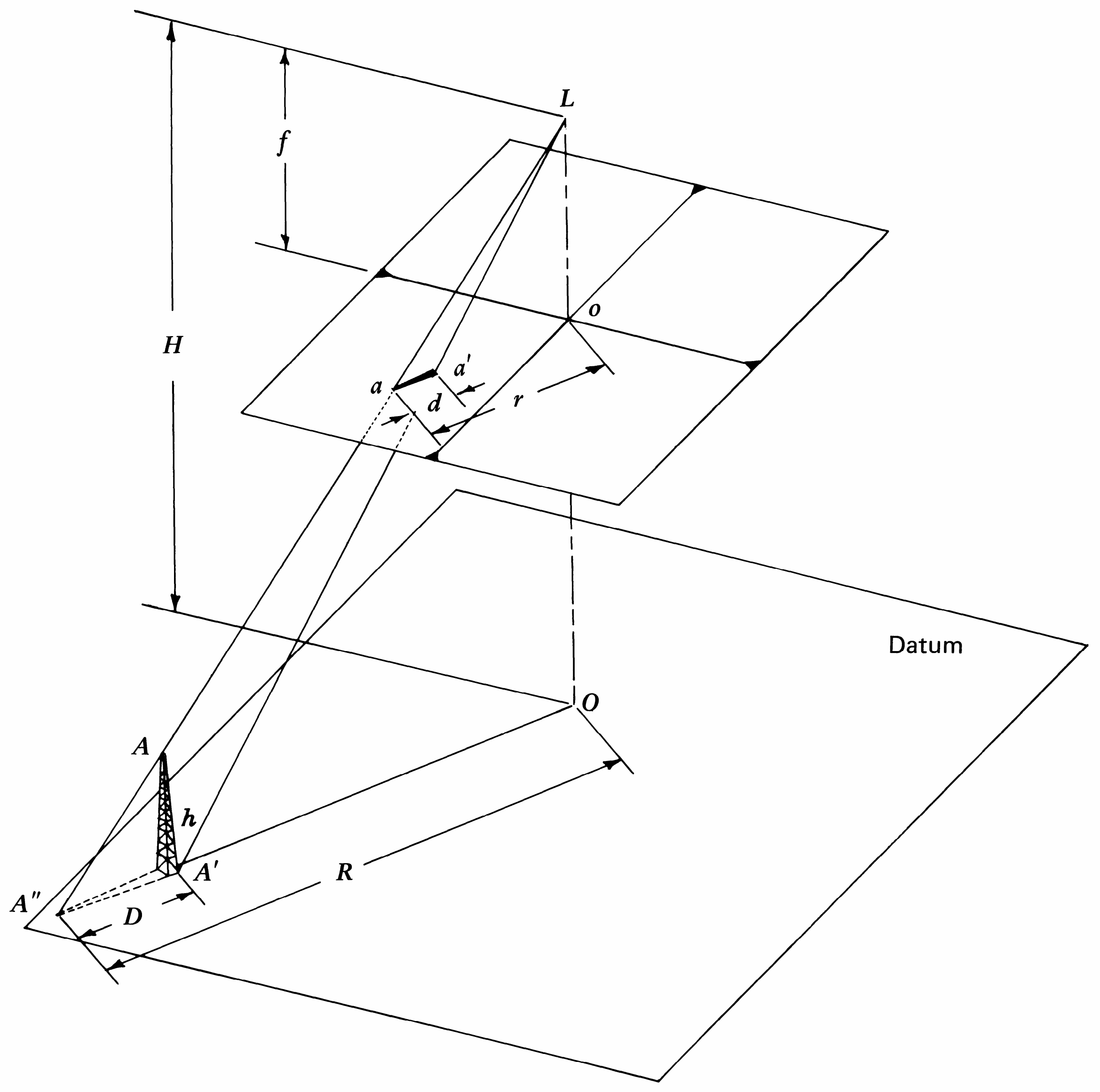


Figure 3.13 Geometric components of relief displacement.

Relief Displacement Equation

In Figure 3.13, $\frac{D}{h} = \frac{R}{H}$ $D = \left(\frac{H}{f}\right)d$ $R = \left(\frac{H}{f}\right)r$

Therefore, $\frac{Hd}{fh} = \frac{Hr}{fH}$ $d = \frac{rh}{H}$

Where

d = relief displacement

r = radial distance on the photograph from the principal point to the displaced image point

h = height above datum of the displaced image point

H = flying height above the same datum chosen to reference h

Characteristics of Relief Displacement

$$d = \frac{rh}{H}$$

- d increases with increase in radial distance from principal point (r)
- d increases with increase in height of object (h)
- d decreases with increase in flying height (H)
- at the principal point, r = 0, d = 0
- can be either outward (for points above datum) or inward (for points below datum)
- causes objects to lean
- causes straight lines on ground to appear crooked on photo

Sample Relief Displacement Calculation

Given: The top of a 100’ tall tower is located a radial distance of 3.60” from the principal point of a vertical photo taken at a flying height of 3000’ above the ground.

Find: The distance (d) the top of the tower is displaced relative to the base of the tower.

Solution:

$$d = \frac{rh}{H} \qquad d = \frac{(3.60'')(100')}{3000'} = 0.12''$$

Using Relief Displacement to Estimate Heights

$$d = \frac{rh}{H}$$

$$\boxed{h = \frac{dH}{r}}$$

Where

d = relief displacement

r = radial distance on the photograph from the principal point to the displaced image point

h = height above datum of the displaced image point

H = flying height above the same datum chosen to reference h

Example 3.7

For the photo shown in Figure 3.13, assume that the relief displacement for the tower at A is 2.01 mm, and the radial distance from the center of the photo to the top of the tower is 56.43 mm. If the flying height is 1220 m above the base of the tower, find the height of the tower.

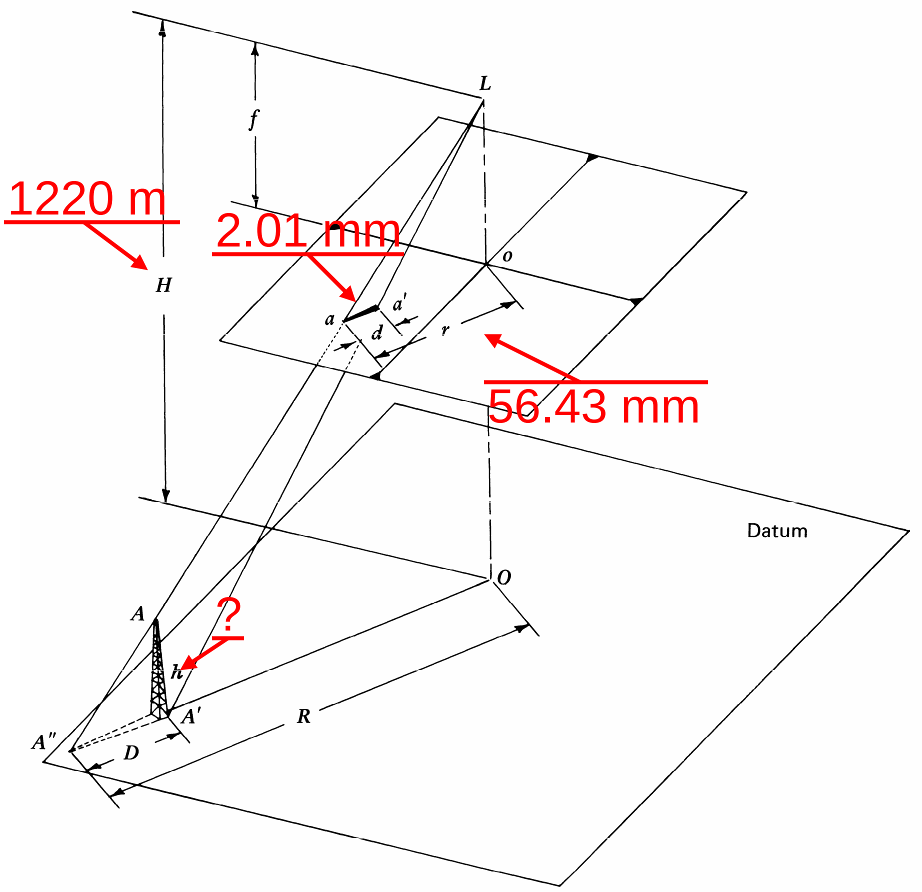


Image Parallax

Definition:

The change in position of an image from one photograph to the next, caused by the aircraft's motion between exposures.

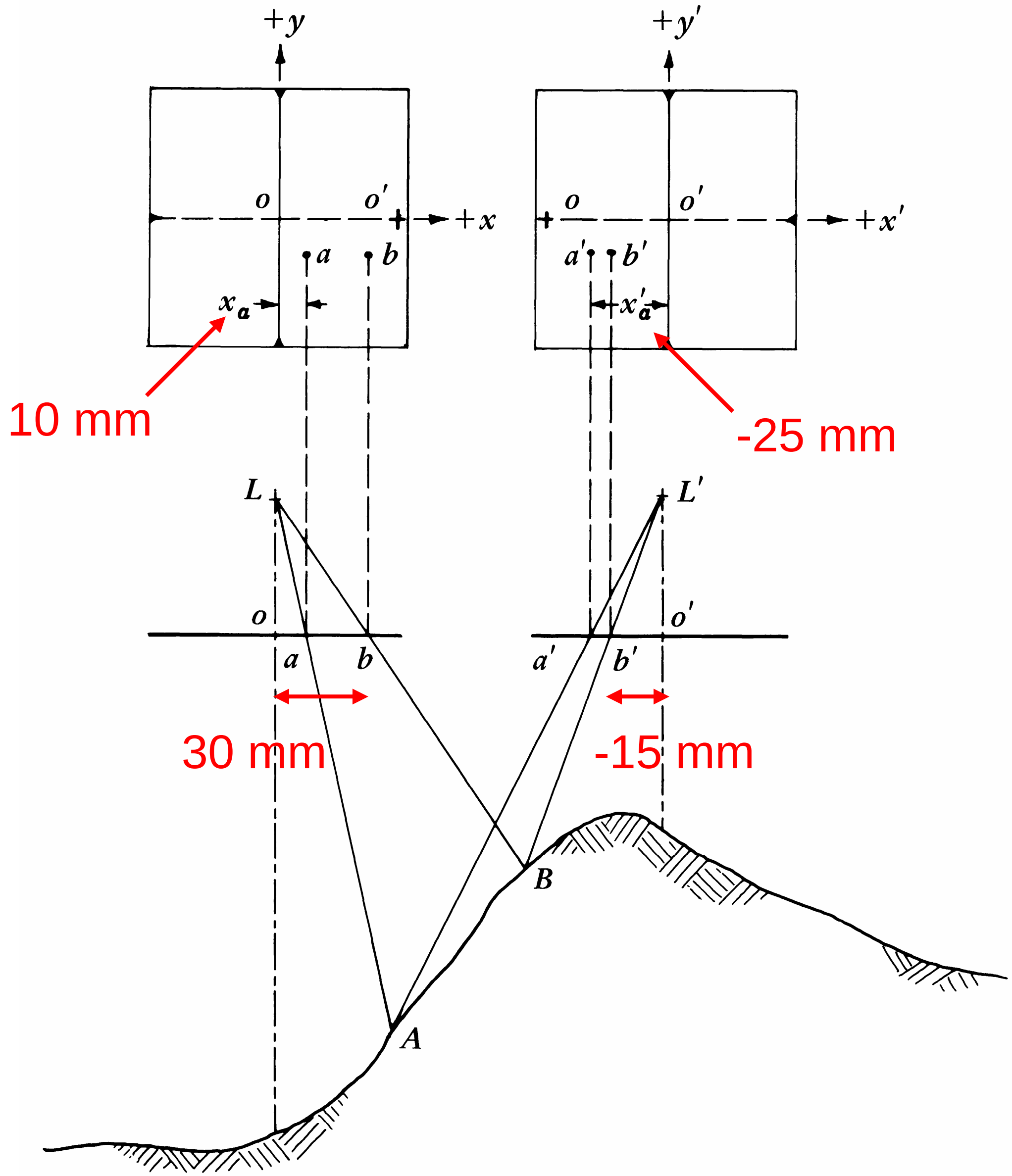
Characteristics:

- Measured parallel to the flight line
- Directly related to the elevation of a ground point
- Basis for viewing and measuring in 3-D
- Need a minimum of 50% overlap to cover all areas stereoscopically

Measurement:

$$p_p = x_p - x_p'$$

measure parallel to flight line.



p_a = 10 mm - (-25 mm)
p_a = 35 mm

p_b = 30 mm - (-15 mm)
p_b = 45 mm

Figure 3.15 Parallax displacements on overlapping vertical photographs.

Using Parallax Measurements To Obtain Elevations

$$h_P = H - \left(\frac{Bf}{P_P} \right)$$

Where

h_P = elevation of any point P

H = flying height

B = air base (between exposures)

f = focal length

p_P = parallax of point P

Parallax Example

Given: A stereopair with the following characteristics

H = 1200 m B = 600 m f = 152.4 mm

Point A has a parallax of 114.06 mm
Point B has a parallax of 126.06 mm

Find: Elevation of points A and B

Parallax Example

Solution:

$$h_P = H - \left(\frac{Bf}{P_P} \right)$$

$$h_A = 1200\ m - \left(\frac{(600\ m)(152.4\ mm)}{114.06\ mm} \right) = 398\ m$$

$$h_B = 1200\ m - \left(\frac{(600\ m)(152.4\ mm)}{126.06\ mm} \right) = 475\ m$$

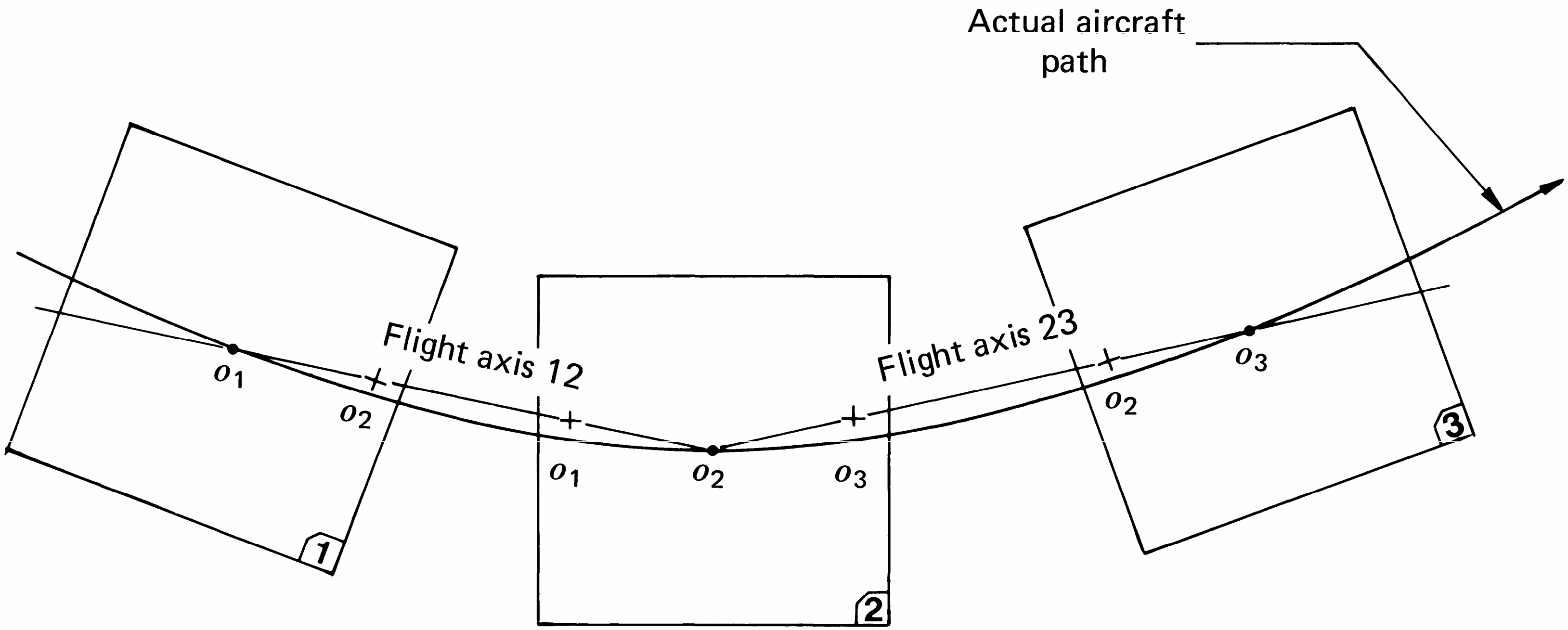


Figure 3.16 Flight line axes for successive stereopairs along a flight strip.

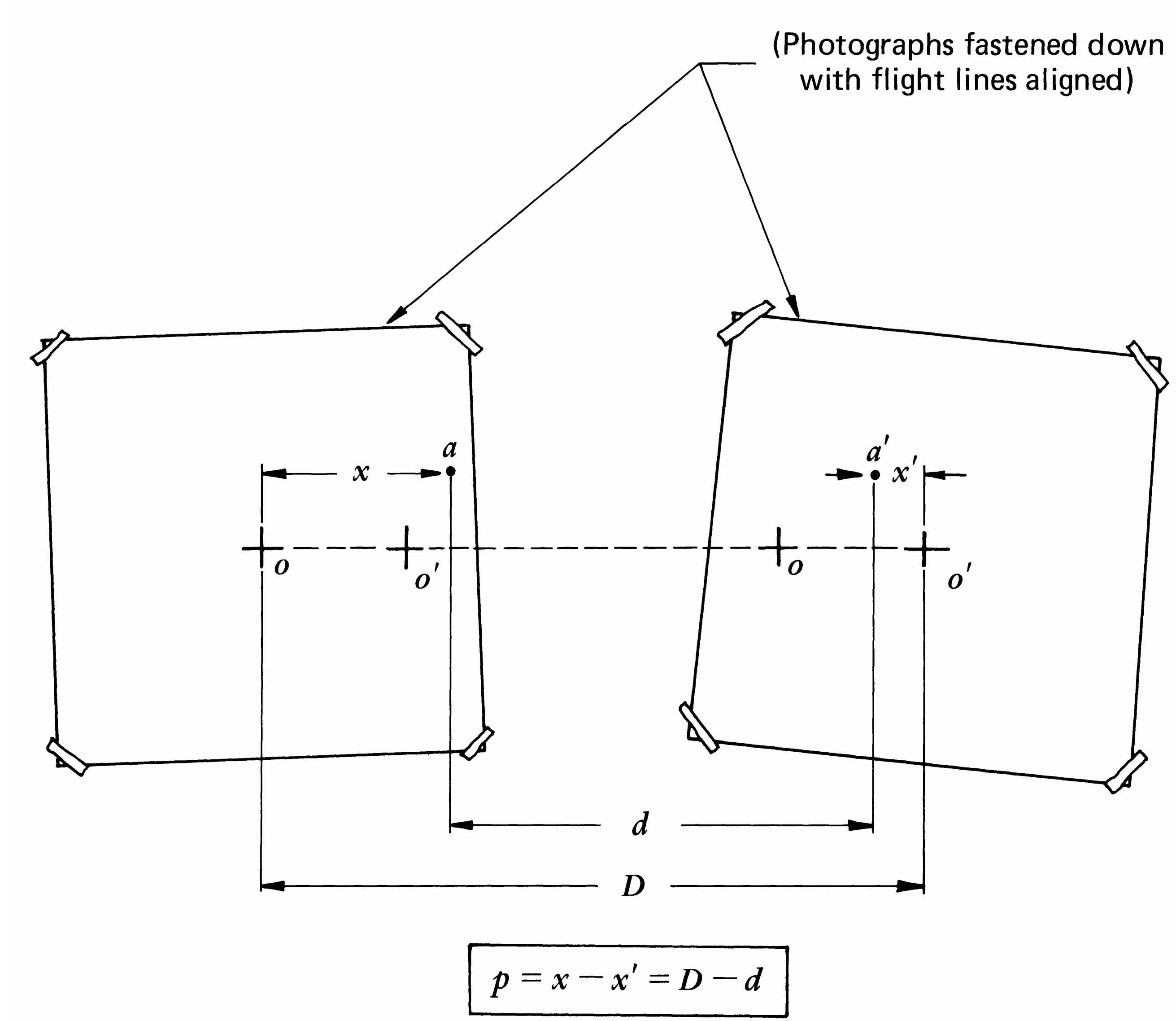


Figure 3.18 Alignment of a stereopair for parallax measurement.

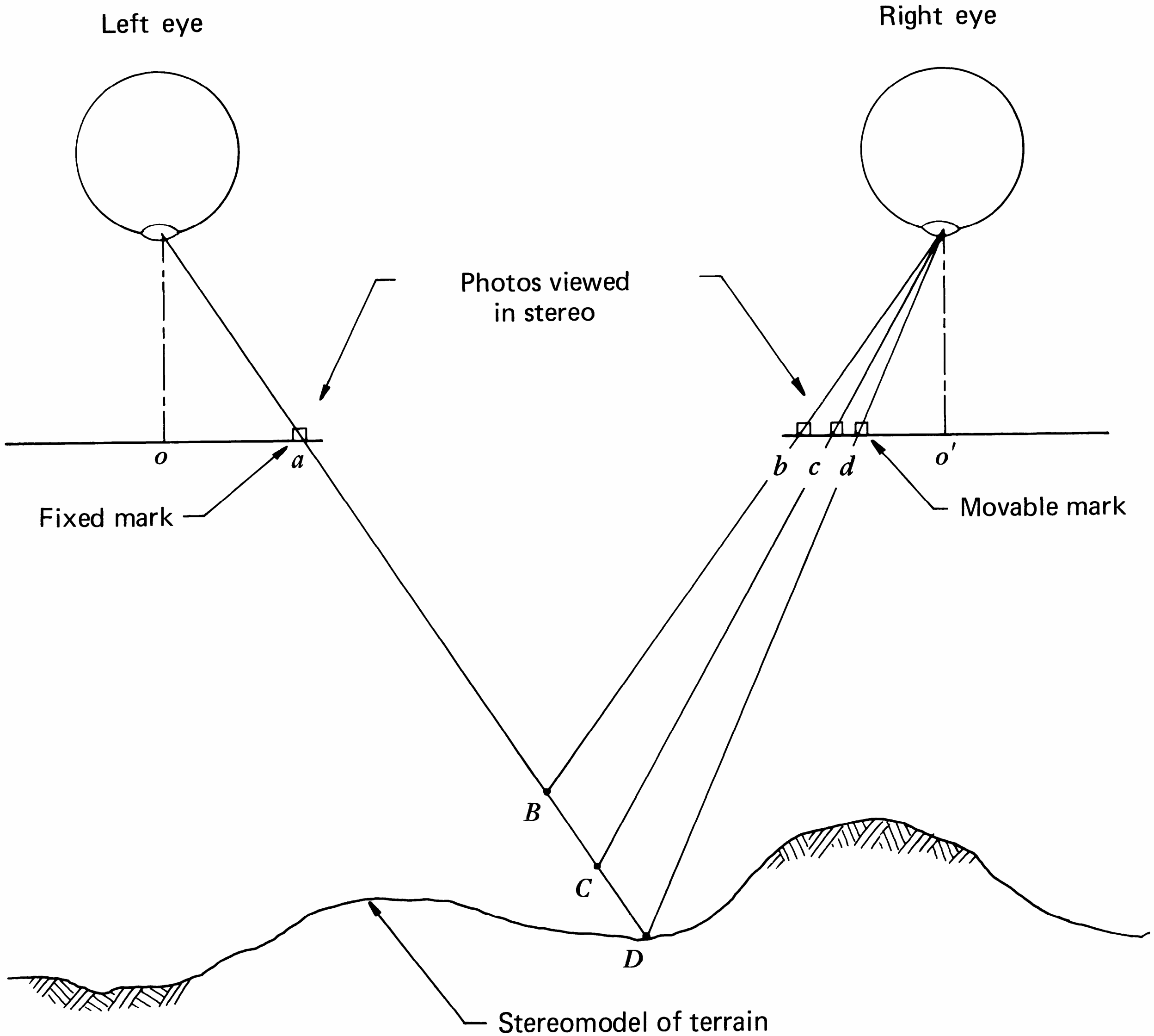


Figure 3.19 Floating-mark principle.

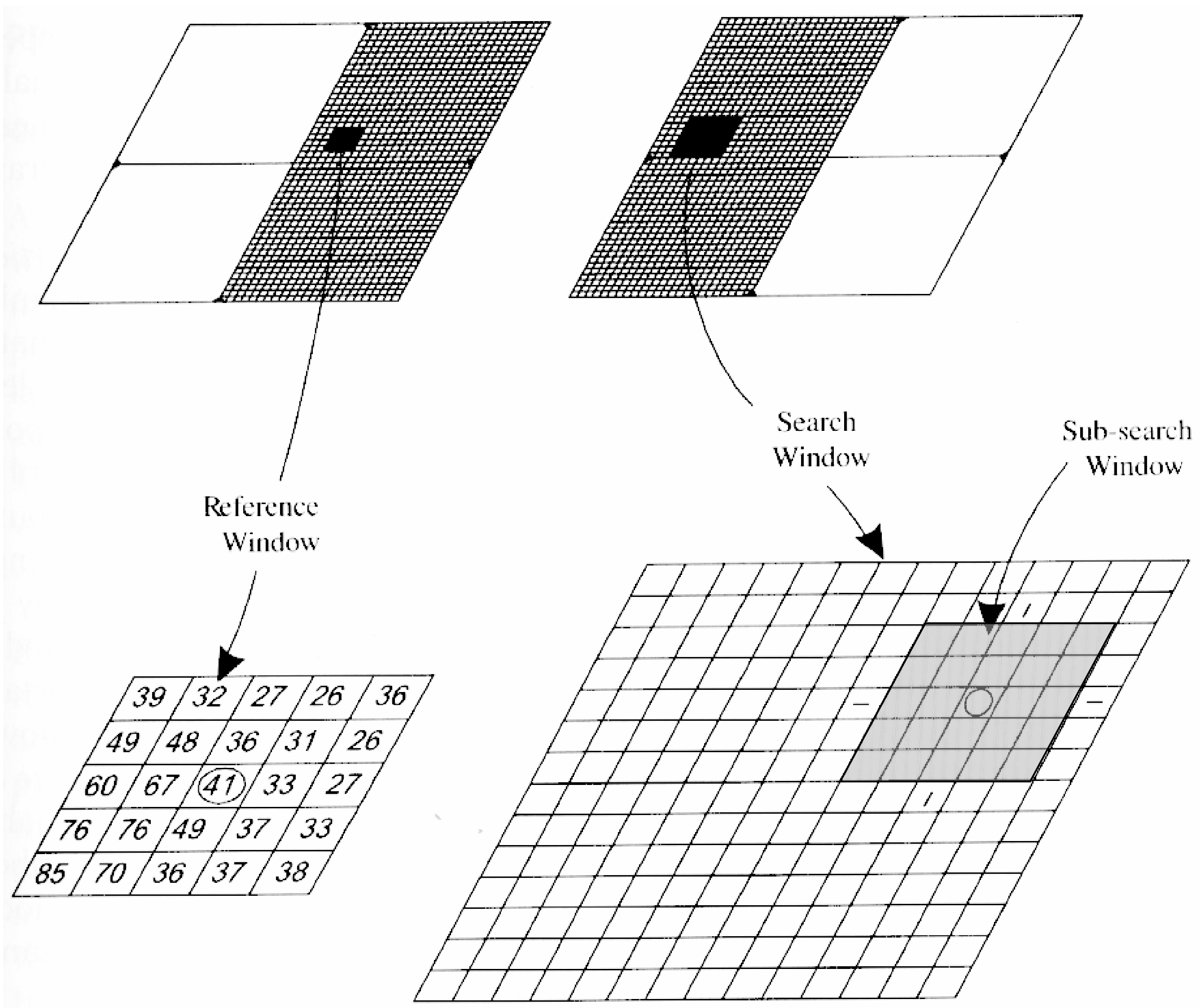


Figure 3.22 Principle of image matching.

Major Sources of Error in Parallax Methods

(1) Locating and marking the flight line on photos

(2) Errors in coordinate and parallax measurement

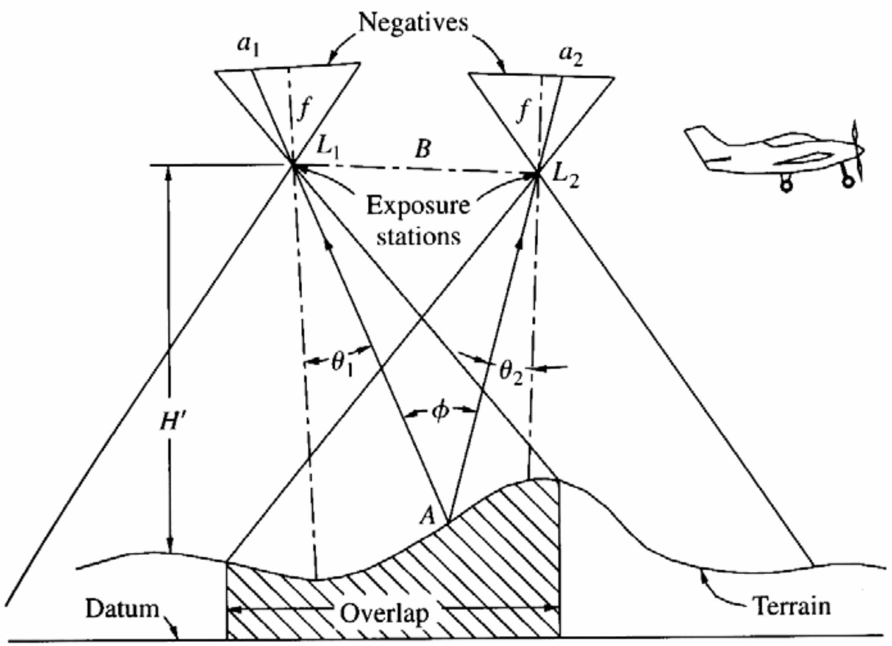
(3) Print shrinkage

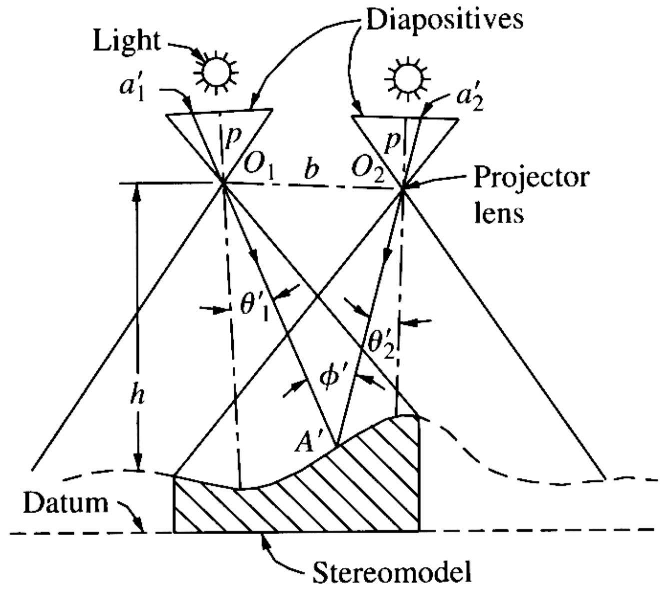
(4) Unequal flying heights

(5) Tilt

(6) Lens distortion

Overcome all these through the use of a stereoplotter





Basic concept of stereoscopic plotting instrument design (Wolf and Dewitt, 2000).

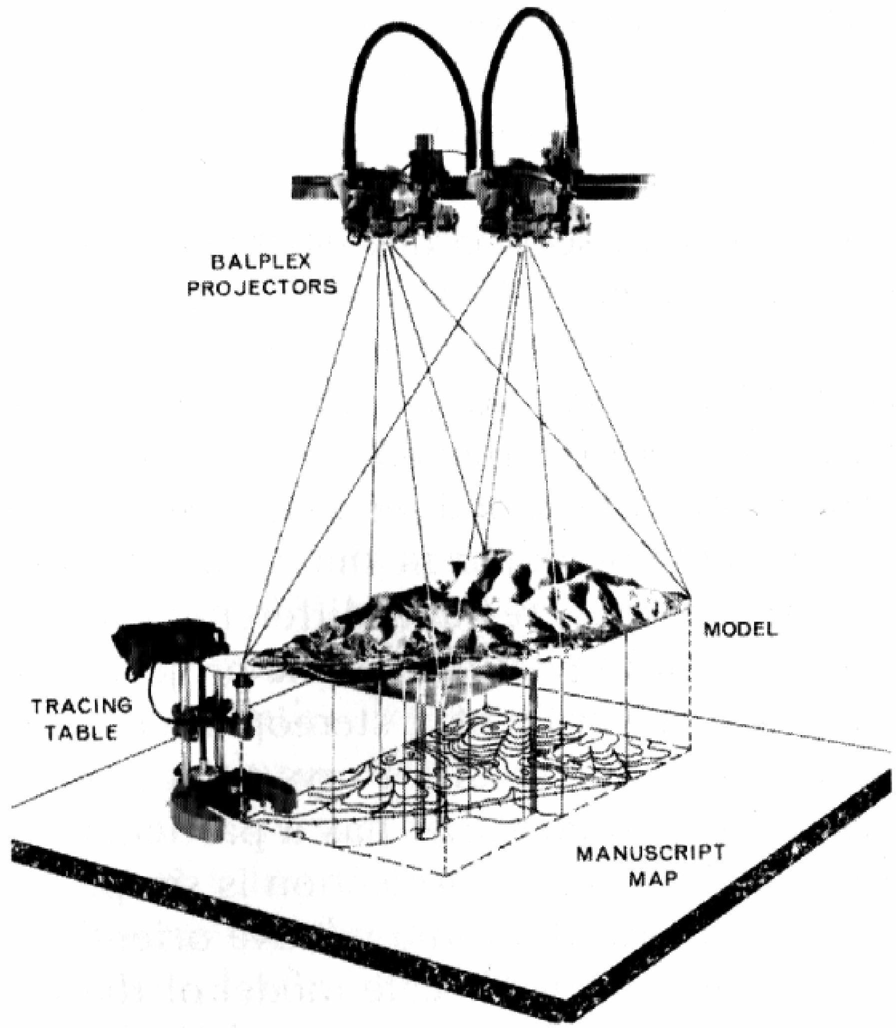
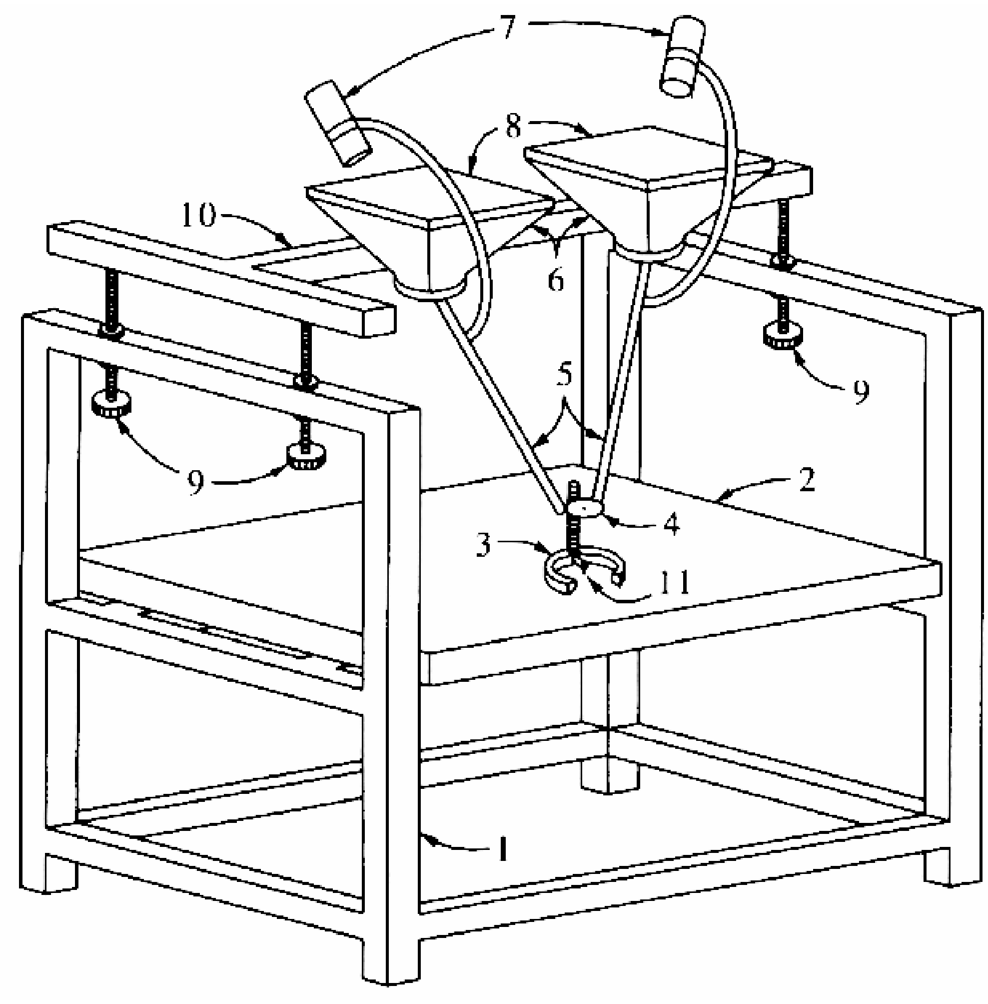


Figure 3.23 Stereomodel projected in a Balplex stereoplotter.



Typical components of a direct optical projection stereoplotter (Wolf and Dewitt, 2000).

Stereoplotter Components (1)

Projection System

(1) Direct optical projection

(2) Mechanical or optical-mechanical projection

(3) Projection of a mathematical model based on precise coordinate measurements of images on both the left and right photos forming the stereopair

(a) Analytical plotters - use stereocomparator measurements at discrete points

(b) Softcopy systems - use digital raster images of all points in overlap area

Stereoplotter Components (2)

Viewing System

(1) Anaglyphic (blue-green and red filters)

(2) Stereo-image alternator (SIA)

(3) Polarized-platen viewing (PPV)

(4) Binocular eyepieces as part of an optical train of lenses, mirrors, and prisms

(5) Computer monitor with provision for stereo

Stereoplotter Components (3)

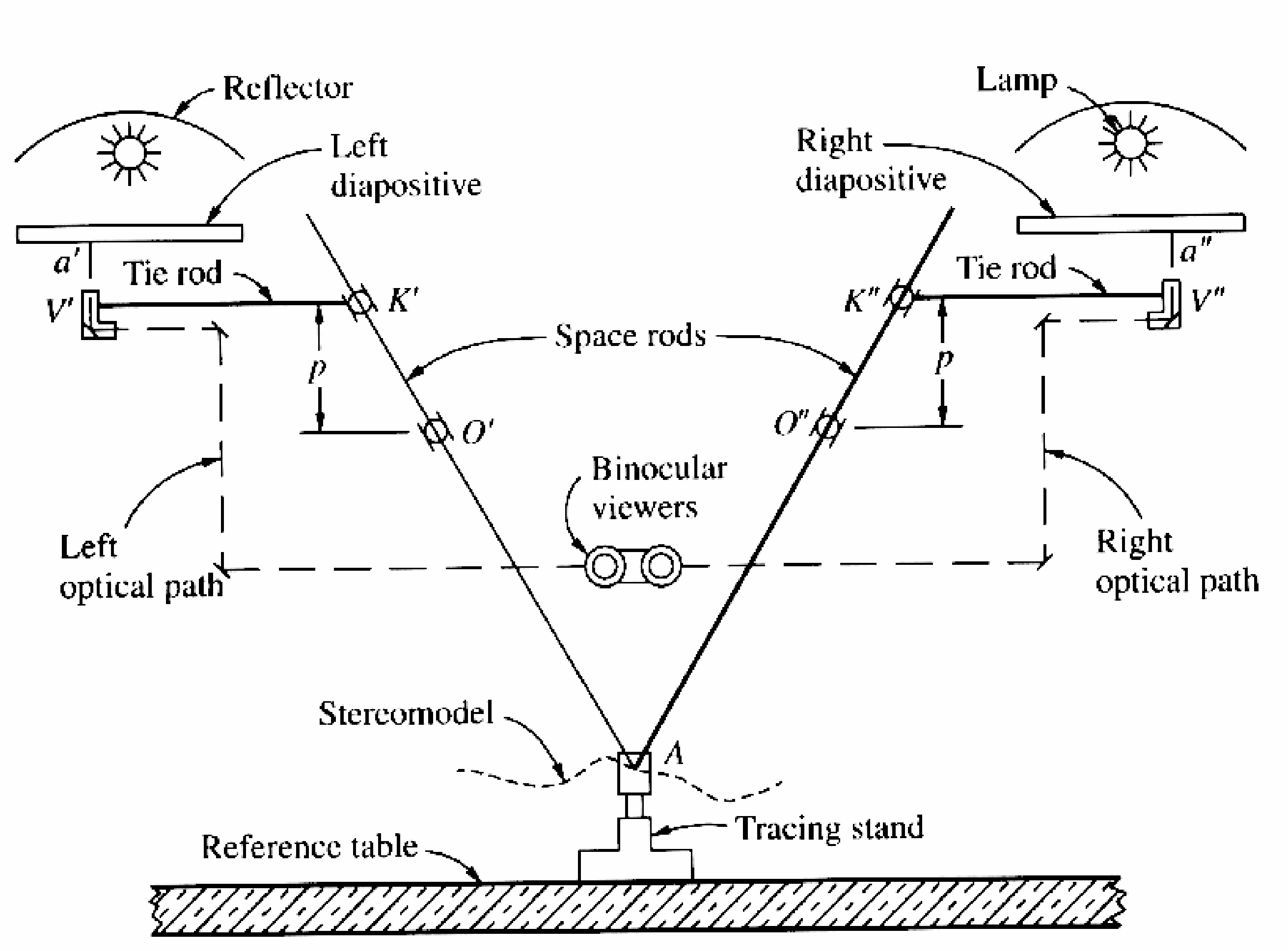
Tracing and Measuring System

(1) Platen and analog measurement

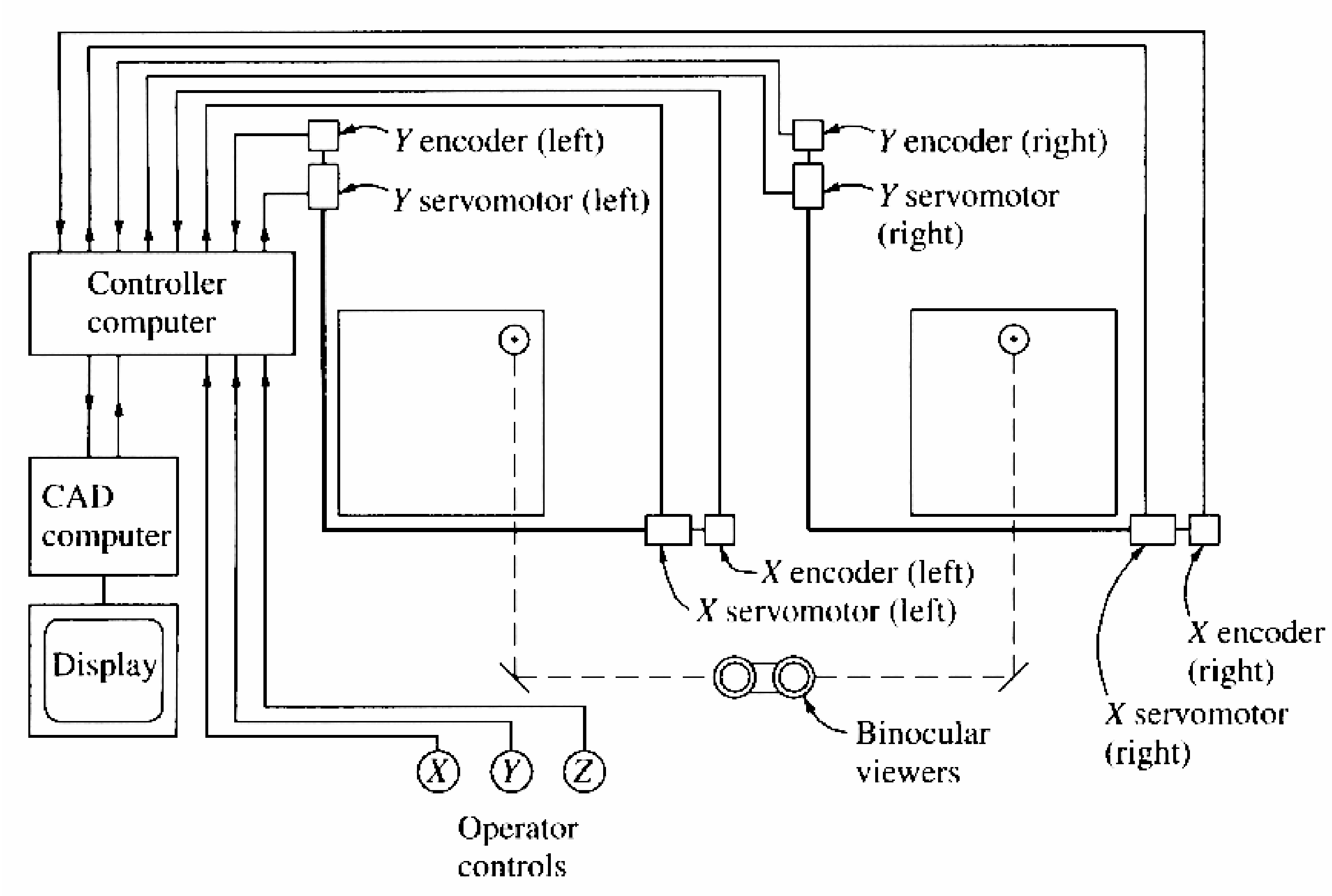
(2) Hand wheel and foot disk system with digital encoders

(3) Electronic image correlation

(4) Digital image correlation



Basic principles of mechanical projection (Wolf and Dewitt, 2000).



Schematic diagram of an analytical plotter (Wolf and Dewitt, 2000).

Orthophotography

Differential Rectification: Compensate for tilt and relief displacement.

Orthophoto = A photograph showing images of objects in their true orthographic projections

“Best of both worlds”: Properties of maps and photographs

- Map property: uniform scale (even in varying terrain)
- Photo property: show actual objects (rather than lines and symbols)

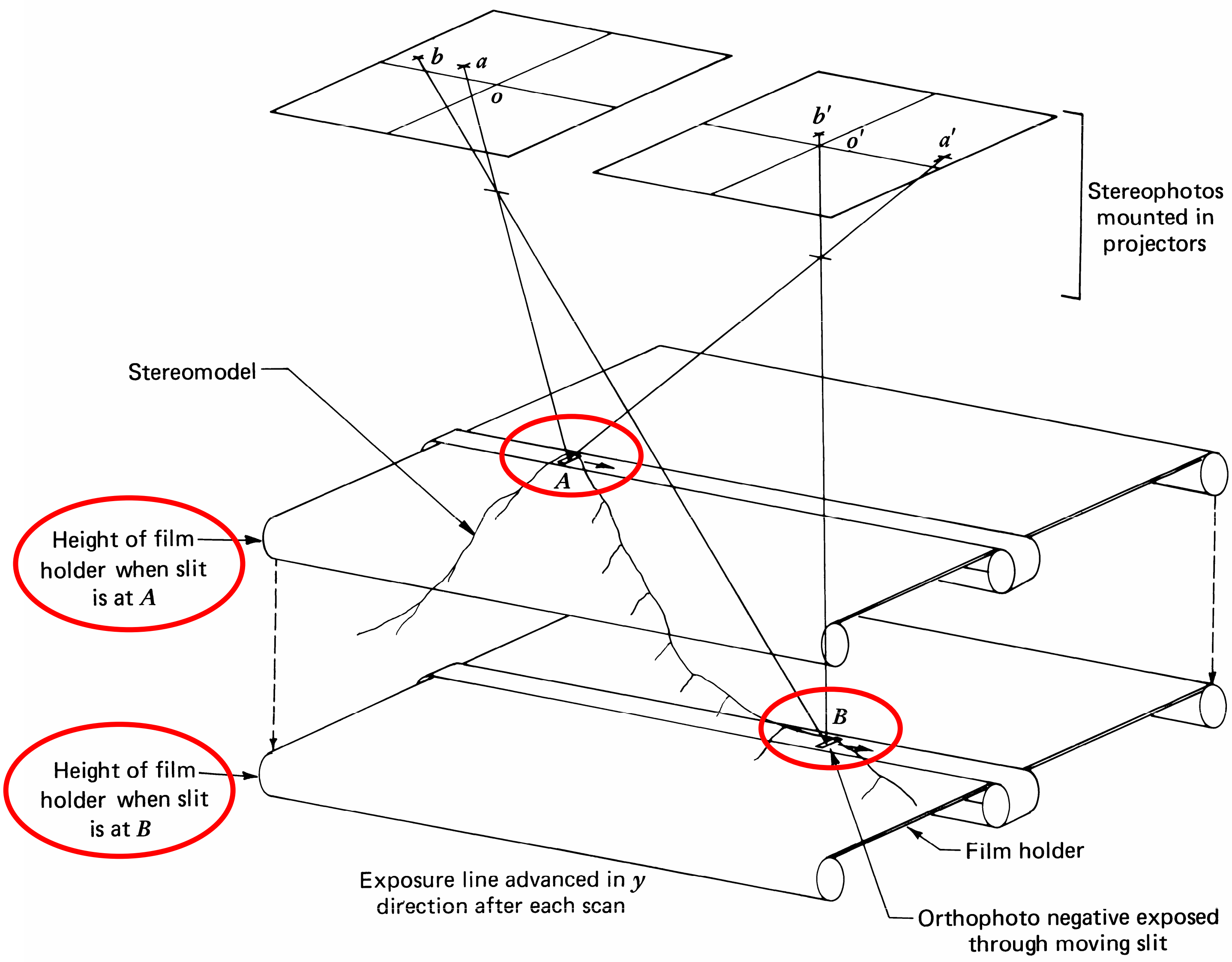


Figure 3.26 Operating principle of a direct optical projection orthophotoscope.

Tilt and Terrain Distortions

Area Distortions:

- 5.54 to 6.91 mm²
- 6.91 to 7.03 mm²
- 7.03 to 7.08 mm²
- 7.08 to 7.13 mm²
- 7.13 to 7.27 mm²
- 7.27 to 7.62 mm²
- 7.62 to 8.73 mm²

Average scale:
1:14,000

Softcopy Photogrammetry

•“Softcopy photogrammetry” is another name for “digital photogrammetry”

•Uses digital images rather than hardcopy images

•Uses principles of “analytical photogrammetry” - analysis of 3-D models that are created mathematically rather than through optical or mechanical projection

Photogrammetric Workstation

- Stereoplotting using various viewing systems (e.g., liquid crystal shutters with IR link)
- DEM preparation
- Orthophoto production (digital and hardcopy)
- Direct capture (in 3-D) of GIS data

Trends

- More and more sources of digital image data
- More image processing and GIS functionality in softcopy systems (lines between photogrammetric, image processing, and GIS systems blurring)

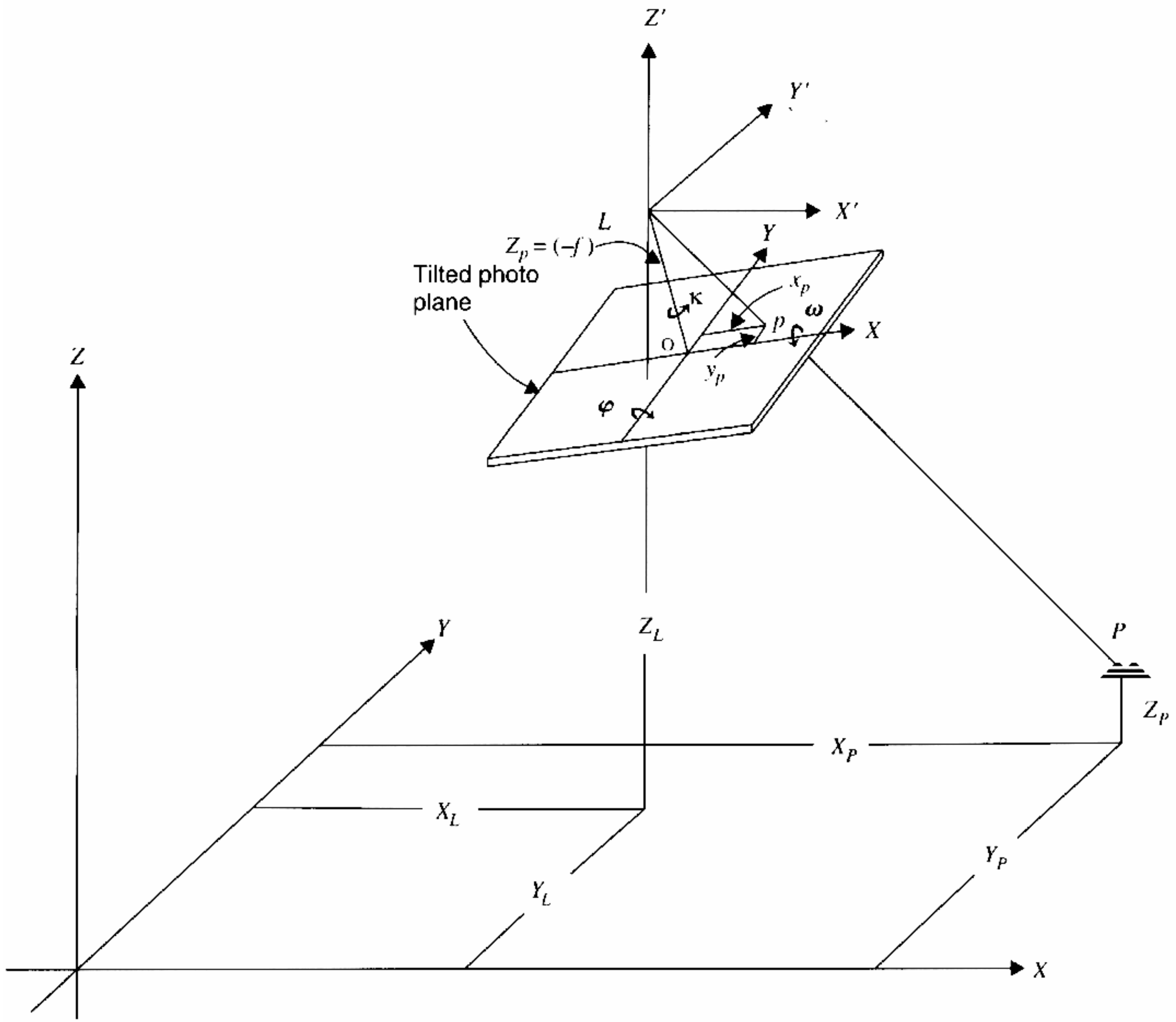


Figure 3.32 The collinearity condition.

Collinearity Equations

$$x_p = -f \left[\frac{m_{11}(X_P - X_L) + m_{12}(Y_P - Y_L) + m_{13}(Z_P - Z_L)}{m_{31}(X_P - X_L) + m_{32}(Y_P - Y_L) + m_{33}(Z_P - Z_L)} \right] \quad (3.14)$$

$$y_p = -f \left[\frac{m_{21}(X_P - X_L) + m_{22}(Y_P - Y_L) + m_{23}(Z_P - Z_L)}{m_{31}(X_P - X_L) + m_{32}(Y_P - Y_L) + m_{33}(Z_P - Z_L)} \right] \quad (3.15)$$

x_P, y_P = image coordinates of any point p

f = focal length

X_P, Y_P, Z_P = ground coordinates of point P

X_L, Y_L, Z_L = ground coordinates of exposure station L

$m_{11}...m_{33}$ = coefficients of a 3x3 rotation matrix defined by the angles ω, ϕ, and κ that transforms the ground coordinate system to the image coordinate system

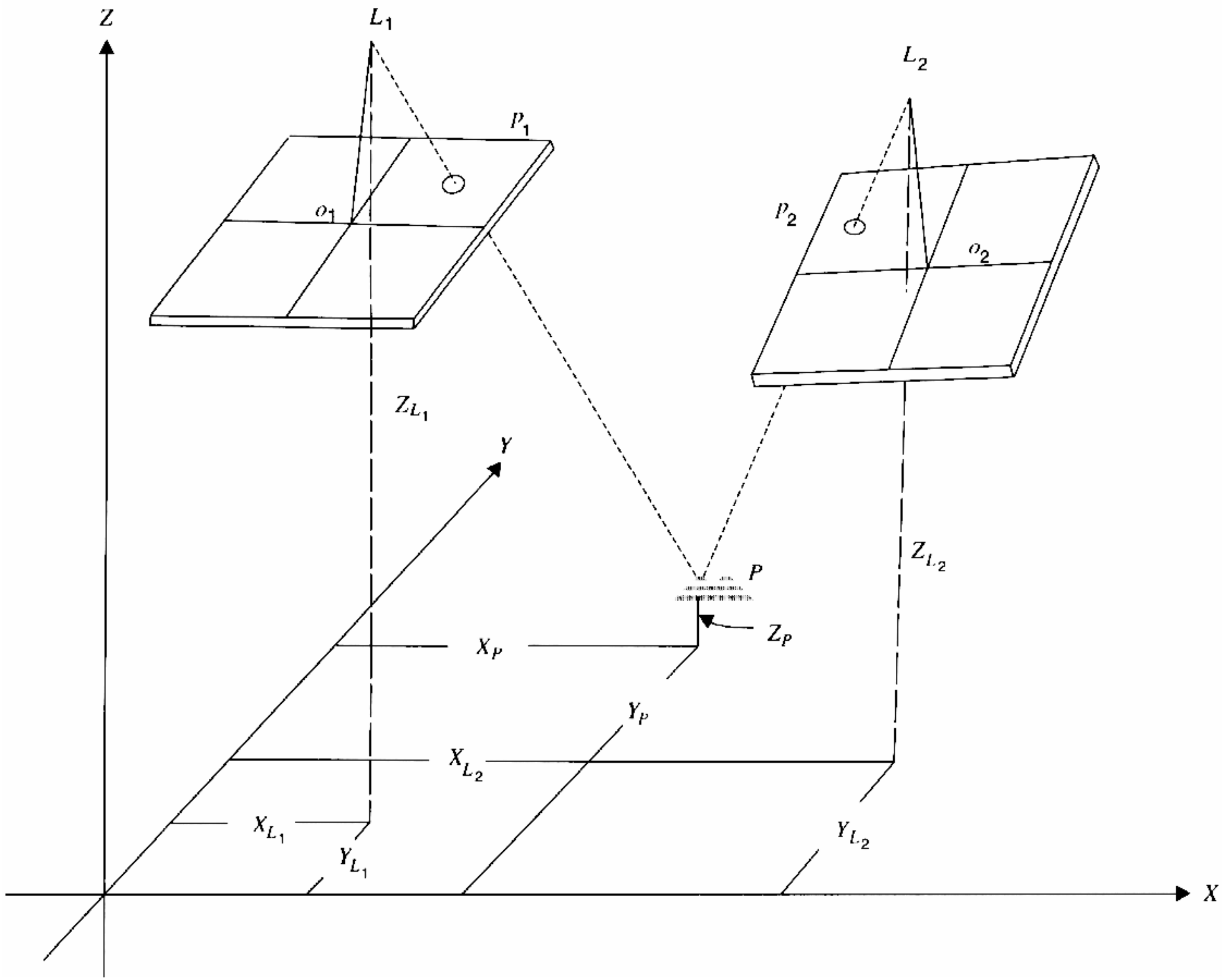
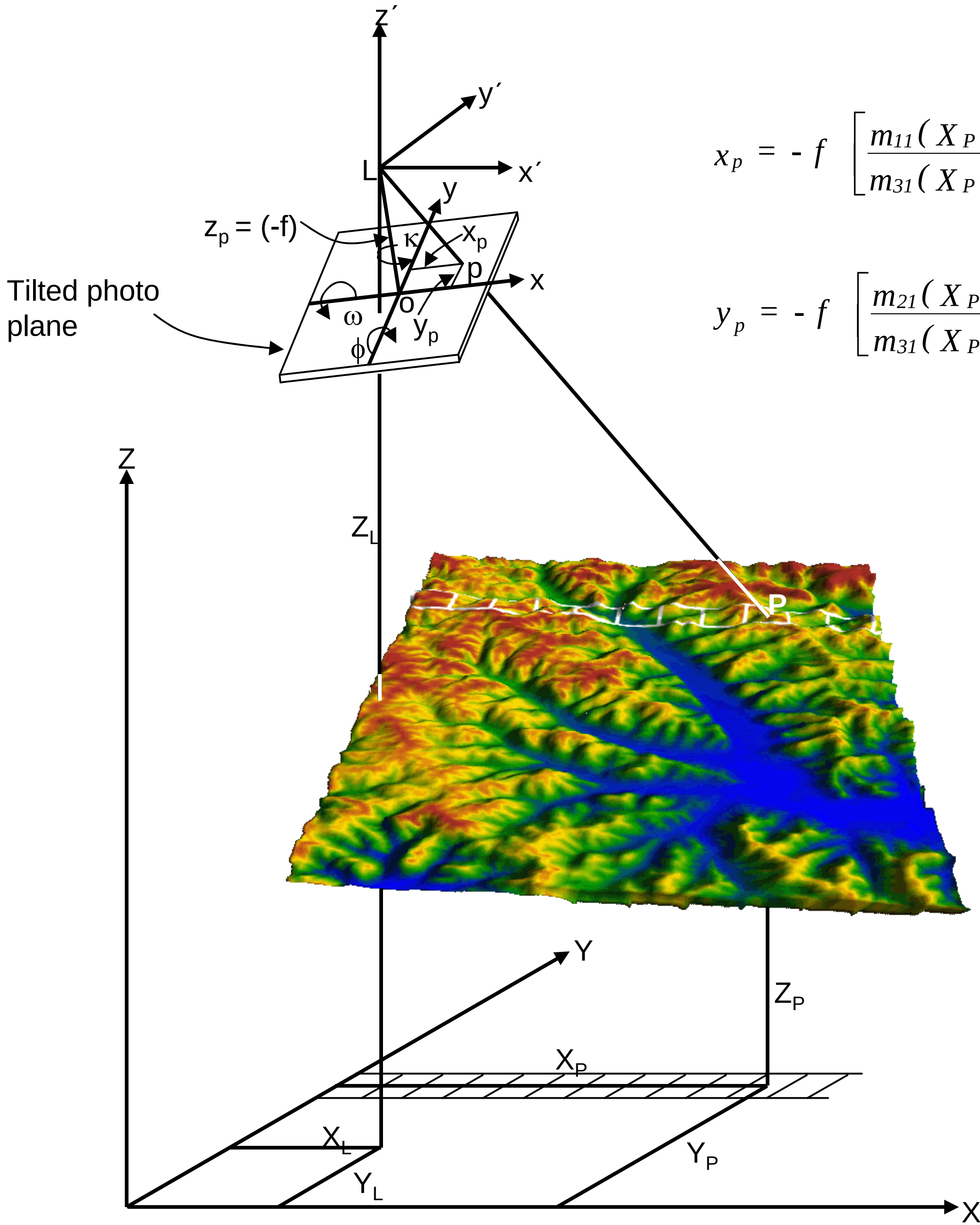


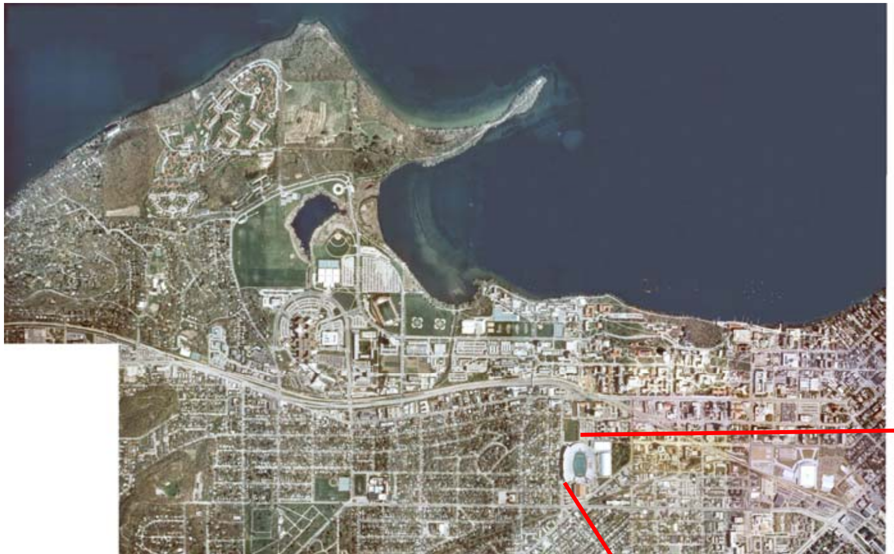
Figure 3.33 Space intersection.



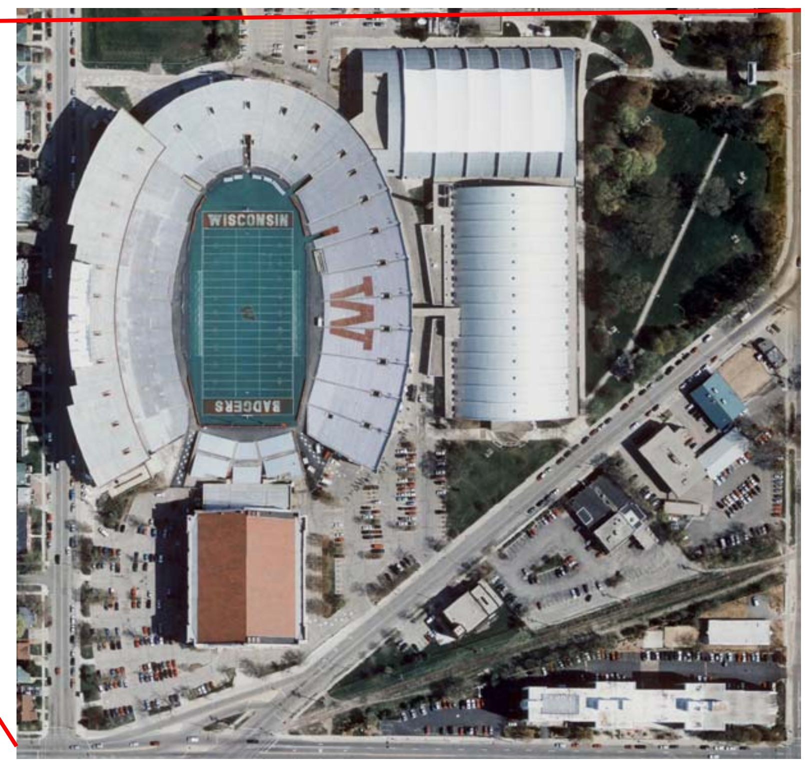
$$x_p = -f \left[\frac{m_{11}(X_P - X_L) + m_{12}(Y_P - Y_L) + m_{13}(Z_P - Z_L)}{m_{31}(X_P - X_L) + m_{32}(Y_P - Y_L) + m_{33}(Z_P - Z_L)} \right]$$

$$y_p = -f \left[\frac{m_{21}(X_P - X_L) + m_{22}(Y_P - Y_L) + m_{23}(Z_P - Z_L)}{m_{31}(X_P - X_L) + m_{32}(Y_P - Y_L) + m_{33}(Z_P - Z_L)} \right]$$

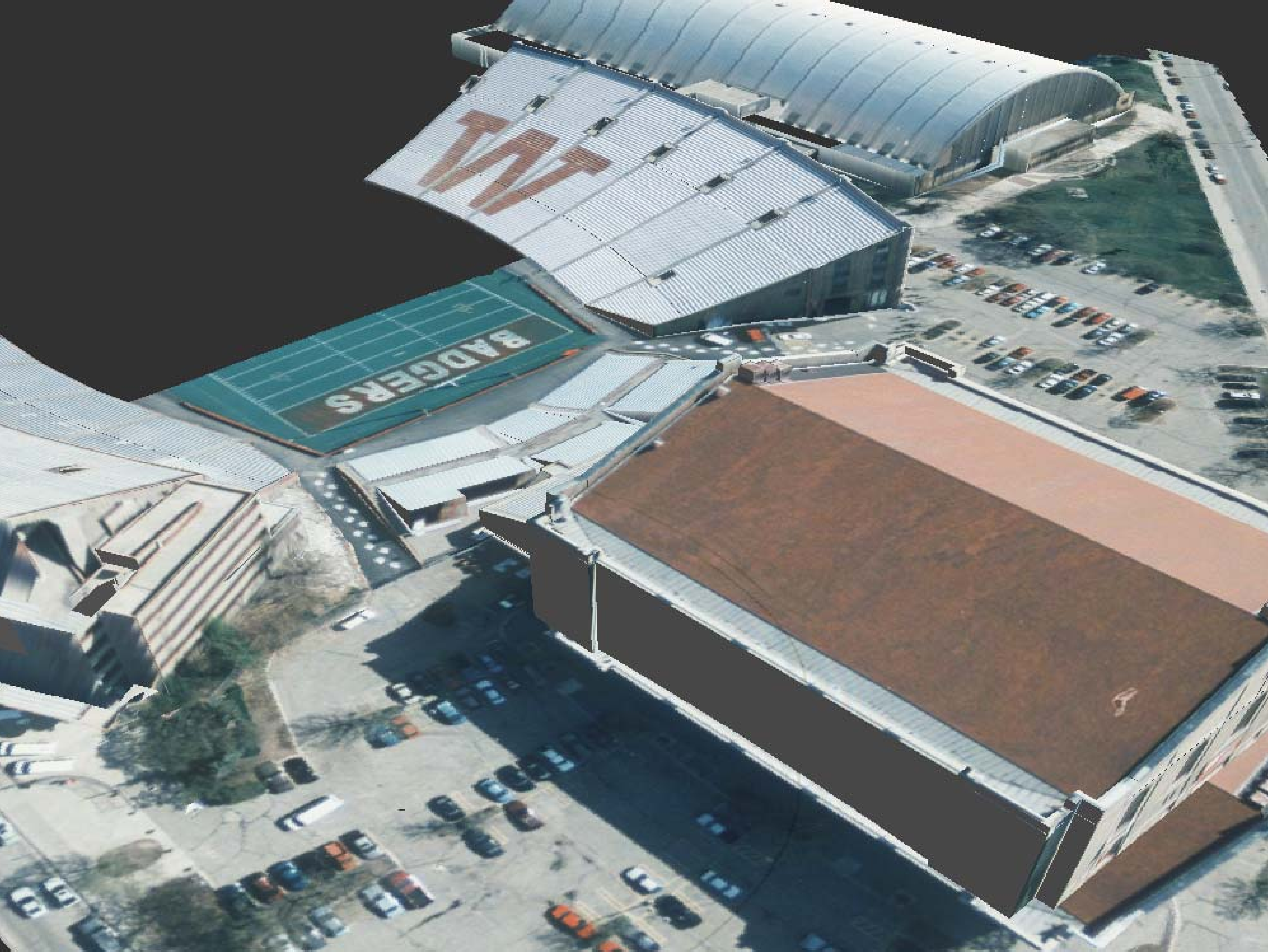
UW Campus Mapping Project



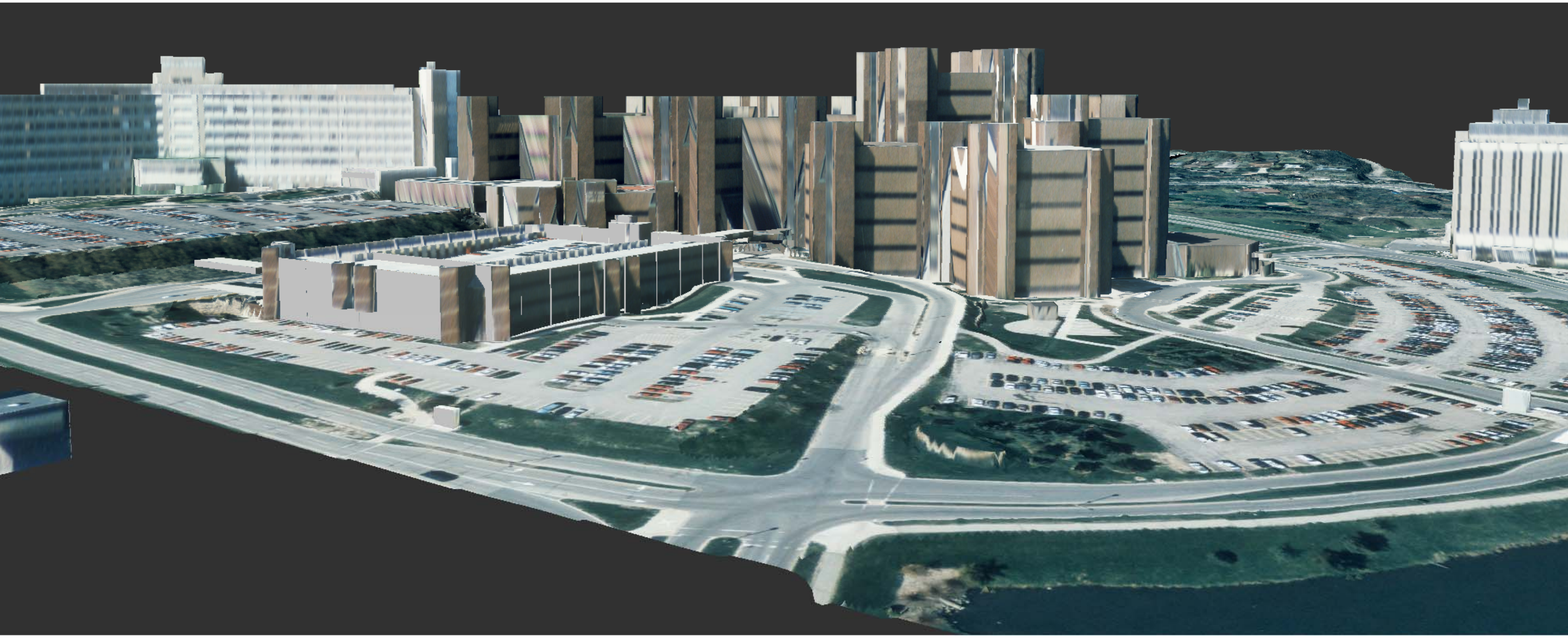
- 8 cm. resolution photography
- 16 cm. positional accuracy



- 3-D infrastructure features
- Orthophoto made over campus


BADGERS

Perspective Scene from the University of Wisconsin Campus Mapping Project



UW Hospital and Clinics