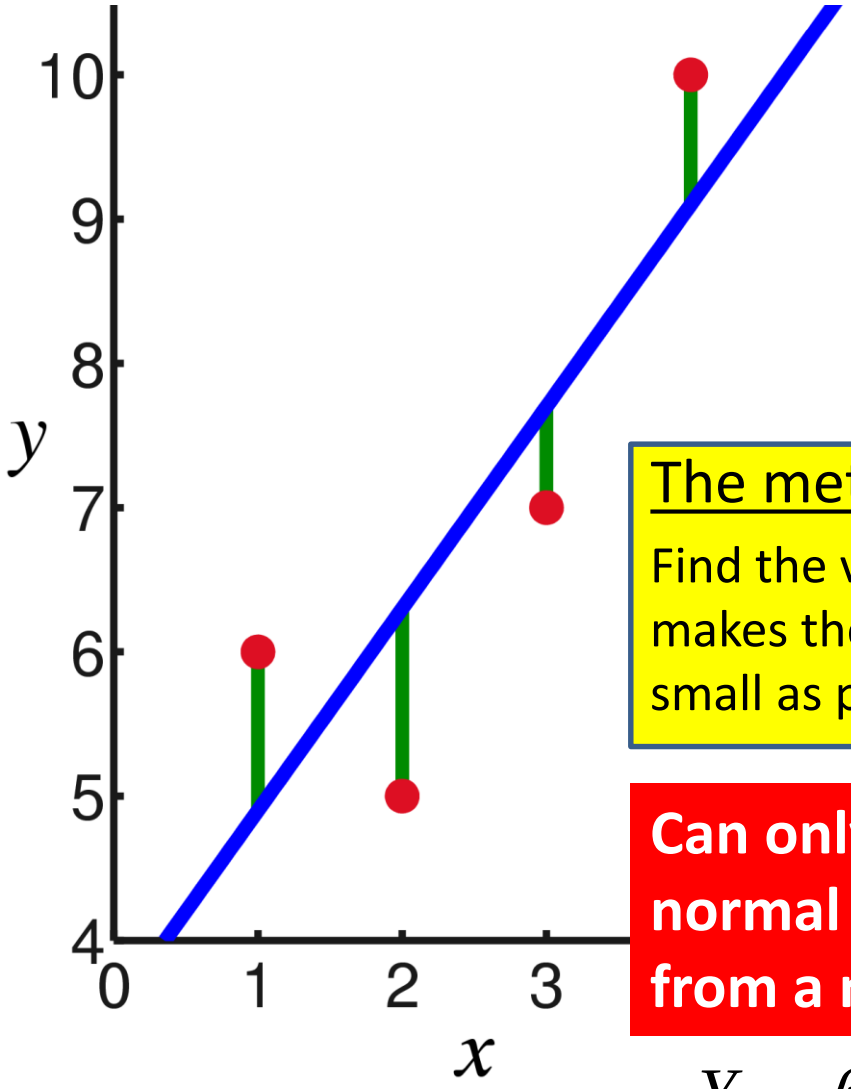
Maximum-likelihood estimation

A general method for estimating parameters in a model

The method of least-squares



Model for the expectation (fixed part of the model):

$$E[Y_i] = \beta_0 + \beta_1 x_i$$

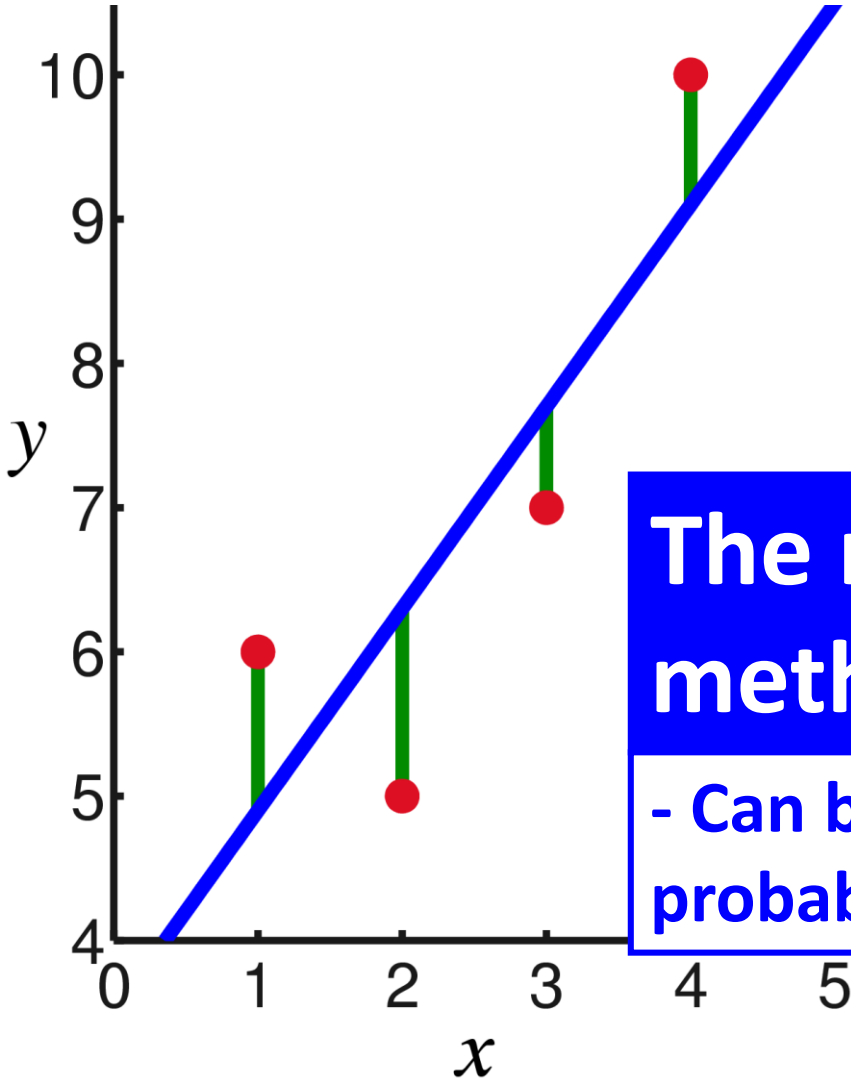
Residuals: $r_i = y_i - E[Y_i]$

The method of least-squares:

Find the values for the parameters (β_0 and β_1) that makes the sum of the squared residuals (Σr_j^2) as small as possible.

Can only be used when the error term is normal (residuals are assumed to be drawn from a normal distribution)

$$Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i, \text{ where } \varepsilon_i \sim N(0, \sigma)$$



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The maximum likelihood method is more general!

- Can be applied to models with any probability distribution

The maximum likelihood

Example:

We want to estimate the probability, p, that individuals are infected with a certain kind of parasite.

Ind.:	Infected:	Probability of observation:
1	1	p
2	0	$1-p$
3	1	p
4	1	p
5	0	$1-p$
6	1	p
7	1	p
8	0	$1-p$
9	0	$1-p$
10	1	p

The maximum likelihood method (discrete distribution):

1. Write down the probability of each observation by using the model parameters
2. Write down the probability of all the data

$$\Pr(\text{Data} \mid p) = p^6(1-p)^4$$

3. Find the value parameter(s) that maximize this probability

The maximum likelihood

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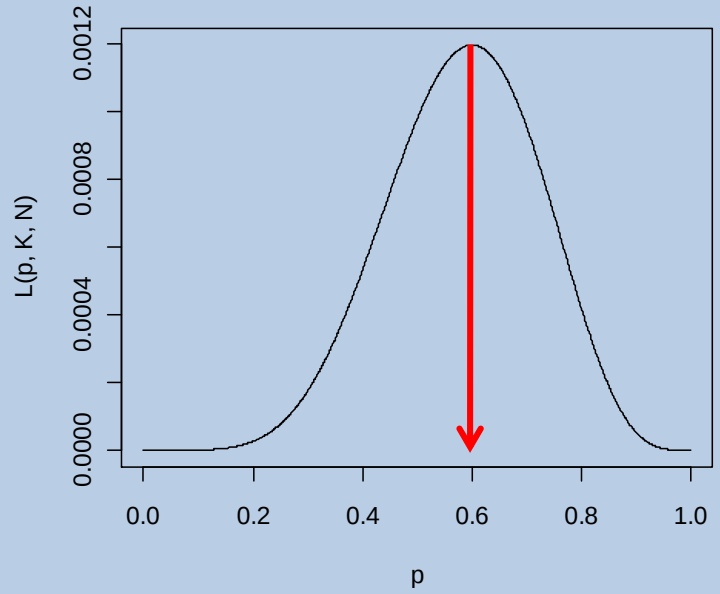
Likelihood function:

$$L(p) = \Pr(\text{Data} \mid p) = p^6(1-p)^4$$

- Find the value parameter(s) that maximize this probability



Exercises!