AMTH 108 Homework
5–9–125

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Question #21

Studies in population genetics indicate that 39% of the available genes for determining the Rh blood factor are negative. Rh negative blood occurs if and only if the individual has two negative genes. One gene is inherited independently from each parent. What is the probability that a randomly selected individual will have Rh negative blood?

(a) We are told that the probabilities of the genes are independent of each other. This means that the probability of having Rh negative blood is:

$$\begin{aligned} P[Rh\ Negative\ Blood] &= P[First\ Negative \cap Second\ Negative] \\ &= P[First\ Negative \cdot Second\ Negative] \\ &= 0.39 \cdot 0.39 = 0.1521 \end{aligned}$$

Question #25

A study of major flash floods that occurred over the last 15 years indicates that the probability that a flash flood warning will be issued is 0.5 and that the probability of dam failure during the flood is 0.33. The probability of dam failure given that a warning is issued is 0.17. Find the probability that a flash flood warning will be issued and a dam failure will occur.

(a) We are given the probability of dam failure given the warning as well as the probability of a warning. Using the theorem of a conditional probability:

$$\begin{aligned} P[Dam\ Fail \mid Warning] &= \frac{P[Dam\ Fail \cap Warning]}{P[Warning]} \\ \therefore P[Dam\ Fail \cap Warning] &= P[Dam\ Fail \mid Warning]P[Warning] \\ &= 0.17 \cdot 0.5 = 0.085 \end{aligned}$$

Question #26

The ability to observe and recall details is important in science. Unfortunately, the power of suggestion can distort memory. A study of recall is conducted as follows: Subjects are shown a film in which a car is moving along a country road. There is no barn in the film. The subjects are then asked a series of questions concerning the film. Half the subjects are asked, "How fast was the car moving when it passed the barn?" The other half is not asked the question. Later each subjects is asked, "Is there a barn in the film?" Of those asked the first question concerning the barn, 17% answer "yes"; only 3% of the others answer "yes." What is the probability that a randomly selected participant in this study claims to have seen the nonexistent barn? Is claiming to see the barn independent of being asked the first question about the barn?

(a) We are given two statistics. People who said yes (to question two) given the first question, and people who said yes (to question two) not given the first question. The probability that someone said yes about seeing the barn is:

$$\begin{aligned} P[Yes] &= P[Yes \mid Barn] + P[Yes \mid No\ Barn] \\ &= P[Yes \mid Barn]P[Barn] + P[Yes \mid No\ Barn]P[No\ Barn] \\ &= 0.175 \cdot 0.5 + 0.03 \cdot (1 - 0.5) \\ &= 0.085 + 0.015 = 0.1 \end{aligned}$$

(b) To test to see if the two events are independent, we check to see if the equivalency:

$$P[Yes \cap Barn] = P[Yes] \cdot P[Barn]$$

$$\begin{aligned} P[Yes \cap Barn] &= P[Yes \mid Barn][P[Barn] \\ &= 0.17 * 0.5 = 0.085 \end{aligned}$$

$$\Rightarrow P[Yes] \cdot P[Barn] = 0.1 * 0.5 = 0.05$$

$$\therefore P[Yes \cap Barn] \neq P[Yes] \cdot P[Barn]$$

Question #27

The probability that an unit of blood was donated by a paid donor is 0.67. If the donor was paid, the probability of contracting serum hepatitis from the unit is 0.0144. If the donor was not paid, this probability is 0.0012. A patient receives an unit of blood. What is the probability of the patient's contracting serum hepatitis from this source?

(a) The probability that someone contracting serum hepatitis is:

$$\begin{aligned} P[Hepatitis] &= P[Hepatitis \mid Donor] + P[Hepatitis \mid Not\ Donor] \\ &= P[Hepatitis \mid Donor]P[Donor] + P[Hepatitis \mid Not\ Donor]P[Not\ Donor] \\ &= 0.67 \cdot 0.0144 + 0.33 \cdot 0.0012 \\ &= 0.0096 + 0.0004 = 0.01 \end{aligned}$$

Question #36

It is reported that 50% of all computer chips produced are defective. Inspection ensures that only 5% of the chips legally marketed are defective. Unfortunately, some chips are stolen before inspection. If 1% of all chips on the market are stolen, find the probability hat a given chip is stolen given that it is defective.

(a) Using Bayes' theorem, the probability from a collection of mutually exclusive events, the probability that a chip that is stolen is also defective is:

$$P[Stolen \mid Defective] \Rightarrow$$
$$= \frac{P[Defective \mid Stolen]P[Stolen]}{P[Defective \mid Stolen]P[Stolen] + P[Defective \mid Not\ Stolen]P[Not\ Stolen]}$$
$$= \frac{0.5 * 0.01}{0.5 * 0.01 + 0.05 * (1 - 0.01)}$$
$$= \frac{0.005}{0.005 + 0.0495} = \frac{0.005}{0.0545} = 0.0917$$

Combinatorics Problem # 3.6

A rat is placed into the set of rooms depicted at left. When the rat hears a bell it moves to the other room, or not, depending on whimsy. The probability that the rat will move to room B given that it is in room A is 1/3. The probability that the rat will move to room A given that it is in room B is 1/2. The rat is initially placed into room B. What is the probability that the rat is in room B after the bell has rung twice?

(a) Using the tree diagram method, starting at room B, there is two paths: move to room A (50%) or stay (50%). After stage one, if the rat is in room A, there is a 1/3 chance it will move back to room B after two rings. If the room initially stayed in room B, then there is another 50% chance it will stay in the room again. Therefore the probability that the rat will be in room B after two rings is:

$$\begin{aligned} P[B] &= P[bab] + P[bbb] \\ &= 0.5 * 0.33 + 0.5 * 0.5 = 0.4167 \end{aligned}$$

Combinatorics Problem # 3.7

With respect to the previous problem, if the rat is observed to be in room B after two bells what is the probability that the rat was in room B after one bell?

(a) The probability that the room was in room B after one bell is:

$$\begin{aligned} P[bb \mid B] &= \frac{P[bb \cap B]}{P[B]} \\ &= \frac{0.5 * 0.5}{0.4167} = 0.6 \end{aligned}$$

Combinatorics Problem # 3.8

Each day our oil market analysts issue their forecast for the change in the price of oil at the close of trading for each of the next two days. (Assume that the daily change in the oil price is limited to one of +$1, $0, or $1.) Early on Monday afternoon, they issue the following two day report:

Tues closing forecast (relative to Monday's close)	**Prob**	**Wed closing forecast** (relative to Tuesday's close)	**Prob**
Up $1	0.60	Up $1	0.25
Unchanged	0.05	Unchanged	0.20
Down $1	0.35	Down $1	0.55

We buy oil today only if the price in two days is more likely to go up than stay the same or go down. Will be buy oil today?

(a) Using the tree diagram method again, there are three branches from Monday to Tuesday, Up, Unchanged, and Down. From each branch, there are another three branches for each one again, Up, Unchanged, and Down. If the prices go up or down by exactly 1$, then the leaves that result in a higher price are:

$$[Up,\ Up],\ [Up,\ Unchanged],\ and\ [Unchanged,\ Up]$$

The probability of these leaves are:

$$\begin{aligned} P[Buy\ Oil\ Today] &= P[Up,\ Up] + P[Up,\ Unchanged] + P[Unchanged,\ Up] \\ &= 0.6 * 0.25 + 0.6 * 0.2 + 0.05 * 0.25 = 0.2825 \end{aligned}$$

It is less likely for the oil price to rise in two days, so we should not buy oil today.

Combinatorics Problem # 3.11

The famed Dutch philosopher H. D. Holland was once cited by a Detroit traffic cop for failing to stop in the name of love – a supreme insult to such a renowned thinker. Professor Holland is a poor driver and the chances that he actually came to a full stop is only 0.15. On the other hand, the cop's ability to see truth is only .70. Given that a ticket was issued, what is the probability that Professor Holland actually stopped?

(a) The probability that he was given a ticket is when he stopped and still got a ticket, or he didn't stop and got a ticket. This is $0.15*0.7+0.85*0.7 = 0.7$. Given that he was given a ticket, the probability that he did stop is: $\frac{0.15*0.7}{0.7} = 0.15$.

Combinatorics Problem # 3.21

A communications circuit is known to have an availability of 0.99 (that is, 99% of the time, the circuit is operational). A total of n such circuits are going to be set up by the FAA between San Francisco and Los Angeles in such a way that the circuits will fail independently of each other. How many such parallel circuits must be set up to attain an overall availability of 0.99999?

(a) The probability for each single parallel (independent) circuit will be down is 0.01. Because these are all independent, the setup for the amount of parallel circuits we need is:

$$\begin{aligned} P[Down] &= P[C_1\ Down]P[C_2\ Down]...P[C_n\ Down] \\ &= 0.01 * 0.01 * ... * 0.01_n = (0.01)^n \end{aligned}$$

We now can setup the equivalency:

$$\begin{aligned} 0.99999 &= 1 - (0.01)^n \\ 0.01^n &= 10^{-5} \\ (10^{-2})^n &= 10^{-5} \\ \therefore n &= 2.5 \end{aligned}$$

Therefore the amount of lines has to be greater than 2.5, which (you can't have half of a circuit) is at least 3.