MATH 51 Homework #19

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Section 9.1 - Relations and Their Properties

1.

a) $R = \{(0,0),(1,1),(2,2),(3,3)\}$

b) $R = \{(1,3),(2,2),(3,1),(4,0)\}$

4.

a) Not reflexive (a can't be taller than a).

Not symmetric (a taller than b, b can't be taller than a).

Transitive (a taller than b, b taller than c, then a taller than c).

b) Reflexive (a born same day as a).

Symmetric (a born same day as b, b born say day as a).

Transitive (a born same day as b, b born same day as c, a born same day as c).

c) Reflexive (a has the same name as a).

Symmetric (a has the same name as b, b has the same name as a).

Transitive (a has the same name as b, b has the same name as c, a has the same name as c).

d) Reflexive (a has the same grandparent as a).

Symmetric (a has the same grandparent as b, b has the same grandparent as a).

Not transitive (if a and b share a common **grandmother** and b and c share a common **grandfather**, its possible that a and c don't share the same grandparent).

6.

a) Not reflexive (check a=1). Symmetric (if x+y=0, then y+x=0). Not transitive (check x=1, y=-1, z=1).

b) Reflexive (a=±a, $\forall a \in \mathbb{R}$). Symmetric (if x=±y, then y=±x). Transitive (if x=±y, and y=±z, then x=±z).

c) Reflexive (any number subtract itself is 0, and 0 is rational). Symmetric (if $x - y$ rational, then $y - x$ rational). Transitive (if $x - y$ rational and $y - z$ rational then $x - z$ rational).

d) Not reflexive (check a=1, 1≠2). Not symmetric (check x=2, y=1, then y=2, x=1). Not transitive (x=2y, y=2z, x≠4y).

e) Reflexive (any number squared is $a^2 \geqslant 0$). Symmetric (if $xy \geqslant 0$, then $yx \geqslant 0$). Not transitive (check $x = -1$, $y = 0$, $z = 1$, $xz \ngeqslant 0$).

f) Not reflexive (check a=1). Symmetric (if $xy = 0$, then $yx = 0$). Not transitive (check $x = 1$, $y = 0$, $z = 1$, $xz \neq 0$).

g) Not reflexive (choose any $a \neq 1$. Not symmetric (check x=1, y=2, then x=2, y=1). Transitive (if $x = 1$, and $y = 1$, then (x, z) is always true).

h) Not reflexive (choose any $a \neq 1$). Symmetric (if x or y = 1, then y or x = 1 is true). Not transitive (check x=2, y=1, z=2. (x, z) does not work, but (x, y) and (y, z) works.).

8.

This proof works on the vacuous fact that the pair $(a, b) \in R$ is always false. It is not possible to move from the nonempty set S to $R = \varnothing$. Therefore:

Reflexive: $\forall a \in S : (a, a) \notin R$, because $R = \varnothing$. This is not reflexive.

Symmetric: If $(a, b) \in R$ then $(b, a) \in R$. Since $(a, b) \in R$ is false, then $(b, a) \in R$ is vacuously true.

Transitive: If $(a, b) \in R \wedge (b, c) \in R$ then $(a, c) \in R$. Since the first part of the implication is false, this is again vacuously true.