AMTH 108 Homework
7–6–70

Tamir Enkhjargal

January 2019

Question #14

(a) The expected value $E[X]$ is

$$\sum x \cdot f(x) = 0.7 * 0 + 0.2 * 1 + 0.05 * 2 + 0.03 * 3 + 0.01 * 4 + 0.01 * 5 = 0.48$$

(b) μ_x is the same as the expected value, which is 0.48

(c) $E[X^2] = \sum x^2 \cdot f(x) = 0.7 * 0^2 + 0.2 * 1^2 + 0.05 * 2^2 + 0.03 * 3^2 + 0.01 * 4^2 + 0.01 * 5^2 = 1.08$

(d) The variance $VarX$ is the same as $E[X^2] - (E[X])^2 = 1.08 - 0.2304 = 0.8496$

(e) σ_x^2 is the same as $VarX$, therefore $\sigma_x^2 = 0.8496$

(f) The standard deviation is just σ so if we square root the variance we find $\sqrt{0.8496} = 0.9217$

(g) The subscript under the sigma is X, which is denoted to be our number of grafts that fail.

Question #17

(a) We are given that the it follows the series:

$$= \sum_{x=1}^{\infty} x(0.7)(0.3)^{x-1} = E[X]$$

$$= (1 * 0.7) + (2 * 0.7 * 0.3) + (3 * 0.7 * 0.3^2) + ... + \infty$$

$$\therefore 0.3E[X] = (1 * 0.7 * 0.3) + (2 * 0.7 * 0.3^2) + (3 * 0.7 * 0.3^3) + ... + \infty$$

$$\therefore E[X] - 0.3E[X] = 0.7 + (0.7 * 0.3) + (0.7 * 0.3^2) + ... + \infty$$

$$\Rightarrow 0.7E[X] = 0.7(1 + 0.3 + 0.3^2 + 0.3^3 + ... + 0.3^{\infty}$$

$$E[X] = 1 + 0.3 + 0.3^2 + 0.3^3 + ... + 0.3^{\infty}$$

We have derived that $E[x]$ can now be a geometric series, so the answer will converge to

$$\frac{a}{1-r} = \frac{1}{1-0.3} = \frac{1}{0.7} = 1.428$$

Question #21

(a) If $E[x] = 3$, $E[Y] = 10$, then $E[3X + Y - 8] = 3 * 3 + 10 - 8 = 11$

(b) $E[2X - 3Y + 7] = 2 * 3 - 3 * 10 + 7 = -17$

(c) $VarX = E[X^2] - (E[X])^2 = 25 - 3^2 = 16$

(d) $\sigma_x = \sqrt{16} = 4$

(e) $VarY = E[Y^2] - (E[Y])^2 = 164 - 10^2 = 64$

(f) $\sigma_y = \sqrt{64} = 16$

(g) $Var[3X + Y - 8] = 3^2 Var[X] + Var[Y] - 8 = 9 * 16 + 64 - 0 = 208$

(h) $Var[2X - 3Y + 7] = 2^2 Var[X] + (-3)^2 Var[Y] + 0 = 4 * 16 + 9 * 64 = 640$

(i) $E[(X - 3)/4] = E[X]/4 - E[3]/4 = 3/4 - 3/4 = 0$

$Var[(X - 3)/4] = Var[X] * 0.25^2 - Var[3]/4 = 1$

(j) $E[(Y - 10)/8)] = 10/8 - 10/8 = 0$

$Var[(Y - 10)/8] = 64/8^2 = 1$

(k) The first part is $E[\frac{X-\mu}{\sigma}] = 0$

The second part is $Var[\frac{X-\mu}{\sigma}] = 1$

Question #31(a, b, e, & g)

(a) For values $x = 3, 4, 5$, $f(x) = 0, 1/5, 4/5$, respectively. The probabilities for these nontrivial values add up to one. This is a proper discrete random variable.

(b) $E[X] = 3 * 0 + 4 * 1/5 + 5 * 4/5 = 4.8$

(c) $E[X^2] = 3^2 * 0 + 4^2 * 1/5 + 5^2 * 4/5 = 23.2$

(d) $\sigma^2 = E[X^2] - (E[x])^2 = 23.2 - 4.8^2 = 0.16$. $\sigma = \sqrt{0.16} = 0.4$

Discrete Distributions Problem # 4.2

Let X be a random variable whose distribution function takes the form:

$$F(x) = \begin{cases} 0.0, & \text{if } x < -1 \\ 0.3, & \text{if } -1 \leq x < 1 \\ 0.4, & \text{if } 1 \leq x < 2 \\ 0.6, & \text{if } 2 \leq x < 4 \\ 0.7, & \text{if } 4 \leq x < 5 \\ 1.0, & \text{if } 5 \leq x \end{cases}$$

(a) The mean of X is:

$$-1 * 0.3 + 1 * 0.1 + 2 * 0.2 + 4 * 0.1 + 5 * 0.3 = 2.1$$

The variance of X is:

$$\begin{aligned} &= E[X^2] - (E[x])^2 \\ &= (-1)^2 * 0.3 + 1^2 * 0.1 + 2^2 * 0.4 + 4^2 * 0.1 + 5^2 * 0.3 - 2.1^2 \\ &= 5.89 \end{aligned}$$

Discrete Distributions Problem # 4.3

The random variable X has a mean of 2 and a variance of 1. What is the expected value of the random variable $Y = (X + 1)^2$?

(a) If $Y = (X + 1)^2$, then $E[Y]$ is:

$$\begin{aligned} E[Y] = E[(X+1)^2] &= E[X^2 + 2X + 1] \\ &= E[X^2] + 2E[X] + E[1] \end{aligned}$$

$$\begin{aligned} Var[X] = 1 &= E[X^2] - (E[X])^2 \\ &= E[X^2] - (2^2) \\ &\therefore E[X^2] = 5 \end{aligned}$$

$$\begin{aligned} &\therefore E[X^2] + 2E[X] + E[1] \\ &= 5 + 2 * 2 + 1 = 10 \end{aligned}$$