MATH 51 Homework #16

Tamir Enkhjargal

May 2019

1.

2.

(a) This is false because of the handshake theorem. If a graph is 5-regular with 15 vertices, that means there is 5*15=75 total "handshakes" or edges. The handshake theorem states that there is $\sum deg(v) = 2n$. Each v would then have to have a number of edges that is divisible by 2. $2 \nmid 75$ so it doesn't work.

(b)

3.

(a) We can use the contrapositive here, if G contains cycles, then there is *at least* two paths $P_1,\ P_2,\ P_i$ that goes from v to w. Therefore the contrapositive is true, and thus we can state that if G is a simple graph and there is an unique path from v to w.

(b) We can prove through induction.

Base case: $P(1)$, so the graph has 1 vertices and there is 0 edges.

Inductive step: We can assume $P(k)$ is true, where $P(k)$ is that G is bipartite. Following the steps, when there is only an one unique single path from v to w, and we can only move one edge by one, then G must be bipartite.

4.

By definition, a tree is a simple connected graph that has no cycles. If there is no cycles, then there is always a single unique path from one vertex to the other. As there is no cycles, there will always be two "endpoint" vertices. These two endpoints only have degree 1, because they don't loop around (no cycles allowed). Since at least two vertices must have degree 1, a tree can not be 2-regular.