MATH 51 Homework #4

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Section 1.4

42.

a) Let domain be x all megabytes on a hard disk, y all users.

   $F(x)$ is "there is less than x megabytes free on a hard disk".

   $W(y)$ is "warning is sent to y users".

   $\forall y \forall x, F(30) \rightarrow W(y)$

b) Let domain be x all directories and y all files.

   $D(x)$ is "x directories in the file system can be opened"

   $F(y)$ is "x files can be closed"

   E is "system errors have been detected"

   $\forall x \forall y, E \rightarrow (\neg D(x) \wedge \neg F(y))$

c) Let domain be x be all users logged on.

   $L(x)$ is "x is currently logged on"

   B is "system can be backed up"

   $\exists x, L(x) \rightarrow \neg B$

d) Let domain be x be all megabytes of memory, y be total connection speed.

   $M(x)$ is "There are at least x megabytes of memory available"

   $C(y)$ is "The connection speed is at least y kilobits per second"

   V is "Video on demand can be delivered"

   $\forall x \forall y, (M(8) \wedge C(56)) \rightarrow V$

Section 1.5

2.

a) There's a number x where $xy = y$ for all y.

b) For all x and y, if both x is equal to or larger than 0, and y is less than 0, then $x - y$ is greater than 0.

c) There's a number z where $x = y + z$ for all permutations x and y.

8.

Let $Q(x, y)$ be "Student x has been a contestant on quiz show y."

Let x be all students at your school and y be all quiz shows on TV.

a) $\exists x \exists y, Q(x, y)$

b) $\neg\exists x \exists y, Q(x, y)$

c) $\exists x \exists y, Q(x, (Jeopardy! \wedge Wheel\ of\ Fortune))$

d) $\forall y \exists x, Q(x, y)$

e) $\exists y \exists x_1 \exists x_2, (x_1 \neq x_2) \wedge (Q(x_1, Jeopardy! \wedge Q(x_2, Jeopardy!)$

18.

a) Let domains x be all consoles and y be all fault conditions.

$A(x, y)$ is "x console must be accessible during y fault conditions".

$\forall y \exists x, A(x, y)$

b) Let domains x be all users and y be all messages sent.

$S(x, y)$ is "y message sent by x users on the system".

$R(x)$ is "The email address of user x can be retrieved".

$\forall x \exists y, S(x, y) \rightarrow R(x)$

c) Let domains x be all mechanisms, y be all breaches, z be all processes.

$D(x, y)$ is "Mechanism x can detect y breach".

$C(z)$ is "Process z has been compromised".

$\forall y \exists x \exists z, D(x, y) \leftrightarrow \neg C(z)$

d) Let x be all paths and y be all distinct endpoints.

$C(x, y)$ is "There is x path connecting every y on the network".

$\forall y_1 \forall y_2 \exists x_1 \exists x_2, (y_1 \neq y_2) \wedge (x_1 \neq x_2) \wedge (C(x_1, (y_1 \wedge y_2) \wedge C(x_2, (y_1 \wedge y_2))$

e) Let domains x be all users, y be all passwords of every user.

$P(x, y)$ is "x knows the password y".

$A(x)$ is "x is a system administrator".

$(\forall x \forall y, \neg A(x) \rightarrow \neg P(x, y)) \wedge (\exists x \forall y, A(x) \rightarrow P(x, y))$

28.

a) True, set $y = x^2$

b) False, x can be negative, and any squared number can't be negative.

c) True, x can be 0.

d) False, this would break the commutative law of addition.

e) True, set $y = \frac{1}{x}$

f) False, would be true if we *could* set $x = \frac{1}{y}$ but x can't depend on y.

g) True, set $y = -x + 1$

h) False, solving for a solution leads to $2 = 5/2$.

i) False, there is only one solution $(1, 1)$, but this statement looks for all x.

j) True, just set $z = \frac{x+y}{2}$, since it can depend on both x and y.

36.

a) Let domain be x all people and y be all dollars playing the lottery.

$L(x, y)$ is "x person has lost y dollars playing the lottery".

$\neg\exists x\exists y, (y > 1000) \wedge L(x, y)$

Negation: $\exists x\forall y, (y > 1000) \wedge L(x, y)$

There exists a person that lost more than 1000 dollars playing the lottery.

b) Let domain be x students in this class.

$C(x_1, x_2)$ is "x_1 student has chatted with x_2 student."

$\exists x_1\exists x_2, (x_1 \neq x_2) \wedge (C(x_1, x_2))$

Negation: $\forall x_1\forall x_2, (x_1 = x_2) \vee \neg C(x_1, x_2)$

Every student has either talked with no one or with themselves.

c) Need help with this problem.

d) Let x be all students, and y be all exercises in the book.

$S(x, y)$ is "x student has solved y exercise".

$\exists x\forall y, S(x, y)$

Negation: $\forall x\exists y, \neg S(x, y)$

All students have not solved some exercise in the book.

e) Let x be all students, and y be all exercises per section and z be all sections in the book.

$S(x, y)$ is "x student has solved each y exercise"

$B(y, z)$ is "y exercises have been solved in z section"

$\neg\exists x\forall z\exists y, (S(x, y) \wedge B(y, z)$

Negation: $\exists x\exists z\forall y, (\neg(S(x, y)) \wedge (\neg(B(y, z))$

There exists a student that has solved all problems in some section in the book.