AMTH 108 Homework
2–11–110

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January 2019

Question #17

Evaluate each of these expressions:

a) ${}_9C_4 = \frac{9!}{(9-4)!4!} = \frac{9*8*7*6}{4*3*2*1} = 126$

b) ${}_8C_3 = \frac{8!}{(8-3)!3!} = \frac{8*7*6}{3*2*1} = 56$

c) ${}_8C_5 = \frac{8!}{(8-5)!5!} = \frac{8*7*6*5*4}{5*4*3*2*1} = 56$

d) ${}_8C_0 = \frac{8!}{(8-0)!0!} = \frac{1}{1} = 1$

Question #19

A chemical engineer has 7 different treatments that she wishes to compare for effectiveness in producing a sand cast to be used in casting molten iron. She wants to compare each treatment to each of the others. How many pairwise comparisons will she have to make? That is, in how many ways can these treatments be selected two at a time?

1. If we do the function ${}_7C_2$ we get 21 different pairs that can be selected at a time.

Question #21

A firm employs 10 programmers, 8 system analysts, 4 computer engineers, and 3 statisticians. A "team" is to be chosen to handle a new long-term project. The team will consist of 3 programmers, 2 systems analysts, 2 computer engineers, and 1 statistician.

a) The total team combinations are ${}_{10}C_3 * {}_8C_2 * {}_4C_2 * {}_3C_1 = 120*28*6*3 = 60480$

b) We put a restriction that one of the engineers be fixed. The total team combinations are now ${}_{10}C_3 * {}_8C_2 * {}_3C_1 * {}_3C_1 = 120*28*3*3 = 30240$

Question #22

A company receives a shipment of 20 hard drives. Before accepting the shipment, 5 of them will be randomly selected and tested. If all 5 meet specifications, then the shipment will be accepted; otherwise all 20 will be returned to the manufacturer. If, in fact, 3 of the 20 drives are defective, what is the probability that the shipment will not be accepted.

a) The probability that the shipment will not be accepted is $P = 1 - \frac{\text{Chance of not pulling a defect}}{\text{Total combinations}}$. The chance of pulling a defect is ${}_{17}C_5$ and the total combinations are ${}_{20}C_5$.

$$= 1 - \frac{\frac{17!}{(17-5)!5!}}{\frac{20!}{(20-5)!5!}}$$

$$= 1 - \frac{\frac{17!}{12!5!}}{\frac{20!}{15!5!}}$$

$$= 1 - \frac{6188}{15504}$$

$$= \frac{137}{228} = 0.3991228 = 39.91128\%$$

Question #23

A control chart is used to monitor the average thread count produced by a machine making spandex cloth. Samples are taken periodically, and each sample is classified into one of 5 categories. These are: in control but above average, in control and average, in control but below average, out of control and high, and out of control and low. In taking a series of 20 samples, in how many ways can we obtain a series in which there are exactly:

a) These series of combinations can be found with ${}_{20}C_5 * {}_{15}C_5 * {}_{10}C_5 * {}_5C_3 * {}_2C_2$ which is functionally the same as $\frac{20!}{5!5!5!3!2!} = 117327450240$.

b) This combination can be found with $\frac{20!}{18!2!} = \frac{20*19}{2} = 190$

Question #25

In studying a chemical reaction, 12 experiments will be conducted. Four different temperatures will be used 3 times, each with the temperatures run in random order. In how many orders can the series of experiments be conducted?

Using the *Permutations of Indistinguishable Objects* formula:

$$= \frac{12!}{3!3!3!3!}$$
$$= 369600$$

Question #31

A project manager has 10 chemical engineers on her staff. Four are women and six are men. These engineers are equally qualified. In a random selection of three workers, what is the probability that no women will be selected? Would you consider it unusual for no women to be selected under these circumstances? Explain.

a) The probability of randomly choosing no women is the same as the chance of choosing exactly 3 men out of the total amount of combinations.

$$= \frac{{}_6C_3 \ * \ {}_4C_0}{{}_{10}C_3}$$

$$= \frac{\frac{6!}{3!3!}}{\frac{10!}{7!3!}}$$

$$= \frac{20}{120} = \frac{1}{6} = 16.667\%$$

b) Having a 1/6th chance of randomly choosing only men is not unusual, in fact running through the numbers of randomly choosing 2 men and 1 woman, 1 man and 2 women, and 3 women only are all similarly small numbers, and will theoretically add up to 1.

Question #34

A flashlight operates on two batteries. Eight batteries are available, but three are dead. In a random selection of batteries, what is the probability that exactly one dead battery will be selected?

1. The probability that you will find exactly only one dead battery out of choice of two batteries is:

$$P(Exactly\ one\ dead) = \frac{Choose\ one\ dead\ battery}{Total\ pair\ combinations}$$

$$= \frac{{}_5C_1 \ * \ {}_3C_1}{{}_8C_2}$$

$$= \frac{5 * 3}{28} = \frac{15}{28} = 53.57\%$$

Combinatorics Problem # 1.3

There are 20 coins in my pocket in this distribution: 7 pennies, 3 nickels, 2 dimes, 5 quarters, and 3 dollar coins. I pull out 10 coins. In how any ways can pull out 6 pennies, 2 dollars, and 2 quarters. What is the probability that a selection of 10 coins will contain all three of the dollar coins?

a) The combinations of exactly pulling 6 pennies, 2 dollars, and 2 quarters is:

$$\begin{aligned} &= {}_7C_6 * {}_3C_2 * {}_5C_2 \\ &= 7 * 3 * 10 = 210 \end{aligned}$$

b) The probability of pulling all three dollar coins is the same as:

$$\begin{aligned} P(Pulling\ three\ dollars) &= \frac{Pulling\ three\ dollars}{Total\ combinations\ of\ coins} \\ &= \frac{{}_3C_3 * {}_{17}C_7}{{}_{20}C_{10}} \\ &= \frac{1 * 19448}{184756} = \frac{2}{19} = 10.526\% \end{aligned}$$

Combinatorics Problem # 1.6

A president, treasurer, and secretary (all different) are elected from a club of 9 members. How many choices of club officers are there if there are no restrictions imposed as to who can serve? How many choices of club officers are there if there are two specific club members who refuse to serve together? How many choices of club officers are there if one club member refuses to serve unless they can be president?

a) If there are no restrictions, then there are ${}_9C_3 = 84$ total combinations. If order/rank matters (permutation) then it is ${}_9P_3 = 504$.

b) If we find the contradiction, ways in which specifically those two *are* in office, then it is ${}_2P_2 * {}_7P_1 = 2*7 = 14$. Therefore there are $504 - 14 = 490$ combinations where two specific people are not in office together.

c) We put a restriction on the president role, so we can choose two roles from the remaining 8 people: ${}_8P_2 = 56$

Combinatorics Problem # 1.15

A byte consists of eight binary digits (called bits) usually labeled $b_7b_6b_5b_4b_3b_2b_1b_0$. Each bit may be represented by either the digit 0 or the digit 1. For example, the eight bits of the byte 011011110 are... How many bytes contain an even number of bits equal to 1? If a byte is selected at random, what is the probability that exactly 3 of the bits are a 0?

a) We can think of having an even number of bits equal to 1 as having either 0, 2, 4, 6, or 8 bits be a 1. This means:

$$\begin{aligned} &={}_8C_0 + {}_8C_2 + {}_8C_4 + {}_8C_6 + {}_8C_8 \\ &= 1 + 28 + 70 + 28 + 1 \\ &= 128 \end{aligned}$$

b) If we set a restriction that exactly 3 of the bits are 0, we can see:

$$\begin{aligned} P(Exactly\ 3\ 0s) &= \frac{Combinations\ of\ having\ exactly\ 3\ 0s}{Total\ combinations} \\ &= \frac{{}_8C_3 * {}_5C_5}{2^8} \\ &= \frac{56 * 1}{256} = \frac{7}{32} = 21.875\% \end{aligned}$$

Combinatorics Problem # 1.21

When *Teen Talk Barbie* uttered those immortal words "Math class is tough!" in 1992 the *American Association of University Women* criticized this so strongly that Mattel deleted the phrase from the doll's vocabulary and offered a swap to anyone who had such a doll. Each doll was programmed to say 4 different phrases randomly selected from a list of 270 phrases such as "Will we ever have enough clothes?", "I love shopping!", and "Wanna have a pizza party?" What percentage of the dolls manufactured at the time uttered the offending phrase about math class?

1. If there was 270 phrases, including the math phrase, then there are 269 phrases that didn't include the math phrase.

   The total amount of configurations of dolls with phrases:

   $${}_{270}C_4 = 216546345$$

   When the phrase was taken out, the amount of configurations of dolls without the phrase was:

   $${}_{269}C_4 = 213338251$$

   Therefore the percentage of dolls with the phrase is:

   $$\frac{216546345 - 213338251}{216546345} \cdot 100 = 1.482\%$$