MATH 51 Homework #12

Tamir Enkhjargal

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Section 2.5

2.

a) This set is countably infinite. For the function, find $f : \mathbb{Z}_+ \to A$ as $f(x) = x + 10$. This is one-to-one.

b) The odd negative integers are countably infinite. The function is $f : \mathbb{Z}_+ \to A$ as $f(x) = 1 - 2x$. This is one-to-one.

c) The integers $|A| < 1000000$ is finite. The range of the function $f : \mathbb{Z}_+ \to A$ is from $-1000000 < x < 1000000$, where $x \in \mathbb{Z}$.

d) The real numbers, from any domain (i.e. $a < x < b$) is uncountable.

e) This is countably infinite. This will consist of a sets in form $\{2, \mathbb{Z}_+\}$ and $\{3, \mathbb{Z}_+\}$. $f : \mathbb{Z}_+ \to A \times \mathbb{Z}_+$ where $f(2, x) = 2x$ and $f(3, x) = 2x-1$. These are one-to-one.

f) The integers that are multiples of 10 are countably infinite. $f : \mathbb{Z}_+ \to A$ where $f(x) = 10 - 10x$ if x *odd* and $f(x) = 5x$ if x *even*.

10.

a) $A \in \mathbb{R},\ A = [-1, 1]$ and $B \in \mathbb{R},\ B = (-1, 1)$. $A - B = \{-1, 1\}$

b) A is all real numbers. B is all real numbers $-\mathbb{Z}$. Therefore $A - B = A - (A - \mathbb{Z}) = \mathbb{Z}$.

c) A all real numbers, B all real numbers from $[0, 1]$. $A - B =$ all real numbers not including $[0, 1]$

18.

If A and B have n elements.

$$|A| = n \tag{1}$$
$$|P(A)| = 2^n \tag{2}$$
$$|B| = n \tag{3}$$
$$|P(B)| = 2^n \tag{4}$$
$$|P(A)| = |P(B)| \tag{5}$$

Question 4.

(a) One way to show a bijective is to graph and use the vertical line test.

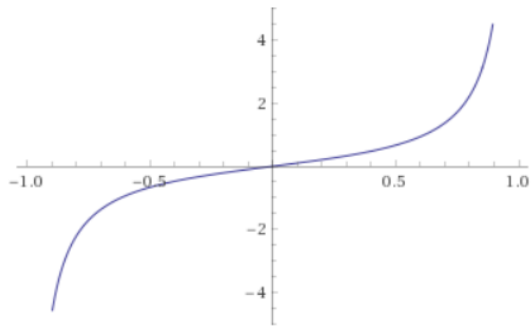

Figure 1: A graph of the function $\frac{x}{1-x^2}$ from -1 to 1

Here, we see that this function is bijective because all x have a unique y and vice versa, all y have a unique x. This is the same definition of a "function" we've been working with since elementary school.

(b) We can say that there is a bijection iff the cardinality of the two sets are the same. We need to show that there is an injection from $(-1,1) \to [-1,1]$ and a surjection from $[-1,1] \to (-1,1)$. If we just plot the numbers $f(x) = x$, all numbers in $(-1,1)$ can map inside $[-1,1]$ uniquely (injective). All numbers from $[-1,1]$ to $(-1,1)$ are also onto. Thus this means there is a bijection between the two sets, and by the *Shroder-Bernstein theorem*, they have the same cardinality.

Question 5.

If S is the union of countably infinite sets, find a surjection from $f : \mathbb{Z}_+ \times \mathbb{Z}_+ \to S$.

If there are $n \in \mathbb{Z}_+$ elements in S. We have previously stated that $\mathbb{Z}_+$ is infinitely countable.

We also know that the cartesian product $\mathbb{Z}_+ \times \mathbb{Z}_+$ is infinitely countable. We can just use an prime factored form, such as $2^n 3^m$.

Therefore, $f : \mathbb{Z}_+ \times \mathbb{Z}_+ \to S,\ n, m \in \mathbb{Z}_+,\ f(n,m) = f_m(n)$, where an surjective infinitely countable set onto a surjective infintely countable set is also countable.

This is an example of Cantor's first diagonal arugment.