MATH 51 Homework #5

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April 2019

Section 1.5

40.

a) $x = 0,\ 0 * y \neq 1$

b) $x = 0,\ y = 11,\ 121 - 0 > 100$

c) $x = 1,\ y = 1,\ 1^2 = 1^3$

44.

$\forall a \forall b \forall c \exists x \exists y,\ a, b, c, x, y \in \mathbb{R},\ [(a \neq 0) \wedge (ax^2 + bx + c = 0) \wedge (ay^2 + by + c = 0) \wedge [\forall z((x = z) \vee (y = z)) \vee (az^2 + bz + c \neq 0)]]$

Section 1.6

20.

a) This is not a valid argument because it needs to be true for all values a. Any example where a is negative works for a^2 being positive, but that doesn't mean a needs to be positive.

b) If $x^2 \neq 0$ means $x \neq 0$, then $a \in \forall x$ where x is a real number is the same as $a^2 \neq 0 \to a \neq 0$

24.

On steps 3 and 5, since it is in the form $P(c) \vee Q(c)$, we can't directly simplify to just $P(c)$ or $Q(c)$, you would have to make sure *both* are true, so if it was $P(c) \wedge Q(c)$ then it would be okay to simplify.

34.

P is "Logic is difficult to many students".
Q is "Many students like logic".
R is "Mathematics is easy".

1. $P \vee \neg Q$
2. $R \to \neg P$

a) $Q \to \neg R$, this is true because it is logically equivalent to $\neg Q \vee \neg R$ which is logically equivalent to the resolution of statement 1. and $\neg R \vee \neg P$

b) $\neg R \to \neg Q$ is false. If $Q \to \neg R$, then $\neg Q \vee \neg R \equiv \neg R \vee \neg Q \equiv R \to \neg Q$, and this statement does not match.

c) $\neg R \vee P \equiv R \to P$, but this contradicts with statement 2., so this is false.

d) $\neg P \vee \neg R \equiv \neg R \vee \neg P \equiv R \to \neg P$ which is the same as statement 2., this is true.

e) $\neg Q \to (\neg R \vee \neg P)$. From problem a, we determined that $\neg Q \vee \neg R$ which is logically equivalent to $\neg R \vee \neg Q$. Bringing in statement 1., $(P \vee \neg Q) \wedge (\neg R \vee \neg Q) \equiv \neg Q \vee (P \wedge \neg R) \equiv Q \to (P \wedge \neg R)$. From this, we can see that the statement above contradicts this, so this is false.