MATH 51 Homework #6

Tamir Enkhjargal

April 2019

Section 1.7

4.

Prove that the negative of an even number is even.

$$\text{Assume } n \in \mathbb{Z} \text{ and is even} \tag{1}$$
$$\exists k \in \mathbb{Z} \tag{2}$$
$$n = 2k \tag{3}$$
$$\therefore -n = (-1)n = 2(-1k) \tag{4}$$
$$\because -1 \in \mathbb{Z} \rightarrow -1 * k \in \mathbb{Z} \tag{5}$$
$$\therefore -n \text{ is even} \tag{6}$$

6.

Prove that the product of two odd numbers is odd.

$$\text{Assume } n, m \in \mathbb{Z} \tag{1}$$
$$\exists k \exists j \in \mathbb{Z} \tag{2}$$
$$n = 2k + 1,\ m = 2j + 1 \tag{3}$$
$$n * m = (2k + 1)(2j + 1) = 4jk + 2k + 2j + 1 \tag{4}$$
$$2(2jk + k + j) + 1 = 2i + 1,\ i \in \mathbb{Z} \tag{5}$$
$$i = 2jk + k + j \tag{6}$$
$$\therefore \text{Product of two odd numbers is odd} \tag{7}$$

16.

Prove that if $x, y, z \in \mathbb{Z}$ and $x + y + z$ is odd, then at least one of x, y, z is odd.

$$\text{Assume } x, y, z \in \mathbb{Z} \text{ and } x, y, z \text{ are } \textit{all even}. \tag{1}$$
$$\exists a \exists b \exists c \in \mathbb{Z}, x = 2a,\ y = 2b,\ z = 2c \tag{2}$$
$$x + y + z = 2a + 2b + 2c = 2(a + b + c) \tag{3}$$
$$\therefore x + y + z \text{ is not odd} \tag{4}$$

By proof of contraposition if x+y+z is odd, then at least one of x, y, and z is odd.

28.

Prove that if n is a positive integer, then n is even if and only if $7n+4$ is even.

$$\text{Assume } n \in \mathbb{Z} \text{ and is positive and even} \tag{1}$$
$$\exists k \in \mathbb{Z},\ n = 2k \tag{2}$$
$$7n+4 = 7(2k)+4 = 14k+4 = 2(7k+2) \tag{3}$$
$$\therefore n \text{ is even} \rightarrow 7n+4 \text{ is even} \equiv \text{true} \tag{4}$$

$$\text{Assume } n \in \mathbb{Z} \text{ is not even} \tag{5}$$
$$\exists k \in \mathbb{Z},\ n = 2k+1 \tag{6}$$
$$7n+4 = 7(2k+1)+4 = 14k+11 = 2(7k+5)+1 \tag{7}$$
$$\therefore n \text{ is not even} \rightarrow 7n+4 \text{ is not even} \equiv \text{ true} \tag{8}$$

$P(x) \rightarrow Q(x)$ proven by direct proof and $Q(x) \rightarrow P(x)$ proven by contraposition.

32.

$a, b \in \mathbb{R}$

(i) a is less than b

(ii) the average of a and b is greater than a

(iii) the average of a and b is less than b

$$-----i \rightarrow ii----- \tag{1}$$
$$a < b \tag{2}$$
$$a+a < b+a \tag{3}$$
$$2a < b+a \tag{4}$$
$$a < (b+a)/2 \tag{5}$$
$$-----ii \rightarrow iii----- \tag{6}$$
$$a < (b+a)/2 \tag{7}$$
$$2a < b+a \tag{8}$$
$$a < b \tag{9}$$
$$a+b < 2b \tag{10}$$
$$(a+b)/2 < b \tag{11}$$
$$-----iii \rightarrow i----- \tag{12}$$
$$(a+b)/2 < b \tag{13}$$
$$a+b < 2b \tag{14}$$
$$a < b \tag{15}$$

We have proved that these three statements are all valid arguments as biconditionals.