

# MATH 51 Homework #3

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## Section 1.4

10.

Let  $C(x)$  be the statement “ $x$  has a cat”

Let  $D(x)$  be the statement “ $x$  has a dog”

Let  $F(x)$  be the statement “ $x$  has a ferret”

- a  $\exists!x, (C(x) \wedge D(x) \wedge F(x))$
- b  $\forall x, (C(x) \vee D(x) \vee F(x))$
- c  $\exists x, (C(x) \wedge F(x) \wedge \neg D(x))$
- d  $\forall x, \neg(C(x) \wedge D(x) \wedge F(x))$
- e  $(\exists x, C(x)) \wedge (\exists x, D(x)) \wedge (\exists x, F(x))$

18.

- a  $P(-2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2)$
- b  $P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2)$
- c  $\neg P(-2) \vee \neg P(-1) \vee \neg P(0) \vee \neg P(1) \vee \neg P(2)$
- d  $\neg P(-2) \wedge \neg P(-1) \wedge \neg P(0) \wedge \neg P(1) \wedge \neg P(2)$
- e  $\neg(P(-2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2))$
- f  $\neg(P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2))$

32.

- a Domain:  $D$  all dogs.  $F(D)$  = “D has fleas”.  $\forall D, F(D)$ .  
Negation:  $\exists D, \neg F(D)$ . There is a dog that doesn’t have fleas.
- b Domain:  $H$  all horses.  $A(H)$  = “H can add”.  $\exists H, A(H)$ .  
Negation:  $\forall H, \neg A(H)$ . All horses can’t add.
- c Domain:  $K$  all koalas.  $C(K)$  = “K can climb”.  $\forall K, C(K)$ .  
Negation:  $\exists K, \neg C(K)$ . There exists a koala that can’t climb.
- d Domain:  $M$  all monkeys.  $F(M)$  = “M can speak French”.  $\forall M, \neg F(M)$ .  
Negation:  $\exists M, F(M)$ . There is a monkey that can speak French.
- e Domain:  $P$  all pigs.  $S(P)$  = “P can swim.”  $F(P)$  = “P can catch fish”.  
 $\exists P, (S(P) \wedge F(P))$   
Negation:  $\forall P, (\neg S(P) \vee \neg F(P))$ . All pigs can’t swim or can’t catch fish.

**36.**

- a  $\exists x, (-2 \geq x \geq 3)$
- b  $\exists x, (0 > x \geq 5)$
- c  $\forall x, (-4 > x > 1)$
- d  $\forall x, (-5 \geq x \geq -1)$

**46.**

These are not logically equivalent. The reasoning comes from when you choose your  $x$ . With  $\forall x, (...)$ , you choose a single  $x$  that is input into both  $P(x)$  and  $Q(x)$ . On the other hand, with  $\forall x, P(x) \leftrightarrow \forall x, Q(X)$  you are stating that for all  $x$  you choose for  $P(x)$  works for any other all  $x$  for  $Q(x)$ . Because of this difference, the outputs for  $P(x)$  or  $Q(x)$  can be different.