MATH 51 Homework #9

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May 2019

Section 2.3: Functions

4.

a) The set of all non-negative integers $Z_+ = \{0, 1, 2...\}$ is the domain and the set of its last digits is $\{n \in \mathbb{Z} | 0 \leqslant n \leqslant 9\} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is the range.

b) The set of all positive integers $Z_{n>0} = \{1, 2, 3, ...\}$ is the domain and the set of all positive integers $Z_{n>1} = \{2, 3, 4, ...\}$ is the range.

c) The set of all possible combinations of bits $n \in \{0, 1\} = \{0, 1, 00, 01, 10, 11, ...\}$ is the domain and the set of all non-negative integers $Z_+ = \{0, 1, 2, ...\}$

d) The set of all possible combinations of bits $n \in \{0, 1\} = \{0, 1, 00, 01, ...\}$ is the domain the range is the amount of bits $Z_{n>0} = \{1, 2, 3, ...\}$

10.

a) Every item from $f : a \to b$ in a has a corresponding element in b. This is **one-to-one**

b) The image b has two elements pointing to it a, b, and the image a has no correspondence. This is not **one-to-one**

c) The image d has two elements pointing from a, d, and the image a has no correspondence. This is not **one-to-one**

12.

A function f from $a \to b$ is one-to-one if $f(x) = f(y) \to x = y$

a) $f(n) = n - 1$. $f(n) = f(m) \equiv n - 1 = m - 1 \equiv n = m$. By definition this is **one-to-one**

b) $f(n) = n^2 + 1$. $f(n) = f(m) \equiv n^2 + 1 = m^2 + 1 \equiv \pm n = \pm m$. n and m don't have to have the same parity, so this is **not one-to-one**

c) $f(n) = n^3$. $f(n) = f(m) \equiv n^3 = m^3 \equiv n = m$. Both of these must have the same parity. This is **one-to-one**.

d) $f(n) = \lceil n/2 \rceil$. $f(n) = f(m) \equiv \lceil n/2 \rceil = \lceil m/2 \rceil$. Because of the way the ceiling function works, n and m don't have to equal each other for the ceiling to be the same. For example $f(6) = 3$ and $f(5) = 3$, and $5 \neq 6$

16.

a) All phone numbers are already unique, so two students can't share the same phone number.

b) Students will need to each have unique student identification grades.

c) Every student will need to end with a different grade from each other for one-to-one.

d) Every student will need to have come from all different unique places.

34.

b) A definition states that if $f \circ g$ is **one-to-one** then f is also **one-to-one**. Proving by contradiction, show that $Q \to P$ doesn't hold. If g is not one-to-one, then $f \circ g$ nor f is neither one-to-one. This contradicts with our assumptions.