MATH 51 Homework #7

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Section 1.7

8.

Solving by proof of contradiction.

$$\begin{aligned}
&\text{Assume } n+2 \text{ and } n \text{ are perfect squares} && (1)\\
&\text{Then } n = a^2,\ n+2 = b^2 \text{ where } a, b \in \mathbb{Z} && (2)\\
&n+2 = b^2 && (3)\\
&a^2+2 = b^2 && (4)\\
&2 = b^2 - a^2 && (5)\\
&2 = (b-a)(b+a) && (6)\\
&b-a = 1,\ b+a = 2 && (7)\\
&(b-a)+(b+a) = 1+2 && (8)\\
&2b = 3 && (9)\\
&b = 3/2,\ \notin \mathbb{Z} && (10)
\end{aligned}$$

We reached a contradiction where it breaks the definition of a perfect square, where an *integer* is squared.

34.

(i)$\leftrightarrow (ii) \leftrightarrow (iii)$

$$\begin{aligned}
(i) &\rightarrow (ii) && (1)\\
\text{Assume } x &\in \mathbb{Q} && (2)\\
x = a|b,\ \text{where } b = ka,\ a, b, k &\in \mathbb{Z} && (3)\\
x/2 = 2a|b \equiv b = 2ka &= (2k)a && (4)\\
x/2 &\in \mathbb{Q} && (5)
\end{aligned}$$

$$\begin{aligned}
(ii) &\rightarrow (iii) && (6)\\
\text{Assume } x/2 &\in \mathbb{Q} && (7)\\
x/2 &= a|b && (8)\\
x &= a|2b && (9)\\
3x-1 = a|6b-1 &= a|(6b-a) && (10)\\
3x-1 &\in \mathbb{Z} && (11)
\end{aligned}$$

$$\begin{aligned}
3x-1 \in \mathbb{Z}3x-1 &= a|b && (12)\\
x &= 3a|(b+a) && (13)\\
x &\in \mathbb{Q} && (14)
\end{aligned}$$

Q.E.D. these three statements are all logically equivalent.

36.

In step (4), when it was found that $(x-1)(x+1) = 0$, the solution $x = -1$ does not work because $\sqrt{2x^2 - 1} = \sqrt{2(-1)^2 - 1} = \sqrt{2-1} = 1 \neq -1$

Section 1.8

2.

Prove using by cases.

$$\text{Assume } x \in \mathbb{Z},\ x > 0 \tag{1}$$
$$\text{Then if } 1 \leq x \leq 3 \tag{2}$$
$$1 \leq x^2 \leq 9 \tag{3}$$
$$\text{Then if } 4 \leq x \tag{4}$$
$$16 \leq x^2 \tag{5}$$

Therefore, 10 is not in the domain of x^2

14.

$x = 65^{1000} - 8^{2001} + 3^{177}$
$y = 79^{1212} - 9^{2399} + 2^{2001}$
$z = 25^{4493} - 5^{8192} + 7^{1777}$

We can choose any two of x, y, or z as possible factors.

x	y	z	Non-negative product
+	+	+	Any two
+	+	-	x*y
+	-	+	x*z
+	-	-	y*z
-	+	+	y*z
-	-	+	x*y
-	+	-	x*z
-	-	-	Any two

Table 1: Cases where x, y, and z are positive or negative

This proof is non constructive.

24.

$$\text{Assume } x \in \mathbb{R},\ x \neq 0 \tag{1}$$

$$(x - \frac{1}{x})^2 \geq 0 \tag{2}$$

$$x^2 + 2 * x * -\frac{1}{x} + (-\frac{1}{x})^2 \geq 0 \tag{3}$$

$$x^2 + \frac{1}{x}^2 - 2 \geq 0 \tag{4}$$

$$x^2 + \frac{1}{x}^2 \geq 2 \tag{5}$$

Therefore, for all nonzero real x, the inequality $x^2 + \frac{1}{x}^2 \geq 2$ is true.