AMTH 108 Homework
12–8–165

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Question #18

(a) Given a basic uniformly distributed random variable X, we can determine the expected value and variance.

$$E[X] = \int_a^b (\frac{1}{b-a})xdx \tag{1}$$

$$= \frac{1}{b-a}[\frac{1}{2}x^2]_a^b \tag{2}$$

$$= \frac{b^2 - a^2}{2(b-a)} \tag{3}$$

$$E[X] \therefore \frac{a+b}{2} \tag{4}$$

$$E[X^2] = \int_a^b (\frac{1}{b-a})x^2dx \tag{5}$$

$$= \frac{1}{b-a}[\frac{1}{3}x^3]_a^b \tag{6}$$

$$= \frac{b^3 - a^3}{3(b-a)} \tag{7}$$

$$VarX \therefore \frac{b^3 - a^3}{3(b-a)} - (\frac{a+b}{2})^2 \tag{8}$$

$$= \frac{a^2}{12} - \frac{ab}{6} + \frac{b^2}{12} \tag{9}$$

$$= \frac{a^2}{12} - \frac{2ab}{12} + \frac{b^2}{12} \tag{10}$$

$$= \frac{a^2 - 2ab + b^2}{12} \tag{11}$$

$$VarX \therefore \frac{(b-a)^2}{12} \tag{12}$$

Question #19

(a) If we set our function to be based off of θ, from the interval 0 to 2π, then the mean is:

$$E[X] = \frac{0 + 2\pi}{2}$$
$$= \pi$$

$$VarX = \frac{(2\pi)^2}{12}$$
$$= \frac{\pi^2}{3}$$

$$\sigma = \sqrt{\frac{\pi^2}{3}}$$
$$= \frac{\pi}{\sqrt{3}}$$

Question #28

(a)

$$= \int_0^\infty \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{\frac{-x}{\beta}} dx$$
$$= \frac{1}{\Gamma(\alpha)} \int_0^\infty (z)^\alpha (z\beta)^{-1} e^{-z} \beta dz$$
$$= \frac{1}{\Gamma(\alpha)} \int_0^\infty (z)^{\alpha-1} e^{-z} \beta dz$$
$$= \frac{1}{\Gamma(\alpha)} \Gamma(\alpha)$$

$$\therefore \int_0^\infty \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{\frac{-x}{\beta}} dx = 1$$

Question #29

(a) The expression is: $\int_0^\infty \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{\frac{-x}{\beta}} dx$. Setting values $\alpha = 3$ and $\beta = 4$, we get $\int_0^\infty \frac{1}{\Gamma(3)4^3} x^2 e^{\frac{-x}{4}} dx$ where $\Gamma(3)$ is the same as $\Gamma(2+1) = 2 * \Gamma(2) = 2 * \Gamma(1+1) = 2 * 1 = 2$. Therefore the complete density function is: $\int_0^\infty \frac{1}{2*4^3} x^2 e^{\frac{-x}{4}} dx$

(b) The moment generating function is: $m_x(t) = (1 - \beta t)^{-\alpha} = (1 - 4t)^{-3}$

(c) The mean is $\alpha\beta = 3 * 4 = 12$. The variance is $\alpha\beta^2 = 3 * 4^2 = 48$. The standard deviation is $\sqrt{48}$

Continuous Distribution Questions #5.3

(a) If T falls within two standard deviations of the mean, then that means the probability is:

$$\sigma = \mu = 1/\lambda$$

$$= P(\mu - 2\sigma < T < \mu + 2\sigma)$$

$$= P(\mu - 2\mu < T < \mu + 2\mu)$$

$$= P(-\mu < T < 3\mu)$$

$$= \int_0^{3\mu} \lambda e^{-\lambda t} dt$$

$$= \int_0^{3/\lambda} \lambda e^{-\lambda t} dt$$

$$= [-e^{-\lambda t}]_0^{3/\lambda}$$

$$= 1 - e^{\frac{-3\lambda}{\lambda}} = 1 - e^{-3} = 0.9502$$

Continuous Distribution Questions #5.5

(a) The exponential function is given by: $f(t) = 0.01e^{-0.01t}$. If we take this from 90 and 110: $\int_{90}^{110} 0.01e^{-0.01t}dt = 0.0737$.

Continuous Distribution Questions #5.6

(a) 10 percent of the time, which means that $P(t \leq T) = 0.1 = P(t < T) = 0.9$. Therefore the exponential function is setup as: $F(t) = 1 - e^{-0.01t} = 0.9$, and we find t to be 230.86 minutes.

Continuous Distribution Questions #5.9

(a) The expected value of an eThing is: $E[x] = B_3E[U] = 25 + 3(\frac{a+b}{2}) = 25 + 3(\frac{10+15}{2}) = 62.5$. The variance is: $VarX = 3^2Var(U) = 9(\frac{(b-a)^2}{12}) = 9(\frac{25}{12}) = 18.75$

(b) The probability that the cost will be larger than 67\$ is: $P(B+3U > 67) = P(3U > 42) = P(U > 14)$. This can be found with: $\int_{14}^{15} \frac{1}{15-10}dx = 0.2$