AMTH 108 Homework
3–9–115

Tamir Enkhjargal

January 2019

Question #2

The theft of precious metals from companies in the United States was and is a serious problem. The estimated probability that such a theft will involve a particular metal is given below:

- Tin: 1/35, Platinum: 1/35, Nickel: 1/35, Steel: 11/35, Gold: 5/35, Zinc: 1/35, Copper: 8/35, Aluminum: 2/35, Silver: 4/35, Titanium: 1/35

(a) The probability that a theft involves gold, silver, or platinum is:

$$\begin{aligned} P[Gold \cup Silver \cup Platinum] &= P[Gold] + P[Silver] + P[Platinum] \\ &= 5/35 + 4/35 + 1/35 \\ &= 10/35 = 28.571\% \end{aligned}$$

(b) The probability that a theft will not involve steel is:

$$\begin{aligned} P[Steel^c] &= 1 - P[Steel] \\ &= 1 - 11/35 = 24/35 = 68.571\% \end{aligned}$$

Question #3

Assuming the blood type distribution to be A: 41%, B: 9%, AB: 4%, O: 46%, what is the probability that the blood of a randomly selected individual will contain the A antigen? That it will contain the B antigen? That it will contain neither the A nor the B antigen?

(a) The probability of containing the A antigen is:

$$\begin{aligned} P[Contains\ the\ A\ Antigen] &= P[A] + P[AB] \\ &= 0.41 + 0.04 = 0.45 = 45\% \end{aligned}$$

(b) The probability of containing the B antigen is:

$$\begin{aligned} P[Contains\ the\ B\ Antigen] &= P[B] + P[AB] \\ &= 0.9 + 0.04 = 0.13 = 13\% \end{aligned}$$

(c) The probability of containing neither the A nor the B antigen is the same as being the "O antigen".

$$\begin{aligned} P[Neither\ A\ nor\ B] = P[O] &= 1 - P[A] - P[AB] - P[B] \\ &= 1 - 0.41 - 0.04 - 0.09 \\ &= 0.46 = 46\% \end{aligned}$$

Question #5

When an individual is exposed to radiation, death may ensue. Factors affecting the outcome are the size of the dose, the length and intensity of the exposure and the biological makeup of the individual. The term LD_{50} is used to denote the dose that is usually lethal for 50% of the individuals exposed to it. Assume that in a nuclear accident 30% of the workers are exposed to the LD_{50} and die; 40% of the workers die; and 68% are exposed to the LD_{50} or die. What is the probability that a randomly selected worker is exposed to the LD_{50}? Use a Venn diagram to find the probability that a randomly selected worker is exposed to the LD_{50} but does not die. Find the probability that a randomly selected worker is not exposed to the LD_{50} but dies.

(a) The probability that a randomly selected worker is exposed to LD_{50} is:

$$\begin{aligned} P(Has\ LD_{50}) &= P(LD_{50} \cup D) + P(LD_{50} \cap D) - P(D) \\ &= 0.68 + 0.3 - 0.4 = 0.58 \end{aligned}$$

(b) The probability that a randomly selected worker is exposed to LD_{50} but does not die is:

$$\begin{aligned} P(A \cap B^C) &= P(LD_{50}) - P(LD_{50} \cap D) \\ &= 0.58 - 0.3 = 0.28 \end{aligned}$$

(c) The probability that a randomly selected worker is not exposed but dies is:

$$\begin{aligned} P({LD_{50}}^C \cap D) &= P(D) - P(LD_{50} \cap D) \\ &= 0.4 - 0.3 = 0.1 \end{aligned}$$

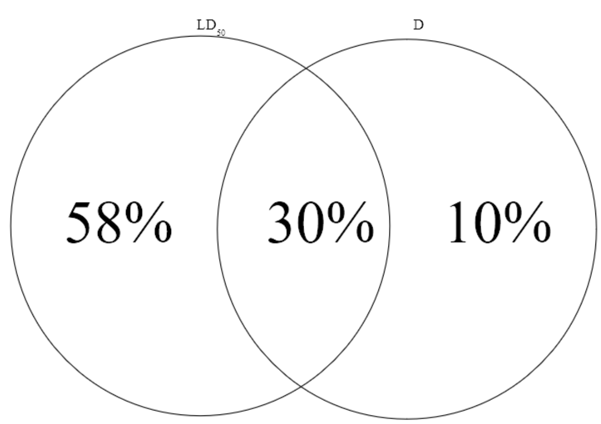

Figure 1: Venn Diagram of Probability of LD_{50} exposure and Death

Question #6

When a computer goes down, there is a 75% chance that it is due to an overload and a 15% chance that it is due to a software problem. There is an 85% chance that it is due to an overlord or a software problem. What is the probability that both of these problems are at fault? What is the probability that there is a software problem but no overload?

(a) If the probability of overloading is 0.75 and the probability of a software problem is 0.15, and the union of overload or software problem is 0.85. Therefore the probability of both is:

$$\begin{aligned} P(Overload \cap Software) &= P(Overload) + P(Software) - P(Overload \cup Software) \\ &= 0.75 + 0.15 - 0.85 = 0.05 = 5\% \end{aligned}$$

(b) The probability of a software problem but no overload is:

$$\begin{aligned} P(Software \cap Overload^C) &= P(Software) - P(Software \cap Overload) \\ &= 0.15 - 0.05 = 0.1 \end{aligned}$$

Question #11

Assume that 1% of all tires of a particular brand are defective due to a problem with a supplier of an important chemical component of the tire. Assume that .5% of this brand of tire will eventually fail due to sidewall blowouts. Also 1.4% of this brand tire experience at least one of these problems. What is the probability that in a future accident involving these tires, a blowout will occur but there will be no problem found with the chemical composition of the tire?

(a) The probability that a blowout will occur but not have a problem with the chemical composition is:

$$\begin{aligned} P(Blowout \cap Chemical^C) &= P(Blowout) - P(Blowout \cap Chemical) \\ &= 0.005 - 0.001 \end{aligned}$$

Where the $P(Blowout \cap Chemical)$ is:

$$\begin{aligned} &= P(Blowout) + P(Chemical) - P(Blowout \cup Chemical) \\ &= 0.005 + 0.01 - 0.014 = 0.001 \end{aligned}$$

Combinatorics Problem # 1.4

In how many ways can Hayes High School, home of the fighting Rutherfords, play an 11 game football season that comprises 5 ties, 2 wins (but not on consecutive games), and 4 losses?

(a) The probability of (in random order) exactly 5 ties, 2 wins, and 4 losses is:

$$\binom{11}{5\ 2\ 4} = \frac{11!}{5!2!4!} = 6930$$

Combinatorics Problem # 1.19

A box of 10 batteries contains 4 batteries that are dead. A portable CD player requires 3 batteries to operate, so three batteries are chosen at random from the lot of 10. In how many ways can the CD player be loaded with batteries? How many of these trios of batteries contain exactly two dead batteries?

(a) There can be $\binom{10}{3} = 120$ ways to choose random batteries for the CD player.

(b) If we set a restriction on specifically two batteries to be dead, we get:

$$\begin{aligned} &= \binom{4}{2}\text{(Exactly 2 dead)} * \binom{6}{1}\text{(Last not dead)} \\ &= 6 * 6 = 36 \end{aligned}$$

Combinatorics Problem # 1.23

To choose a password for our newest computer system, 26 tiles (each containing a different letter of the Latin alphabet) are placed into a bag. At each stage of the selection process, two tiles are chosen from the bag. From these two tiles one tile is chosen at random and becomes the first letter in the password. Both tiles are then discarded (so neither can be chosen again). This procedures continues until a 6 character password has been formed.

(a) From 26 characters we choose 2 and then we choose 1 from the pair, and this process is repeated until we have 6 characters:

$$\begin{aligned} &= \binom{26}{2} * 2 * \binom{24}{2} * 2 * \binom{22}{2} * 2 * \binom{20}{2} * 2 * \binom{18}{2} * 2 * \binom{16}{2} * 2 \\ &= 26 * 25 * 24 * 23 * 22 * 21 * 20 * 19 * 18 * 17 * 16 * 15 \\ &= 4,626,053,752,320,000 \end{aligned}$$

(b) `AMOEBA` is not able to be used because it uses the letter A two times, when it gets doesn't get replaced.

Combinatorics Problem # 1.24

The Post Office created Zip-codes during the 1960s. How many five digit Zip-codes would there have been if the Post Office decided to require that all Zip-codes contain at least three different digits?

(a) We understand that we are not replacing at least three of the numbers after choosing, but the remaining two can be any of 8 numbers.

$$
\begin{aligned}
&= \binom{10}{9} * \binom{9}{8} * \binom{8}{7} * 8 * 8 \\
&= 10 * 9 * 8 * 8 * 8 = 46080
\end{aligned}
$$