AMTH 108 Homework
4–7–140

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Question #14

Use the data of Exercise 4 to answer these questions.

(a) The probability that the backup engine will work if the main engine fails is:

$$\begin{aligned} P[B|A^C] &= \frac{P[B \cap A^C]}{P[A^C]} \\ &= \frac{P[B \cap A - P[A]}{1 - P[A]} \\ &= \frac{0.99 - 0.95}{1 - 0.95} = \frac{0.04}{0.05} = 0.8 \end{aligned}$$

(b) $P[Backup\ Functions] = P[B]$ and $P[Backup\ Functions \mid Main\ Fails] = P[B|A^C]$. We can determine that since they both hold the same probability (from answer (a) and the given data), $P[B] = P[B|A^C] \to 0.8 = 0.8$. Therefore the events B and A are independent of each other. Logically considering this, the backup engine working should not have any influence if the main engine works or not.

Question #15

In a study of waters ear power plants and other industrial plants that release wastewater into the water system it was found that 5% showed signs of chemical and thermal pollution, 40% showed signs of chemical pollution, and 35% showed evidence of thermal pollution. Assume that the results of the study accurately reflect the general situation. What is the probability that a stream that shows some thermal pollution will also show signs of chemical pollution? What is the probability that a stream showing chemical pollution will not show signs of thermal pollution?

(a) The probability of a stream showing thermal pollution also showing chemical pollution is:

$$\begin{aligned} P[Chemical \mid Thermal] &= \frac{P[Chemical \cap Thermal]}{P[Thermal]} \\ &= \frac{0.05}{0.35} = \frac{1}{7} = 0.14286 \end{aligned}$$

(b) The probability of a stream showing not thermal given chemical is:

$$\begin{aligned} P[Thermal^C \mid Chemical] &= \frac{P[Thermal^C \cap Chemical]}{P[Chemical]} \\ &= \frac{P[Chemical] - P[Thermal \cap Chemical]}{P[Chemical]} \\ &= \frac{0.4 - 0.05}{0.4} = \frac{0.35}{0.4} = 0.875 \end{aligned}$$

Combinatorics Problem # 2.5

I roll a pair of dice 24 times. Should I bet for or against a 12 appearing on one of the rolls? How about if I roll 25 times?

(a) The probability of rolling exactly a 12 from a pair of dice is 1/36. This means that rolling *not* a 12 exactly is 35/36. The amount of rolls that don't have 12 is 35^n4 in 24 rolls. Therefore the probability of a 12 appearing once is:

$$P[Rolling\ A\ 12] = \frac{36^{24} - 35^{24}}{36^{24}} = 0.4914$$

(b)

$$P[Rolling\ A\ 12] = \frac{36^{25} - 35^{25}}{36^{25}} = 0.5055$$

Both have around a 50% chance of rolling a 12, so there's a good chance of both.

Combinatorics Problem # 2.6

I roll a die 4 times. I win if a six appears. To make this game more interesting, I decide to add a second die and target the appearance of a double six. I reason as follows: a double six is one-sixth as likely as a six – 1/36 compared to 1/6. I should be able to increase the number of rolls by a factor of 6 (now 24 rolls) and still maintain the same probability of winning. Is this true? (Probabilistic justification is required here!)

(a) From the previous problem, rolling for a 12 in 24 rolls has 0.4914 probability. Using the same logic that we were lead to, the probability of rolling exactly a 6 with a single die is 1/6, or $1 - \frac{5}{6}$. Therefore in four rolls, the chance of seeing at least a single 6 is:

$$P[Rolling\ A\ 6] = \frac{6^4 - 5^4}{6^4} = 0.5177$$

We can see that going for the single die 4 times has a slightly better chance than two die 24 times.

Combinatorics Problem # 2.9

Three powders, A: Eye of Newt, B: Black Cat Bone, and C: Powdered Instant Polyjuice Potion, are to be mixed with water and poured into a cauldron. Initially, the solutions, in pairs, react to form three new compounds (called AB, AC, and BC). These reactions are incomplete in the sense that after all reactions have stopped, the cauldron contains water and some amount of A, B, C, AB, AC, and BC, and nothing else. But not for long. Shortly, a third reaction occurs between AB, AC¡ and BC to produce ABC: non-dairy coffee creamer. Again, the reactions are incomplete. When the dust settles, the cauldron contains water and seven types of molecules in these proportions:

- 8% residual A, 54% combined A, 5% residual B, 49% combined B, 5% residual C, 45% combined C, 43% combined AB, the remainder being water.

What percentage of the cauldron is water? After a day, the heavy molecules have settled to the bottom of the cauldron. What is the probability that a heavy molecule is non-dairy creamer?

(a) Using the Venn Diagram method, three circles are setup as A, B, and C, with 8, 5, and 5 inside percent each, respectively. Setting up arbitrary variable naming conventions:

AC = a, AB = b, BC = c, and ABC = d. The water outside of the three circles is denoted e.

We find that using the percentages that were given to us, we setup the system of linear equations as (and after reducing the rows):

$$M = \left[\begin{array}{ccccc|c} a & b & c & d & e & n \\ \hline 1 & 1 & 0 & 1 & 0 & 46 \\ 0 & 1 & 1 & 1 & 0 & 44 \\ 1 & 0 & 1 & 1 & 0 & 40 \\ 0 & 1 & 0 & 1 & 0 & 43 \\ 1 & 1 & 1 & 1 & 1 & 82 \end{array}\right] = \left[\begin{array}{ccccc|c} a & b & c & d & e & n \\ \hline 1 & 0 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 36 \\ 0 & 0 & 0 & 0 & 1 & 35 \end{array}\right]$$

Variable e represents the percent of water in the cauldron, which is 35%

(b) The probability of a heavy molecule being non-dairy creamer (i.e. ABC) is:

$$P[Creamer \mid Heavy\ Molecules] = \frac{d}{a+b+c+d}$$
$$= \frac{0.36}{0.03+0.07+0.01+0.36} = 0.766$$

Combinatorics Problem # 2.12

Everything printed in the *Daily Screamer* is in (1) boldface or (2) italics or (3) both boldface and italics. In today's edition, I notice that if something is in boldface, then the chances of it also being in italics is 30%. Moreover, if something is in italics, then the chances of it also being in boldface is 20%. What is the probability that something in the *Daily Screamer* is in boldface?

$$P[I|B] = \frac{P[B \cap I]}{P[I]} = 0.2$$
$$P[B|I] = \frac{P[B \cap I]}{P[B]} = 0.3$$

(a) The probability of something in the *Daily Screamer* is in boldface is:

$$P[B] = \frac{P[B \cap I]]}{P[B|I]} = \frac{P[B \cap I]]}{0.3}$$

We need to find a way to substitute out $P[B \cap I]]$. We can find that through the relationship of:

$$\begin{aligned}
P[B \cup I] &= P[B] + P[I] - P[B \cap I] \\
&= \frac{P[B \cap I]}{0.3} + \frac{P[B \cap I]}{0.2} - P[B \cap I] \\
&= \frac{22P[B \cap I]}{3} \\
\therefore P[B] &= \frac{\frac{22}{3}}{0.3} = \frac{5}{11} = 0.4545
\end{aligned}$$

Combinatorics Problem # 2.15

My local theatre company is going to stage *Cat on a Streetcar named Iguana*. After the first round of auditions, it is determined that 30% of the applicants can act, 60% look like Marlon Brando, and 52% can simultaneously tap dance while singing. There are 10% of the applicants who possess all three of these attributes and an additional 10% who possess none of these qualities. The casting director says that she'll be happy with someone who possesses at least 2 of these traits. If 100 people audition, how many satisfactory choices will the casting director have?

(a) We begin by setting up the systems of equations and known equivalencies. If we have three events, A, B, and C, with the 7 possible sections a (only A), b (only B), c (only C), d (only AC), e (only AB), f (only BC), and g (only ABC).

$$\begin{aligned}
0.3 &= a + d + e + g \\
0.6 &= b + e + f + g \\
0.52 &= c + d + f + g \\
1 &= 0.1 + a + b + c + d + e + f + g \\
0.1 &= g
\end{aligned}$$

$$\begin{aligned}
P[A \cup B \cup C] &= P[A] + P[B] + P[C] - P[AB] - P[BC] - P[AC] + P[ABC] \\
0.9 &= 0.3 + 0.6 + 0.52 - (e + g) - (f + g) - (d + g) + 0.1 \\
-0.62 &= -d - e - f - 3g \\
-0.32 &= -d - e - f \\
d + e + f &= 0.32
\end{aligned}$$

$$\begin{aligned}
P[\mathit{At\ least\ two}] &= P[\mathit{Exactly\ two}] + P[\mathit{Exactly\ Three}] \\
&= (d + e + f) + (g) \\
&= 0.32 + 0.1 = 0.42
\end{aligned}$$