MATH 51 Homework #20

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1. Determine whether $\sim$ is an equivalence relation. If yes, give a proof and if no, determine what property fails (with an explicit counterexample).

   a) This is not transitive. Check $x = 2,\ y = 1,\ z = 0$. We find that $|2-1| < 2$, $|1-0| < 2$, but $|2-0| \nless 2$

   b) This is not symmetric. Check that $x = 2, y = 1$. We see that $2-1 \geqslant 0$, but $1-2 \ngeqslant 0$

   c) $\sim$ is an equivalence relation. Reflexive $a^2 = a^2$ is true. Symmetric, if $x^2 = y^2$, then $y^2 = x^2$ is also true. Transitive. If $x^2 = y^2$, and $y^2 = z^2$, then $x^2 = z^2$ is true, after substituting $y^2 = z^2$.

   d) This is an equivalence relation. Reflexive $f(1) - f(1) = 0$ is true for any function. Symmetric, if $f(1) - g(1) = 0$, then $g(1) - f(1) = 0$ is true as $g(1) = f(1)$. Transitive, if $f(1) - g(1) = 0$, $g(1) - h(1) = 0$, then $f(1) - h(1) = 0$, as $g(1) = h(1)$.

   e) This is not reflexive. Check $f(0) - f(0) = 0$. This is also not symmetric. Check if $f(0) = 1$, $g(0) = 0$, then $f(0) - g(0) = 1$, but $g(0) - f(0) = 0 - 1 = -1$. This is also not transitive. If $f(0) = 1$, $g(0) = 0$, $h(0) = -1$, then $f(0) - g(0) = 1$, $g(0) - h(0) = 1$, but $f(0) - h(0) = 2$.

   f) This is not transitive. Lets have $f(0) = g(0)$ true and $g(1) = h(1)$ true. Then it does not necessarily hold that $f(0) = h(1)$ or WLOG $f(1) = h(0)$ is not necessarily true. Lets have $f(0) = 0,\ g(0) = 0,\ g(1) = 1,\ h(1) = 1$, then $f(0) \neq h(1)$

2. Let R be the relation on the set of all sets of real numbers such that SRT if and only if S and T have the same cardinality. Show that R is an equivalence relation. What are the equivalence classes of the sets $\{0, 1, 2\}$ and $\mathbb{Z}$?

   Let $R = \{(S, T) \mid |S| = |T|\}$.

   We can see that $(S, S) \in R$, as $|S| = |S|$. So this relation is reflexive.

   We can see that if $(S, T) \in R$, then $|S| = |T|$, and symmetrically $|T| = |S|$.

   We can see that if $(S, T) \in R$ and $(T, U) \in R$, then we can assume $|S| = |T|$ and $|T| = |U|$. Therefore $|S| = |U|$. This shows that R is also transitive.

   Since R possesses all three properties, R is an equivalence relation.

   $[\{0, 1, 2, \}]$=$\{S \in R \mid |S| = 3\}$. Therefore the equivalency class is any set of real numbers containing *exactly* 3 real numbers.

   $[\mathbb{Z}] = \{S \in R \mid |S| = |\mathbb{Z}|\}$. The equivalence class of $\mathbb{Z}$ is any set of real numbers that are countably infinite.