MATH 51 Homework #2

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Section 1.3

16.

(a) For the first example, I will also give the name of the identity used per step.

$$
\begin{array}{rr}
[\neg p \wedge (p \vee q)] \rightarrow q & \textit{Given} \quad (1)\\
\neg([\neg p \wedge (p \vee q)]) \vee q & \textit{Implication} \quad (2)\\
[p \vee \neg(p \vee q)] \vee q & \textit{DeMorgan's \& Double Negation} \quad (3)\\
[p \vee (\neg p \wedge \neg q)] \vee q & \textit{DeMorgan's} \quad (4)\\
[(p \vee \neg p) \wedge (p \vee \neg q)] \vee q & \textit{Distributive} \quad (5)\\
[T \wedge (p \vee \neg q)] \vee q & \textit{Negation} \quad (6)\\
(T \vee q) \wedge ((p \vee \neg q) \vee q) & \textit{Distributive} \quad (7)\\
T \wedge (p \vee (\neg q \vee q)) & \textit{Domination \& Associative} \quad (8)\\
T \wedge (p \vee T) & \textit{Negation} \quad (9)\\
T \wedge T & \textit{Domination} \quad (10)\\
T & \textit{Q.E.D.} \quad (11)
\end{array}
$$

(b)

$$
\begin{array}{rr}
[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r) & (1)\\
[(\neg p \vee q) \wedge (\neg q \vee r)] \rightarrow (\neg p \vee r) & (2)\\
\neg[(\neg p \vee q) \wedge (\neg q \vee r)] \vee (\neg p \vee r) & (3)\\
[(p \wedge \neg q) \vee (q \wedge \neg r)] \vee (\neg p \vee r) & (4)\\
[(p \vee q) \wedge (p \vee \neg r)) \wedge ((\neg q \vee q) \wedge (\neg q \vee \neg r))] \vee (\neg p \vee r) & (5)\\
[((p \vee q) \wedge (p \vee \neg r)) \wedge (T \wedge (\neg q \vee \neg r))] \vee (\neg p \vee r) & (6)\\
[((p \vee q) \wedge (p \vee \neg r)) \wedge (\neg q \vee \neg r)] \vee (\neg p \vee r) & (7)\\
[((p \vee q) \wedge (p \vee \neg r)) \vee (\neg p \vee r)] \wedge [(\neg q \vee \neg r) \vee (\neg p \vee r)] & (8)\\
[((p \vee q) \wedge (p \vee \neg r)) \vee (\neg p \vee r)] \wedge [(\neg q \vee \neg p) \vee (\neg r \vee r)] & (9)\\
[((p \vee q) \wedge (p \vee \neg r)) \vee (\neg p \vee r)] \wedge [(\neg q \vee \neg p) \vee T] & (10)\\
[((p \vee q) \wedge (p \vee \neg r)) \vee (\neg p \vee r)] \wedge T & (11)\\
[((p \vee q) \wedge (p \vee \neg r)) \vee (\neg p \vee r)] & (12)\\
[(p \vee q) \vee (\neg p \vee r)] \wedge [(p \vee \neg r) \vee (\neg p \vee r)] & (13)\\
[(p \vee \neg p) \vee (q \vee r)] \wedge [(p \vee \neg p) \vee (\neg r \vee r)] & (14)\\
[T \vee (q \vee r] \wedge [T \vee T] & (15)\\
T \wedge T & (16)\\
T & (17)
\end{array}
$$

(c)

$$\begin{align}
[p \wedge (p \rightarrow q] \rightarrow q \tag{1}\\
[p \wedge (\neg p \vee q)] \rightarrow q \tag{2}\\
\neg[p \wedge (\neg p \vee q)] \vee q \tag{3}\\
[\neg p \vee (p \wedge \neg q)] \vee q \tag{4}\\
[(\neg p \vee p) \wedge (\neg p \vee \neg q)] \vee q \tag{5}\\
[T \wedge (\neg p \vee \neg q] \vee q \tag{6}\\
(\neg p \vee \neg q) \vee q \tag{7}\\
\neg p \vee (\neg q \vee q) \tag{8}\\
\neg p \vee T \tag{9}\\
T \tag{10}
\end{align}$$

(d)

$$\begin{align}
[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r \tag{1}\\
[(p \vee q) \wedge (\neg p \vee r) \wedge (\neg q \vee r)] \rightarrow r \tag{2}\\
\neg[(p \vee q) \wedge (\neg p \vee r) \wedge (\neg q \vee r)] \vee r \tag{3}\\
[(\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee (q \wedge \neg r)] \vee r \tag{4}\\
[((\neg p \vee p) \wedge (\neg p \vee \neg r)) \vee (q \wedge \neg r)] \vee r \tag{5}\\
[((T \wedge (\neg p \vee \neg r)) \vee (q \wedge \neg r)] \vee r \tag{6}\\
[\neg p \vee \neg r \vee (q \wedge \neg r)] \vee r \tag{7}\\
\neg p \vee (q \wedge \neg r) \vee (\neg r \vee r) \tag{8}\\
\neg p \vee (q \wedge \neg r) \vee T \tag{9}\\
T \tag{10}
\end{align}$$

24.

$p \leftrightarrow q$ is a true statement only when they are the same truth value. On the other hand, $p \oplus q$ is only true when they are the opposite truth value. Negating the XOR would mean that these two are logically equivalent.

30.

$$\begin{align}
\neg p \rightarrow (q \rightarrow r) &\equiv p \vee \neg q \vee r \tag{1}\\
&\equiv \neg q \vee p \vee r \tag{2}
\end{align}$$

$$q \rightarrow (p \vee r) \equiv \neg q \vee p \vee r \tag{3}$$

$$\therefore \neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r) \tag{4}$$

32.

$p \leftrightarrow q$ is a true statement only when they are the same truth value. Even if you negate both p and q, the statement $\neg p \leftrightarrow \neg q$ is true when they are the same truth value.

p	q	$\neg p$	$\neg q$	$p \leftrightarrow q$	$\neg p \leftrightarrow \neg q$
F	F	T	T	T	T
F	T	T	F	F	F
T	F	F	T	F	F
T	T	F	F	T	T