AMTH 108 Homework
8–4.5–85

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Question #31(b, c, d, & f)

(a) $E[X] = 3 * 0 + 4 * \frac{1}{5} + 5 * \frac{4}{5} = 4.8$

(b) The moment generating function is $m_x(t) = E[e^{tx}]$.

Evaluating the function, $E[e^{tx}] = \frac{1}{5}e^{4t} + \frac{4}{5}e^{5t}$

(c) $E[X] = \mu_1 = m'_x(0) = 4 * \frac{1}{5}e^{4t} + 5 * \frac{4}{5}e^{5t} = \frac{4}{5} + 4 = 4.8$

(d) $E[X^2] = \mu_2 = m''_x(0) = 4 * 4 * \frac{1}{5}e^{4t} + 5 * 5 * \frac{4}{5}e^{5t} = \frac{16}{5} + 20 = 23.2$

Question #32

(a) If the moment generating function is $m_x(t) = e^{2(e^t-1)}$, then $E[X] = m'_x(t)$

$m'_x(t) = 2e^{2e^t+t-2}$. If we take $t = 0, E[X] = 2e^{2*1+0-2} = 2$

(b) $E[X^2] = m''_x(t) = e^{2e^t} * (4e^{2t-2} + 2e^{t-2})$

(c)

$$
\begin{aligned}
\sigma^2 &= E[X^2] - (E[X])^2 \\
&= e^{2e^t} * (4e^{2t-2} + 2e^{t-2}) - (2e^{2e^t+t-2})^2 \\
&= e^{2e^t} * (4e^{2t-2} + 2e^{t-2}) - 4e^{4e^t+2t-4}
\end{aligned}
$$

$$
\begin{aligned}
\sigma &= \sqrt{\sigma^2} \\
&= \sqrt{e^{2e^t} * (4e^{2t-2} + 2e^{t-2}) - 4e^{4e^t+2t-4}}
\end{aligned}
$$

Question #34

(a) $E[e^{xt}] = \sum e^{xt} * f(x) = \sum \frac{1}{n}e^{nt} = \frac{1}{n}\sum e^{nt} = \frac{1}{n} * \frac{e^t(1-e^{nt})}{1-e^t}$

Question #35

(a)

$$\sum_{n=0}^{\infty} ce^{-x} = 1$$
$$c\sum_{n=0}^{\infty} e^{-x} = 1$$
$$c(e^{-1} + e^{-2} + e^{-3} + ... + e^{-\infty}) = 1$$
$$ce^{-1}(1 + e^{-1} + e^{-2} + ... + e^{-\infty}) = 1$$
$$\frac{ce^{-1}}{1 - e^{-1}} = 1$$
$$\frac{c}{e - 1} = 1$$
$$\therefore c = e - 1$$

(b) $E[e^{xt}] = \sum e^{xt} * (e-1)(e^{-x}) = \sum (e-1)(e^{xt-x}) = (e-1)\sum e^{xt-x}$
$(e-1)(e^{t-1} + e^{2t-2} + e^{3t-3} + ...) = (e-1)(e^{t-1})(1 + e^{t-1} + e^{2t-2} + ...)$
$= (e-1)(e^{t-1}) * \frac{1}{1-e^{t-1}}$

(c) $E[X] = m'_x(0) = \frac{(e-1)(e^{t+1})}{(e^t-e)^2} = \frac{(e-1)(e)}{(1-e)^2} = \frac{e}{e-1}$

Discrete Distributions Problem # 4.28

Let X be a discrete random variable. Are the following statements true or false. Explain each.

1. Let X take on the values -1.3, -0.3, 0.0, 0.4, 0.7, 2.0. Then $E[X] = 2.3$.

   False. Regardless of the various probabilities of the nontrivial values, the expected value can not exceed the greatest nontrivial number. $E[X] \leq M$

2. Same X values in last item. The variance of X is -1.2.

   False. Variance can never be false.

3. The random variable X (shown below) has a mean of 0.

x	-1.3	-0.3	-0.1	0.0	0.4	0.7	2.0
$f(x)$	0.1	0.1	0.3	0.2	0.05	0.1	0.05

   False. The expected value or mean is 0.0.

4. A geometrically distributed random variable can have a mean of 0.

   False. A geometrically distributed random variable starts at $X = 1, 2, 3....$ Because 0 is less than the minimum non-trivial value, a mean of 0 is not possible.

5. BART trains arrive at San Bruno exactly every 20 minutes on weekends. The number of trains arriving in a 30 minute period is Poisson Distributed with $\lambda = 1/3$ trains per hour.

   False. The Poisson distribution is at $\lambda = 1/2$ trains per hour, if trains come in a 30 minute period.