MATH 51 Homework #11

Tamir Enkhjargal

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Section 2.4

12.

a) $\{a_n\}$, $a_n = 0, n \geqslant 2 \rightarrow \; -3(0) + 4(0) = 0$. Therefore $a_n = 0$ is a solution.

b) $\{a_n\}$, $a_n = 1, n \geqslant 2 \rightarrow \; -3(1) + 4(1) = 1$. Therefore $a_n = 1$ is a solution.

c) $\{a_n\}$, $a_n = (-4)^n, n \geqslant 2 \rightarrow -3(-4)^{n-1} + 4(-4)^{n-2} = (-4)^{n-2}(-4)^2 = a_n$. Therefore $a_n = (-4)^n$ is a solution.

d) $\{a_n\}$, $a_n = 2(-4)^n + 3, n \geqslant 2$:

$$
\begin{aligned}
&= -3(2(-4)^n + 3) + 4(2(-4)^n + 3) && (1)\\
&= 2(-4)^{n-2}(-3(-4) + 4) + 3(-3 + 4) && (2)\\
&= 2(-4)^{n-2}(-4)^2 + 3 && (3)\\
&= a_n && (4)
\end{aligned}
$$

Therefore. $2(-4)^n + 3$ is a solution.

16.

a) $a_n = -a_{n-1}, a_0 = 5, a_1 = -5, a_2 = 5 \rightarrow a_n = 5(-1)^n$

c) $a_n = a_{n-1} - n, a_0 = 4, a_1 = 4 - 1, a_2 = 4 - 1 - 2, a_n = 4 - \frac{n(n+1)}{2}$

e) $a_n = (n+1)a_{n-1}, a_0 = 2, a_1 = 2 * 2, a_2 = 3 * 2 * 2, a_3 = 4 * 3 * 2 * 2 = a_n = 2(n+1)!$

32.

1. When j even, (i.e. 0,2,4,6,8) the round is equal to 2. When j odd (i.e. 1,3,5,7) the round is equal to 0. Therefore there are 5 even rounds $5 * 2 = 10$. This sum is 10.
2. $\sum_{j=0}^{8} 3^j - 2^j = \sum_{j=0}^{8} 3^j - \sum_{j=0}^{8} 2^j = 9841 - 511 = 9330$
3. $\sum_{j=0}^{8}(2*3^j + 3*2^j) = \sum_{j=0}^{8}(2*3^j) + \sum_{j=0}^{8}(3*2^j) = 2\sum_{j=0}^{8} 3^j + 3\sum_{j=0}^{8} 2^j = 21215$
4. $\sum_{j=0}^{8}(2^{j+1} - 2^j) = \sum_{j=0}^{8}(2^j(2^1 - 2^0)) = \sum_{j=0}^{8}(2^j) = 511$

35.

$$
\begin{aligned}
\sum_{j=1}^{n}(a_j - a_{j-1}) &= (a_1 - a_0) + (a_2 - a_1) + (a_3 - a_2) + ... + (a_n + a_{n-1}) && (1)\\
&= a_n - a_0 && (2)
\end{aligned}
$$

This is true because all of the elements a_1 to a_{n-1} cancel out.

38.

$$a_k = k^3 \to \sum_{k=0}^{n} k^3 = \sum_{k=0}^{n} (k+1)^3 - (n+1)^3 \tag{1}$$

$$= \sum_{k=0}^{n} k^3 + 3\sum_{k=0}^{n} k^2 + 3\sum_{k=0}^{n} k + \sum_{k=0}^{n} 1 - (n-1)^3 \tag{2}$$

$$3\sum_{k=0}^{n} k^2 = (n+1)^3 - 3\sum_{k=0}^{n} k - \sum_{k=0}^{n} 1 \tag{3}$$

$$3\sum_{k=0}^{n} k^2 = n^3 + 3n^2 + 3n + 1 - \frac{3}{2}(n(n+1)) - (n+1) \tag{4}$$

$$3\sum_{k=0}^{n} k^2 = n^3 + \frac{3}{2}n^2 + \frac{n}{2} \tag{5}$$

$$\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \tag{6}$$