

Problem Set: Investment Return Measures

1: Suppose you invest \$50,000 at a semiannually compounded interest rate of 7%. Suppose your investment horizon is 10 years. Calculate your P&L, gross return, return, and annualized return.

2: If you invest \$40,000 in a private business, and 3 years later your share is worth \$105,000, calculate your P&L, gross return, return, and annualized return.

3: (a) Show that the annualized return has the form of an average of the annual returns over the period of the investment by showing (i) that it is approximately the arithmetic average of the 1 year returns of the asset, and (ii) by showing that the one year gross return it implies is the geometric mean of the 1 year gross returns. (**Hint:** write

$$\frac{V(N)}{V(0)} = \frac{V(1)}{V(0)} \cdot \frac{V(2)}{V(1)} \cdots \frac{V(N)}{V(N-1)}$$

and use the Taylor expansion $\log(1+x) \approx x$ for small x .)

(b) Show that we may similarly define average returns using different bases of time. For instance, we may define a "semiannualized" return ρ as

$$(1+\rho)^{2N} = \frac{V(N)}{V(0)}$$

Show that this measure of return bears the same relationship to 6 month returns over the life of the investment as the annualized return does to 1 year returns outlined in part (a).

(c) Define similar quarterly and monthly returns and show they have similar average properties.

4: (a) Let r be an annually compounded interest rate. If R is the "semi-annualized" return defined in (b) of problem 3, show that

$$R = \sqrt{1+r} - 1$$

(b) If r is a semiannually compounded interest rate, show that

$$R = \frac{r}{2}$$

so that in this case, r is an annualized version of R , in the same sense as the "annualization" of an APR. Show also that the annualized return U is related to r by

$$U = r + \frac{r^2}{4}$$

(c) Show similar relationships as in parts (a) and (b) for quarterly and monthly compounded interest rates and corresponding cumulative return measures.

5: Consider investing \$10,000 for 1 year at an interest rate of 17%, but consider annually, semiannually, quarterly, monthly, daily, and continuous compounding. Calculate the annualized return you will realize on your investment for each different compounding convention. Can you explain the trend you observe in your annualized return as the compounding frequency increases?

6: (a) What APR must you earn on a bank account with daily compounding in order to earn an annualized return of 6%? What is your EAR? Can you generalize this finding?

(b) Calculate the effective annual rate for an APR of 9% if it is annually, semiannually, and monthly compounded. Verify this by calculating the annualized return realized on an investment of \$25,000 for 5 years at these APRs.