Chapter 4

Allocation Problems in Parallel Computers

4.1 Overview

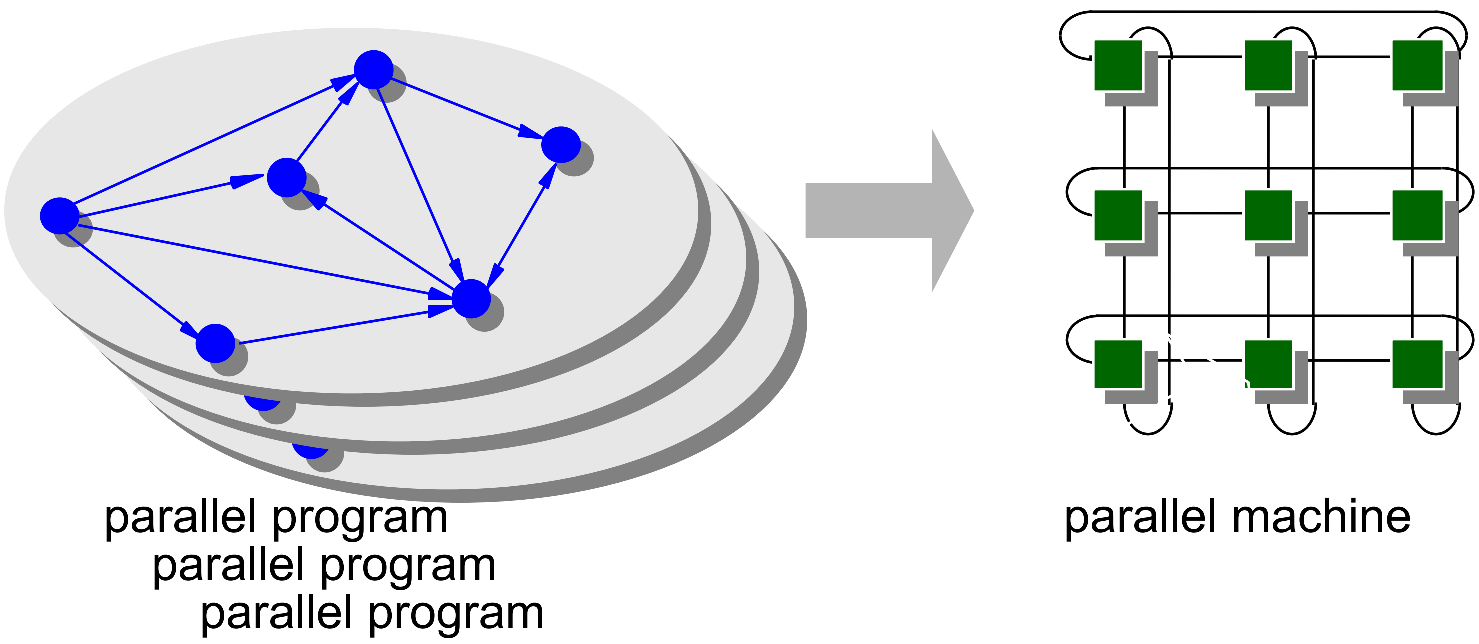
- In the early nineties parallel computing was characterized by the following properties:
  - **Machine dependent programming**

    The programmer had to explicitly consider size, type and architecture of the target machine.
  - **Manual allocation**

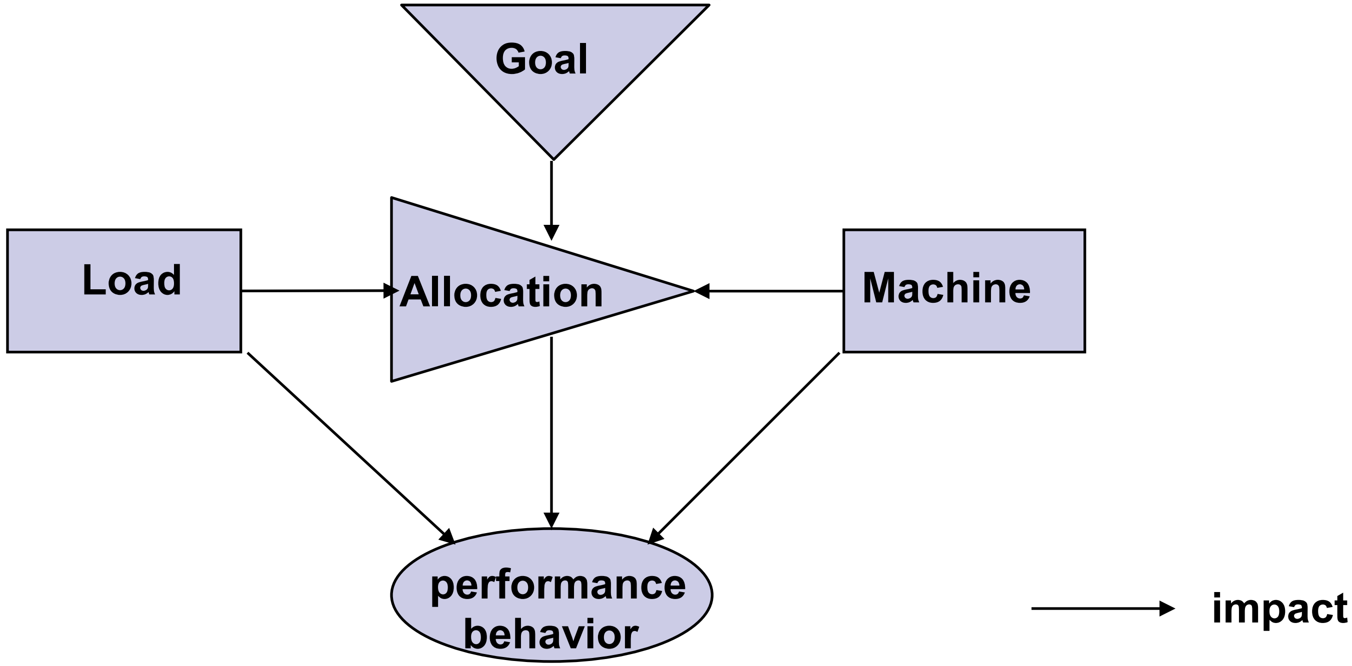
    The programmer himself was responsible for the mapping of logical objects to physical objects.
  - **Monoprogramming**

    At any point in time only one parallel programming could be executed occupying the entire machine.
- This characterization corresponds to the situation of sequential programming in the sixties.
- The goal is by support of system software to make parallel computing as efficient and comfortable as conventional sequential programming.

Allocation Problem



Problem and Problem components



- An allocation problem is described by four components:
  - Machine model M
  - Load model L
  - Allocation relation R
  - Allocation goal G

4.2 Machine model

- A parallel computer system can be described by a graph, with the processors as the vertices and the direct processor links as the edges:

  (P,E^P) with

P	set of processors as vertices ($\lvert P \rvert = n$)
E^P	set of links as edges

  Both vertices and edges can have weights:

$\mu_i: P \rightarrow R$	vertex weight processor speed (e.g. MFlops)
$\gamma_i: E^P \rightarrow R$	edge weight transmission speed (e.g. Mbit/sec)

4.3 Load model

- Load can be described at two levels:
  - Program level set of parallel programs
  - Thread level set of interacting threads of a program
- At thread level a parallel program can be represented (analogously to the machine) as a graph:
- $L = (T, E^T)$ program graph with

  T set of parallel threads (tasks, threads) as vertices ($|T| = m$)

  E^T set of interaction relations as edges
- Vertex and edge weights are also possible:

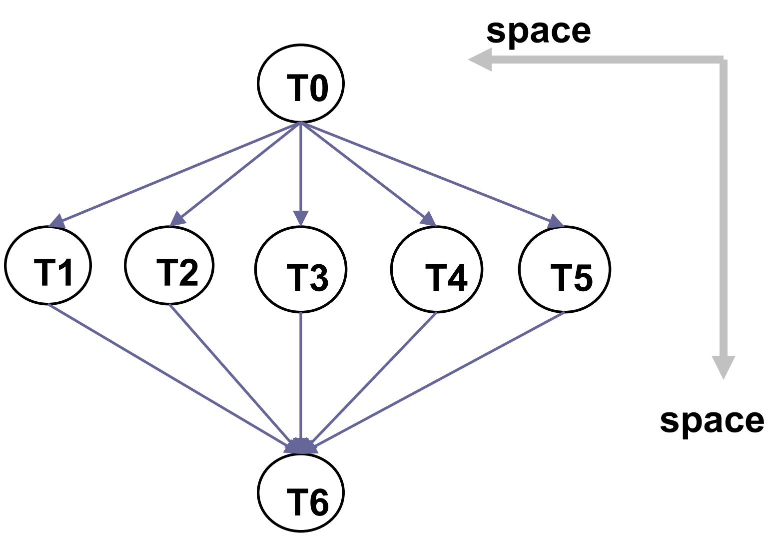
  $b_i: T \rightarrow R$ vertex weight length of thread (e.g. #instructions)

  $a_i: E^T \rightarrow R$ edge weight communication intensity (e.g. bits or packets)

Program Graph

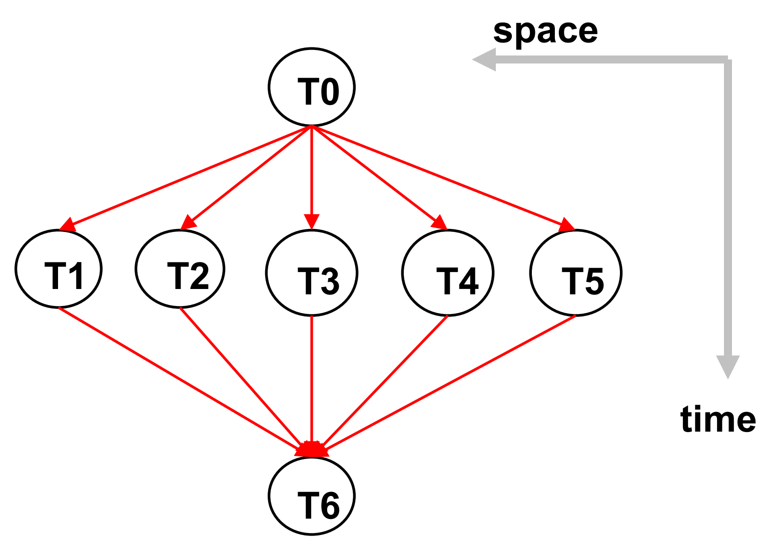
- Two types of program graphs
  - TIG task (=thread) interaction graph
  - TPG task (=thread) precedence graph

TIG



Arrows define
communication flow

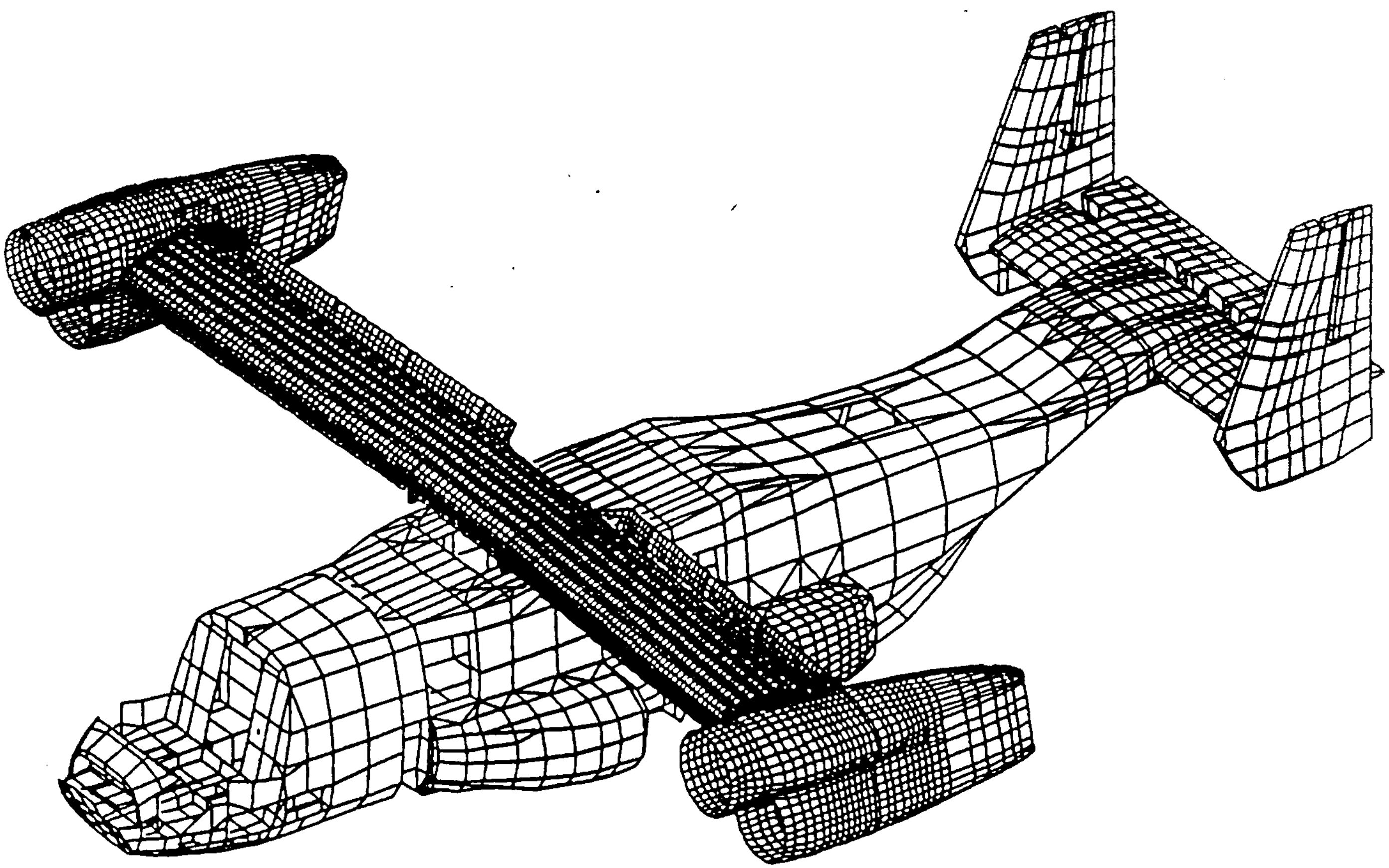
TPG



Arrows define
precedence relation

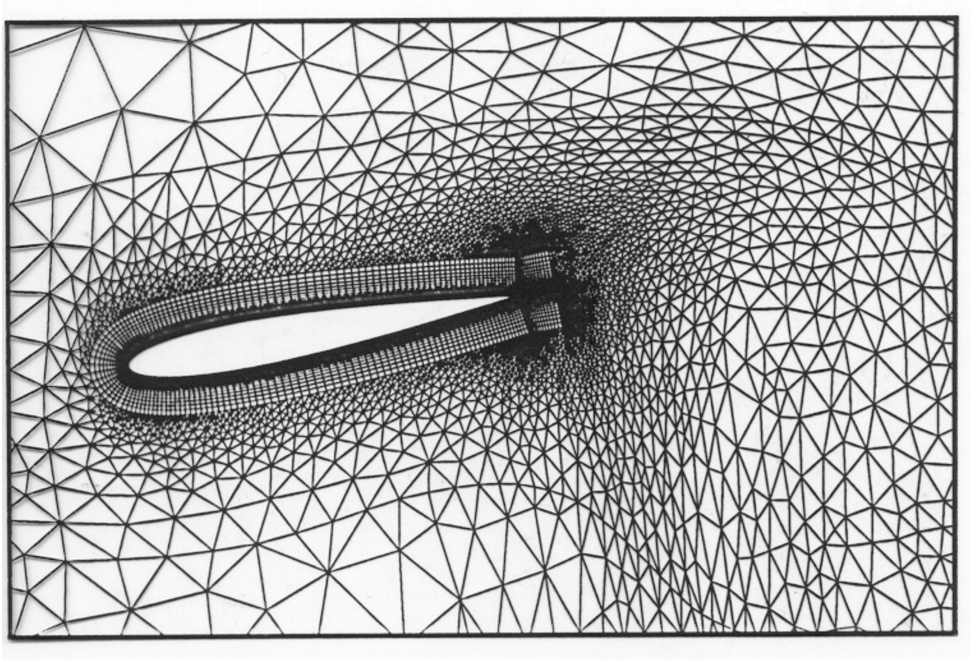
Example TIG

Aircraft engineering: Finite-Element-method



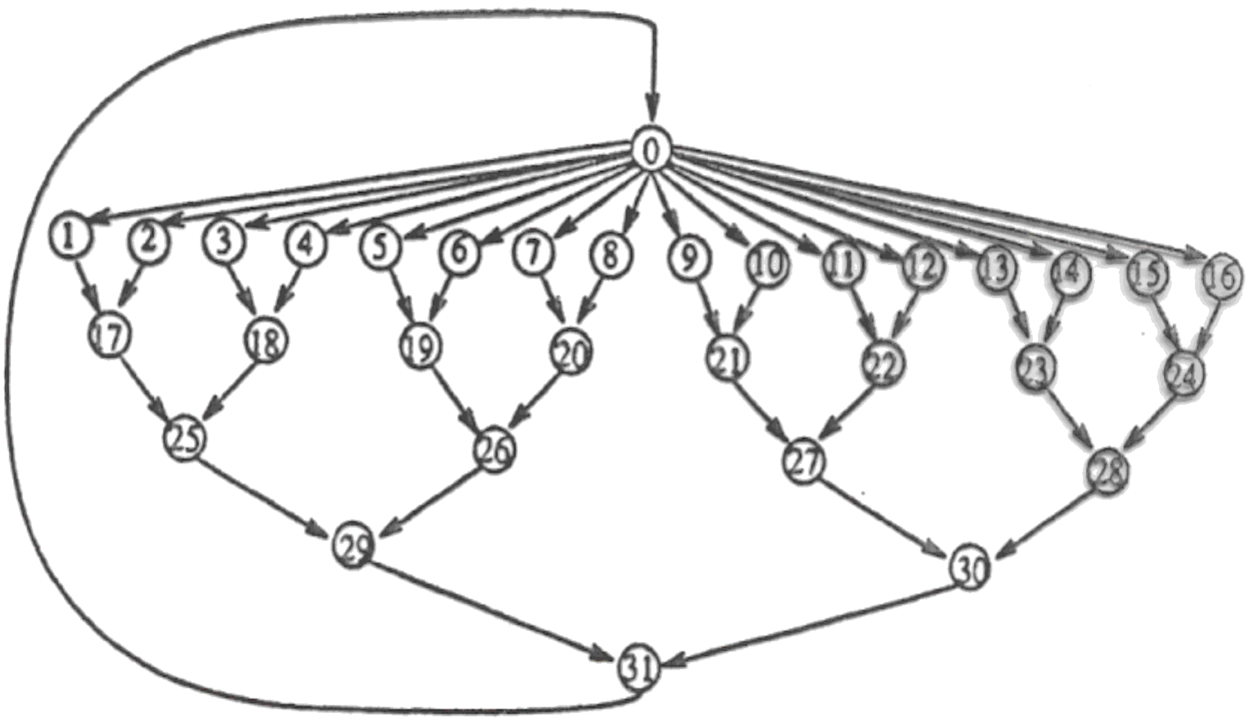
Example TIG

Airfoil (Finite-Element Method)



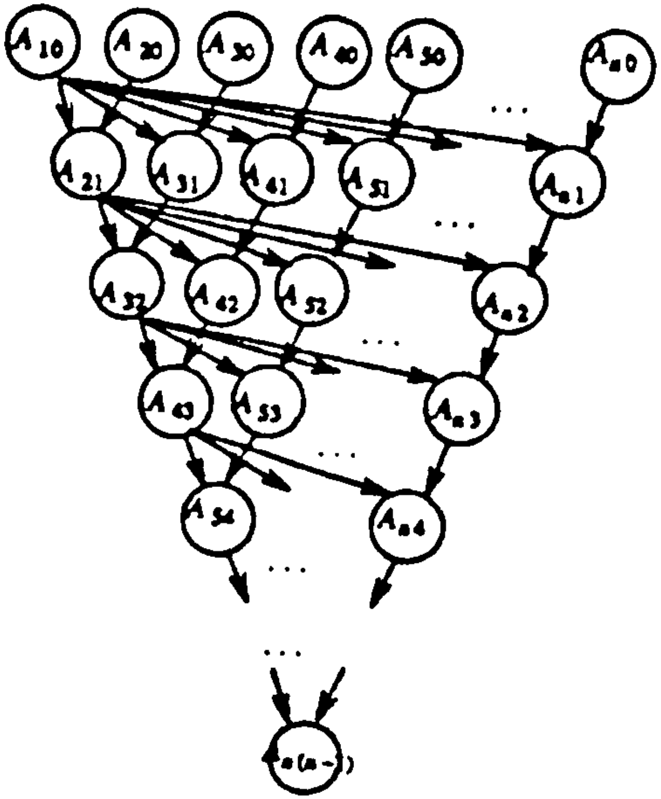
Example TPG

- Sieve of Erathostenes (Calculation of primes)



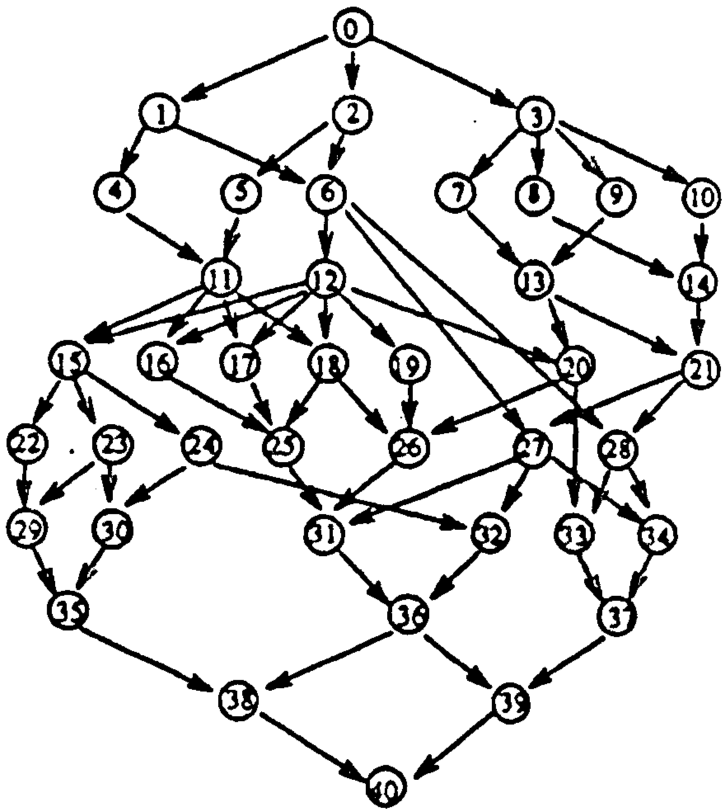
Example TPG

- Gaussian Elimination Method (LES)



Example TPG

- Application from Molecular Biology



Program Phase Graph (formal)

- Program phase graph

  PPG := (S, E^S) with

  S Set of Phases

  E^S Phase transitions

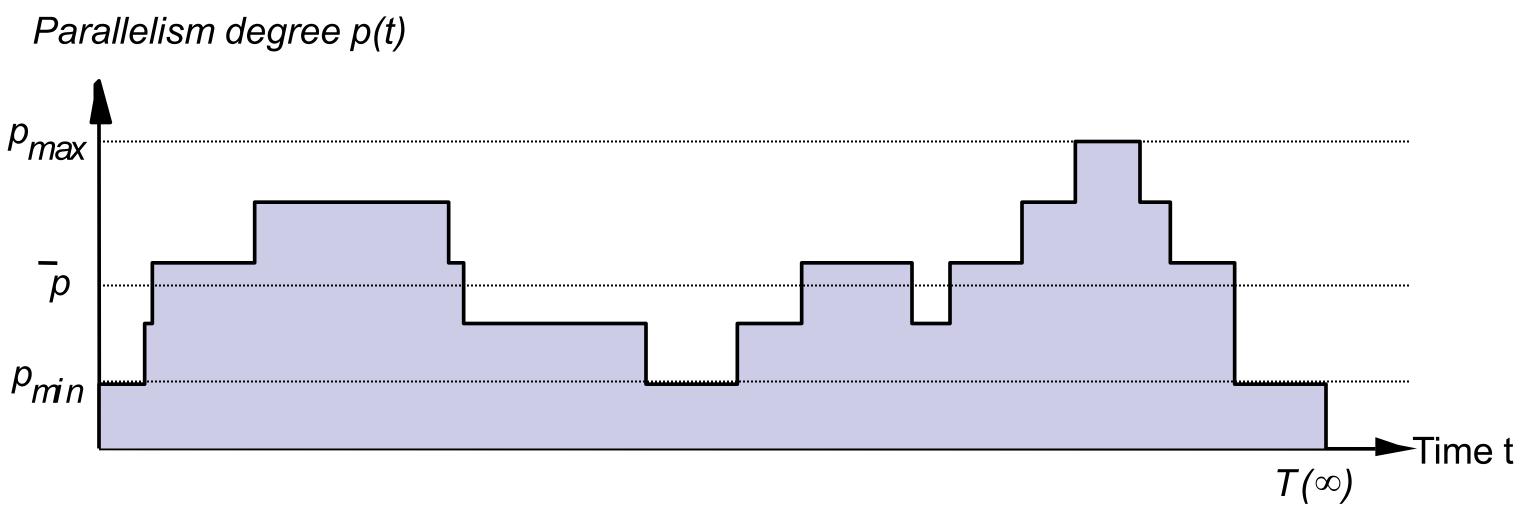
  p_{ij} transition probabilities
- Each phase consists of a TIG:

  $s_i := (T_i, E^{T_i}) \;\; \forall\, s_i \in S$
- To make sure that the phases are connected to each other, we request that two adjacent phases have at least one thread in common.:

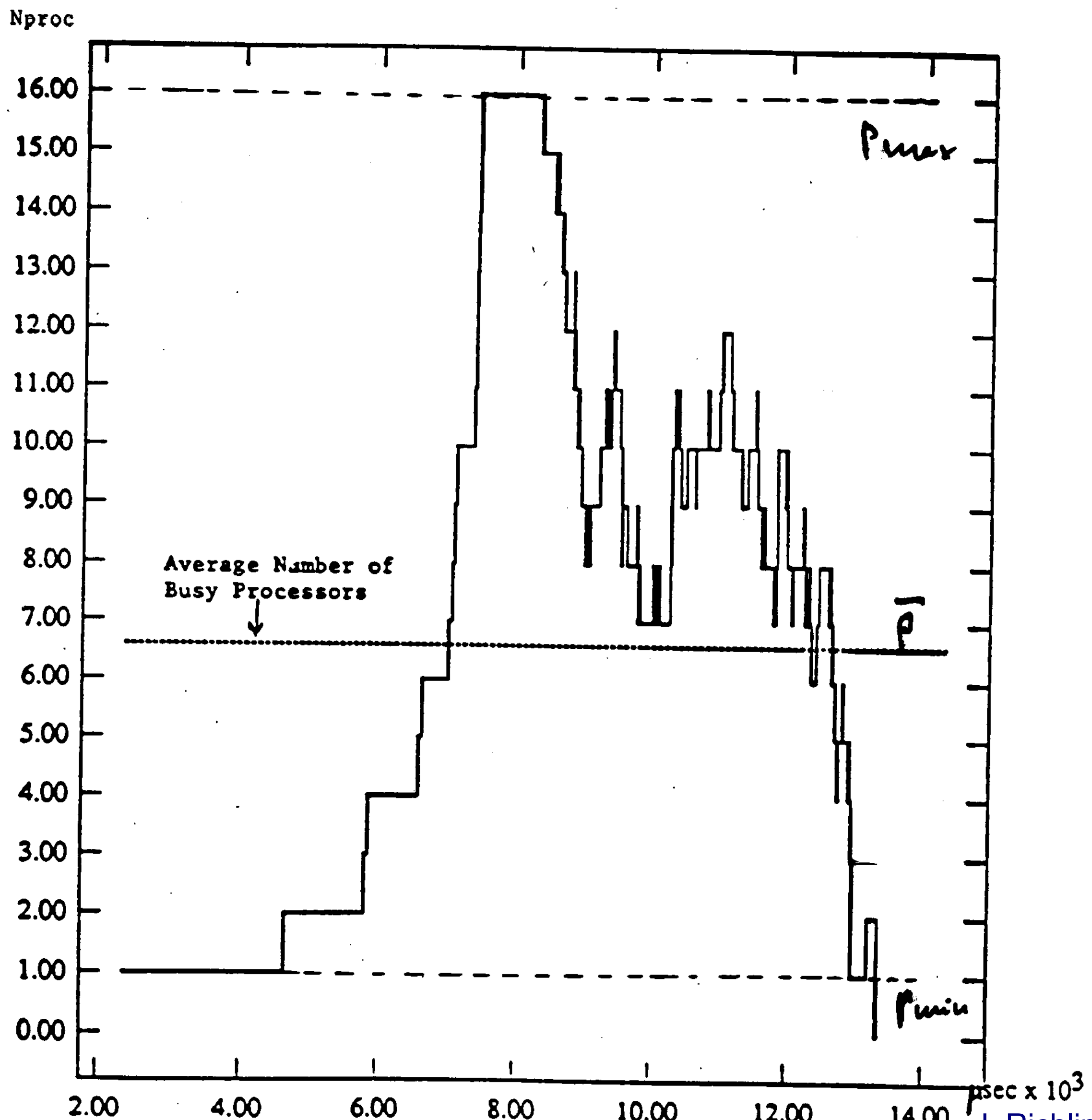
  $(s_i, s_j) \in E^S \Rightarrow \exists\, t: t \in T_i \wedge t \in T_j$

Parallelism profile

- If the communication behavior is unknown or irrelevant, the program description is reduced to the (dynamic) number of threads.
- If in turn the threadsare distinguished from each other, the number of threads (parallelism degree) is sufficient.
- For a dynamic parallelism degree we obtain the parallelism profile (known from chapter 4).



Example: Quicksort on 16 Processors



Example: Fine grain Parallelism

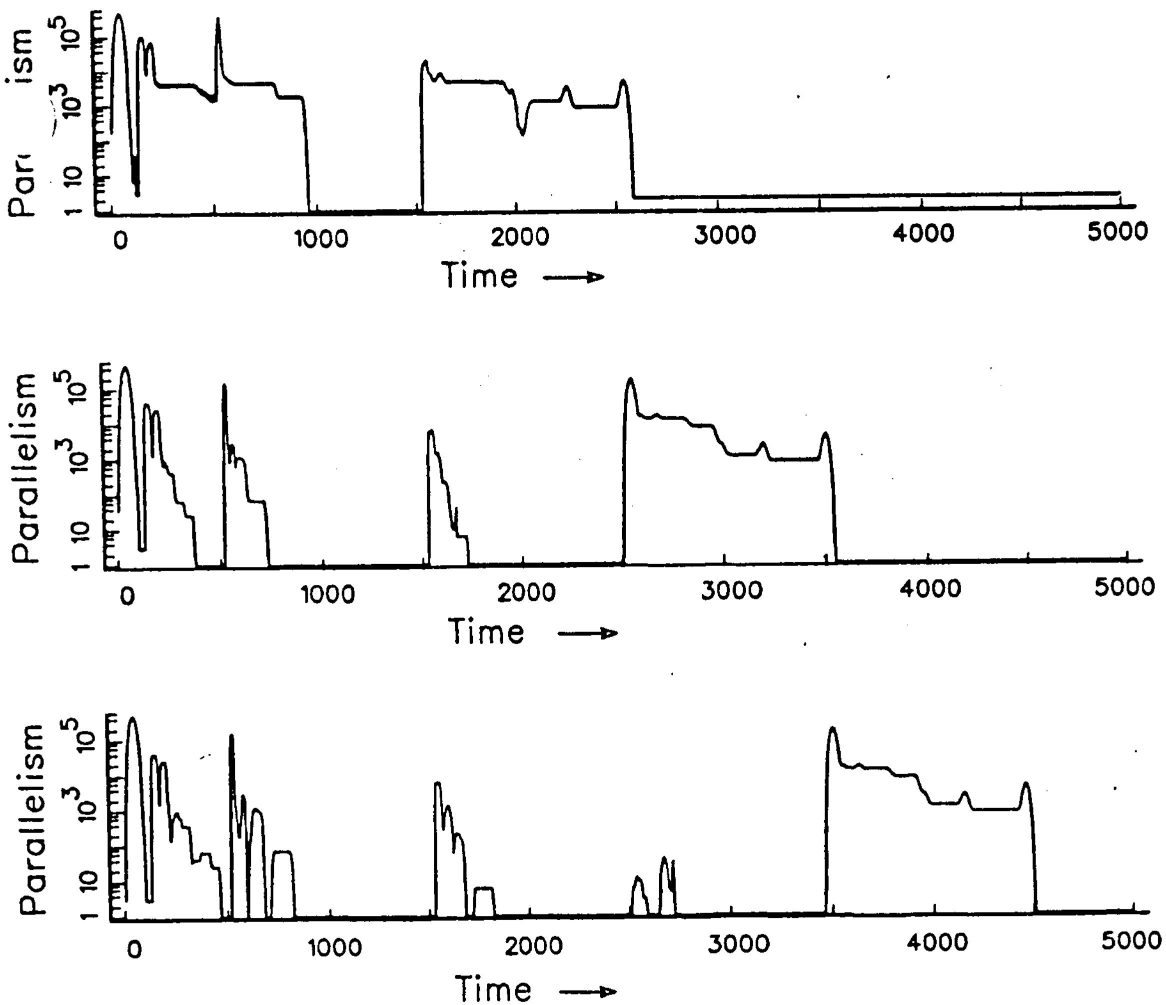


Fig. 7. Parallelism in three consecutive iterations of the VA3D program.

4.4 Allocation

Let be

- PCG = (P, EP) — The processor connection graph with P set of processors, $|P| = n$
- $A := \{A_1, A_2, \ldots, A_q\}$ — the load consisting of a set of parallel programs
- T_i — the set of threads of program A_i

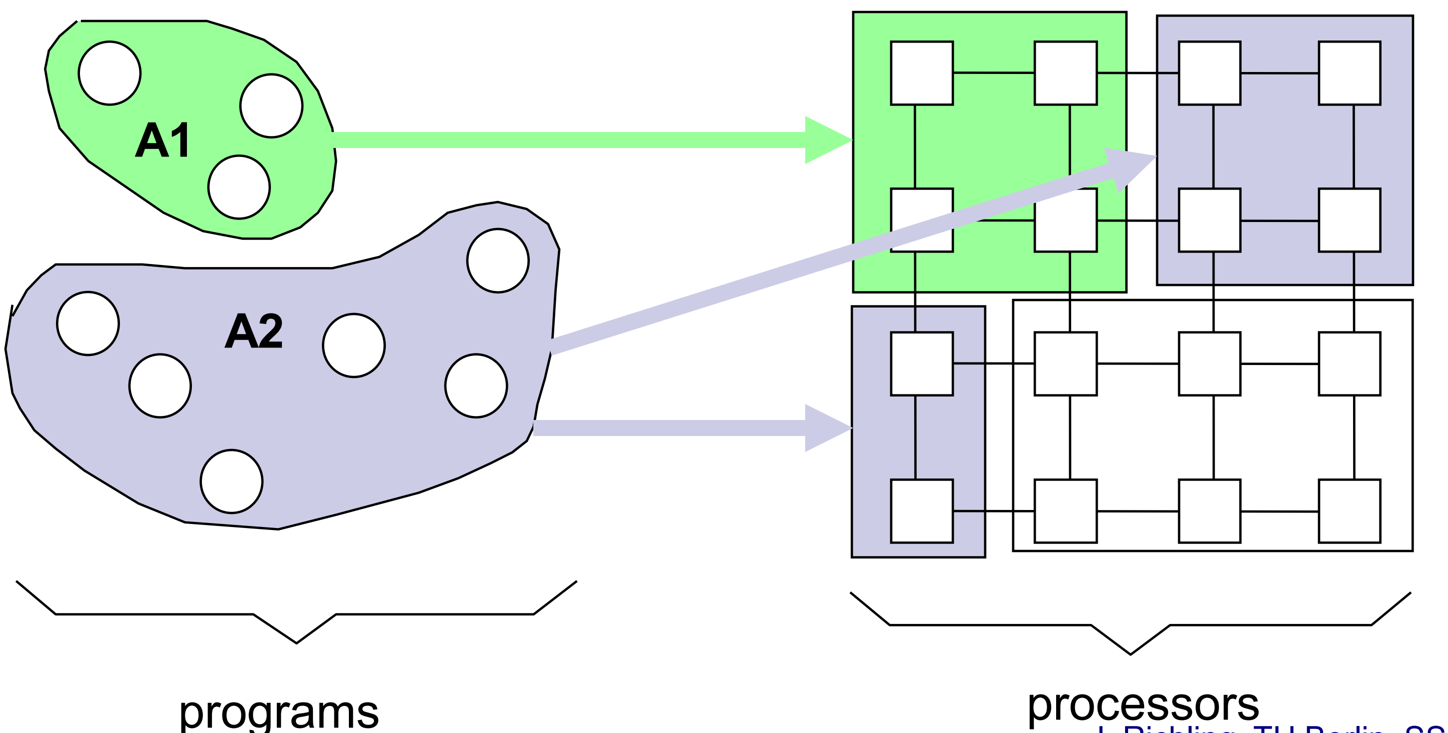
An allocation can take place on the program level or on the thread level.

Program Allocation

φ: $A \rightarrow \wp(P)$ mapping of programs to subsets of processors

$\varphi(A_i)$ is the processor set allocated to program A_i . It is called the **Territory** of A_i .

φ s called disjoint, if $\forall\ i \neq k : \varphi(A_i) \cap \varphi(A_k) = \emptyset$



Program Allocation

- A disjoint program allocation is called **partitioning**.

  (The processors not allocated by φ form the so-called **free partition**).
- A territory $\varphi(A_i)$ is called **contiguous**, if the subgraph of the PCG defined by the territory is connected.
- A program allocation φ is called contiguous, if $\varphi(A_i)$ is contiguous for all i = 1,.., q .

Sometimes topological aspects are irrelevant:

A **quantitative partitioning** only decides, **how many** processors each program obtains:

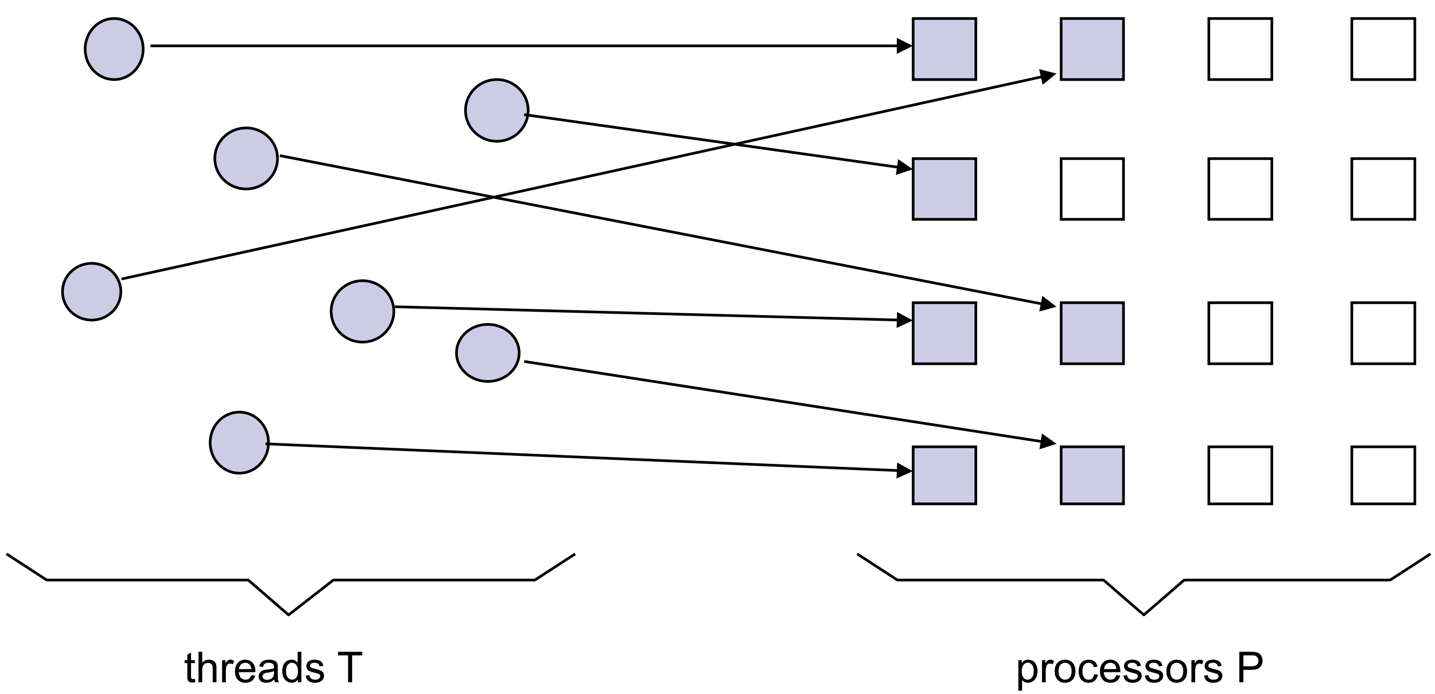
$$\chi : A \to \{1,...,n\} \;\; \textit{with} \;\; \sum_{i=1}^{q} \chi(A_i) \le n$$

Allocation at Thread Level (Mapping)

Within each program each thread must be assigned to exactly one processor:

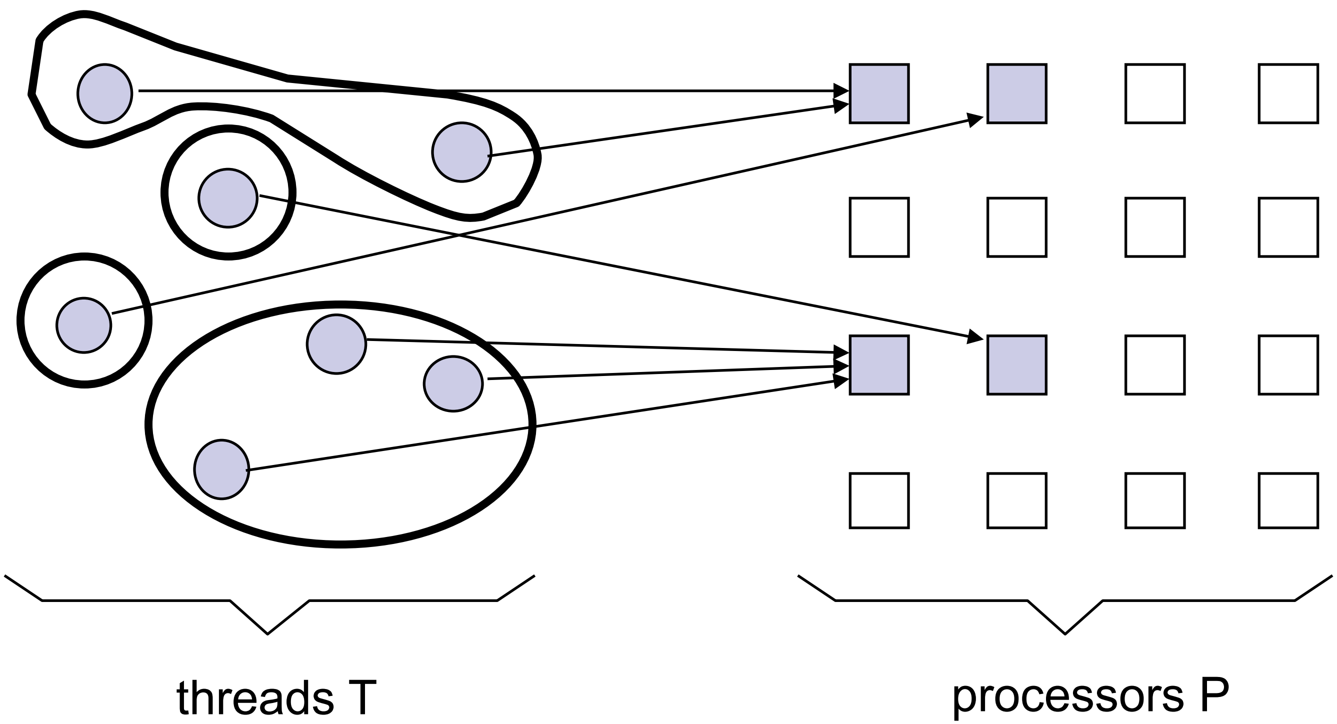
$\pi: \; T \rightarrow P$

If π is injective, the allocation is called **injective** (one-to-one), otherwise **contractive** (many-to-one).



Thread Allocation

For a contractive allocation there is often an intermediate step which determines which threads are mapped to the same processor (Contraction, Grouping, Clustering).



Contractive allocation

Allocation Problem

In multiprogramming operation, an allocation problem can consist of four steps that have to be solved one after the other:

- Quantitative Partitioning:
  - Which program obtains how many processors?
- Qualitative Partitioning
  - Which program obtains which processors?
- Clustering (Contraction) within the programs
  - Which threads are grouped together?
- Injective Allocation
  - Which thread group is mapped to which processor?

4.5 Goals

List of typical objective functions

- response time RT $\rightarrow$ min
- execution time ET $\rightarrow$ min
- communication cost CC $\rightarrow$ min
- utilization UT $\rightarrow$ max
- Speed-up SU $\rightarrow$ max
- throughput TP $\rightarrow$ max
- load unbalance $\rightarrow$ min
- .....

Since some quantities are contained in others and some are contradictory, it is reasonable to define combinations :

- Arithmetic combination, e.g. weighted sum
- Logical combination using restrictions
  - E.g.. $ET \rightarrow \min \mid LU < 2$

4.6 Allocation Algorithms

- An allocation algorithm is described by the problem it is supposed to solve and some additional properties :
- Optimality:
  - An algorithm is called **optimal**, if the optimality of the solution is guaranteed.
  - Otherwise it is called **suboptimal**.
  - Suboptimal algorithms can be divided into two classes:
    - An algorithm is **approximate**, if it finds an optimal solution only approximately. However, an error bound must be provided.
    - If we are unable neither to guarantee optimality nor to specify an error bound, the algorithm is called **heuristic.**
- Structure
  - If there is only one instance that has global information and decides about the global allocation then the algorithm is called **central**.
  - **Decentralized** or **distributed** algorithms can be further subdivided into
    - **hierarchical algorithms**
    - **cooperative algorithms** (peer-to-peer)

4.7 Application Areas

Another aspect is the question, at what time the allocation is taking place.

- Offline allocation
  - Optimization problem is formulated explicitly and solved.
- Allocation at compile time
  - Compiler knows the communication and data dependency structure of the parallel program.
- Allocation at start time
  - At this point of time the current load situation is known and can be taken into account.
- Allocation at run-time
  - Data dependent behavior can be collected during program execution (monitoring) resulting in an adaptive dynamic allocation (start new threads, migrate threads).

Further References:

- Heiss, H.-U.: Processor Allocation in Parallel Computers (in German) *Prozessorzuteilung in Parallelrechnern, Bibliographic Institute, Mannheim, 1994*
- T.L. Casavant and J.G. Kuhl, A Taxonomy of Scheduling in General-Purpose Distributed Computing Systems, *IEEE Transactions on Software Engineering,* Vol. SE-14, No. 2, February 1988, pp. 141-154.