PROJECT

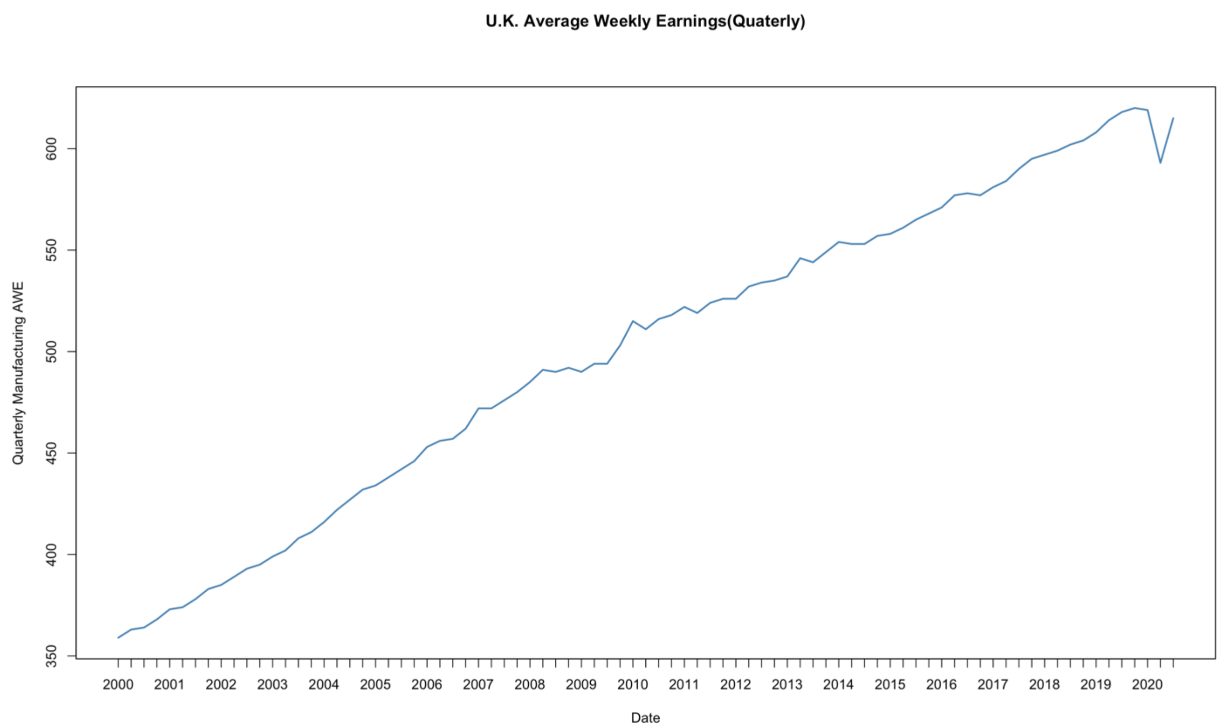
1. Descriptive Analysis

1.1 Data description

This paper uses the Average Weekly Earnings (AWE) on the base of the Manufacturing level in the UK as the outcome variable. This data is seasonally adjusted total pay excluding arrears, which is collected from the Office of National Statistics. The primary explanatory variable is the total actual weekly hours worked in the UK, which also comes from the same sources as the outcome variable. The assignment involved in three main stages: identification, estimation, and diagnostic checking to find out the relationship and economic implications behind the result of analyzing the dependent and independent variables.

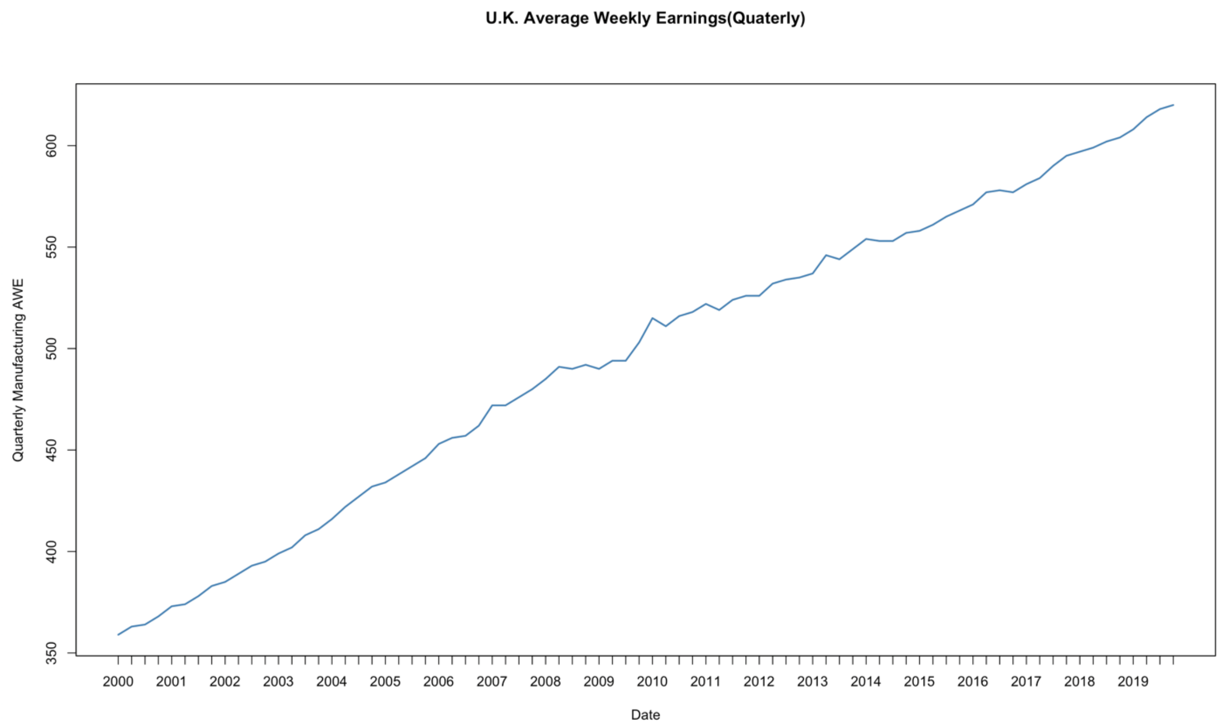
1.2 *Time series plot*

Figure 1. Average Weekly Earnings in UK (Quarterly)



The outcome variable is set as quarter frequency from the first quarter of 2000 to the third quarter of 2020. There is an increasing trend in the graph, Average Weekly Earnings increase gradually over time. It seems that the year 2020 has outliers so the paper decides to exclude the data from the last three quarters. Y_t names UK_M_AWE_Q_xts in the R studio.

Figure 2. Average Weekly Earnings in UK excluded outliers (Quarterly)



2. Autoregression Analysis of a Time Series

2.1 *Estimate an Autoregression Model*

BIC computation is used to identify the lag length for the AR model.

$$BIC(p) = \ln\left[\frac{SSR(p)}{T}\right] + (p+1)\frac{\ln(T)}{T}$$

Where: SSR(p) is the sum of squared residual for an estimated AR(p)

p_{max} is the maximum lag length we consider

T is the number of periods in our data

The table below shows the result of BIC with a maximum of four lags:

Table 1. BIC computation for AR(1) model

p	**BIC**	$\mathbf{R^2}$
1	2.1123	0.9987
2	2.1349	0.9987

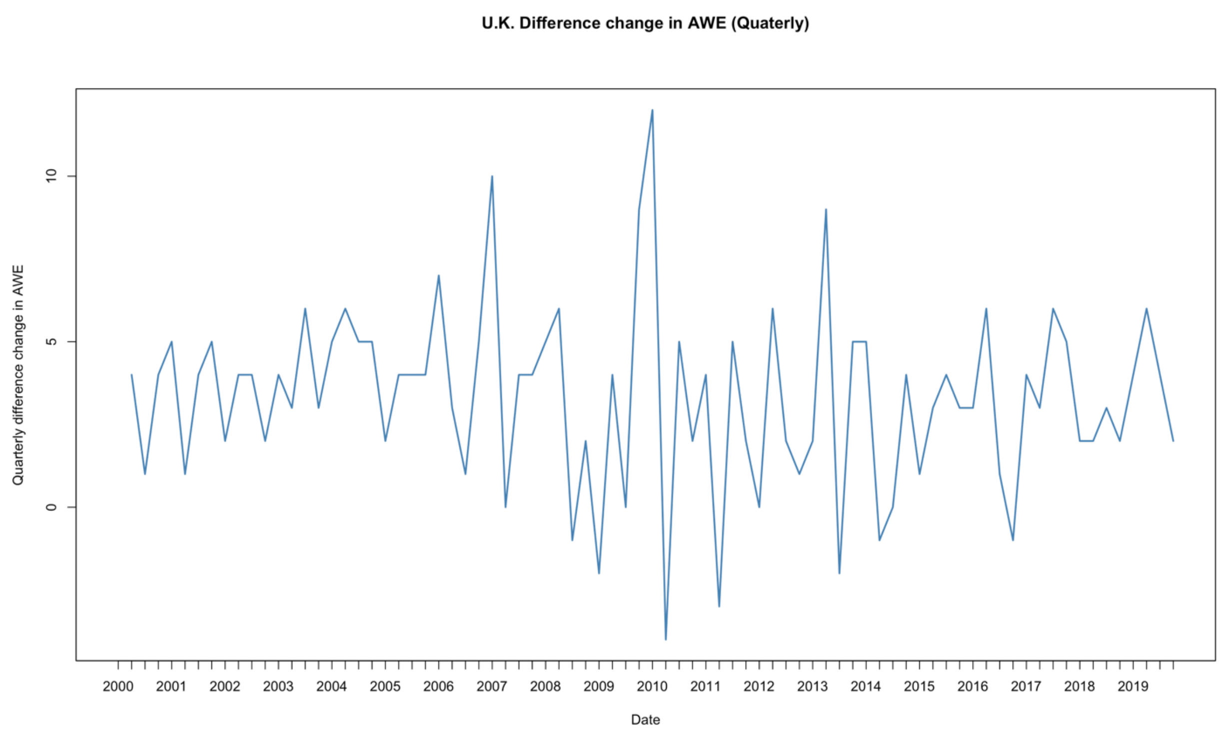
3	2.1604	0.9987
4	2.2312	0.9986

To balance the additional information and the cost of increasing error when adding more lags, BIC with the smallest value will be chosen with p =1, then AR(1) is the selecting model.

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + u_t$$

Average Weekly Earnings tend to increase over time due to several factors such as demand for labor, productivity, change in the labor market in institutions, inflation, and government policies. For example, inflation is one of the largest contributors. When the cost-of-living increases due to a rise in goods and services, workers may demand higher wages to maintain their standard of living. Wages of workers in the same occupation or industry can also be raised by labor market institutions, such as unions and minimum wage laws. Besides, the earnings depend on the need for labor, employers are willing to pay higher wages to attract and retain workers in economic growth, or the development of technology can enhance employee productivity. The data then seems to exhibit a linear time trend. The AR(1) model will be tested by Dickey-Fuller Test for the null hypothesis is that it has a unit root, and the alternative hypothesis is that it is stationary around a time trend.

Figure 3. Change of AWE difference in the UK (Quarterly)



The AR test statistic is t = -1.816. The corresponding 5% critical value of the ADF test is -3.45. As the result, we cannot reject the null hypothesis that the AR(1) model has a unit root. In dealing with the unit root problem, this paper will take differences to change the model into a stationary process and run BIC computation again on the transformed time series.

Table 2. BIC computation for AR(1) transformed model

p	BIC	R^2
1	2.1010	0.0404
2	2.1413	0.0605
3	2.2099	0.0626
4	2.2788	0.0620

The model will be tested for a break in time series by using the QLR test, which is suitable for the unknown date of the break. The QLR test statistic is 1.9657, which is estimated in the first quarter of 2010 with 515£ of average earnings a week. Compared to the critical value (5.86) of two restrictions with 15% trimming QLR Statistic at 5%, it is concluded that there is not enough evidence to confirm there is a break in the model.

Figure 4. QLR test for a break

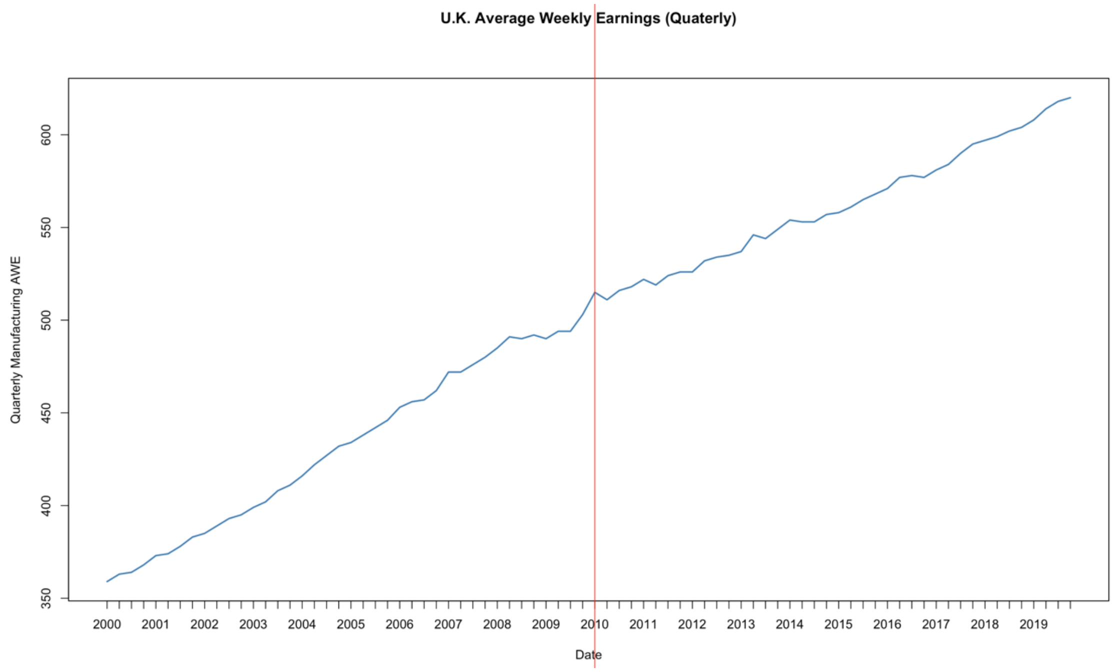


Table 3. AR(1) model and transformed AR(1) model

	Dependent variable:	
	UK_M_AWE_Q_xts	diff(UK_M_AWE_Q_xts)
	AR(1)	Modified AR(1)
lag(UK_M_AWE_Q_xts, 1)	0.995^{***}	
	(0.004)	
lag(diff(UK_M_AWE_Q_xts), 1)		-0.201^{*}
		(0.112)
Constant	5.662^{***}	3.963^{***}
	(2.077)	(0.485)
Observations	79	78
R^2	0.999	0.040
Adjusted R^2	0.999	0.028
Residual Std. Error	2.756 (df = 77)	2.739 (df = 76)
F Statistic	$57{,}495.160^{***}$ (df = 1; 77)	3.203^{*} (df = 1; 76)
Note:		*p**p***p<0.01

The coefficient of Y_{t-1} in the AR(1) model is significant at the 1% level. One pound increase in the first lag of AWE (quarterly) will increase 0.995 pounds in the current value AWE, all else is constant. The model also has adjusted R^2 is 0.999. It tells us that the regression model is a very good predictor of the dependent variable and the average weekly earnings at the present quarter can be explained by the previous quarter. This is because the employees will rely on the value of past earnings as a base to require an increase in their salary. Past earnings can be a useful predictor of future earnings, as it is often an indicator of an individual's skills, experience, and qualifications.

The modified AR(1) has a t statistic is t = $\frac{-0.201}{0.112}$ = -1.79, which its coefficient is significant at the 10% level. One pound increase in the previous difference of AWE will decrease 0.201 pounds in the current difference of AWE, ceteris paribus. Adjusted R^2 equals only 0.028, which is very small and implies that the regression model is not a good fit for the data. A small R^2 value may be acceptable if the goal of the analysis is simply to understand the relationship between the independent and dependent variables.

2.2. Estimate an Autoregressive Distributed Lag Model

The explanatory variable is added for estimating the ADL(1,1) with the continuous use of the modified model above and the data length is set the same as Y_t, which comes from the first quarter of 2000 to the third quarter of 2019. Before that, X_t is tested for a unit root by Dickey-Fuller Test. The AR test statistic is t = 0.4148 and the corresponding 5% critical value of the ADF test is -2.89 so we cannot reject the null hypothesis that the AR(1) model has a unit root. In order to avoid unit root, we will transform the X_t into a stationary process by taking differences. As a result, our ADL(1,1) model will be:

$$\Delta Y_t = \beta_0 + \beta_1 \Delta Y_{t-1} + \delta \Delta X_{t-1} + u_t$$

BIC computation is used again to identify the lag length for the ADL(1,1) model.

Table 4. BIC computation for ADL(1,1) model

p	BIC	R^2
1	2.1568	0.0405
2	2.2535	0.0610

3	2.3803	0.0632
4	2.3316	0.2146

The lag length p = 1 will be chosen as the most appropriate lag length. However, the paper uses ADL(2,2) to conduct the Granger causality test for the purpose of showing how jointly significant independent variable. With R^2 of p = 4 being significantly high compared to R^2 of p = 1, 2, 3, we also test Granger causality for ADL(4,4). The ADL(2,2) returns the F statistic of 0.0195, which is not jointly significant. In contrast, the joint test of ADL(4,4) is significant at the 5% level with the F statistic being 3.205.

Table 5. ADL(1,1) and ADL(4,4)

	Dependent variable:	
	diff(UK_M_AWE_Q_xts)	
	ADL(1,1)	ADL(4,4)
lag(diff(UK_M_AWE_Q_xts), 1)	-0.201*	-0.218*
	(0.113)	(0.113)
lag(diff(UK_M_AWE_Q_xts), 2)		-0.147
		(0.116)
lag(diff(UK_M_AWE_Q_xts), 3)		0.023
		(0.117)
lag(diff(UK_M_AWE_Q_xts), 4)		-0.021
		(0.115)
lag(diff(UK_weekly_hour_2000_2019), 1)	-0.003	0.036
	(0.055)	(0.056)
lag(diff(UK_weekly_hour_2000_2019), 2)		0.033
		(0.055)
lag(diff(UK_weekly_hour_2000_2019), 3)		-0.019
		(0.055)
lag(diff(UK_weekly_hour_2000_2019), 4)		-0.199***
		(0.056)
Constant	3.970***	4.821***
	(0.504)	(0.970)
Observations	78	75
R^2	0.040	0.215

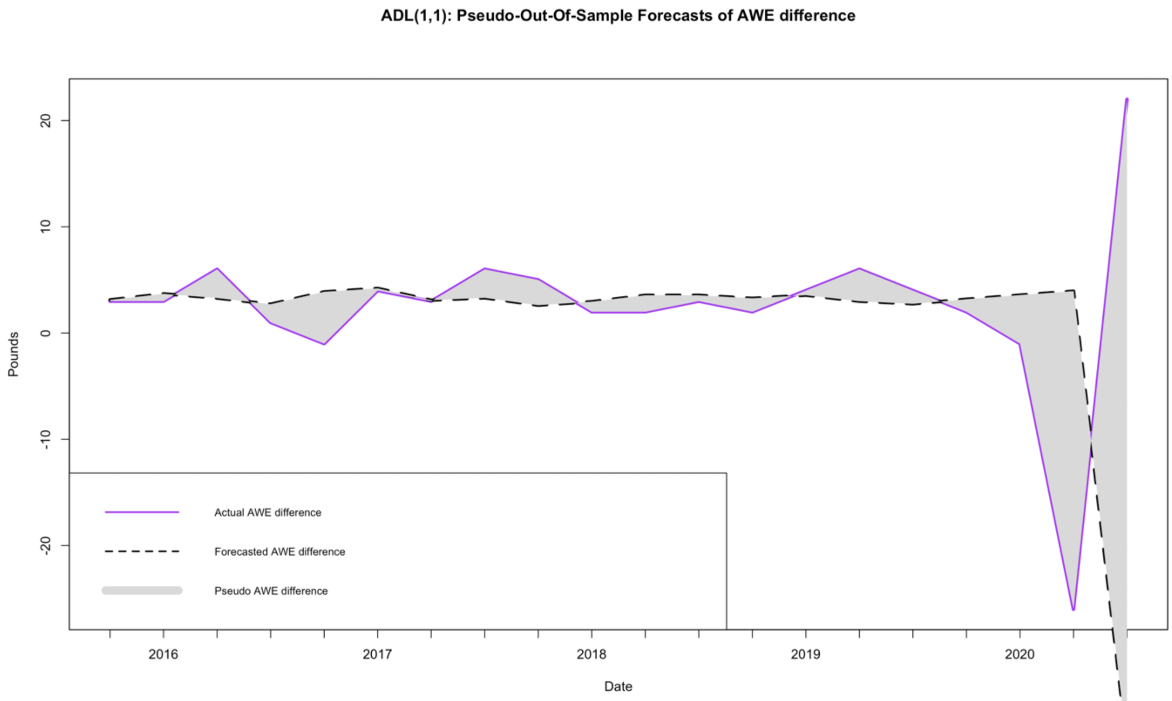
Adjusted R^2	0.015	0.119
Residual Std. Error	2.757 (df = 75)	2.640 (df = 66)
F Statistic	1.582 (df = 2; 75)	2.254** (df = 8; 66)
Note:		*p**p***p<0.01

The coefficients of ΔY_{t-1} on both ADL models above are significant at the 10% level. In the ADL(4,4) model, one pound increase in the first lag of difference of AWE will decrease by 0.218 pounds in the current difference of AWE, ceteris paribus. The coefficients of ΔX_{t-1} are not significant in the two models, which are -0.003 and 0.036 respectively. The coefficient of ΔX_{t-4} in the ADL(4,4) model is significant at the 1% level, and adjusted R^2 is quite high (0.119) in comparison with adjusted R^2 (0.015) of the ADL(1,1) model. It means that the ΔX_{t-4} can have a quite good prediction for the Y_t where the model also includes the lags of ΔY_t. This means one additional million hours worked in the fourth lag of the difference between two consecutive quarters will decrease by 0.199 pounds for the current difference AWE between two consecutive quarters in the ADL(4,4) model, ceteris paribus.

2.3 Check Out-Of-Sample Forecast Performance

The ADL(1,1) model will be used to conduct Out-Of-Sample Forecast Performance, with 25% of the final data length will be cut for the use of excluded sample. The model also uses the original data, which spans from the first quarter of 2000 to the third quarter of 2020.

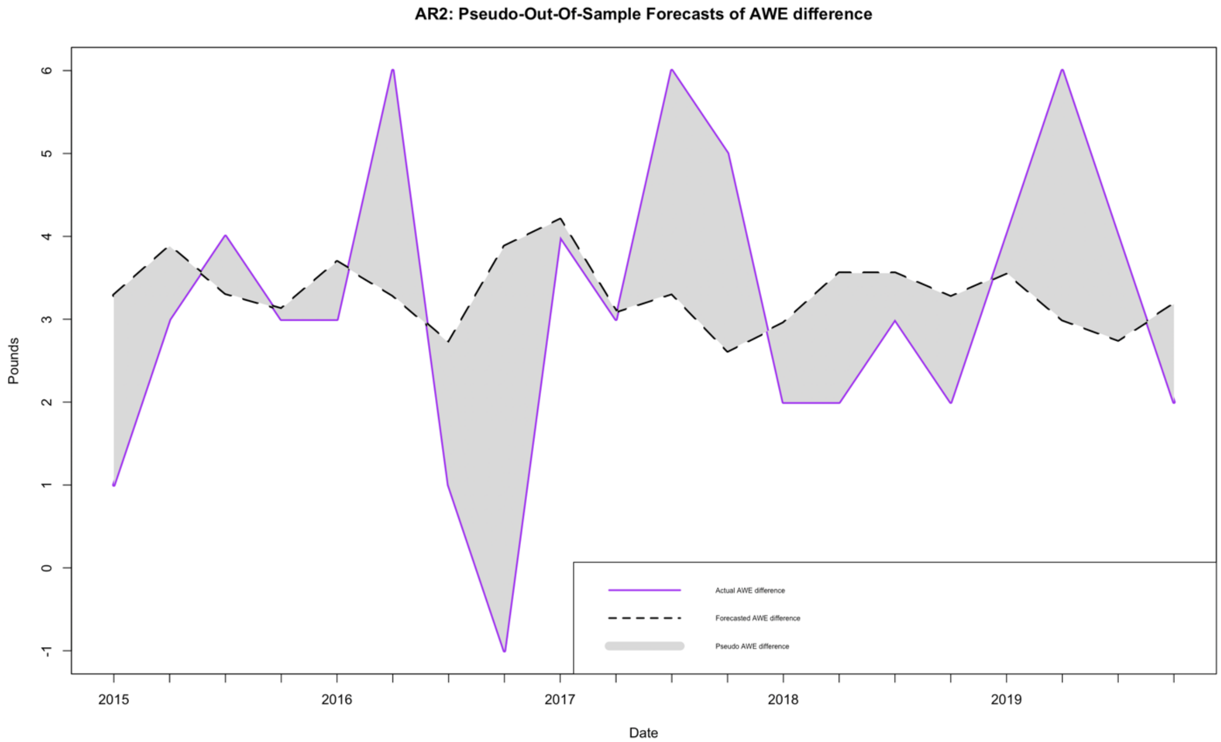
Figure 5. Pseudo Out-Of-Sample Forecast Errors of AWE difference (original data)



We estimate the Within-Sample Errors is 2.957998, while the Estimated RMSFE is 15.6472. This means that the estimated standard deviation of our forecast around the correct value is approximately 15.6472. The RMSFE is considerably larger than the Within-Sample Errors, which means the model is bad for predicting the out-of-sample. However, the test statistic of t = 0.01827065 and p-value = 0.7188 for the test on the mean out-of-sample errors. If the t statistic is smaller than the p-value, we fail to reject the null hypothesis, so that is not enough evidence of systematic errors.

To evaluate the effect of outliers, we also run the Out-Of-Sample Forecast for the data without predetermined outliers.

Figure 6. Pseudo Out-Of-Sample Forecast Errors of AWE difference (no outliers)



The Within-Sample Errors is 3.018643 and the Estimated RMSFE is 1.932177, which means that out-of-sample errors are even better than the estimated sample. The t statistic is -0.01873095 and the p-value is 0.7121, this also means not enough evidence of systematic errors.

The difference in output between the two types of data is caused by the outliers. If the out-of-sample experiences a heavily sudden change (outliers), it will make a lag on the prediction as seen in figure 5, where the great decrease in the second quarter of 2022 of actual difference AWE is reflected in the third quarter of 2022 of forecasted difference AWE. The huge error of outlier is the contributor of big RMSFE.

3. Dynamic Causal Effects

Our distributed lag model where r = 3 will be used for estimating the dynamic multipliers.

$$\Delta Y_t = \beta_0 + \beta_1 \Delta X_t + \beta_2 \Delta X_{t-1} + \beta_3 \Delta X_{t-2} + u_t$$

For the estimation of dynamic causal effects, β needs to be constant overtime and the lags of independent variables need to be uncorrelated with the error term u_t. Under our distributed lag model, ΔY_t and ΔX_t both are difference stationary and do not have outliers. The assumption that needs to be defined is whether the explanatory variable is exogenous . There are two types of exogeneity: strict exogeneity and exogeneity. Under exogeneity, u_t is only exogenous to past and present regressors ($E[u_t|X_t, X_{t-1},...] = 0$) but in strict exogeneity, u_t need to be exogenous to past, present and future regressors ($E[u_t|...,X_{t+1},X_t,X_{t-1},...] = 0$).

As we mentioned above, one of the factors that affect the average earnings weekly is the demand for labor. In economic growth or industry growth, companies are willing to increase salaries to attract and retain workers, which impacts the difference in earnings between two quarters. Let's look at our distributed lag model, this variable is omitted and will be in the residual u_t. An increase in demand for labor will make u_t unusually high ($u_t > 0$). This change can also cause more people to join the labor market, which leads to an increase in the total hours worked and then an increase in the difference in total hours worked between two consecutive quarters. As the result, corr(u_t, ΔX_t) is positive. For that reason, we cannot assume the model is exogeneity or strictly exogeneity.

To understand more about two types of exogeneity, let's take a simple example. The volatility of weather can affect the supply and demand for various types of energy products, including natural gas. It is clear to see that humans cannot control the weather, so from the view of the energy market, the weather is randomly assigned, and the weather is exogenous. If an exceptional cold is forecasted, the current price of natural gas will be above the predicted value from the population regression, which makes u_t uncommonly high. If the weather forecast is precise, the future weather turns out to be cold, and it creates a correlation between the residual of the current price of natural gas and future weather. We cannot conclude there is a strictly exogeneity in this case.

Let's consider our case of strictly exogeneity. In a garden of peppers, the fertilizer is assigned randomly by a robot throughout time. The amount of fertilizer is a random application, out of human control, so it is exogenous. It is understandable to see that the pepper yield today does not rely on the amount of fertilizer used in the future, this makes fertilizer time series satisfy strictly exogeneity assumption.

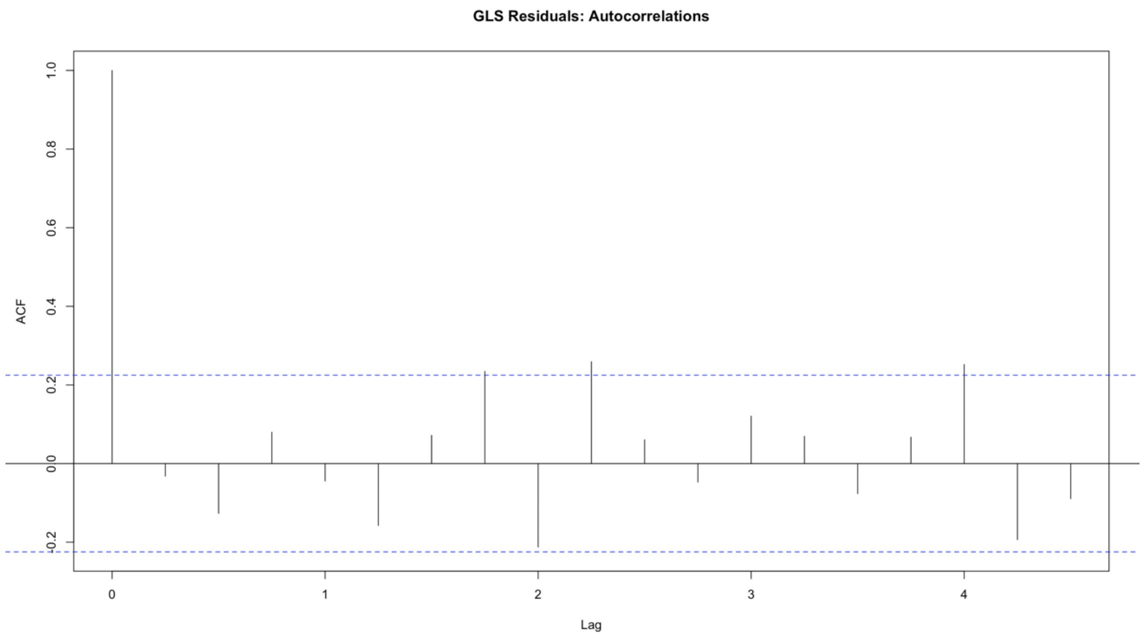
With the conclusion that the explanatory variable is not exogenous, we still use the Cohcrane-Orcutt method to estimate the dynamic multipliers. Violation of strictly exogeneity assumption can lead to biased and inconsistent estimates of the regression coefficients.

Table 6. Dynamic Causal Effects – DL(3,3) & Feasible GLS

	Dependent variable:	
	diff(UK_M_AWE_Q_xts)	Y_tilde_xts
	(1)	(2)
diff(UK_weekly_hour_2000_2019)	-0.008	
	(0.058)	
lag(diff(UK_weekly_hour_2000_2019), 1)	0.006	
	(0.058)	
lag(diff(UK_weekly_hour_2000_2019), 2)	-0.003	
	(0.058)	
X_tilde_xts		-0.001
		(0.058)
Xlag1_tilde_xts		0.005
		(0.058)
Xlag2_tilde_xts		-0.006
		(0.058)
Constant	3.334^{***}	3.978^{***}
	(0.382)	(0.381)
Observations	77	76
R^2	0.0005	0.0002
Adjusted R^2	-0.041	-0.041
Residual Std. Error	2.840 (df = 73)	2.802 (df = 72)
F Statistic	0.012 (df = 3; 73)	0.006 (df = 3; 72)
Note:		*p**p***p<0.01

For the DL(3,3) model, the coefficient of all explanatory variables is not significant. The dynamic multiplier of ΔX_t is strongest and the effect decreases over time. After estimating by a process known as Generalized Least Square, the model error is non-serially correlated.

Figure 7. GLS Residuals: Autocorrelation



4. Multiperiod Forecasts

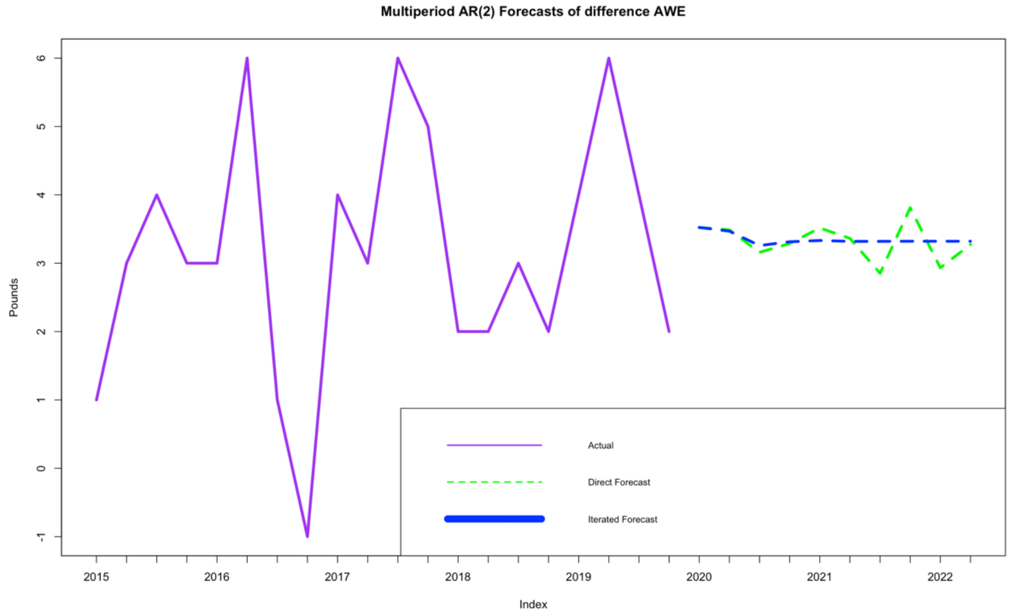
Because ΔX_{t-1} is not significant in the ADL(1,1) model and the adjusted R^2 is very low, we decide to conduct the forecast ten periods after the end of the third of 2019 by using the AR(2) model, which is:

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + u_t$$

Table 7. Forecasted value in ten periods time (pounds)

Time period	Iterated Multiperiod Forecasts	Direct Multiperiod Forecasts
t+1	2.773775	3.523015
t+2	3.346519	3.487760
t+3	3.395547	3.158945
t+4	3.300444	3.287874
t+5	3.314953	3.515746
t+6	3.325578	3.363505
t+7	3.321029	2.851916
t+8	3.320510	3.811664
t+9	3.321295	2.932352
t+10	3.321192	3.273659

Figure 8. Multiperiod AR(2) Forecasts of difference AWE

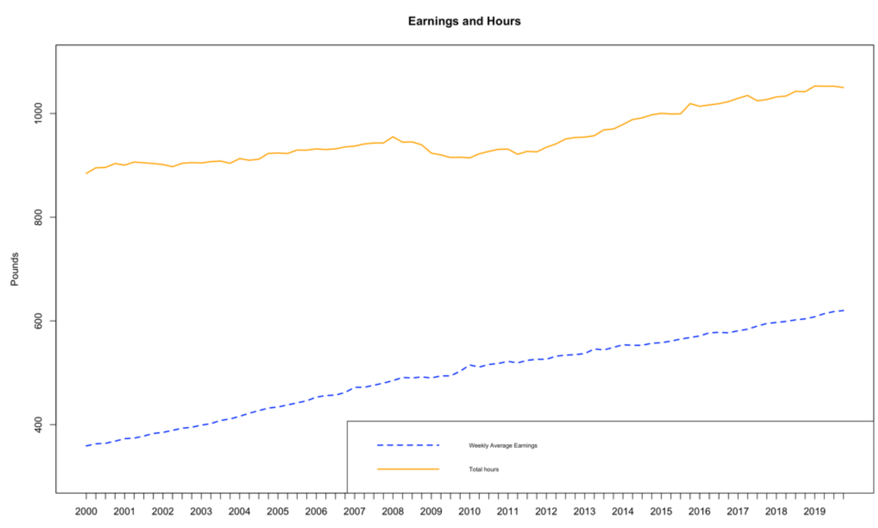


The Iterated Multiperiod Forecasts are very smooth with predicted values is around 3.3 £, which means that the difference AWE of 10 periods in the future will be the sum of 3.3£ and the previous one. The Direct Multiperiod Forecasts vary quite more, but still is an easier prediction compared to the actual value.

5. Cointegration

In the test of cointegration, we do not need to use the differenced data. So that Y_t here is the average weekly earnings at time t and X_t is the total weekly hours worked (million).

Figure 9. Average weekly earnings and total weekly hours worked



We compute GLS Dickey-Fuller Test, which is more powerful than the standard Dickey-Fuller Test to check whether the two time series above are nonstationary. The test statistics of Y and X are -1.2193 and -1.1233 respectively, in comparison with the 5% critical value for both is -3.03. We cannot reject the null hypothesis of nonstationary for both Y_t and X_t. That is reasonable for us to then test for cointegration with unknown θ. From the regression: $Y_t = \alpha t + \theta X_t + z_t$, $\hat{\theta}$ will be estimated and used to compute the residual:

$$\widehat{z_t} = Y_t - \hat{\theta} X_t$$

Using the Dickey-Fuller Test, the time series of $\widehat{z_t}$ has a test statistic of -0.6798 and the critical value of the ADF test is -3.41 for 5%. As a result, we cannot reject the null hypothesis, so Y_t and X_t are not cointegrated.

6. Volatility Clustering Analysis

The paper uses ARMA(1,1)-GARCH(1,1) model on the differenced data.

$$\Delta Y_t = \mu + \sigma_t \varepsilon_t + \varphi_1 \Delta Y_{t-1} + \theta_1 \varepsilon_{t-1}$$

$$\sigma_t^2 = \omega + \alpha_1 \sigma_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

Figure 10. ARMA(1,1)-GARCH(1,1)

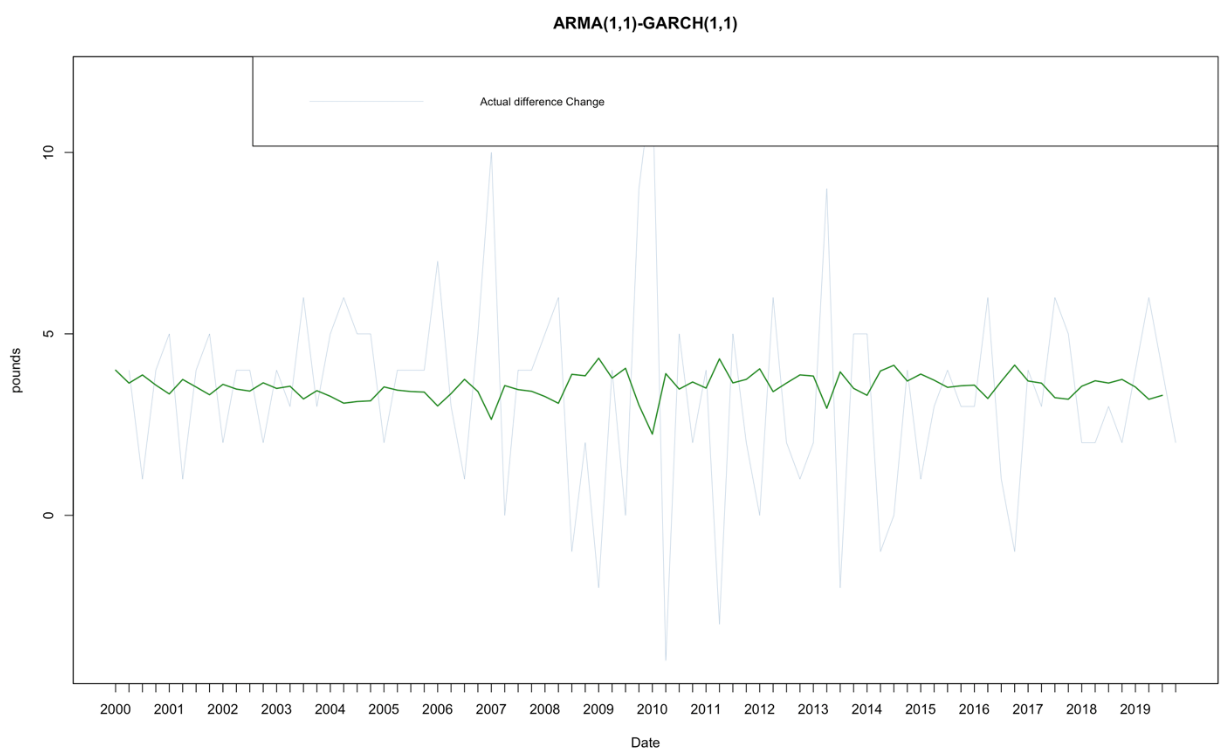
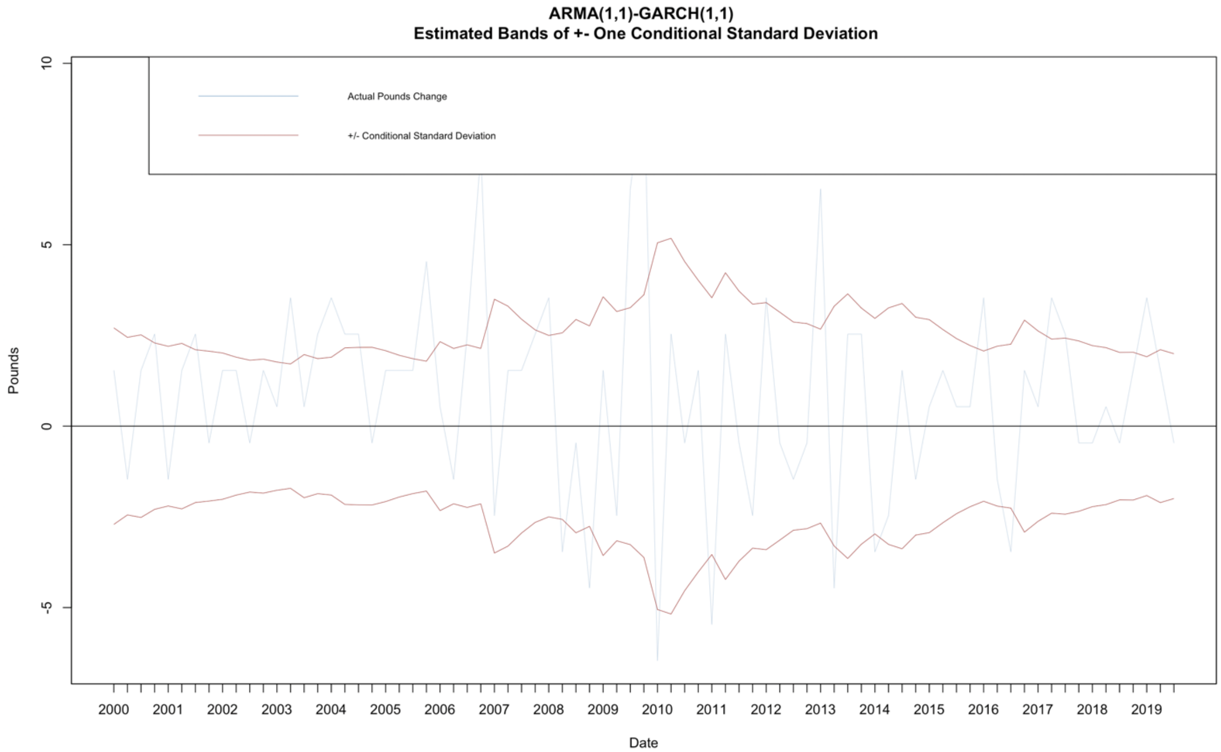


Table 8. ARMA(1,1)-GARCH(1,1)

	Dependent variable:
	[
mu	2.464
	(1.709)
ar1	0.295
	(0.481)
ma1	-0.420
	(0.465)
omega	0.589
	(0.547)
alpha1	0.190
	(0.119)
beta1	0.737^{***}
	(0.140)
Observations	79
Log Likelihood	185.773
Akaike Inf. Crit.	4.855
Bayesian Inf. Crit.	5.035
Note:	*p**p***p<0.01

β_1 is significant at the 1% level while α_1 is not significant. It means that the movement of σ_t^2 is determined by the coefficient of σ_{t-1}^2. The value of this coefficient is quite high, this suggests that changes in the conditional variance are highly persistent, indicating prolonged periods of increased instability. In the context of weekly average earnings, if the volatility is caused by a sudden shock like shifts in the economy, for example, during times of high demand for skilled workers, wages may increase due to a shortage of labor and eventually adjust to the new conditions. If volatility is driven by technological advances, and changes in demographics, it may persist for a longer period. Technological advancements may increase demand for skilled workers and lead to higher wages, while changes in demographics, such as an aging population, may reduce the labor supply and increase wages.

Figure 11. ARMA(1,1)-GARCH(1,1) +- One Conditional Standard Deviation



The bands of the estimated conditional standard deviations track the observation in the series of the quarterly change in the AWE difference quite well.

7. Conclusion

In summary, Average Weekly Earnings at the manufacturing level of the previous quarter is a good predictor for the current one. Taking differences is conducted to deal with unit root for both Average Weekly Earnings and Weekly hours worked, and the transformed model seems not to fit the differenced data. Then the AR(1) model and ADL(1,1) have the appropriate lag length for the analysis. Without outliers, the ADL(1,1) has out-of-sample errors which are better than the estimated sample in Out-Of-Sample Forecast compared to ADL(1,1) with outliers. The explanatory variable is not exogenous and not cointegrated with the outcome variable in the original data. The paper considers that conditional variance is highly persistent in the modified model.