Assignment

1. A businessman goes to hotels x, y, z at 20%, 50%, 30% of the time respectively. It is known that 5%, 4%, 8% of the rooms in x, y, z hotels having faculty plumbing. What is the probability that the business man's room having faculty plumbing in the hotels x and y.

Sol: Let E_1, E_2, E_3 be the events that the businessman goes to hotels x, y, z respectively then,

$$P(E_1) = 20\% = \frac{20}{100} = 0.2$$

$$P(E_2) = \frac{50}{100} = 0.5$$

$$P(E_3) = 30\% = \frac{30}{100} = 0.3$$

Let A be the event that the hotel having faculty plumbing.

then, $P(A|E_1) = \frac{5}{100} = 0.05$ $P(A|E_2) = \frac{4}{100} = 0.04$

$p(A|E_3) = \frac{8}{100} = 0.08$

From Baye's theorem we know that,

$$p(E_k/A) = \frac{p(E_k)\,p(A/E_k)}{\sum_{i=1}^{n} p(E_i)\,p(A/E_i)}.$$

the probability that the business man's room having faculty plumbing in the hotel

① x is

$$p(E_1|A) = \frac{p(E_1) \cdot p(A/E_1)}{p(E_1) \cdot p(A/E_1) + p(E_2) \cdot p(A/E_2) + p(E_3) \cdot p(A/E_3)}$$

$$= \frac{(0.2)(0.05)}{(0.2)(0.05) + (0.5)(0.04) + (0.3)(0.08)}$$

$$= \frac{10 \times 10^{-3}}{(10+20+24) \times 10^{-3}} = \frac{10}{54} = 0.1851$$

$\therefore \boxed{p(E_1|A) = 0.1851}$

② y is

$$p(E_2/A) = \frac{p(E_2) \cdot p(A/E_2)}{p(E_1) \cdot p(A/E_1) + p(E_2) \cdot p(A/E_2) + p(E_3) \cdot p(A/E_3)}$$

$$= \frac{(0.5)(0.04)}{54 \times 10^{-3}} = \frac{20 \times 10^{-3}}{54 \times 10^{-3}}$$

$= 0.3703$

$\therefore \boxed{p(E_2/A) = 0.3703}$

2(a) calculate the Rank correlation from the following data:

x	12	9	8	10	11	13	12
y	14	8	6	9	14	12	13

sol: Given, $n=7$.

x	r_1 = Rank of (x)	y	r_2 = Rank of (y)	$d = r_1 - r_2$	d^2
12	5.5	14	6.5	-1	1
9	2	8	2	0	0
8	1	6	1	0	0
10	3	9	3	0	0
11	4	14	6.5	-2.5	6.25
13	7	12	4	3	9
12	5.5	13	5	0.5	0.25
					$\Sigma d^2 = 16.50$

1st term:- (12) 2nd term:- (14)

$\frac{5+6}{2} = 5.5$ $\frac{6+7}{2} = 6.5$ $\therefore \Sigma d^2 = 16.5$

$m_1 = 2$ $m_2 = 2$

We know that,

$$r = 1 - \frac{6\left[\Sigma d^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2)\right]}{n(n^2-1)}$$

$$r = 1 - \frac{\left[6\left(16.5 + \frac{1}{12}(8-2) + \frac{1}{12}(8-2)\right)\right]}{7(49-1)}$$

$$r = 1 - \frac{\cancel{6}\left(16.5 + \frac{12}{12}\right)}{7(48)^{8}} = 1 - \frac{16.5+1}{56}$$

$$= 1 - \frac{17.50}{56} = 1 - 0.3125$$

$$= 0.6875$$

$$\boxed{r = 0.6875}$$

7)b) Find the two regression line's for the following data:-

	x	y
Average	30	500
S.D	5	100
Coefficient of correlation		0.8

Sol:- Given, $\bar{x} = 30$; $\bar{y} = 500$

$\sigma_x = 5$; $\sigma_y = 100$

$r = 0.8$

① Regression line y on x

$$(y - \bar{y}) = b_{yx}(x - \bar{x})$$

$$b_{yx} = r\frac{\sigma_y}{\sigma_x} = 0.8\left(\frac{\cancel{100}^{20}}{\cancel{5}}\right) = 16$$

$b_{yx} = 16$.

$(y-500) = 16(x-30)$

$y = 16x - 480 + 500$

$$\boxed{y = 16x + 20}$$

(2) Regression line x on y.

$(x-\bar{x}) = b_{xy}\,(y-\bar{y})$

$(x-30) = r\dfrac{\sigma_x}{\sigma_y}(y-500)$

$(x-30) = 0.8\left(\dfrac{\cancel{5}}{\cancel{100}_{20}}\right)(y-500)$

$(x-30) = 0.04\,(y-500)$

$x = 0.04y - 20 + 30$

$$\boxed{x = 0.04y + 10}$$

∴ The two Regression lines are:-

(1) Regression line y on x is $\boxed{y = 16x + 20}$

(2) Regression line x on y is $\boxed{x = 0.04y + 10}$

Q).a) Find the mean and variance of density function $f(x) = \frac{1}{2}\sin x$

$0 \le x \le \pi$.

Sol: We know that,

Mean and variance of density function as

$$\text{Mean } (\mu) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$\text{variance } (\sigma^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2$$

Given, $f(x) = \frac{1}{2}\sin x \quad 0 \le x \le \pi$

$$\text{Mean } (\mu) = \int_{-\infty}^{\infty} x \frac{1}{2}\sin x\, dx$$

$$= \underbrace{\int_{-\infty}^{0} x \frac{1}{2}\sin x\, dx}_{(0)} + \int_{0}^{\pi} x \frac{1}{2}\sin x\, dx + \underbrace{\int_{\pi}^{\infty} x \frac{1}{2}\sin x\, dx}_{(0)}$$

$$\mu = \frac{1}{2}\int_{0}^{\pi} x \sin x\, dx$$

$$\left[\int uv\, dx = u\int v\, dx - \int\left(u' \int v\, dx\right) dx\right]$$

$$\mu = \frac{1}{2}\left[x \int \sin x\, dx - \int\left(\frac{d}{dx}(x) \cdot \int \sin x\, dx\right) dx\right]_0^{\pi}$$

$$\mu = \frac{1}{2}\left[\left(x(-\cos x)\right)_0^{\pi} - \left[\int -\cos x\, dx\right]_0^{\pi}\right]$$

$\mu = \frac{1}{2}\left[(-x\cos x)_0^{\pi} + (\sin x)_0^{\pi}\right]$

$= \frac{1}{2}\left[-(\pi(-1)-0) + 0\right]$

$= \frac{1}{2}(\pi) = \pi/2$

$\therefore \boxed{(\mu) = \pi/2}$

Variance $(\sigma^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

$= \int_0^{\pi} x^2\left(\frac{1}{2}\sin x\right)dx - (\pi/2)^2$ $\quad\therefore [0 \le x \le \pi]$

$\sigma^2 = \frac{1}{2}\int_0^{\pi} x^2 \sin x\,dx - \frac{\pi^2}{4}$

$\left[\int uv\,dx = u\int v\,dx - \int(u'\int v\,dx)\,dx\right]$

$\sigma^2 = \frac{1}{2}\left[x^2\int \sin x - \left(\int 2x.\int \sin x\,dx\right)dx\right]_0^{\pi} - \frac{\pi^2}{4}$

$= \frac{1}{2}\left[(x^2(-\cos x))_0^{\pi}\right] - \frac{1}{2}\cdot\left[\int 2x(-\cos x)dx\right] - \frac{\pi^2}{4}$

$= \frac{1}{2}\left[(-x^2\cos x)_0^{\pi} + \int x\cos x\,dx\right] - \frac{\pi^2}{4}$

$= \frac{1}{2}\left[-[\pi^2(-1)-0]\right] + \left[x\int \cos x\,dx - \int \sin x\,dx\right]_0^{\pi} - \frac{\pi^2}{4}$

$= \frac{\pi^2}{2} - \frac{\pi^2}{4} + (x\sin x)_0^{\pi} - (\sin x)_0^{\pi} + (\cos x)_0^{\pi}$

$= \frac{\pi^2}{4} + (0-0) + (\cos x)_0^{\pi}$

$\sigma^2 = \frac{\pi^2}{4} + (-1-1)$

$\sigma^2 = \frac{\pi^2}{4} - 2$.

$\therefore$ $\boxed{\begin{array}{l} \text{Mean}(\mu) = \pi/2 \\ \text{Variance}(\sigma^2) = \frac{\pi^2}{4} - 2 \end{array}}$

2) b) Find (i) $p(x<3)$ (ii) $p(0<x<5)$ given if $p(0) = 3c^3$, $p(1) = 4c - 10c^2$, $p(2) = 5c - 1$ for $c > 0$.

Sol: Given, $p(0) = 3c^3$

$p(1) = 4c - 10c^2$

$p(2) = 5c - 1$.

so, Range of $x/\alpha = \{0, 1, 2\}$

$\sum_{i=0}^{n} p(x_i) = 1$.

$\therefore p(0) + p(1) + p(2) = 1$

$3c^3 + 4c - 10c^2 + 5c - 1 = 1$

$3c^3 - 10c^2 + 9c - 2 = 0 \rightarrow (1)$

put $c = 1$,

$3 - 10 + 9 - 2 = 0$.

$12 - 12 = 0$

$0 = 0$

$\therefore c = 1$ is a root of eq-(1)

From synthetic division:-

1	3	-10	9	-2
	0	3	-7	2
	3	-7	2	0

$3c^2 - 7c - 2 = 0$

$$c = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{7 \pm \sqrt{49 - 4(3)(-2)}}{2(3)}$$

$$\Rightarrow \frac{7 \pm \sqrt{25}}{6} \Rightarrow \frac{7 \pm 5}{6},$$

$$c = \frac{7+5}{6}, \quad c = \frac{7-5}{6}$$

$$c = \frac{12}{6} = 2, \quad c = \frac{2}{6} = 1/3$$

$$\therefore \quad c = 1, \ 2, \ 1/3$$

$\boxed{c = 1/3.}$ $\because$ c should be in range of 1&2.

and $p(x=3) = p(x=4) = 0$ since $x/d = \{0, 1, 2\}$.

(i) $p(x<3) = p(x=0) + p(x=1) + p(x=2)$

$$= 3c^3 + 4c - 10c^2 + 5c - 1$$

$$= 3c^3 - 10c^2 + 9c - 1$$

$$= 3\left(\frac{1}{3}\right)^3 - 10\left(\frac{1}{3}\right)^2 + 9\left(1/3\right) - 1$$

$$= \frac{3}{27} - \frac{10}{9} + 9/3 - 1$$

$$= \frac{1}{9} - \frac{10}{9} + 3 - 1 = -1 + 2 = +1$$

$$\therefore \boxed{p(x<3) = 1}$$

(ii) $p(0<x<5) = p(x=1) + p(x=2) + p(x=3) + p(x=4)$

$= 4c - 10c^2 + 5c - 1 + 0 + 0$

$= -10c^2 + 9c - 1$

$= -10\left(\frac{1}{3}\right)^2 + 9\left(\frac{1}{3}\right) - 1$

$= \frac{-10}{9} + \frac{9}{3} - 1$

$= \frac{-10+27-9}{9}$

$= \frac{-19+27}{9} = 8/9.$

$\therefore \boxed{p(0<x<5) = 8/9}$

3)a) the mean of the Binomial distribution is 3 and variance is 9/4 then find (i) $P(x \geq 7)$ ii) $P(1<x<6)$

sol:- Given, Mean $(np) = 3$

variance $(npq) = 9/4$

$$\frac{\cancel{np}q}{\cancel{np}} = \frac{\cancel{9}^{3}/4}{\cancel{3}}$$

$q = 3/4$, $p = 1-q = 1/4$

$np = 3$

$n\left(\frac{1}{4}\right) = 3$

$\boxed{n = 12}$

$\therefore p = \frac{1}{4},\ q = 3/4,\ n = 12.$

$P(x=r) = n_{c_r} p^r q^{n-r}$
in BD

(i) $p(x \geq 7) = 1 - p(x<7)$

$= 1 - p(x=0) + p(x=1) + p(x=2) + p(x=3) + p(x=4) + p(x=5) + p(x=6)$

$p(x=0) = 12_{c_0} \cdot \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{12} = 0.0316$

$p(x=1) = 12_{c_1} \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^{11} = 0.1267$

$p(x=2) = 12_{c_2} \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^{10} = 0.2322$

$p(x=3) = 12_{c_3} \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^{9} = 0.2581$

$p(x=4) = 12_{c_4} \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^{8} = 0.1935$

$p(x=5) = 12_{c_5} \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^{7} = 0.1030$

$p(x=6) = 12_{c_6} \left(\frac{1}{4}\right)^6 \left(\frac{3}{4}\right)^{6} = 0.0401$

$p(x \geq 7) = 1 - \sum_{i=0}^{6} p(x=r)$

$= 1 - (0.0316 + 0.1267 + 0.2322 + 0.2581 + 0.1935 + 0.1030 + 0.0401)$

$= 1 - 0.9852$

$= 0.0148.$

$\therefore \boxed{p(x \geq 7) = 0.0148}$

ii) $p(1 < x < 6) = p(x=2) + p(x=3) + p(x=4) + p(x=5)$

$= 0.2 + 0.2 + 0.19 + 0.1$

$= 0.69$

$\boxed{p(1<x<6) = 0.69}$

3) b) Fit the poission distribution for the following data

x	0	1	2	3	4	5
f	147	147	74	25	6	1

sol:- In poission distribution we know that, $\lambda = \mu = \frac{\sum f_i x_i}{\sum f_i}$ & $\sum f_i = N$

$$\lambda = \frac{\sum f_i x_i}{\sum f_i} = \frac{147(0) + 147(1) + 74(2) + 25(3) + 6(4) + 5(1)}{147 + 147 + 74 + 25 + 6 + 1}$$

$$= \frac{399}{400} = 0.99$$

$\boxed{N = 400}$ $\boxed{\lambda = 0.99}$

Expected frequency:-

$f(x=r) = N * p(x=r)$

where,

$$p(x=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!}$$

Now,

$$f(0) = N * p(x=0)$$

$$= 400 * \frac{e^{-0.99} \cdot (0.99)^0}{0!}$$

$$= 400 * 0.371$$

$$= 148.4 \simeq 148$$

$$\boxed{f(0) = 148}$$

$$f(1) = N * p(x=1)$$

$$= 400 * \frac{e^{-0.99} \cdot (0.99)^1}{1!}$$

$$= 400 * 0.371 \cdot (0.99)$$

$$= 148.4 \times 0.99$$

$$= 146.917 \simeq 147$$

$$\boxed{f(1) = 147}$$

$$f(2) = N * p(x=2)$$

$$= 400 * \frac{e^{-0.99} (0.99)^2}{2!}$$

$$= 400 * 0.371 \frac{(0.99)^2}{2}$$

$$= 148.4 \times \frac{(0.99)^2}{2}$$

$$= 72.72 \simeq 73$$

$$\boxed{f(2) = 73}$$

$f(3) = N * p(x=3)$

$= 400 * \frac{e^{-0.99} (0.99)^3}{3!}$

$= 400 * 0.371 \frac{(0.99)^3}{6}$

$= 148.4 \times \frac{(0.99)^3}{6}$

$= 23.99 \simeq 24$

$\boxed{f(3) = 24}$

$f(4) = N * p(x=4)$

$= 400 * \frac{e^{-0.99} (0.99)^4}{4!}$

$= 400 * 0.371 \times \frac{(0.99)^4}{24}$

$= 148.4 \times \frac{(0.99)^4}{24}$

$= 5.93 \simeq 6$

$\boxed{f(4) = 6}$

$f(5) = N * p(x=5)$

$= 400 * \frac{e^{-0.99} (0.99)^5}{5!}$

$= 148.4 \times \frac{(0.99)^5}{120}$

$= 148.4 \times 0.0079$

$= 1.17 \simeq 1$

$\boxed{f(5) = 1}$

Result:

x	0	1	2	3	4	5
f	147	147	74	25	6	1
Expected frequency	148	147	73	24	6	1

4.
a) Derive the Mean and variance of the Binomial distribution.

sol: the binomial distribution is

$P(X=x) = n_{C_x} p^x q^{n-x}$; $x = 0, 1, 2 \ldots n$

Mean $= np$, variance $= npq$.

Mean of $x = \sum x P(x)$

$$= \sum_{x=0}^{n} x \cdot \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)!} p^x q^{n-x}$$

$$= n \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^x q^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} q^{n-x}$$

$$= np (p+q)^{n-1} \quad \because [p+q=1]$$

$= np$.

$\therefore \boxed{\text{Mean} = np}$

* variance $= E(x^2) - (E(x))^2$

$= E(x(x-1)+x) - (np)^2$

$= E(x(x-1)) + E(x) - n^2p^2$

$= \sum_{x=0}^{n} x(x-1)\left(\frac{n!}{x!\,(n-x)!} p^x q^{n-x}\right) + np - n^2p^2$

$= \sum_{x=2}^{n} \frac{n!}{(x-2)!\,(n-x)!} p^x q^{n-x} + np - n^2p^2$

$= n(n-1) \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!\,(n-x)!} p^x q^{n-x} + np - n^2p^2$

$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!\,(n-x)!} p^{x-2} q^{n-x} + np - n^2p^2$

$= n(n-1)p^2 (p+q)^{n-2} + np - n^2p^2$

$= n(n-1)p^2 + np - n^2p^2$

$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$

$= np(1-p)$ $\quad \because \boxed{p+q=1}$

$= np(q) = npq$

$\boxed{\text{Variance} = npq}$

Hence derived

4)b) If the probability of defective fuse from a manufactures unit is 2% in box of 200 fuses, find the probability that

(i) exactly 4 fuses are defective

(ii) more than 3 fuses are defective.

Sol: Given: $\lambda = \frac{2}{100} \times 200 = 4$

$\boxed{\lambda = 4}$

(i) $P(x=4)$.

In poission distribution

$$P(x=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!}$$

$$\therefore P(x=4) = \frac{e^{-4} (4)^4}{4!}$$

$$= \frac{e^{-4} (16 \times 16)}{24} = \frac{e^{-4}(256)}{24}$$

$$= \cancel{\frac{e^{-4}}{10.66}} = e^{-4}(10.66)$$

$$= 0.195$$

$$\boxed{P(x=4) = 0.195}$$

(ii) $P(x>3) = 1 - P(x<3)$

$$= 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3)]$$

$$= 1 - \left[\frac{e^{-4}(4)^0}{0!} + \frac{e^{-4}(4)^1}{1!} + \frac{e^{-4}(4)^2}{2!} + \frac{e^{-4}(4)^3}{3!}\right]$$

$$= 1 - [e^{-4}(1 + 4 + 8 + 10.66)]$$

$$= 1 - e^{-4}(23.66) = 1 - e^{-4}(23.66)$$

$= 1 - 0.433$

$= 0.567$

$\therefore \boxed{P(x>3) = 0.567}$

8)a) The Mean of two random samples of sizes 7 and 6 are 31.28 and 28.16 respectively. The sum of the squares of the deviation from the means are 31.43 and 26.83. Can the sample be considered to have drawn from the same normal population at 5% level.

sol: Given, $n_1 = 7$, $n_2 = 6$.

$\bar{x} = 31.28$, $\bar{y} = 28.16$.

$\sum_{i=1}^{n} (x-\bar{x})^2 = 31.43$

$\sum_{i=1}^{n} (y-\bar{y})^2 = 26.83$, $\alpha = 0.05$.

$$S = \sqrt{\frac{\sum_{i=1}^{n}(x-\bar{x})^2 + \sum_{i=1}^{n}(y-\bar{y})^2}{n_1+n_2-2}}$$

$$S = \sqrt{\frac{31.43+26.83}{7+6-2}} = \sqrt{\frac{58.26}{11}}$$

$S = \sqrt{5.296} = 2.301.$

$\boxed{S = 2.301}$

$$t = \frac{\bar{x} - \bar{y}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{31.28 - 28.16}{2.3\sqrt{\frac{1}{7} + \frac{1}{6}}}$$

$$= \frac{3.12}{2.3\,(0.555)}$$

$$= \frac{3.12}{1.2765} = 2.414.$$

$\therefore \boxed{t_{cal} = 2.414}$

* Null Hypothesis (H_0) :- $\mu_1 = \mu_2$
* Alternate hypothesis (H_1) :- $\mu_1 \neq \mu_2$
* Level of significance

$\alpha = 0.05$

* Test the statistic :-

$t_{cal} = 2.414$

$t_{table} \Rightarrow \nu = n_1 + n_2 - 2 = 11.$

$t_{0.05} = 1.796$

* $t_{cal} > t_{table}$

H_0 i.e. Null hypothesis is rejected.
Alternate Hypothesis is accepted.

8) b) In a sample of 600 students of a certain college 400 are found to use ball pens. In another college from a sample of 900 students 450 are found to use ball pens. Test whether two colleges are significantly different with respect to the habit of using ball pens at 1% level.

Sol: Given, $n_1 = 600$, $P_1 = \frac{400}{600} = 0.66$

$n_2 = 900$, $P_2 = \frac{450}{900} = 0.5$

$\alpha = 0.01$.

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{600\left(\frac{400}{600}\right) + 900\left(\frac{450}{900}\right)}{600+900}$$

$$= \frac{850}{1500} = 0.566$$

$\boxed{P = 0.566}$

$q = 1 - P = 1 - 0.566 = 0.434$

$\boxed{q = 0.434}$

$$z = \frac{P_1 - P_2}{\sqrt{Pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.66 - 0.5}{\sqrt{(0.566)(0.434)\left(\frac{1}{600} + \frac{1}{900}\right)}}$$

$$z = \frac{0.16 \times 10}{\sqrt{0.0682}} = \frac{1.6}{\sqrt{0.0682}}$$

$$= \frac{1.6}{0.261} = 6.15$$

$$\boxed{z = 6.15}$$

* Null Hypothesis (H_0) :- $P_1 = P_2$
* Alternate Hypothesis (H_1) :- $P_1 \neq P_2$
* Level of significance (α) :- $\alpha = 0.01$
* Test for statistic :-

$z_{cal} = 6.15$

* $z_{table} = 2.33$

$z_{cal} > z_{table}$

Null Hypothesis (H_0) is rejected.