ENGR 102
Engineering Lab I: Computation

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Linear Interpolation

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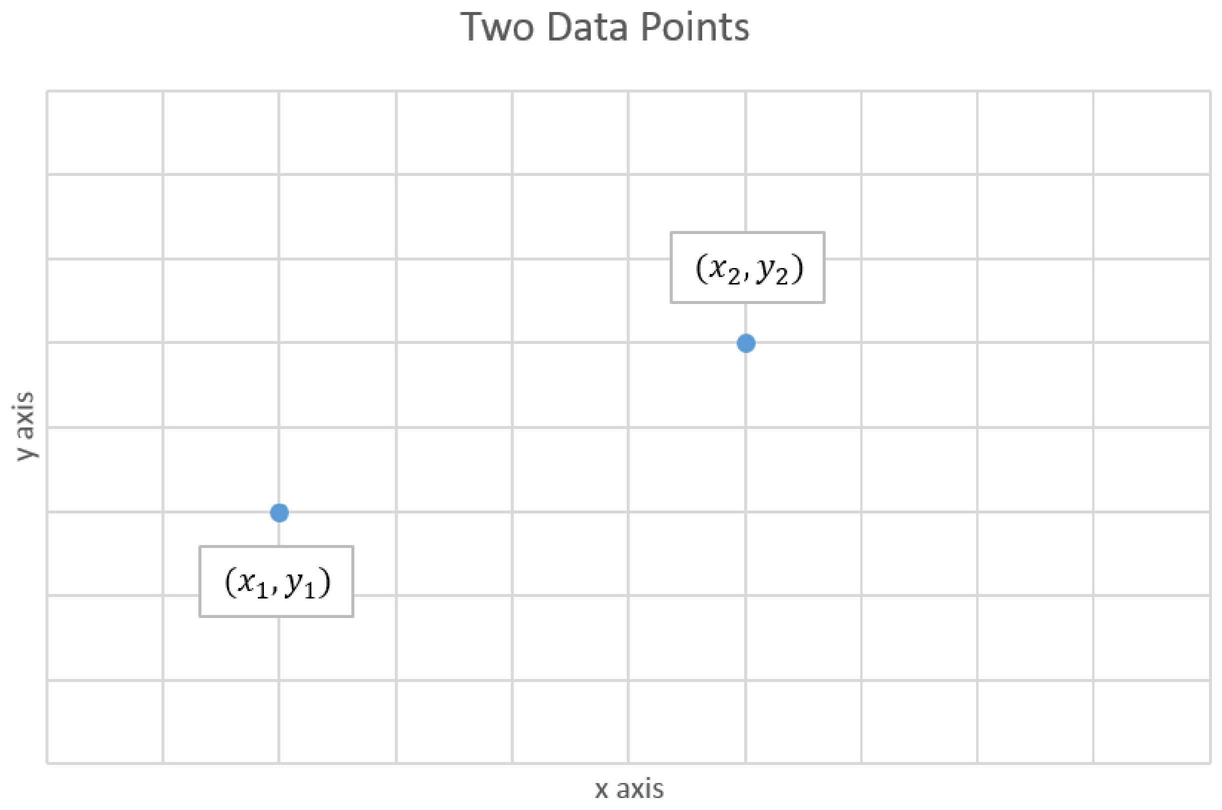
Linear Interpolation

- We are going to use something called “Linear Interpolation” in several of our Lab Assignments this semester.
  - It may also make an appearance on a Quiz or Exam.
- As with any concept, we want to fully understand what is meant by the term “Linear Interpolation” before we attempt to write code to implement it.
  - We also need to understand what IS NOT meant by the term “Linear Interpolation” so that we don’t exert any more effort than required

Here's the scenario . . .

- We have two data points (x_1, y_1) and (x_2, y_2)
- Let's assume that they are not the same point
- When we plot on (x, y) axes we see the two points as shown below



Here's the scenario . . .

- Linear Interpolation is a process that will allow us to predict or estimate values of y between y_1 and y_2 for a given value of x between x_1 and x_2
  - Notice, we use the word "predict" or "estimate" not simply "calculate"
    - More explanation on why this is important is coming up
  - Also notice the restriction that $x_1 \le x \le x_2$ or $x_1 \ge x \ge x_2$
    - This is required for Linear Interpolation
    - If our value of x is not between x_1 and x_2, that would require Linear "Extrapolation"
      - Danger! Danger!
      - More on that later

Here's the scenario . . .

- Linear Interpolation also requires a judgment call on the part of the user
- If someone uses Linear Interpolation, they have either explicitly or implicitly decided that y varies linearly with x in the x domain
  - . . . or at least close enough for the situation at hand
- That means that for this process, we can draw a straight line segment in an x-y coordinate system to represent changing values of y as x changes

Here's the scenario . . .

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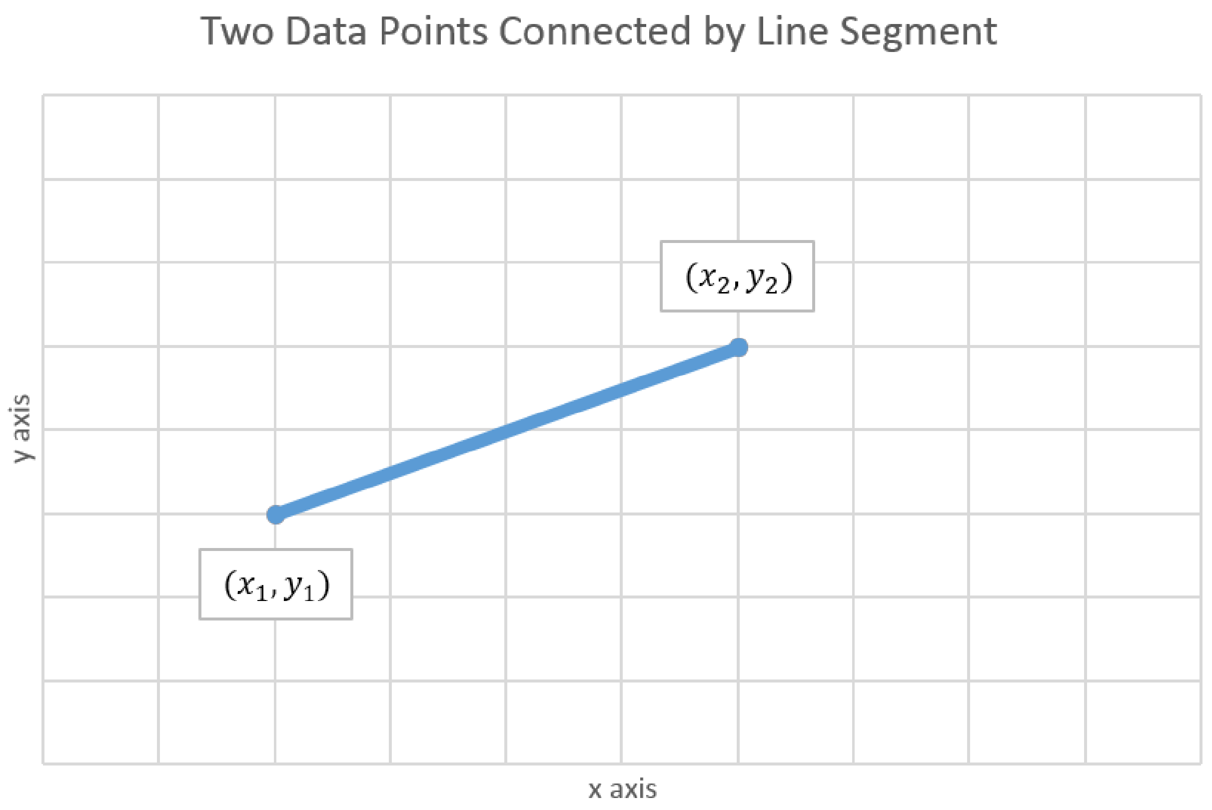

Here’s the scenario . . .

- Now it should be clear what we are trying to do
- Given an x value between x_1 and x_2. . .
  - We can find x along the horizontal axis
  - Project vertically until we intersect the line segment
  - Project the intersection horizontally to the vertical axis
  - Read the y value associated with x
  - Simple as that!
  - (Illustration on next page)

Here's the scenario . . .

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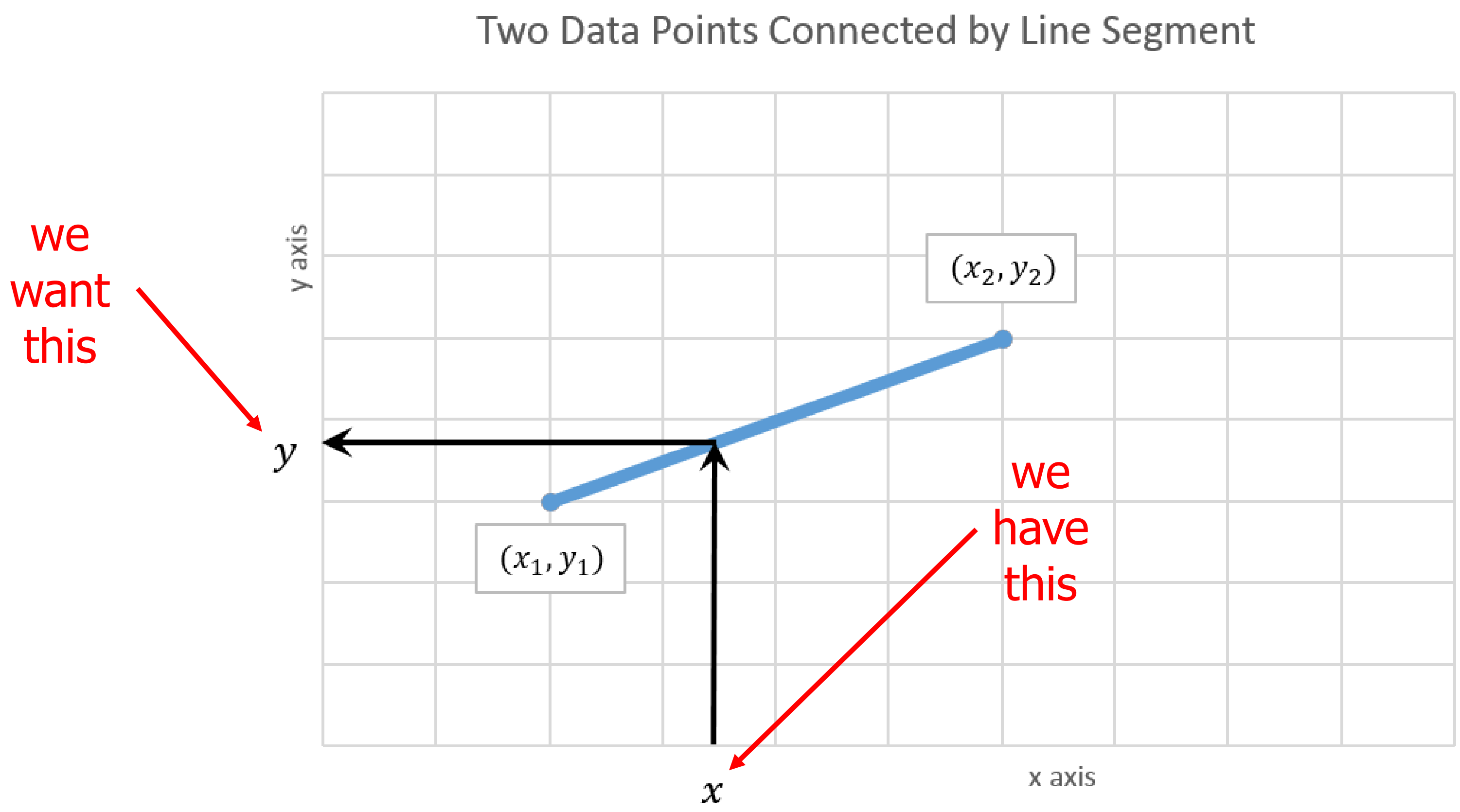


How do we do this analytically?

- This concept is very simple
  - If we had a sketch on a piece of paper, we could easily do this
  - But, how can we solve for the y value analytically?

- One thought that comes to mind is to write the equation of the line through the two points in the form . . .

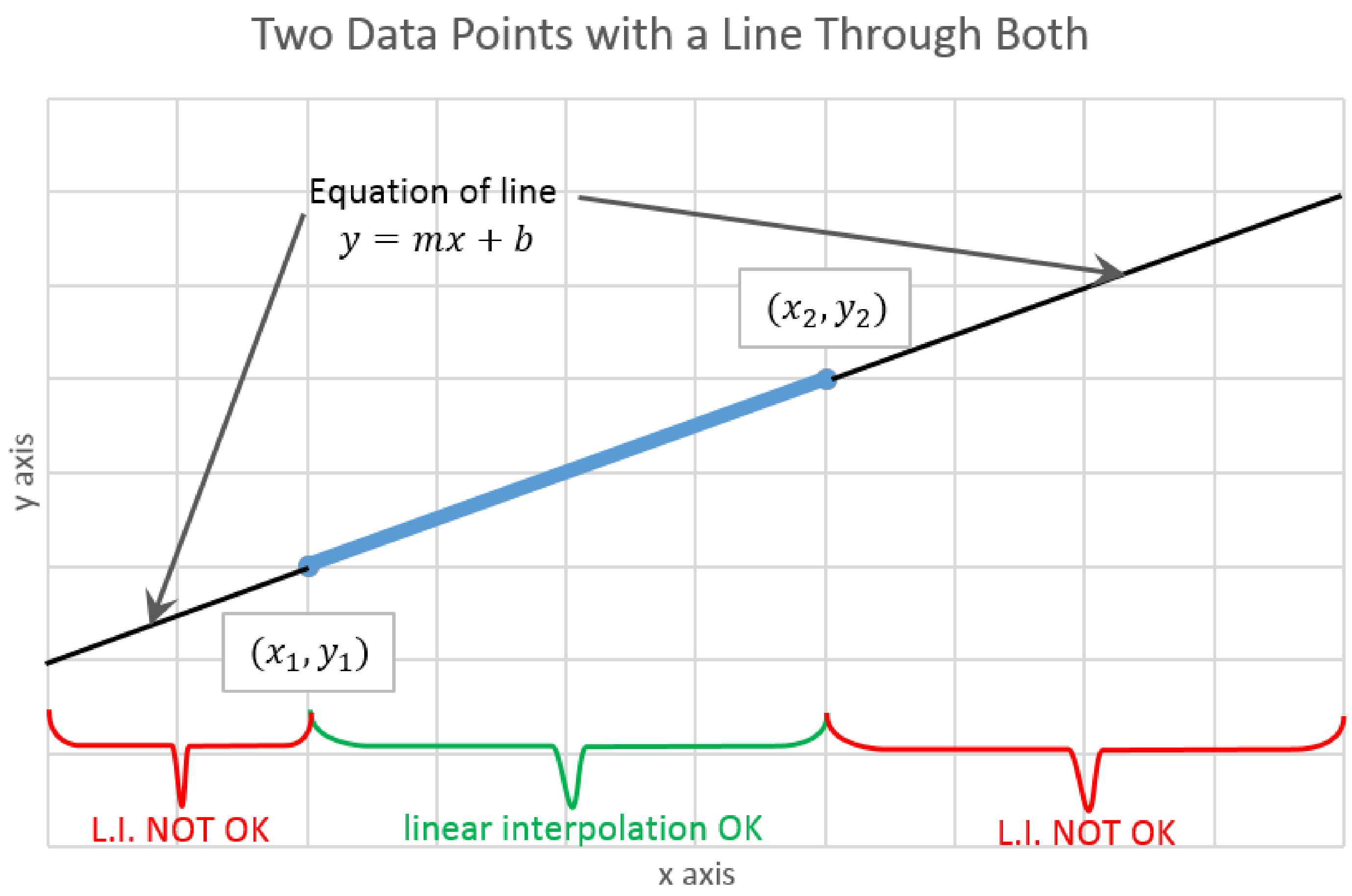
$$y = mx + b$$

  - We could calculate the slope m and the y-intercept b based on the two given points
  - Then, we could simply enter our value of x into the equation and the value of y would be the result

How do we do this analytically?

- This is true and it will work, but this is not Linear Interpolation
  - First, you don’t need the equation of the line to get what you’re after, so you are working a bit too hard
  - Second, having the equation of the line gives you the impression that the relationship is valid for all x
    - It is (almost certainly) not
      - Exception: you are certain that y varies linearly with x for all x
  - (Illustration on next page)

How do we do this analytically?



How do we do this analytically?

- Let's be very clear about something
  - Using Linear Interpolation between two data points is usually just an assumption of convenience
    - It often happens to be a very useful assumption, but it is only that
  - The underlying physics of a particular problem may tell you that there is some relationship between x and y other than a linear one
    - y could vary with x^2 or e^x or $\sin(x)$ or whatever
  - It turns out that for just about any relationship between y and x, if the interval of x is small enough, assuming linear variation produces reasonably small errors
  - What is a reasonably small error? – that's another judgment call

How do we do this analytically?

- All this is to say that we should be careful of using interpolation outside the domain of x's given in the problem
  - There are also possible problems using x between x_1 and x_2 for some cases
  - Examples coming up
- If we choose to use Linear Interpolation, then we are only concerned with the line segment between Point 1 and Point 2
- If we write the equation of the line that includes that line segment, we can certainly get the answer we seek, but we may also be tempted to use that equation for points not lying on the line segment
- So, let's develop a way to get what we want and only use the line segment between Points 1 and 2

How do we do this analytically?

- Similar to writing the equation of the line, let's use the slope of the line segment to develop the equation we need
- If we were writing the equation of the line, we know from experience that we can use any two points (that are not co-located) to determine the slope
- First, using the known point coordinates, the slope is given by the following equation:

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

- For a given x between x_1 and x_2, we know that y is between y_1 and y_2
- Let's write another expression for the slope of the line segment in terms of <u>known</u> x and <u>unknown</u> y:

$$slope = \frac{y - y_1}{x - x_1}$$

How do we do this analytically?

- We can set these two expressions for the slope equal to each other

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

- Note that there is only one unknown in this equation: y
- Solving for y, we have an expression for unknown y in terms of the (x, y) coordinates of Point 1 and Point 2 and in terms of the given value of x

$$y = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1) + y_1$$

Or

$$y = (slope)(x - x_1) + y_1$$

- Note that there is only one unknown in this equation: y . . .
- . . . and this is what we set out to find

How do we implement in Python?

- What variables do we need?
  - Let's use x_1 and y_1 for Point 1 location
  - Let's use x_2 and y_2 for Point 2 location
  - Let's use x_0 for the given x value of the point of interest
  - Let's use y_0 for the unknown y value that we seek
  - We may decide to use other variables, but this will get us going
- What steps do we take?
  - Assign values to x_1 and y_1
  - Assign values to x_2 and y_2
  - Assign x_0 its value
  - Calculate y_0 based on the expression developed above

How do we implement in Python?

- Is that all we need?
  - Strictly speaking – yes.
- However, let's make sure that we are performing Linear Interpolation not Linear Extrapolation
- To do that, check to make sure that x_0 is between x_1 and x_2 prior to performing the other calculations

How do we implement in Python?

- Now the program sequence looks like this:
  - Assign values to x_1 and y_1
  - Assign values to x_2 and y_2
  - Assign x_0 its value
  - If x_0 is between x_1 and x_2 (inclusive), do the following:
    - Calculate y_0 based on the expression we developed
    - Output or store the value of y_0
    - End program
  - If x_0 IS NOT between x_1 and x_2 (inclusive), do the following:
    - Print error message to screen
    - End program

Assumption of Linearity

- As we stated earlier, when someone uses Linear Interpolation between two data points, the assumption is that either . . .
  - The relationship between x and y is truly linear in which case we are calculating an exact y_0 based on a given x_0
  - Or . . .
  - For the x domain chosen, assuming a linear relationship between x and y produces results with an acceptable amount of error

- Let's see some examples

Lab 2a Activity 3

- Race track
  - What shape is it?
  - Does the shape matter?
- We are told to use Linear Interpolation
  - The decision to use LI is made for us in this case
- How to get started . . .
- Let's answer the following question:
  - What two quantities have a linear relationship?

Lab 2a Activity 3

- We are given the following:
  - At time=30 sec, vehicle is 50 m beyond the start
  - At time=45 sec, vehicle is at 615 m beyond the start
  - The speed of the vehicle is assumed constant
  - In the time considered, the vehicle has not passed the start
- We are asked to find how far beyond the start is the vehicle at time=37 sec?
- Does the shape of the track matter?
  - Not for this problem.
- Time looks like a good choice for the independent variable
  - So, time (t) will replace x in our previous analysis

Lab 2a Activity 3

- What varies linearly with time?
- Distance traveled from the starting line varies linearly with time

- Plot of time on horizontal axis and distance on vertical axis with two known points
- Show a time between t1 and t2 and two projections to find d0

- Same as we had before with x and y