$\boxed{1}$

- 余事象 (Complementary Event).
  A^c で表す.

- 条件付き独立.
  $P(A \cap B | C) = P(A \cap C) \cdot P(B \cap C)$.

$\boxed{例題}$

$\boxed{1.1}$ [1] $P(女) = 0.4$.
$P(合|女) = 0.5$
$P(合|男) = 0.4$.

$P(合) = P(女)P(合|女) + P(男)P(合|男)$.
$= 0.4 \cdot 0.5 + 0.6 \cdot 0.4$.
$= 0.44$. 答

[2]

$$P(女|合) = \frac{P(合|女) \cdot P(女)}{P(合)} = \frac{0.5 \cdot 0.4}{0.44}$$

$\boxed{1.2}$

[1] 面のパターン.

	1, 2	3
①	1つ	4つ
②	2つ	2つ

◀ $P[X=1] = P[X=2]$ より

①

$\frac{1}{6}(1 + 2 + 4 \cdot 3) = \frac{15}{6} > 2$.

②

$\frac{1}{6}(1 \cdot 2 + 2 \cdot 2 + 3 \cdot 2) = 2$. < 2 より不適.

$E[X] = \frac{5}{2}$, $V[X] = E[X^2] - E[X]^2 = \frac{7}{12}$ 答

[2] $P(Y=3) =$ ← $1 - P(X \neq 3)$ の方がカンタン.

$$P(X=3)^2 + {}_2C_1\, P(X=3)\cdot P(X \neq 3)$$

$$= \left(\frac{4}{6}\right)^2 + 2\cdot\frac{4}{6}\cdot\frac{2}{6} = \frac{16}{36} + \frac{16}{36} = \frac{8}{9}$$

$\boxed{1.8}$ 病 : A

$P(A) = 0.01$.

検1 : ①

$P(①|A) = 0.99$.

$P(①|A^c) = 0.02$

検2 : ②

$P(②|A\cap①) = 0.9$.

$P(②|A^c\cap①) = 0.1$.

$$[1]\ P(A|①) = \frac{P(①|A)\cdot P(A)}{P(A)\cdot P(①|A) + P(A^c)\cdot P(①|A^c)} = \frac{0.99^{-2}}{0.99^{-2} + 0.0198}$$

$$= \frac{99}{297} = \frac{1}{3}$$

$$[2].\ P(A\cap①|②) = \frac{P(A\cap①)\cdot P(②|A\cap①)}{\underset{0.99^{-2}}{P(A\cap①)}\cdot \underset{0.9}{P(②|A\cap①)} + \underset{0.0198}{P(A^c\cap①)}\cdot \underset{0.1}{P(②|A^c\cap①)}}$$

$$= \frac{99\cdot 9}{99\cdot 9 + 198} \simeq 0.818.$$

X : 病 Y_1 : ① Y_2 : ②.

[2]は $P(X|Y_1, Y_2)$ を求める.

$$\boxed{P(X|Y_1, Y_2) = \frac{P(Y_2|X, Y_1)\cdot P(X|Y_1)}{P(Y_2|Y_1)}}$$

$$, \quad P(X \mid Y_1, Y_2) \cdot P(Y_2 \mid Y_1) \cdot P(Y_1) \qquad = P(X, Y_1, Y_2).$$

$$= P(Y_2 \mid X, Y_1) \cdot P(X \mid Y_1) \cdot P(Y_1)$$

Y_2の尤度.

Y_1を観測した後の事後分布

$$\therefore \quad P(X \mid Y_1, Y_2) = \frac{P(Y_2 \mid X, Y_1) \cdot P(X \mid Y_1)}{P(Y_2 \mid Y_1)}$$

変数が増えても同様.

$$P(Y \mid X_1, X_2, X_3) = \frac{P(X_3 \mid Y, X_1, X_2) \cdot P(Y \mid X_1, X_2)}{P(X_3 \mid X_1, X_2)}$$