1

- 余事象 (Complementary Event).

  A^c で表す.

- 条件付き独立.

  $$P(A \cap B \mid C) = P(A \cap C) \cdot P(B \cap C).$$

例題

1.1 [1] $P(女) = 0.4.$

$P(合 \mid 女) = 0.5$

$P(合 \mid 男) = 0.4.$

$$P(合) = P(女)P(合 \mid 女) + P(男)P(合 \mid 男).$$

$$= 0.4 \cdot 0.5 + 0.6 \cdot 0.4.$$

$$= 0.44. \quad 答$$

[2]

$$P(女 \mid 合) = \frac{P(合 \mid 女) \cdot P(女)}{P(合)} = \frac{0.5 \cdot 0.4}{0.44}$$

1.2

[1] 面のパターン.

	1, 2	3
①	1つ	4つ
②	2つ	2つ

$P[X=1] = P[X=2]$ より

①

$$\frac{1}{6}(\underbrace{1+2+4 \cdot 3}_{15}) = \frac{15}{6} > 2.$$

②

$$\frac{1}{6}(1 \cdot 2 + 2 \cdot 2 + 3 \cdot 2) = 2. < 2 \text{ より不適}.$$

$$E[X] = \frac{5}{2}, \quad V[X] = E[X^2] - E[X]^2 = \frac{7}{12} \quad 答$$

[2] $P(Y=3)=$

$\leftarrow 1-P(X\neq 3)$ の方がカンタン.

$$P(X=3)^2 + {}_2C_1\, P(X=3)\cdot P(X\neq 3).$$

$$=\left(\frac{4}{6}\right)^2 + 2\cdot\frac{4}{6}\cdot\frac{2}{6} = \frac{16}{36}+\frac{16}{36} = \frac{8}{9}$$

$\boxed{1.3}$ 病 : A

$P(A)=0.01.$

検1 : ①

$P(①|A)=0.99.$

$P(①|A^c)=0.02$

検2 : ②

$P(②|A\cap①)=0.9.$

$P(②|A^c\cap①)=0.1.$

[1] $$P(A|①)=\frac{P(①|A)\cdot P(A)}{P(A)\cdot P(①|A)+P(A^c)\cdot P(①|A^c)} = \frac{0.99^{-2}}{0.99^{-2}+0.0198}$$

$$=\frac{99}{297}=\frac{1}{3}$$

[2]. $$P(A\cap①|②)=\frac{P(A\cap①)\cdot P(②|A\cap①)}{\underset{0.99^{-2}}{P(A\cap①)}\cdot\underset{0.9}{P(②|A\cap①)}+\underset{0.0198}{P(A^c\cap①)}\cdot\underset{0.1.}{P(②|A^c\cap①)}}$$

$$=\frac{99\cdot 9}{99\cdot 9+198}.\simeq 0.818.$$

X : 病　Y_1 : ①　Y_2 : ②.

[2]は $P(X|Y_1,Y_2)$ を求める.

$$\boxed{P(X|Y_1,Y_2)=\frac{P(Y_2|X,Y_1)\cdot P(X|Y_1)}{P(Y_2|Y_1)}}$$

$$, \quad P(X \mid Y_1, Y_2) \cdot P(Y_2 \mid Y_1) \cdot P(Y_1) \qquad = P(X, Y_1, Y_2)$$

$$= P(Y_2 \mid X, Y_1) \cdot P(X \mid Y_1) \cdot P(Y_1)$$

$$\therefore \quad P(X \mid Y_1, Y_2) = \frac{\underbrace{P(Y_2 \mid X, Y_1)}_{Y_2\text{の尤度}} \cdot \underbrace{P(X \mid Y_1)}_{Y_1\text{を観測した後の事後分布}}}{P(Y_2 \mid Y_1)}$$

変数が増えても同様.

$$P(Y \mid X_1, X_2, X_3) = \frac{P(X_3 \mid Y, X_1, X_2) \cdot P(Y \mid X_1, X_2)}{P(X_3 \mid X_1, X_2)}$$