

1.

○ 余事象 (Complementary Event).

$A^c$  で表す.

○ 条件付き独立.

$$P(A \cap B | C) = P(A \cap C) \cdot P(B \cap C).$$

例題

[1.1] [1]  $P(\text{女}) : 0.4.$

$$P(\text{合} | \text{女}) : 0.5$$

$$P(\text{合} | \text{男}) : 0.4.$$

$$P(\text{合}) = P(\text{女})P(\text{合} | \text{女}) + P(\text{男})P(\text{合} | \text{男}).$$

$$= 0.4 \cdot 0.5 + 0.6 \cdot 0.4.$$

$$= 0.44. \quad \square$$

[2] 
$$P(\text{女} | \text{合}) = \frac{P(\text{合} | \text{女}) \cdot P(\text{女})}{P(\text{合})} = \frac{0.5 \cdot 0.4}{0.44}$$

1.2

[1] 面の117-2.

	1, 2	3
①	1, 3	4, 3
②	2, 3	2, 3

$$\triangleleft P[X=1] = P[X=2] \text{ 否}$$

① 
$$\frac{1}{6}(1+2+4+3) = \frac{10}{6} > 2.$$

② 
$$\frac{1}{6}(1+2+2+3+2) = 2. < 2 \text{ 否} \text{ 不適.}$$

$$E[X] = \frac{5}{2}, \quad V[X] = E[X^2] - E[X]^2 = \frac{7}{12} \quad \square$$

$$[2] P(Y=3):$$

$$4-1-P(X \neq 3) \text{ 求める}$$

$$P(X=3)^2 + 2C_1 P(X=3) \cdot P(X \neq 3).$$

$$= \left(\frac{4}{6}\right)^2 + 2 \cdot \frac{4}{6} \cdot \frac{2}{6} = \frac{16}{36} + \frac{16}{36} = \frac{32}{36} = \frac{8}{9}$$

$$[1.8] \text{ 病} = A$$

$$P(A) = 0.01.$$

検査1: ①

$$P(\textcircled{1}|A) = 0.99.$$

$$P(\textcircled{1}|A^c) = 0.02$$

検査2: ②

$$P(\textcircled{2}|A \cap \textcircled{1}) = 0.9.$$

$$P(\textcircled{2}|A^c \cap \textcircled{1}) = 0.1.$$

$$[1] P(A|\textcircled{1}) = \frac{P(\textcircled{1}|A) \cdot P(A)}{P(A) \cdot P(\textcircled{1}|A) + P(A^c) \cdot P(\textcircled{1}|A^c)} = \frac{0.99 \cdot 0.01}{0.99 \cdot 0.01 + 0.0198} = \frac{99}{297} = \frac{1}{3}$$

$$[2] P(A \cap \textcircled{1} \cap \textcircled{2}) = \frac{P(A \cap \textcircled{1}) \cdot P(\textcircled{2}|A \cap \textcircled{1})}{P(A \cap \textcircled{1}) \cdot P(\textcircled{2}|A \cap \textcircled{1}) + P(A^c \cap \textcircled{1}) \cdot P(\textcircled{2}|A^c \cap \textcircled{1})} = \frac{0.99 \cdot 0.01 \cdot 0.9}{0.99 \cdot 0.01 \cdot 0.9 + 0.0198 \cdot 0.1} = \frac{99 \cdot 9}{99 \cdot 9 + 198} \approx 0.818.$$

$X$ : 病  $Y_1$ : ①  $Y_2$ : ②

[2]は  $P(X|Y_1, Y_2)$  を求める。

$$P(X|Y_1, Y_2) = \frac{P(Y_2|X, Y_1) \cdot P(X|Y_1)}{P(Y_2|Y_1)}$$

$$, \quad P(X | Y_1, Y_2) \cdot P(Y_2 | Y_1) \cdot P(Y_1) \quad \Delta = P(X, Y_1, Y_2)$$

$$= P(Y_2 | X, Y_1) \cdot P(X | Y_1) \cdot P(Y_1)$$

$Y_2$  先變。

$$\therefore P(X | Y_1, Y_2) = \frac{P(Y_2 | X, Y_1) \cdot P(X | Y_1)}{P(Y_2 | Y_1)}$$

$Y_1$  是觀測出來後再分佈

變數增加也同樣。

$$P(Y | X_1, X_2, X_3) = \frac{P(X_3 | Y, X_1, X_2) \cdot P(Y | X_1, X_2)}{P(X_3 | X_1, X_2)}$$