

1.

o 余事象 (Complementary Event).

 A^c 表す.

9 条件付独立.

$$P(A \cap B | C) = P(A|C) \cdot P(B|C).$$

[例題]

$$\begin{aligned} [1.1] (1) P(\text{女}) &= 0.4 & P(\text{合}) &= P(\text{女})P(\text{合}|\text{女}) + P(\text{男})P(\text{合}|\text{男}) \\ &&&= 0.4 \cdot 0.5 + 0.6 \cdot 0.4 \\ P(\text{合女}) &= 0.5 && \\ P(\text{合男}) &= 0.4. && \end{aligned}$$

$$(2) P(\text{女合}) = \frac{P(\text{合女}) \cdot P(\text{女})}{P(\text{合})}, \quad \frac{0.5 \cdot 0.4}{0.44}$$

1.2

(1)

面の割合 - - -

	1, 2	3
①	1, 2	4, 3
②	2, 1	2, 2

$$\Leftrightarrow P[X=1] = P[X=2] \text{ で}$$

$$\textcircled{1} \quad \frac{1}{6}(1+2+4 \cdot 3) = \frac{15}{6} > 1, 2.$$

$$\textcircled{2} \quad \frac{1}{6}(1 \cdot 2 + 2 \cdot 2 + 3 \cdot 2) = 2. < 2 \text{ も不適}.$$

$$E[X] = \frac{5}{2}, \quad V[X] = E[X^2] - E[X]^2 = \frac{7}{12}$$

$$[2] P(Y=3) =$$

$$1 - P(X \neq 3) = 1 - 0.56 = 0.44$$

$$P(X=3)^2 + 2 \sum_{i=1}^2 P(X=i) \cdot P(X \neq 3) \\ = \left(\frac{4}{6}\right)^2 + 2 \cdot \frac{4}{6} \cdot \frac{2}{6} = \frac{16}{36} + \frac{16}{36} = \frac{16}{9}$$

[1.8] 病:A

$$P(A) = 0.09.$$

検査①: ①

$$P(\text{①}|A) = 0.99.$$

$$P(\text{①}|A^c) = 0.02$$

検査②: ②

$$P(\text{②}|A \cap \text{①}) = 0.9.$$

$$P(\text{②}|A^c \cap \text{①}) = 0.1.$$

$$[1] P(A|\text{①}) = \frac{P(\text{①}|A) \cdot P(A)}{P(A) \cdot P(\text{①}|A) + P(A^c) \cdot P(\text{①}|A^c)} = \frac{0.99^2}{0.99^2 + 0.01 \cdot 0.02} \\ = \frac{99}{299} = \frac{1}{3}$$

$$[2]. P(A \cap \text{①} \cap \text{②}) = \frac{P(A|\text{①}) \cdot P(\text{②}|A \cap \text{①})}{P(A \cap \text{①}) \cdot P(\text{②}|A \cap \text{①}) + P(A^c \cap \text{①}) \cdot P(\text{②}|A^c \cap \text{①})} \\ = \frac{\frac{99}{299} \cdot 0.9}{\frac{99}{299} \cdot 0.9 + 0.01 \cdot 0.1} \approx 0.818.$$

X: 病 Y₁: ① Y₂: ②

[2] は $P(X|Y_1, Y_2)$ を求めろ。

$$P(X|Y_1, Y_2) = \frac{P(Y_2|X, Y_1) \cdot P(X|Y_1)}{P(Y_2|Y_1)}$$

$$P(X|Y_1, Y_2) \cdot P(Y_2|Y_1) \cdot P(Y_1)$$

$$1 = P(X, Y_1, Y_2)$$

$$= P(Y_2|X, Y_1) \cdot P(X|Y_1) \cdot P(Y_1)$$

Y_2 尤度.

$$\therefore P(X|Y_1, Y_2) = \frac{P(Y_2|X, Y_1) \cdot P(X|Y_1)}{P(Y_2|Y_1)}$$

Y_1 観測レート後事後確

変数が増えても同様。

$$P(Y|X_1, X_2, X_3) = \frac{P(X_3|Y, X_1, X_2) \cdot P(Y|X_1, X_2)}{P(X_3|X_1, X_2)}$$