(9.4) Hierarchical Decision Analysis for Radon Measurement

Weihang Ren

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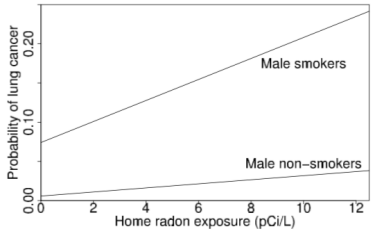
- Distribution of home radon concentrations is approximately log-normal
- measuring the radon concentration and using this information to help decide whether to take steps to reduce the risk from radon.
- If the radon level in a home is sufficiently high, then an individual may take action to control the risk due to radon. remediation techniques

Background


Probability of lung cancer
0.00
0.10
0.20
0
2
4
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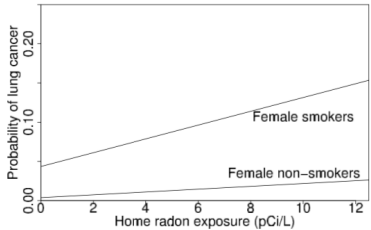
8
10
12
Home radon exposure (pCi/L)
Male smokers
Male non−smokers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The individual decision problem.

- **Remediate without monitoring**: Spend $2000 to remediate the home.
- **Do nothing and accept the current radon exposure**.
- **Take some measurements**: Remediate based on the result.
  1. Short Term: Cost of $15; Biased/ High standard deviation.
  2. Long Term: Cost of $50; More accurate/ Lower standard deviation.

The measurement/remediation decision must generally be made under uncertainty, because most houses have not been measured for radon. Even after measurement, the radon level is not known exactly.

- deciding whether to remediate if the radon exposure were known
- deciding whether it is worth it to measure radon exposure given the current state of knowledge about home radon

Decision-making under certainty

- D_d dollar value associated with a reduction of 10^{-6} in probability of death from lung cancer
- D_r dollar value associated with a reduction of 1 pCi/L in home radon level for a 30-year period
- R_{a}, the home radon level above which you should remediate if your radon level is known.
- R_{r}, the radon level that remediation takes down to.

$$\text{benefit of remediation} = D_r(R_{\mathrm{a}} - R_{\mathrm{r}}) = \$2000$$

$$R_{\mathrm{a}} = \frac{\$2000}{D_r} + R_{\mathrm{r}}$$

Bayesian inference for county radon levels

- House Hold Level

$$y_i \sim N(X_i\beta + \alpha_{j(i)}, \sigma_i^2) \text{ for } i = 1, \ldots, n$$

  - y_i: $\log(\text{randon measurement})$
  - X_i: predictor for each house.

- County Level

$$\alpha_j \sim N(W_j\gamma + \delta_{k(j)}, \tau^2) \text{ for } j = 1, \ldots, J$$

  - $\alpha_{j(i)}$: county effect
  - W_j: predictor for each county.

- Hyperparameter

$$\delta_k \sim N(0, \kappa^2) \text{ for } k = 1, \ldots, K$$

Bayesian inference for the radon level in an individual house

- R_i = radon concentration in house i.
- $\theta_i = \log(R_i)$.

$$\theta_i \sim N(M_i, S_i^2),$$

- $M_i = X_i\hat{\beta} + \hat{\alpha}_{j(i)}$.
- the state of knowledge about the radon level in a house given only its county and basement information.
- it serves as a prior distribution for the homeowner.

$$\theta_i \mid M, y \sim N(\Lambda, V),$$

where

$$\Lambda = \frac{\frac{M}{S^2} + \frac{y}{\sigma^2}}{\frac{1}{S^2} + \frac{1}{\sigma^2}}, \quad V = \frac{1}{\frac{1}{S^2} + \frac{1}{\sigma^2}}$$

Decision analysis for individual homeowners

If $z \sim N(\mu, \sigma^2)$

- $\mathrm{E}(e^z) = e^{\mu+\frac{1}{2}\sigma^2}$
- $\mathrm{E}(e^z|z < a)\mathrm{P}(z < a) = e^{\mu+\frac{1}{2}\sigma^2}(1 - \Phi(\frac{\mu+s^2-a}{s}))$

1.Remediate without monitoring. Expected loss is remediation cost + equivalent dollar cost of radon exposure after remediation:

$$
\begin{aligned}
L_1 =& 2000 + D_r\mathrm{E}(\min\{R, R_\mathrm{r}\}) \\
=& 2000 + D_r[R_\mathrm{r}\mathrm{Pr}(R \geq R_\mathrm{r}) + \mathrm{E}(R \mid R < R_\mathrm{r})\mathrm{Pr}(R < R_\mathrm{r})] \\
=& 2000 + D_r[R_\mathrm{r}\Phi(\frac{M - \log(R_\mathrm{r})}{S}) + e^{M+\frac{1}{2}S^2}(1 - \Phi(\frac{M + S^2 - \log(R_\mathrm{r})}{S}))]
\end{aligned}
$$

Decision analysis for individual homeowners

2.Do not monitor or remediate. Expected loss is the equivalent dollar cost of radon exposure:

$$L_2 = D_r \mathrm{E}(R) = D_r e^{M+\frac{1}{2}S^2}$$

Decision analysis for individual homeowners

3. Take a measurement y. The immediate loss is the measurement cost (assumed to be \$50) and, in addition, the radon exposure during the year that you are taking the measurement (which is 1 of the 30-year exposure). The inner decision has two branches:
   - Remediate.

$$\begin{aligned} L_{3a} =& 50 + \frac{1}{30} D_r e^{M+\frac{1}{2}S^2} + 2000 \\ &+ D_r[R_{\mathrm{r}}\Phi(\frac{\Lambda - \log(R_{\mathrm{r}})}{\sqrt{V}}) + e^{\Lambda+\frac{1}{2}V}(1 - \Phi(\frac{\Lambda + V - \log(R_{\mathrm{r}})}{\sqrt{V}}))] \end{aligned}$$

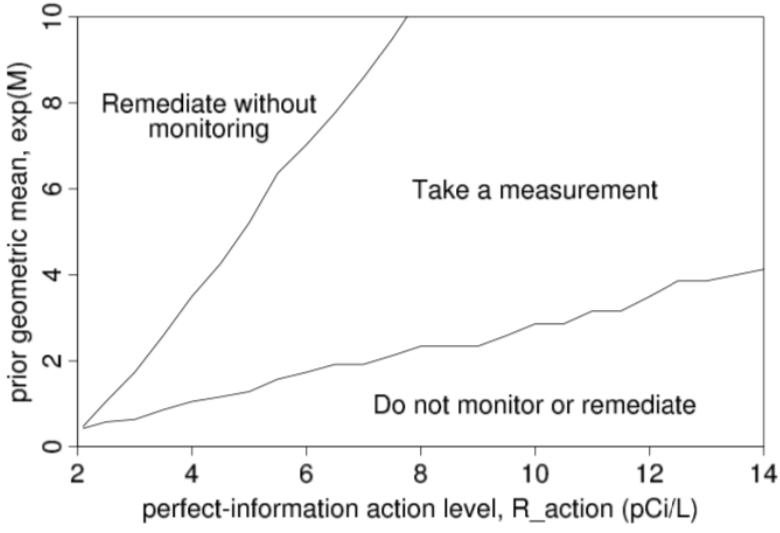
   - Do not remediate

$$L_{3b} = 50 + \frac{1}{30} D_r e^{M+\frac{1}{2}S^2} + D_r e^{\Lambda+\frac{1}{2}V}$$

   -

$$L_3 = \mathrm{E}(\min\{L_{3a}, L_{3b}\})$$

Deciding among the three branches.



Applying the recommended decision strategy to the entire country.