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Disscussion 1C

PROBLEM 1

Generalized product rule:

Pr(A,B | K) = Pr(A | B,K) Pr(B | K)

Pr(A ^ B | K) = P(A | B ^ K) P(B | K)   Convert commas to ANDs (caret)

Pr(A^ B | K) = [P(A ^ B ^ K) / P(B ^ K)] * [Pr(B ^ K) / Pr(K)]   Use Bayes' Conditioning on right side

Pr(A ^ B ^ K) / Pr(K) = Pr(A ^ B ^ K) / Pr(K)   Cross cancel on right and Bayes' Condition on left

Generalized Bayes' Rule:

Pr(A | B,K) = Pr(B | A,K) Pr(A | K) / Pr(B | K)

Pr(A | B ^ K) = Pr(B | A ^ K) Pr(A | K) / Pr(B | K)   Convert commas to ANDs (caret)

Pr(A ^ B ^ K)/Pr(B ^ K) = [Pr(B ^ A ^ K)/Pr(A ^ K)]*[Pr(A ^ K)/Pr(K)]/[Pr(B ^ K)/Pr(K)]   Bayes' Conditioning

Pr(A ^ B ^ K) / Pr(B ^ K) = Pr(B ^ A ^ K) / Pr(B ^ K)   Cross cancel and multiply right sie

Pr(A ^ B ^ K) / Pr(B ^ K) = Pr(A ^ B ^ K) / Pr(B ^ K)   Reorder right hand side

PROBLEM 2

P(Oil | Positive) = P(Oil ^ Positive) / P(Positive)

P(Positive) = .5(.9) + .2(.3) + .3(.1) = .45 + .06 + .03 = .54

P(Oil ^ Positive) = .5(.9) = .45

P(Oil | Positive) = P(Oil ^ Positive) / P(Positive)

P(Oil | Positive) = .45/.54 = 0.83333

There is approximately a 83.333% chance that oil is present given the test comes back positive.

PROBLEM 3

α1: the object is black;
α2: the object is square;
α3: if the object is one or black, then it is also square.

α1 = P(Black) / Total = 9 / 13

α2 = P(Square) / Total = 8 / 13

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α3 = P(Square | 1 or Black) = P(Square ^ (1 or Black)) / P(1 or Black) = 7 / 11

PROBLEM 4

a. List the Markovian assumptions asserted by the DAG

I(x, z, y) -> x is independent of y given z

I(A, 0, BE)

I(B, 0, AC)

I(C, A, BDE)

I(D, AB, CE)

I(E, B, ACDFG)

I(F, CD, ABE)

I(G, F, ABCDEH)

I(H, EF, ABCDG)

b.

(x,z,y) -> x is d separate for y given z

d_separated(A, BH, E) = False, ACFHE is not d-separated since FHE is a convergent type and H is in Z, while no descendants of H are in Z.

d_separated(G, D, E) = True -> EBDFG is false since FDB is sequential and D is in Z, EHFG is false since FHE is convergent and H is not in Z.

d_separated(AB, F, GH) = False -> BEH is a path that is not d-separated. E is not part of z and it is a sequential type.

c. Express Pr(a,b,c,d,e,f,g,h) in factored form using the chain rule for Bayesian networks

P(x1…xn) = PRODUCT from 1 to n (Xi | Parents(Xi))

Pr(a,b,c,d,e,f,g,h) = Pr(A) Pr(B) Pr(C | A) Pr(D | A,B) Pr(E | B) Pr(F | C,D) Pr(G | F) Pr(H | E,F)

d.

Pr(A = 0, B = 0) = Pr(A = 0) * Pr(B = 0) = .8 * .3 = .24

A and B are independent, so Pr(A,B) = Pr(A) * Pr(B)

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Pr(E = 1 | A = 1) = Pr(E = 1) = Pr(E = 1 ^ B = 0) * Pr(E = 0 ^ B = 1) = .9(.3) + .1(.7) = .27 + .07 = .34

Since E and A are independent, Pr(E | A) = Pr(E). Then, to find Pr(E), we use conditional probability with B since E's only parent is B.