CS 343 Homework 7 Solutions

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A Representing knowledge in logical formalisms

1. Represent the sentence, "All Germans speak the same languages" in predicate calculus. Use $Speaks(x, l)$ to mean that person x speaks language l.

   $\forall x, y, l\ German(x) \wedge German(y) \wedge Speaks(x, l) \rightarrow Speaks(y, l)$

   Note that the sentence in question makes a very strong statement about what languages Germans speak; the sentence is certainly false in the real world. It says that the *set* of languages that every German speaks is the same! In this problem, we don't represent sets as explicit objects, so we reason instead about their members. Our logical translation says essentially that any language one German speaks, any other German also speaks.

2. Write a general set of facts and axioms to represent the assertion, "Wellington heard about Napoleon's death" and to correctly answer the question, "Did Napolean hear about Wellington's death?

   Some axiomatizations are more general than others. Here is an extremely general one.

   - $\exists t\ Heard(Wellington, DeathOf(Napoleon), t$
   - $\forall x, e, t\ Heard(x, e, t) \rightarrow Alive(x, t)$
   - $\forall x, e, t_2\ Heard(x, e, t_2) \rightarrow \exists t_1\ Occurred(e, t_1) \wedge t_1 < t_2$
   - $\forall t_1\ Occurred(DeathOf(x), t_1) \rightarrow \forall t_2\ t_1 < t_2 \rightarrow \neg Alive(x, t_2)$

   Technically, we would also need to axiomatize certain basic properties of the less-than relation, such as its transitivity.

3. What problems would be encountered in attempting to represent the following statements in FOPC? It should be possible to deduce the final statement from the others.

   (a) John only likes to see French movies.
   (b) It's safe to assume a movie is American unless explicitly told otherwise.
   (c) The Playhouse rarely shows foreign films.
   (d) People don't do things that will cause them to be in situations that they don't like.
   (e) John doesn't go to the Playhouse very often.

   Without going into detail, some of these sentences contain concepts that are awkward to express properly in FOPC, such as defaults and frequency. Others are ambiguous or have literal translations that depart from the likely intended meaning.

B Inference methods and algorithms for reasoning with knowledge that is represented in logical formalisms

Propositional logical

1. Decide whether each of the following sentences is valid, unsatisfiable, or neither. You may use truth tables or any of the standard sound rules for propositional inference. Show your argument.

   (a) $Smoke \to Smoke$: valid

   (b) $Smoke \to Fire$: neither

   (c) $(Smoke \to Fire) \to (\neg Smoke \to \neg Fire)$: neither

   (d) $Smoke \lor Fire \lor \neg Fire$: valid

   (e) $((Smoke \land Heat) \to Fire) \Leftrightarrow ((Smoke \to Fire) \lor (Heat \to Fire))$: valid

   (f) $(Smoke \to Fire) \to ((Smoke \land Heat) \to Fire)$: valid

   (g) $Big \lor Dumb \lor (Big \to Dumb)$: valid

   (h) $(Big \land Dumb) \lor \neg Dumb$: neither

2. Given the following, can you prove that the unicorn is mythical? Magical? Horned? Show your work.

   If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

   Since the unicorn must be either mythical or not mythical, the unicorn must be either immortal or a mammal, according to the first sentence above. Then by the second sentence, the unicorn must be horned. By the third sentence, it is also magical. However, we have no way of proving whether or not the unicorn is mythical. We reasoned over two cases to show that the unicorn is immortal or a mammal, but we don't know which case actually holds.

3. Show that every propositional logic clause with at least one positive literal and one negative literal can be written in the form $(P_1 \land \cdots \land P_m) \to (Q_1 \lor \cdots \lor Q_n)$, where the Ps and Qs are proposition symbols. A knowledge base consisting of such sentences is in implicatie normal form or Kowalski form.

   Let $(L_1 \lor \cdots \lor L_k)$ be an arbitrary clause, where each L is a literal (a propositional symbol that may or may not be negated). Since $\lor$ is cumutative and associative, we may assume without loss of generality that for some m between 1 and k (inclusive), L_1 through L_m are negated and L_{m+1} through L_k are positive. Then we may write the clause as $(L_1 \lor \cdots \lor L_m) \lor (L_{m+1} \lor \cdots \lor L_k)$. From the definition of $\to$, we have $\neg(L_1 \lor \cdots \lor L_m) \to (L_{m+1} \lor \cdots \lor L_k)$. Applying de Morgan's Law, we obtain $(\neg L_1 \land \cdots \land \neg L_m) \to (L_{m+1} \lor \cdots \lor L_k)$. Since the first m literals are negated propositional symbols and the remaining $n = k - m$ literals are unnegated propositional symbols, we can now express the clause as $(P_1 \land \cdots \land P_m) \to (Q_1 \lor \cdots \lor Q_n)$, as desired.

4. Consider the following KB:

   - $winter \lor hot$
   - $winter \to \neg summer \land \neg spring \land \neg fall$
   - $rainy \to spring \lor winter$
   - $pollen \to winter$
   - $rivers \to rainy$
   - $spring \to bluebonnets$
   - $\neg bluebonnets$

- $rivers$

(a) Convert each of the assertions to clause form.

1.	$winter \vee hot$
2.	$\neg winter \vee \neg summer$
3.	$\neg winter \vee \neg spring$
4.	$\neg winter \vee \neg fall$
5.	$\neg rainy \vee spring \vee winter$
6.	$\neg pollen \vee winter$
7.	$\neg rivers \vee rainy$
8.	$\neg spring \vee bluebonnets$
9.	$\neg bluebonnets$
10.	$rivers$

(b) Use resolution to prove $winter$.

11.	$\neg winter$	negated conclusion
12.	$\neg rainy \vee spring$	5 and 11
13.	$\neg rivers \vee spring$	7 and 12
14.	$spring$	10 and 13
15.	$bluebonnets$	8 and 14
16.	$\perp$	9 and 15

FOPC

5. Write down a logical sentence such that every world in which it is true contains exactly one object.

   $\exists x \forall y\ x = y$

   This sentences says that some individual is the same as any given individual. This is only true if only one individual exists (for appropriate meanings of "same").

6. Show the result of applying the unification algorithm given in class to each of the following pairs of clauses.

(a)	$P(A, B, B)$	$P(x, y, z)$	$A/x,\ B/y,\ B/z$
(b)	$Q(y, G(A, B))$	$Q(G(x, x), y)$	failure: can't unify A and B
(c)	$Older(Father(y), y)$	$Older(Father(x), John)$	$Joh(n/x,\ John/y$
(d)	$Knows(Father(y), y)$	$Knows(x, x)$	failure: can't unify y and $Father(y)$
(e)	$F(Marcus)$	$F(Caesar)$	failure: can't unify $Marcus$ and $Caesar$
(f)	$F(x)$	$F(G(y))$	$G(y)/x$
(g)	$F(Marcus, G(x, y))$	$F(x, G(Caesar, Marcus))$	failure: can't unify $Marcus$ and $Caesar$

7. Consider the following KB:

   - $\forall x\ Married(x) \rightarrow \exists y\ Spouse(x, y)$
   - $\forall x \exists y\ Spouse(x, y) \rightarrow Married(x)$
   - $\forall x \forall y\ Spouse(x, y) \rightarrow Spouse(y, x)$
   - $\forall x \forall y\ JointTaxFilers(x, y) \rightarrow Spouse(x, y)$
   - $JointTaxFilers(John, Mary)$
   - $\neg \exists y\ Spouse(Sue, y)$

(a) Convert each of these formulas to clause form.

1. $\neg Married(x) \vee Spouse(x, S_1(x))$
2. $\neg Spouse(x, y) \vee Married(x)$
3. $\neg Spouse(x, y) \vee Spouse(y, x)$
4. $\neg JointTaxFilers(x, y) \vee Spouse(x, y)$
5. $JointTaxFilers(John, Mary)$
6. $\neg Spouse(Sue, y)$

(b) Use resolution and this KB to prove $Married(Mary)$.

7.	$\neg Married(Mary)$	negated conclusion
8.	$\neg Spouse(Mary, y)$	2 and 7
9.	$\neg Spouse(y, Mary)$	3 and 8
10.	$\neg JointTaxFilers(y, Mary)$	4 and 9
11.	$\perp$	5 and 10

(c) Use resolution and this KB to prove $\neg Married(Sue)$.

12.	$Married(Sue)$	negated conclusion
13.	$Spouse(Sue, S_1(Sue))$	1 and 12
14.	$\perp$	6 and 13

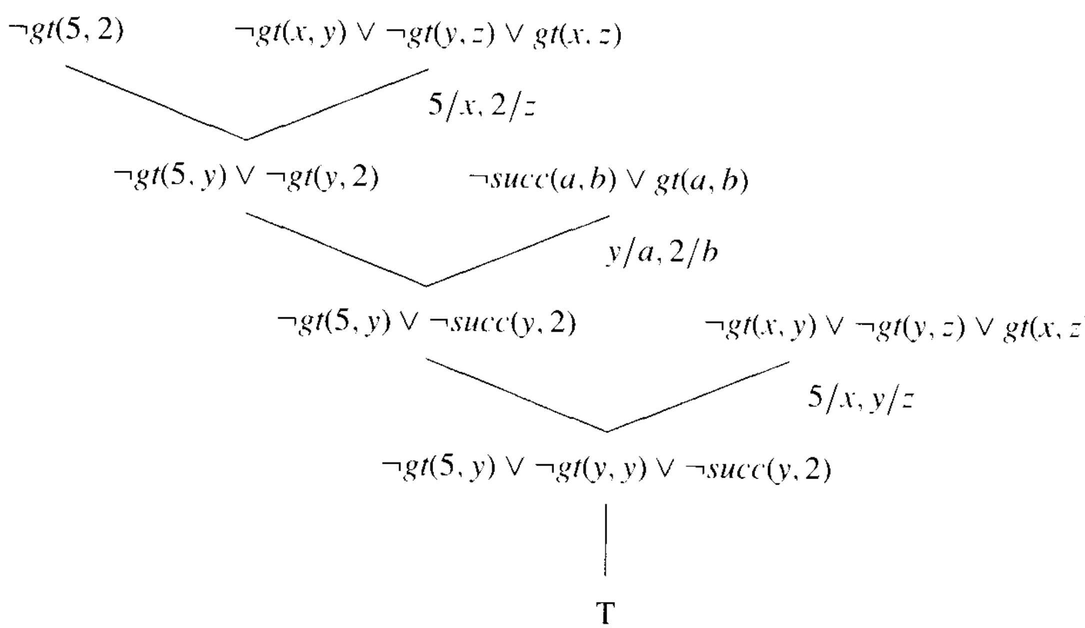
8. Suppose you are given the following facts:

(α) $\forall x, y, z\ gt(x, y) \wedge gt(y, z) \rightarrow gt(x, z)$

(β) $\forall a, b\ succ(a, b) \rightarrow gt(a, b)$

(γ) $\forall x\ \neg gt(x, x)$

Using these facts, we want to prove $gt(5, 2)$, which we should be able to do with resolution. Consider the following attempt at a resolution proof:



(a) What went wrong?

Fact (α) was used in two separate places in the proof, but the same variables were used each time. The y from the first instance isn't supposed to refer to the same variable as the y in the second instance!

(b) What needs to be added to the resolution procedure we described in class to make sure that this problem does not occur?

Every time we use a fact, we must rename all the variables in the fact to avoid conflicts with already used variables. Only then can we safely attempt to unify the terms from each clause.

C Representing facts and reasoning with them

1. Consider the following sentences:

   - John likes all kinds of food.
   - Apples are food.
   - Chicken is food.
   - Anything anyone eats and isn't killed by is food.
   - Bill eats peanuts and is still alive.
   - Sue eats everything Bill eats.

   (a) Translate these sentences into formulas in FOPC.

   1. $\forall x\ Food(x) \rightarrow Likes(John, x)$
   2. $Food(Apples)$
   3. $Food(Chicken)$
   4. $\forall x \exists y\ Eats(y, x) \wedge \neg KilledBy(y, x) \rightarrow Food(x)$
   5. $Eats(Bill, Peanuts) \wedge \neg KilledBy(Bill, Peanuts)$
   6. $\forall x\ Eats(Bill, x) \rightarrow Eats(Sue, x)$

   (b) Use backward chaining to prove that John likes peanuts.

   To prove $Likes(John, Peanuts)$, we just need to prove $Food(Peanuts)$, according to sentence 1. By sentence 4, we just need to prove $\exists y\ Eats(y, Peanuts) \wedge \neg KilledBy(y, Peanuts)$. Sentence 5 allows us to prove this directly.

   (c) Convert the formulas of part (a) into clause form.

   1. $\neg Food(x) \vee Likes(John, x)$
   2. $Food(Apples)$
   3. $Food(Chicken)$
   4. $\neg Eats(y, x) \vee KilledBy(y, x) \vee Food(x)$
   5. $Eats(Bill, Peanuts)$
   6. $\neg KilledBy(Bill, Peanuts)$
   7. $\neg Eats(Bill, x) \vee Eats(Sue, x)$

   (d) Use resolution to prove that John likes peanuts.

8.	$\neg Likes(John, Peanuts)$	negated conclusion
9.	$\neg Food(Peanuts)$	1 and 8
10.	$\neg Eats(y, Peanuts) \vee KilledBy(y, Peanuts)$	4 and 9
11.	$KilledBy(Bill, Peanuts)$	5 and 10
12.	$\perp$	6 and 11

   (e) Use resolution to answer the question, "What food does Sue eat?"

13.	$\neg Eats(Sue, x)$	negated query
14.	$\neg Eats(Bill, x)$	7 and 13
15.	$\perp$	5 and 14, unifying $Peanuts/x$

2. Consider the following facts:

- The members of the Elm St. Bridge Club are Joe, Sally, Bill and Ellen.
- Joe is married to Sally.
- Bill is Ellen's brother.
- The spouse of every married person in the club is also in the club.
- The last meeting of the club was at Joe's house.

(a) Represent these facts in FOPC.

1. $\forall x\ Member(x) \rightarrow x = Joe \lor x = Sally \lor x = Bill \lor x = Ellen$
2. $Married(Joe, Sally)$
3. $Brother(Ellen, Bill)$
4. $\forall x\ Married(x) \rightarrow Member(Spouse(x))$
5. $LastMeeting(House(Joe))$

(b) From the facts given above, most people would be able to decide on the truth of the following additional statements:

- The last meeting of the club was at Sally's house.
- Ellen is not married.

Can you construct resolution proofs to demonstrate the truth of each of these statements given the facts above? Do so if possible. Otherwise, add the facts you need and then construct the proofs.

To prove these statements, we will need some additional facts.

6. $\forall x, y\ Married(x, y) \rightarrow House(x) = House(y)$
7. $\forall x, y\ Married(x, y) \rightarrow Married(y, x)$
8. $\forall x, y\ Brother(x, y) \rightarrow \neg Married(x, y)$
9. $\forall x, y\ Married(x, y) \Leftrightarrow Spouse(x) = y$

We will prove each statement using backward chaining. To prove $LastMeeting(House(Sally))$, it suffices to show that $House(Joe) = House(Sally)$, due to sentence 5. By sentence 6, we must then prove $Married(Joe, Sally)$, but this is exactly sentence 2.

Proving $\neg Married(Ellen)$ will be more involved. From the contrapositive of sentence 4, it suffices to prove $\neg Member(Spouse(Ellen))$. Note that the Skolem function $Spouse$ is defined for all individuals, including those who are not married. Since in actuality Ellen is not married, the value of $Spouse(Ellen)$ is completely arbitrary. Still, we can prove $\neg Member(Spouse(Ellen))$ using sentence 1: we must prove all four of the following: $Spouse(Ellen) \neq Joe$ and $Spouse(Ellen) \neq Sally$ and $Spouse(Ellen) \neq Bill$ and $Spouse(Ellen) \neq Ellen$.

To show $Spouse(Ellen) \neq Joe$, we can show $\neg Married(Ellen, Joe)$, by sentence 9. This goal can be reduced to $\neg Married(Joe, Ellen)$ by sentence 7 and then to $Spouse(Joe) \neq Ellen$ by sentence 9 again. Since $Sally \neq Ellen$, it suffices to show that $Spouse(Joe) = Ellen$, which follows from sentences 2 and 9.

Similar lines of reasoning show that Ellen's hypothetical spouse cannot be any of the members of the bridge club.

3. What is wrong with the following argument:

- Men are widely distributed over the earth.
- Socrates is a man.
- Therefore, Socrates is widely distributed over the earth.

The first statement gives a property of the *set* of men (taken collectively), not a property that applies to each individual member of the set.