

PROBLEM 1

$$\Pr(D) = 1/1000$$

$$\Pr(T \mid \sim D) = 2/100$$

$$\Pr(\sim T \mid D) = 5/100$$

$$\Pr(\sim T \mid \sim D) = 98/100$$

$$\Pr(T \mid D) = 95/100$$

$$P(T) = P(T \mid D) * P(D) + P(T \mid \sim D) * P(\sim D) = 95/100 * 1/1000 + 2/100 * 999/1000 = .02093$$

$$\Pr(D \mid T) = P(T \mid D) * P(D) / P(T) = (95/100 * 1/1000) / .02093 = .04539$$

(Up to 3 decimal places), we ensure that $\Pr(D \mid T) \geq 0.3$ for:

Prior probability of having the disease: .009 (9/1000)

The false positive for the test: .002 (0.2%)

The false negative for the test: It is impossible to get a false negative for a $\Pr(D \mid T) \geq 0.3$. Even with a 0% false negative rate, the maximum $\Pr(D \mid T)$ is 4.77%.

PROBLEM 2

1. Given that Sambot has sensed lights on and no bark:

This answer was obtained by going into query mode and setting LightSensor to On and SoundSensor to Off. Then, we can look at the updated probabilities for each variable and choose the most probable answer for each.

ExpectingGuests: No

OutdoorLight: On

FamilyHome: No

DogOutside: Yes

DogBowelTrouble: Yes

DogBarking: Yes

HearableBarking: No

SoundSensorHealth: OK

Battery: OK

LightSensorHealth: OK

LightSensor: On

SoundSensor: Off

2. Given that family is home and no guests are expected:

Similar to 1, we go into query mode and set FamilyHome to Yes and ExpectingGuests to No. Then, we look at each sensor and choose the most probable answer for each.

LightSensor: Off

SoundSensor: On

3. $Z = \{\text{Battery}, \text{DogOutside}\}$ d-separates Light and Sound sensor

Battery separates them since LightSensor, Battery, and SoundSensor are divergent and Battery is in Z. DogOutside would separate them with FamilyHome, DogOutside, DogBarking (sequential and DogOutside is in Z).

Note: There are several other variable sets Z of length two that can make the two sensors independent. (i.e. {Battery, DogBarking}, {Battery, HearableBarking}, {Battery, FamilyHome})

4. Multiply-Connected Graph: ExpectingGuests, FamilyHome, and OutdoorLight create a loop if it were an undirected graph