***PROBLEM 1***

1. P ⇒ ¬Q, Q ⇒ ¬P

P => ¬Q

|  |  |  |
| --- | --- | --- |
| P | Q | P => Q |
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | T |

Q => ¬P

|  |  |  |
| --- | --- | --- |
| P | Q | Q => P |
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | T |

1. P ⇔ ¬Q, ((P ∧ ¬Q) ∨ (¬P ∧ Q))

P ⇔ ¬Q

|  |  |  |
| --- | --- | --- |
| P | Q | P ⇔ ¬Q |
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

((P ∧ ¬Q) ∨ (¬P ∧ Q))

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| P | Q | P ∧ ¬Q | ¬P ∧ Q | ((P ∧ ¬Q) ∨ (¬P ∧ Q)) |
| T | T | F | F | F |
| T | F | T | F | T |
| F | T | F | T | T |
| F | F | F | F | F |

***PROBLEM 2: Prove using truth table with all possible worlds.***

1. (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire) is neither

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Smoke | Fire | Smoke ⇒ Fire | ¬Smoke ⇒ ¬Fire | (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire) |
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | T | F | F |
| F | F | T | T | T |

1. (Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire) is neither

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Smoke | Fire | Heat | Smoke ⇒ Fire | Smoke ∨ Heat | (Smoke ∨ Heat) ⇒ Fire | (Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire) |
| T | T | T | T | T | T | T |
| T | T | F | T | T | T | T |
| T | F | T | F | T | F | T |
| T | F | F | F | T | F | T |
| F | T | T | T | T | T | T |
| F | T | F | T | F | T | T |
| F | F | T | T | T | F | F |
| F | F | F | T | F | T | T |

1. ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire)) is valid

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Smoke | Fire | Heat | Smoke ∧ Heat | ((Smoke ∧ Heat) ⇒ Fire) | Smoke ⇒ Fire | Heat ⇒ Fire | ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire)) | ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire)) |
| T | T | T | T | T | T | T | T | T |
| T | T | F | F | T | T | T | T | T |
| T | F | T | T | F | F | F | F | T |
| T | F | F | F | T | F | T | T | T |
| F | T | T | F | T | T | T | T | T |
| F | T | F | F | T | T | T | T | T |
| F | F | T | F | T | T | F | T | T |
| F | F | F | F | T | T | T | T | T |

Note: A and B are satisfiable, meaning they were true in some models.

***PROBLEM 3***

*If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal*

*mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is*

*magical if it is horned.*

Variable Names:

Mythical = Mythical

Mortal = Mortal

Mammal = Mammal

Horned = Horned

Magical = Magical

a.

1. Mythical => ¬Mortal

2. ¬Mythical => Mortal ∧ Mammal

3. ¬Mortal ∨ Mammal => Horned

4. Horned => Magical

b.

1. ¬ Mythical ∨ ¬Mortal

2. Mythical ∨ (Mortal ∧ Mammal)

(Mythical ∨ Mortal) ∧ (Mythical ∨ Mammal)

Mythical ∨ Mortal, Mythical ∨ Mammal

3. ¬(¬Mortal ∨ Mammal) ∨ Horned

(Mortal ∧ ¬Mammal) ∨ Horned

(Mortal ∨ Horned) ∧ (¬Mammal ∨ Horned)

Mortal ∨ Horned, ¬Mammal ∨ Horned

4. ¬Horned ∨ Magical

c.

*Mythical*

1. ¬ Mythical ∨ ¬Mortal

2. Mythical ∨ Mortal

3. Mythical ∨ Mammal

4. Mortal ∨ Horned

5. ¬Mammal ∨ Horned

6. ¬Horned ∨ Magical

7. ¬ Mythical

8. Mortal; 2, 7

9. Mammal; 3, 7

10. Horned; 5, 9

11. Magical; 6, 10

We cannot prove that the unicorn is mythical since there is no contradiction when we use resolution.

*Magical*

1. ¬ Mythical ∨ ¬Mortal

2. Mythical ∨ Mortal

3. Mythical ∨ Mammal

4. Mortal ∨ Horned

5. ¬Mammal ∨ Horned

6. ¬Horned ∨ Magical

7. ¬ Magical

8. ¬Horned; 6, 7

9. Mortal; 4, 8

10. ¬Mammal; 5, 8

11. ¬ Mythical; 1, 9

12. Mythical; 3, 10

13. Contradiction; 11, 12

Since we find a contradiction, we prove that the unicorn must be magical.

*Horned*

1. ¬ Mythical ∨ ¬Mortal

2. Mythical ∨ Mortal

3. Mythical ∨ Mammal

4. Mortal ∨ Horned

5. ¬Mammal ∨ Horned

6. ¬Horned ∨ Magical

7. ¬ Horned

8. Mortal; 4, 7

9. ¬Mammal; 5, 7

10. ¬ Mythical; 1, 8

11. Mythical; 3, 9

12. Contradiction; 10, 11

Since we find a contradiction, we prove that the unicorn must be horned.