%用微分方程解龙头位置

```
figure;
hold on;
axis equal; % 保持坐标比例一致
% 定义常数
b = 0.55 * 16; % 起始半径
a = 0.55 / (2 * pi); % 每圈增加的半径
% 微分方程定义
odefun = @(t, y) 1 / sqrt((b - a * y)^2 + a^2);
tspan = [0 300]; % 时间跨度
                  % 初始条件
y0 = 0;
% 使用ode45求解微分方程
options = odeset('RelTol',1e-5,'AbsTol',1e-7);
[t, y] = ode45(odefun, tspan, y0, options);
% 创建每秒一个时间点的向量
t interp = 0:1:300; % 每秒一个点
% 插值得到每秒的解
y interp = -interp1(t, y, t interp, 'linear');
% 计算等距螺旋线的坐标
theta spiral = linspace(0, -32*pi, 8000);
r spiral = b + a * theta_spiral;
x spiral = r spiral .* cos(theta spiral);
y_spiral = r_spiral .* sin(theta_spiral);
% 将插值解映射到螺旋线的坐标上
theta_solution = y_interp; % 使用插值解作为 theta
r solution = b + a * theta solution; % 根据解的角度计算半径
x_solution = r_solution .* cos(theta solution); % <math>x \le \pi
y_solution = r_solution .* sin(theta_solution); % y坐标
% 绘制等距螺旋线
plot(x spiral, y spiral, 'b')
% 绘制插值解的点
plot(x solution, y solution, 'ro')
% 现有螺旋线的参数
b = 0.55 * 16; % 起始半径
a = 0.55 / (2 * pi); % 每圈增加的半径
% 现有螺旋线的计算和绘制
theta_spiral = linspace(0, -32*pi, 4000);
r_spiral = b + a * theta_spiral;
x spiral = r spiral .* cos(theta spiral);
y spiral = r spiral .* sin(theta spiral);
plot(x spiral, y spiral, 'b-'); % 绘制蓝色实线
% 更大螺旋线的计算和绘制
b larger = b + 8.8; % 增大起始半径
r spiral larger = b larger + a * (theta spiral - 20 * pi);
x_spiral_larger = r_spiral_larger .* cos(theta_spiral);
```

```
y_spiral_larger = r_spiral_larger .* sin(theta_spiral);
plot(x_spiral_larger, y_spiral_larger, 'k--'); % 绘制黑色虚线
% 绘制原始的散点(圆心)
plot(x_solution, y_solution, 'ro'); % 使用红色圆圈标记

hold off; % 解除保持状态
xlabel('X');
ylabel('Y');
grid on; % 添加网格以便更清楚地查看
```

```
问题一:模拟舞龙队盘入的过程
```

응

(运行前请clear工作区,避免出现错误!)

```
% 参数设置
```

```
benches num = 224; % 板凳数量
head long = 3.41; % 龙头长度
body long = 2.20; % 龙身和龙尾长度
benches width = 0.3; % 板宽
hole to head = 0.275; % 板凳孔距离最近板凳头距离
                 % 螺距
p = 0.55;
                % 龙头速度
v head = 1.0;
T = 300;
                 % 模拟时间
                 % 时间步长
dt = 0.01;
% 第一个把手的初始位置和角度设置
theta0 = 2*pi*16;
r0 = p*16;
% 初始化龙头的位置
positions(1, :, 1) = [r0 * cos(theta0), r0 * sin(theta0)];
% 计算每个板凳孔相对于上一节的偏移
L = [head long - 2 * hole to head; ...
   repmat(body long - 2 * hole to head, benches num-1, 1)]; % 每节的长度
% 初始化龙身和龙尾位置
initial theta = theta0;
initial r = r0;
%计算龙身的初始位置
for i = 2:benches_num
   delta_theta(i) = L(i-1) / initial_r; % 每节之间的角度差 弧长=半径×角度
   initial theta = initial theta + delta theta(i);
   initial_r = p / (2 * pi) * initial_theta; % 半径变化
   positions(i, 1, 1) = initial r * cos(initial theta); % x位置
   positions(i, 2, 1) = initial r * sin(initial theta); % y位置
end
%初始化速度矩阵
velocities = zeros(224, 301);
%初始化角度和半径
current theta = theta0;
current r = r0;
%每一个dt时刻的龙位置
for j = dt:dt:T
   t = round(j / dt);
   % 计算龙头位置
   theta head = current theta - v head * dt / current r;
   r head = p / (2 * pi) * theta head;
   positions(1, :, t+1) = [r_head * cos(theta_head), ...
                         r head * sin(theta head)];
   % 更新龙头前把手极坐标的角度和半径
   current theta = theta head;
   current r = r head;
   % 更新第一节龙身前把手的角度和半径
   initial_theta = current_theta;
   initial r = current r;
```

```
% 计算当前时刻龙身和龙尾位置
    for i = 2:benches num
       delta theta(i) = L(i-1) / initial r; % 近似: 角度=弧长/半径
       initial theta = initial theta + delta theta(i);
       initial r = p / (2 * pi) * initial theta;
       positions(i, 1, t+1) = initial r * cos(initial theta); % x位置
       positions(i, 2, t+1) = initial r * sin(initial theta); % y位置
    end
    % 计算每节的速度
    if t > 0
       a = (positions(:, 1, t+1) - positions(:, 1, t)) / dt; % vx
       b = (positions(:, 2, t+1) - positions(:, 2, t)) / dt; % vy
       velocities(:, t+1) = sqrt(a.^2 + b.^2);
    end
    % %图像显示占用电脑资源影响运行速度,可以选择注释提高运行效率
    % % 绘制当前时刻龙的位置
   % pause(0.01);
   % clf;
   % hold on;
   % axis equal;
   % xlabel('X (米)');
   % ylabel('Y (米)');
    % % 设置坐标轴范围
   % xlim([-12, 12]);
   % ylim([-12, 12]);
   % title(['板凳龙行进示意图 (t = ', num2str(j), 's)']);
    % grid on;
   % % 画背景螺线图
    % theta spiral = linspace(0, -32*pi, 10000);
    % r spiral = 0.55 * 16 + (0.55 / (2 * pi)) * theta spiral;
    % x_spiral = r_spiral .* cos(theta_spiral);
    % y spiral = r spiral .* sin(theta spiral);
    % plot(x_spiral, y_spiral, 'LineWidth', 0.5, 'Color', 'm');
    % %画龙
    % plot(positions(1, 1, t+1), positions(1, 2, t+1), 'ro-', ...
         'MarkerSize', 4, 'LineWidth', 2, 'MarkerFaceColor', 'r');
    % plot(positions(2:end, 1, t+1), positions(2:end, 2, t+1),...
       'co-', 'MarkerSize', 4, 'LineWidth', 2, 'MarkerFaceColor', 'b');
    % line([positions(1, 1, t+1), positions(2, 1, t+1)],...
         [positions(1, 2, t+1), positions(2, 2, t+1)],...
          'Color', 'red', 'LineWidth', 2, 'LineStyle', '-');
    % hold off;
end
% 输出0s - 300s数据
output times = 0:1:300;
positions output = zeros(benches num, 2, length(output times));
velocities output = zeros(benches num,length(output times));
%存入数据
for i = 1:length(output times)
    t idx = round(output times(i)/dt) + 1;
   positions_output(:, :, i) = positions(:, :, t_idx);
```

% 函数功能: 检测两条线段是否相交,后续问题需要判断是否相交

```
(运行前请clear工作区,避免出现错误!)
```

```
% 参数设置
benches num = 224;
head long = 3.41;
body long = 2.20;
benches width = 0.3;
hole to head = 0.275;
p = 0.55;
v head = 1.0;
%由问题一知前300s不会碰撞,假设再过200秒之内会碰撞
T = 500; % 模拟时间
%缩小时间步长可以提高精度
dt = 0.01; % 时间步长
%需要实现龙在螺旋线上的动态显示,与第一问类似,不再注释
theta0 = 2*pi*16;
r0 = p*16;
positions(1, :, 1) = [r0 * cos(theta0), r0 * sin(theta0)];
L = [head long - 2 * hole to head; ...
    repmat(body long - 2 * hole_to_head, benches_num-1, 1)];
initial theta = theta0;
initial_r = r0;
for i = 2:benches num
    delta theta(i) = L(i-1) / initial r;
    initial_theta = initial_theta + delta_theta(i);
    initial r = p / (2 * pi) * initial theta;
   positions(i, 1, 1) = initial r * cos(initial theta);
   positions(i, 2, 1) = initial r * sin(initial theta);
end
velocities = zeros(224, 301);
current_theta = theta0;
current r = r0;
%初始化标志位为false
stop signal = false;
for j = dt:dt:T
    %如果停止标志出现,立刻停止循环
    if stop signal
       break;
   end
    t = round(j / dt);
    theta head = current theta - v head * dt / current r;
    r head = p / (2 * pi) * theta head;
   positions(1, :, t+1) = [r_head * cos(theta_head), ...
               r head * sin(theta head)];
   current theta = theta head;
   current r = r head;
    initial theta = current theta;
    initial_r = current_r;
    for i = 2:benches num
       delta_theta(i) = L(i-1) / initial_r;
```

```
initial theta = initial theta + delta theta(i);
   initial_r = p / (2 * pi) * initial_theta;
   positions(i, 1, t+1) = initial r * cos(initial theta);
   positions(i, 2, t+1) = initial r * sin(initial theta);
end
if t > 0
   a = (positions(:, 1, t+1) - positions(:, 1, t)) / dt; % vx
   b = (positions(:, 2, t+1) - positions(:, 2, t)) / dt; % vy
   velocities(:, t+1) = sqrt(a.^2 + b.^2);
end
% %图像显示占用电脑资源影响运行速度,可以选择注释提高运行效率
% pause(0.01);
% clf;
% hold on;
% axis equal;
% xlabel('X (米)');
% ylabel('Y (米)');
% xlim([-12, 12]);
% ylim([-12, 12]);
% title(['板凳龙行进示意图 (t = ', num2str(j), 's)']);
% grid on;
% % 画背景螺线图
% theta_spiral = linspace(0, -32*pi, 10000);
% r_{spiral} = 0.55 * 16 + (0.55 / (2 * pi)) * theta_spiral;
% x spiral = r spiral .* cos(theta spiral);
% y spiral = r spiral .* sin(theta spiral);
% plot(x spiral, y spiral,'LineWidth', 2, 'Color', [0 0.4470 0.7410]);
% key:考虑龙的宽度
%绘制出带有宽度的板凳龙
for i = 1:(benches num-1)
   % 计算方向向量
   dx = positions(i+1, 1, t+1) - positions(i, 1, t+1);
   dy = positions(i+1, 2, t+1) - positions(i, 2, t+1);
   length = sqrt(dx^2 + dy^2);
   ux = -dy / length; % 垂直方向的x分量
   uy = dx / length; % 垂直方向的y分量
   % 计算每段线段两侧的四个顶点
   x = 1 + 1 = positions(i, 1, t+1) + ux * benches width / 2;
   y left1 = positions(i, 2, t+1) + uy * benches width / 2;
   x right1 = positions(i, 1, t+1) - ux * benches width / 2;
   y_right1 = positions(i, 2, t+1) - uy * benches_width / 2;
   y_left2 = positions(i+1, 2, t+1) + uy * benches_width / 2;
   x right2 = positions(i+1, 1, t+1) - ux * benches width / 2;
   y right2 = positions(i+1, 2, t+1) - uy * benches width / 2;
   % 计算龙头方向向量
   dx = positions(2, 1, t+1) - positions(1, 1, t+1);
   dy = positions(2, 2, t+1) - positions(1, 2, t+1);
   length = sqrt(dx^2 + dy^2);
   ux = -dy / length; % 垂直方向的x分量
```

```
uy = dx / length; % 垂直方向的y分量
   % 定义延长的倍数
   m = (3.41/2.86-1)/2; % 延长1.5倍长度
   % 计算延长后的新的两个端点
   new_x1 = positions(1, 1, t+1) - m * dx; % 起点向外延长
   new y1 = positions(1, 2, t+1) - m * dy;
   new x2 = positions(2, 1, t+1) + m * dx; % 终点向外延长
   new y2 = positions(2, 2, t+1) + m * dy;
   % 计算延长后的四个顶点
   new x left1 = new x1 + ux * benches width / 2;
   new y left1 = new y1 + uy * benches width / 2;
   new_x_right1 = new_x1 - ux * benches_width / 2;
   new_y_right1 = new_y1 - uy * benches_width / 2;
   %龙头的两个顶点
   P1 head=[new x left1, new y left1]; %龙头的起点
   P2 head=[new x right1, new y right1]; %龙头的终点
   %龙身的四个顶点
   Q1 body=[x left1, y left1];
   Q2 body=[x right1,y right1];
   Q3 body=[x left2,y left2];
   Q4 body=[x right2, y right2];
   new x left2 = new x2 + ux * benches width / 2;
   new y left2 = new y2 + uy * benches width / 2;
   new_x_right2 = new_x2 - ux * benches_width / 2;
   new y right2 = new y2 - uy * benches width / 2;
   if check intersection (P1 head, P2 head, Q1 body, Q2 body) || ...
      check intersection (P1 head, P2 head, Q2 body, Q4 body) || ...
      check intersection (P1 head, P2 head, Q4 body, Q3 body) || ...
      check intersection (P1 head, P2 head, Q3 body, Q1 body)
       disp(['龙头线段与龙身矩形相交,停止模拟 (t = ' num2str(j) 's)']);
       stop signal = true; % 设置标志
       break;
   end
   8 8图像显示占用电脑资源影响运行速度,可以选择注释提高运行效率
   % % 使用 patch 函数绘制每段的宽线段
   % patch([x left1, x left2, x right2, x right1], ...
   % [y left1, y left2, y right2, y right1], 'b');
end
8 8图像显示占用电脑资源影响运行速度,可以选择注释提高运行效率
% % 绘制延长后的宽线段
% patch([new x left1, new x left2, new x right2, new x right1], ...
     [new y left1, new y left2, new y right2, new y right1], 'r');
% %画龙
% plot(positions(1, 1, t+1), positions(1, 2, t+1), 'ro-', ...
     'MarkerSize', 0.3, 'LineWidth', 1, 'MarkerFaceColor', 'r');
% plot(positions(2:end, 1, t+1), positions(2:end, 2, t+1), 'go-', ...
    'MarkerSize', 3, 'LineWidth', 1, 'MarkerFaceColor', 'r');
% hold off;
```

```
disp(['当前运行' num2str(j) 's']);
end
```

%保存结果到Excel文件

```
writematrix(positions(:, 1,end),'result2.xlsx', 'Range', 'B2:B225');%x数据writematrix(positions(:, 2,end),'result2.xlsx', 'Range', 'C2:C225');%y数据writematrix(velocities(:,end),'result2.xlsx', 'Range', 'D2:D225');%v数据disp('数据已存入result2.xlsx');
```

```
问题三: 掉头空间
```

```
응
   (运行前请clear工作区,避免出现错误!)
% 参数设置
num benches = 30; % 只看30个板凳
head length = 3.41;
body length = 2.20;
benches width = 0.3;
hole to head distance = 0.275;
T=20;
v head = 1.0;
dt = 0.01;
% 设置板凳孔的初始位置和角度设置
cnt=1; %计数
p=0.55;
                     %螺距每次减少0.01
dp=-0.0001;
                  % 初始时半径 4.5
r0 = 4.5;
stop signal1 = false ;
stop signal = false ;
for k = p:dp:0.3
    if stop signal
       break; % 退出主循环
   stop signal1 = false;
   cnt=cnt+1;
   theta0 = 9*pi/k;
   theta start=theta0+pi; %初始的角度增加180度
   r start=theta start*k/(2*pi); %初始时刻龙头位置
    % 初始化龙头的位置
   positions(1,: , cnt) = [r_start * cos(theta_start), ...
       r start * sin(theta start)];
   L = [head_length - 2 * hole_to_head_distance;...
       repmat(body_length - 2 * hole_to_head_distance, num_benches-1, 1)];
    % 初始化龙身和龙尾位置
    initial_theta = theta_start;
    initial r = r start;
    for i = 2:num benches
       delta theta(i) = L(i-1) / initial r;
       initial_theta = initial_theta + delta_theta(i);
       initial r = k / (2 * pi) * initial theta;
       positions(i, 1, cnt+1) = initial r * cos(initial theta);
       positions(i, 2, cnt+1) = initial r * sin(initial theta);
       current theta=theta start;
       current_r = r_start;
    for j = dt:dt:T
       %出现终止信号退出循环
       if stop signal1
           break;
       end
```

theta_head = current_theta - v_head * dt / current_r;

t = round(j / dt);

% 计算龙头位置

```
r head = k / (2 * pi) * theta head;
positions(1, :, cnt+1) = [r_head * cos(theta_head), r_head * sin(theta_head)];
% 更新龙头前把手极坐标的角度和半径
current theta = theta head;
current r = r head;
% 更新第一节龙身前把手的角度和半径
initial theta = current theta;
initial r = current r;
% 计算龙身和龙尾位置
for i = 2:num benches
   delta theta(i) = L(i-1) / initial r;
   initial theta = initial theta + delta theta(i);
   initial_r = k / (2 * pi) * initial_theta;
   positions(i, 1, cnt+1) = initial r * cos(initial theta);
   positions(i, 2, cnt+1) = initial_r * sin(initial_theta);
end
for i = 1:(num benches-1)
   % 计算方向向量
   dx = positions(i+1, 1, cnt+1) - positions(i, 1, cnt+1);
   dy = positions(i+1, 2, cnt+1) - positions(i, 2, cnt+1);
   length = sqrt(dx^2 + dy^2);
   ux = -dy / length; % 垂直方向的x分量
   uy = dx / length; % 垂直方向的y分量
   % 计算每段线段两侧的四个顶点
   x_left1 = positions(i, 1, cnt+1) + ux * benches_width / 2;
   y_left1 = positions(i, 2, cnt+1) + uy * benches_width / 2;
   x right1 = positions(i, 1, cnt+1) - ux * benches width / 2;
   y right1 = positions(i, 2, cnt+1) - uy * benches width / 2;
   y left2 = positions(i+1, 2, cnt+1) + uy * benches width / 2;
   x_right2 = positions(i+1, 1, cnt+1) - ux * benches_width / 2;
   y_right2 = positions(i+1, 2, cnt+1) - uy * benches_width / 2;
   % 计算龙头方向向量
   dx = positions(2, 1, cnt+1) - positions(1, 1, cnt+1);
   dy = positions(2, 2, cnt+1) - positions(1, 2, cnt+1);
   length = sqrt(dx^2 + dy^2);
   ux = -dy / length; % 垂直方向的x分量
   uy = dx / length; % 垂直方向的y分量
   % 定义延长的倍数
   scale factor = (3.41/2.86-1)/2; % 延长1.5倍长度
   % 计算延长后的新的两个端点
   new x1 = positions(1, 1, cnt+1) - scale factor * dx; % 起点向外延长
   new y1 = positions(1, 2, cnt+1) - scale factor * dy;
   new_x2 = positions(2, 1, cnt+1) + scale_factor * dx; % 终点向外延长
   new y2 = positions(2, 2, cnt+1) + scale factor * dy;
   % 计算延长后的四个顶点
   new x left1 = new x1 + ux * benches width / 2;
   new y left1 = new y1 + uy * benches width / 2;
   new_x_right1 = new_x1 - ux * benches_width / 2;
   new_y_right1 = new_y1 - uy * benches_width / 2;
   new_x_left2 = new_x2 + ux * benches_width / 2;
```

```
new y left2 = new y2 + uy * benches width / 2;
   new x right2 = new x2 - ux * benches width / 2;
   new y right2 = new y2 - uy * benches width / 2;
   % %图像显示占用电脑资源影响运行速度,可以选择注释提高运行效率
    % % 绘制延长后的宽线段
    % patch([x left1, x left2, x right2, x right1], ...
         [y left1, y left2, y right2, y right1], 'b');
    % patch([new x left1, new x left2, new x right2,...
    % new x right1], [new y left1, new y left2, ...
         new_y_right2, new_y_right1], 'r');
    %龙头的线段
   cnt1 head=[new x left1, new y left1]; %龙头的起点
   cnt2 head=[new x right1, new y right1]; %龙头的终点
   %龙身的矩形
   Q1 body=[x left1, y left1];
   Q2 body=[x right1,y right1];
   Q3 body=[x left2,y left2];
   Q4 body=[x right2,y right2];
   if check intersection(cnt1 head, cnt2 head, Q1 body, Q2 body) || ...
      check intersection(cnt1 head, cnt2 head, Q2 body, Q4 body) || ...
      check intersection(cnt1 head, cnt2 head, Q4 body, Q3 body) || ...
      check intersection(cnt1 head, cnt2 head, Q3 body, Q1 body)
       stop signal1 = true; % 设置标志
   end
 end
 if sqrt(positions(1, 1, cnt+1).^2+positions(1, 2, cnt+1).^2)<4.5</pre>
    stop signal = false;
 else
    stop signal = true;
 end
disp(['p = ' num2str(k) 'm)']);
disp(['t = ' num2str(j) 's)']);
% %图像显示占用电脑资源影响运行速度,可以选择注释提高运行效率
% pause(0.01);
% clf;
% hold on;
% axis equal;
% xlabel('X (米)');
% ylabel('Y (米)');
% xlim([-12, 12]);
% ylim([-12, 12]);
% grid on;
% title(['板凳把手位置示意图 (p = ', num2str(k), 'm)(t = ',...
        num2str(j), 's)']);
% plot(positions(1:end, 1, cnt+1), positions(1:end, 2, cnt+1), ...
      'go-', 'MarkerSize', 3, 'LineWidth', 1, 'MarkerFaceColor', 'r');
% hold off;
```

```
end
```

end

```
disp(['龙头线段与龙身相交, 停止模拟 (p = ' num2str(k-dp) 'm)']);
```

```
% 计算 4.5/(1.7/(2*pi)) 的结果
theta=4.5/(1.7/(2*pi));
ro=4.5;
dy=(1.7/(2*pi))*sin(theta)+(1.7/(2*pi))*theta*cos(theta);
dx=(1.7/(2*pi))*cos(theta)-(1.7/(2*pi))*theta*sin(theta);
kk=dy/dx;
k=-1/kk;
y=ro*sin(theta);
x=ro*cos(theta);
% 计算直线方程的参数
bb= y - kk*x;
b = y - k * x; % 直线方程为 y = kx + b, b为截距
% 计算直线到原点的距离
dd = abs(bb) / sqrt(1 + kk^2); %切线
d = abs(b) / sqrt(1 + k^2);
% 显示结果
disp(['垂线距离为: ', num2str(d)]);
disp(['切线距离为: ', num2str(dd)]);
% 定义目标函数和约束条件
objective = @(a) 2*d*a./sin(pi-a);
constraint = @(a) (2*d*a./sin(a)).*(1 + cos(pi-a))./a - 2*dd;
% 定义范围
a min = pi/2;
a max = pi;
% 设置优化选项
options = optimoptions('fmincon', 'Display', 'off');
% 初始猜测
a0 = (a_min + a_max) / 2;
% 使用 fmincon 进行优化
[a_{opt}, s_{opt}] = fmincon(@(a) 2*d*a./sin(pi-a), a0, [], [], [], ...
   a min, a max, @(a) deal([], constraint(a)), options);
% 输出结果
disp(['s最小值为: ', num2str(s opt)]);
disp(['对应的phi值为: ', num2str(a opt)]);
%求题目给出比例的圆弧长
syms 1 theta0
assume(theta0 > pi/2 & theta0 < pi);
eq1 = 1*(1+(cos(pi-theta0))) == 2*dd;
eq2 = 1*sin(pi-theta0) == 2*d;
sol = solve([eq1, eq2], [1, theta0]);
x sol = sol.1;
y sol = sol.theta0;
fprintf('题设条件下 m = %.4f\n', x_sol);
fprintf('题设条件下 phi = %.4f\n', y_sol);
fprintf('题设条件下 s = %.4f\n', x sol*y sol);
```