Cryptology

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Lecture 06

Pseudo-Random Function, Pseudo-Random Permutation and Block Cipher

Stream Cipher

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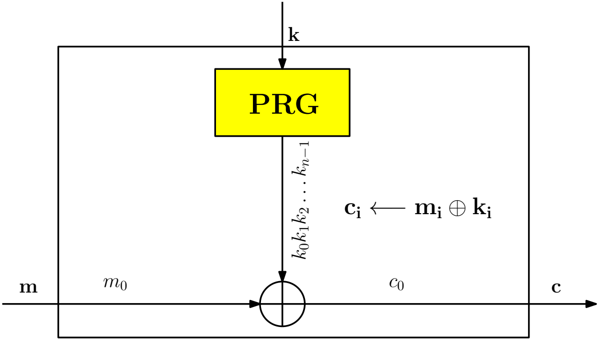

At time $t = 1$

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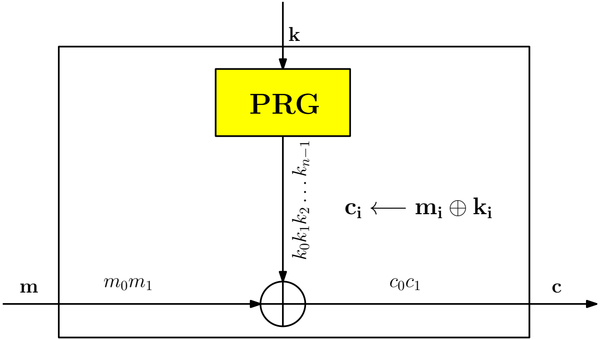

At time $t = 2$

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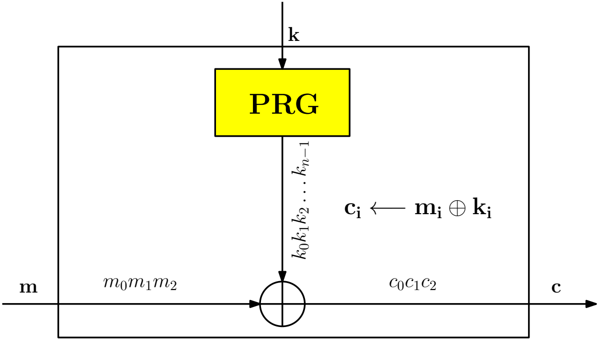

At time $t = 3$

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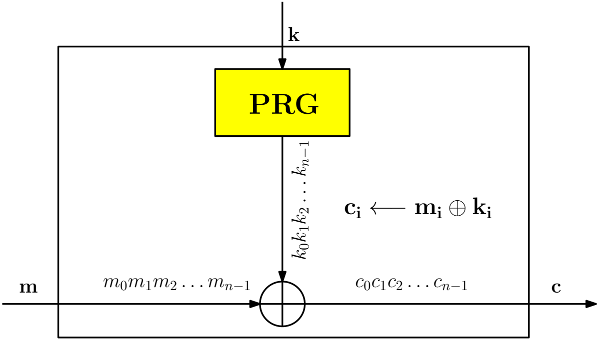

At time $t = n$

Block Cipher

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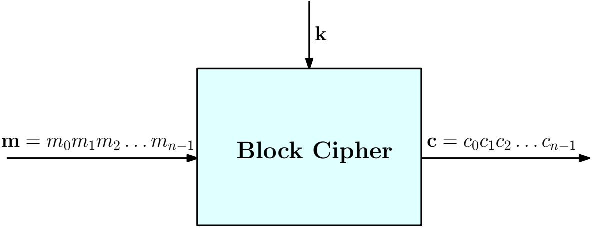

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A deterministic, polynomial-time cipher $\mathfrak{E} = (\mathcal{E}, \mathcal{D})$ whose message space and ciphertext space are the same (finite) set $\mathcal{X}$. If the key space of $\mathfrak{E}$ is $\mathcal{K}$, then $\mathfrak{E}$ is defined over $(\mathcal{K}, \mathcal{X})$.

- We call an element $x \in \mathcal{X}$ a data block, and
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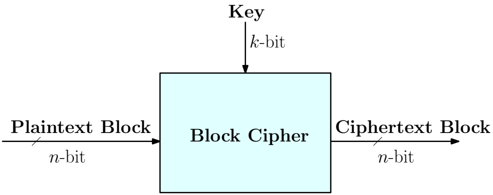

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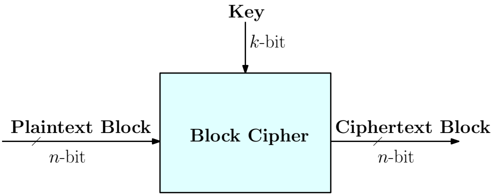

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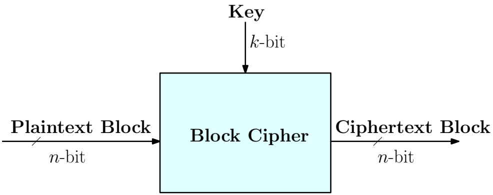

Example

- DES: $n = 64$ and $k = 56$
- AES: $n = 128$ and $k = 128, 192, 256$

Performance

Crypto++ (Wei Dai)

	Cipher	Block/Key Size	Speed (mbps)
Steam	RC4		126
	Salsa20/12		643
	Sosemanuk		727
Block	DES	64/56	39
	AES	128/128	109

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- Merit:
  - Analysis the block cipher in terms of correct construction and security.
- PRP is a subset of a more generalized class called Pseudorandom Function (PRF).
- PRF can be used to design
  - CPA-secure encryption,
  - PRG and many more cryptographic primitives.

Pseudorandom Function (PRF)

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Random Function

Let $\mathsf{Func}[\mathcal{X},\mathcal{Y}]$ be the set of all functions from the domain $\mathcal{X}$ to range $\mathcal{Y}$. We choose a function f uniformly at random from $\mathsf{Func}[\mathcal{X},\mathcal{Y}]$. We call f a random function.

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- Size of $\mathsf{Func}[\mathcal{X}, \mathcal{Y}]$, $|\mathsf{Func}[\mathcal{X}, \mathcal{Y}]| = |\mathcal{Y}|^{|\mathcal{X}|}$.

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  - Each row of the look-up table stores the value of $f(x_i)$ for some $x_i \in \mathcal{X}$.

Pseudorandom Function (PRF)

Description of a Random function

	f
x_1	$f(x_1)$
x_2	$f(x_2)$
$\vdots$	$\vdots$
$x_{\lvert X \rvert}$	$f\left(x_{\lvert X \rvert}\right)$

Pseudorandom Function (PRF)

Description of a Random function

$$
\begin{array}{c|c|}
 & \color{purple}{f} \\
\cline{2-2}
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Alternative view of Random function

Choosing f uniformly at random from $\mathsf{Func}[\mathcal{X}, \mathcal{Y}]$ is equivalent of choosing each row of look-up table uniformly at random from $\mathcal{Y}$.

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Keyed Function

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- We say F is efficient if there is a deterministic, polynomial-time algorithm that computes $F(k, x)$ given k and x as input.

Intuition on Pseudorandom Function (PRF)

- $S_F = \left\{ F_k(\cdot) \mid k \xleftarrow{R} \mathcal{K} \right\} \subseteq \mathsf{Func}[\mathcal{X}, \mathcal{Y}]$.

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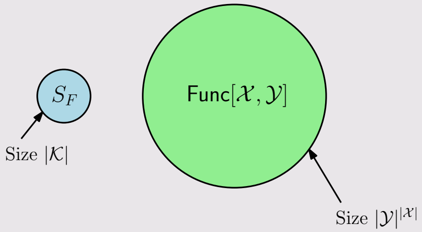

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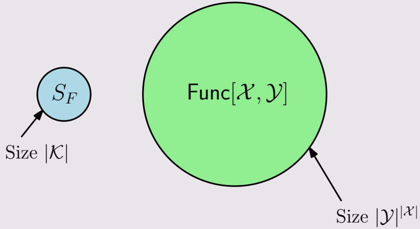

- Choosing F_k uniformly at random from S_F is equivalent of choosing k uniformly at random from $\mathcal{K}$.

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  - Equivalently, F is pseudorandom if no polynomial-time adversary can distinguish whether it is interacting with F_k (for randomly-chosen key k) or f (where f is chosen at random from $\mathsf{Func}[\mathcal{X}, \mathcal{Y}]$).

Pseudorandom Function (PRF)

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- Let $y := F(k,x)$
- x sometimes is referred as input data block, and
- y sometimes is referred as output data block.

PRF Indistinguishability Game

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   - For $i = 1, 2, \ldots$ the i-th query is an input data block $x_i \in \mathcal{X}$.
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   - The queries are adaptive.
3. The adversary computes and outputs a bit $\hat{b} \in \{0, 1\}$.

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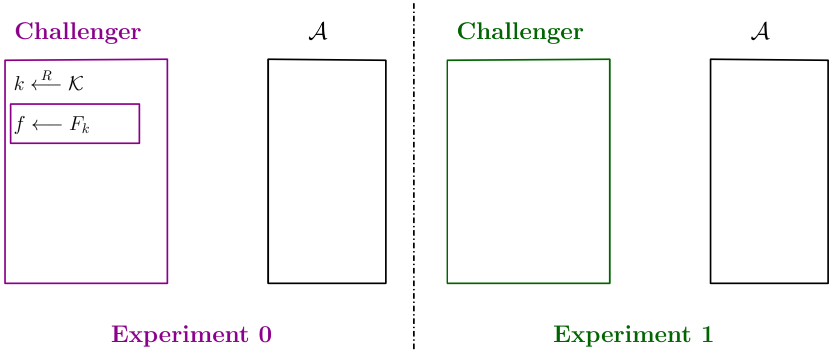

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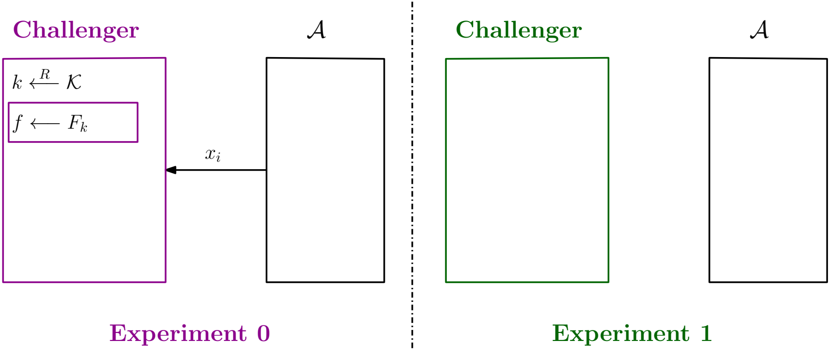

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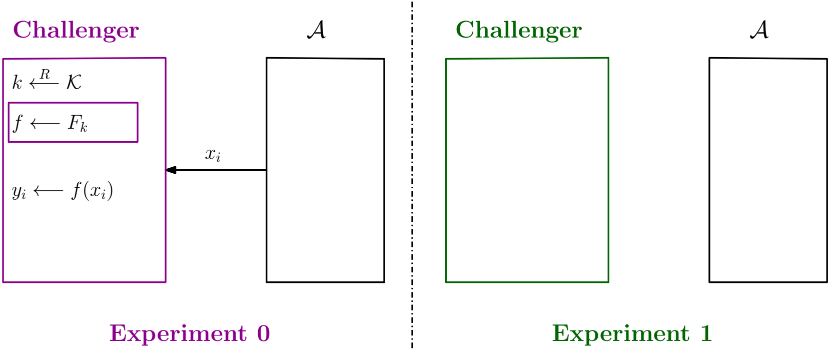

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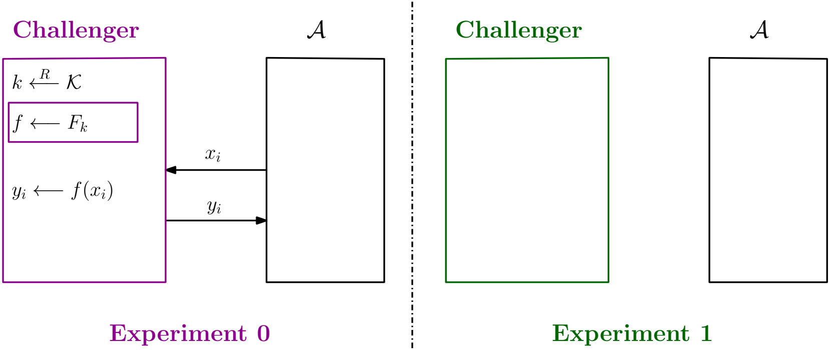

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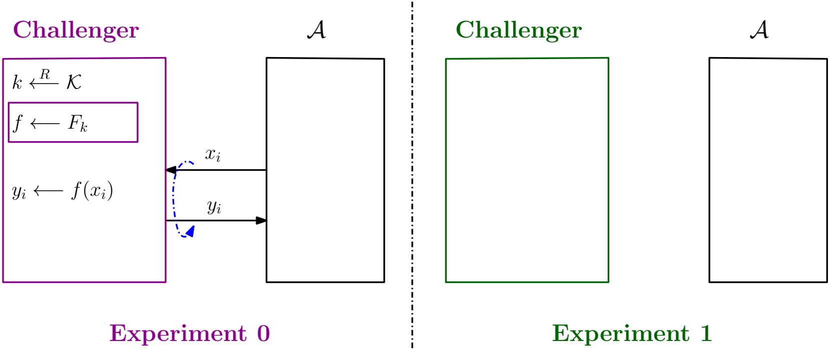

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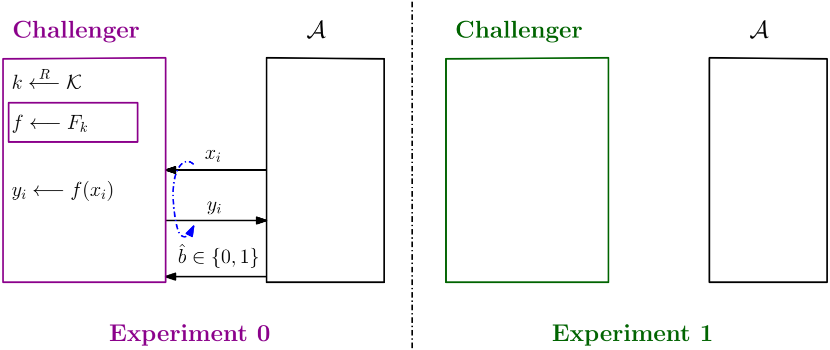

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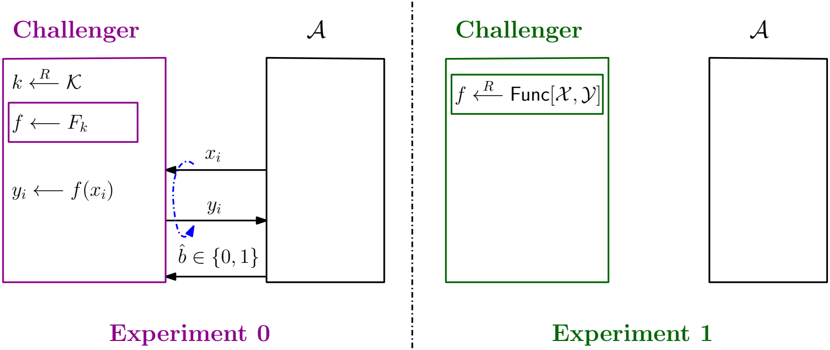

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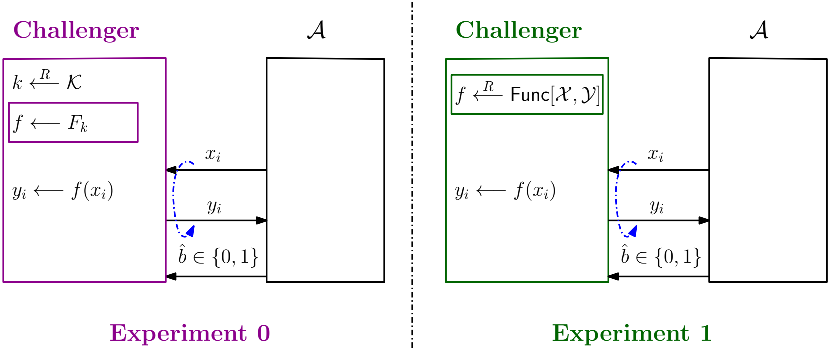

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For $b = 0, 1$, let W_b be the event that $\mathcal{A}$ outputs 1 in Experiment b. We define $\mathcal{A}$'s advantage with respect to F as

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Secure PRF

A PRF F is secure if for all efficient adversaries $\mathcal{A}$, the value $\mathsf{PRFadv}[\mathcal{A}, F]$ is negligible.

PRF Advantage: Bit Guessing Version

PRF Indistinguishability Game

For a given PRF F, defined over $(\mathcal{K}, \mathcal{X}, \mathcal{Y})$, and for a given adversary $\mathcal{A}$, we define Experiment as:

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   - if $b = 1$: $f \xleftarrow{R} \mathsf{Func}[\mathcal{X}, \mathcal{Y}]$.
3. The adversary submits a sequence of queries to the challenger.
   - For $i = 1, 2, \ldots$ the i-th query is an input data block $x_i \in \mathcal{X}$.
   - The challenger computes the output data block $y_i \longleftarrow f(x_i) \in \mathcal{Y}$, and gives y_i to the adversary.
   - The queries are adaptive.
4. The adversary computes and outputs a bit $\hat{b} \in \{0,1\}$.

PRF Advantage: Bit Guessing Version

Challenger $\qquad\qquad$ $\mathcal{A}$

$b \xleftarrow{R} \{0,1\}$

$\text{if } (b == 0)\{$

$\quad k \xleftarrow{R} \mathcal{K}$

$\quad f \longleftarrow F_k$

$\}$

$\text{if } (b == 1)\{$

$\quad f \xleftarrow{R} \mathsf{Func}[\mathcal{X}, \mathcal{Y}]$

$\}$

$y_i \longleftarrow f(x_i)$

$\mathcal{A} \xrightarrow{x_i}$ Challenger

Challenger $\xrightarrow{y_i} \mathcal{A}$

$\mathcal{A} \xrightarrow{\hat{b} \in \{0,1\}}$ Challenger

Experiment

PRF Advantage: Bit Guessing version

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Let W be the event that where $\mathcal{A}$ wins if $\mathcal{A}$ outputs $\hat{b} = b$. We define the advantage of $\mathcal{A}$ in the attack game with respect to F as

$$\mathsf{PRFadv}^*[\mathcal{A}, F] = \left| \Pr[\hat{b} = b] - \frac{1}{2} \right|.$$

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Theorem

For every PRF F and every PPT adversary $\mathcal{A}$, we have

$$\mathsf{PRFadv}[\mathcal{A}, F] = 2 \cdot \mathsf{PRFadv}^*[\mathcal{A}, F].$$

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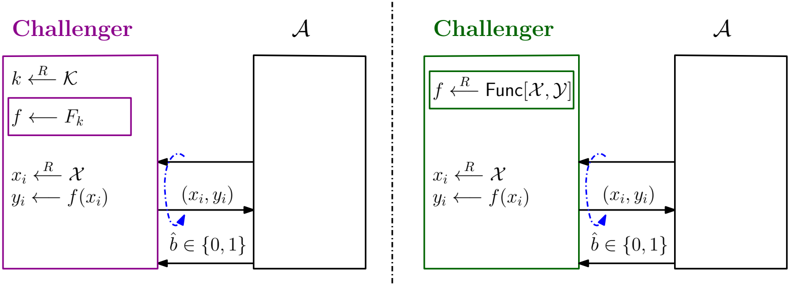
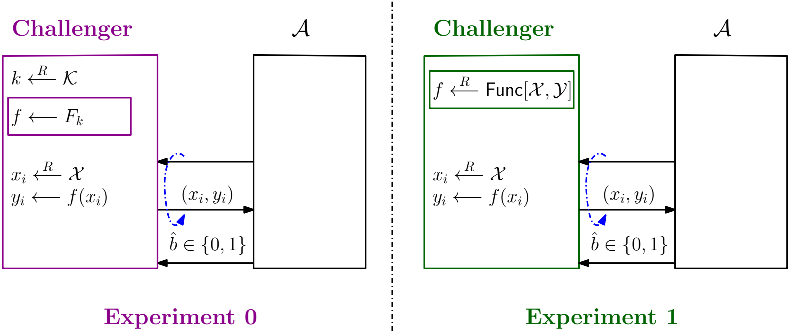
- It can only query the function at random points in the domain.
- Whenever the adversary queries the function, the challenger chooses a random $x_i \in \mathcal{X}$ and sends both x_i and $f(x_i)$ to the adversary.

Weak PRF Advantage



Experiment 0 **Experiment 1**



PRF Advantage

For $b = 0, 1$, let W_b be the event that $\mathcal{A}$ outputs 1 in Experiment b. We define $\mathcal{A}$'s advantage with respect to F as

$$\mathsf{weakPRFadv}[\mathcal{A}, F] = \left| \Pr[W_0] - \Pr[W_1] \right|.$$

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4. else {
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7. }

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send y_i to $\mathcal{A}$.

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 & f \\
\cline{2-2}
x_1 & f(x_1) \\
\cline{2-2}
x_2 & f(x_2) \\
\cline{2-2}
\vdots & \vdots \\
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Alternative view of Random Permutation

Choosing f uniformly at random from $\mathsf{Prem}[\mathcal{X}]$ is equivalent of choosing each row of look-up table uniformly at random from $\mathcal{X}$ without replacement.

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$$E_k(x) \stackrel{\triangle}{=} E(k, x).$$

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  - a deterministic, polynomial-time algorithm to compute $E^{-1}(k, x)$,

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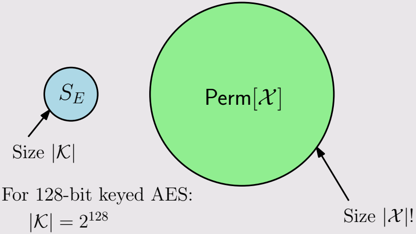

For 128-bit keyed AES:

$|\mathcal{K}| = 2^{128}$

$|\mathcal{X}|! = 2^{128}! \approx 2^{2^{135}}$

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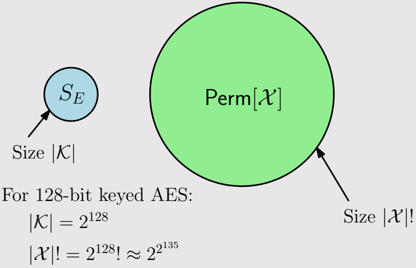

- Choosing E_k uniformly at random from S_E is equivalent of choosing k uniformly at random from $\mathcal{K}$.

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  - E is pseudorandom if the permutation E_k (for a randomly-chosen key k) is indistinguishable (for all practical purposes) from a permutation f chosen uniformly at random from $\mathsf{Prem}[\mathcal{X}]$.

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- Let $y := E(k,x)$
- x sometimes is referred as input data block, and
- y sometimes is referred as output data block.

PRP or Block Cipher Indistinguishability Game

For a given PRP E, defined over $(\mathcal{K}, \mathcal{X})$, and for a given adversary $\mathcal{A}$, we define two experiments, Experiment 0 and Experiment 1. For $b = 0, 1$, we define Experiment b as:

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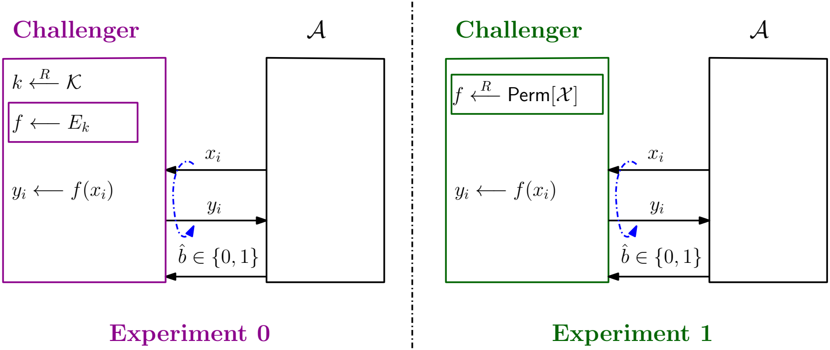

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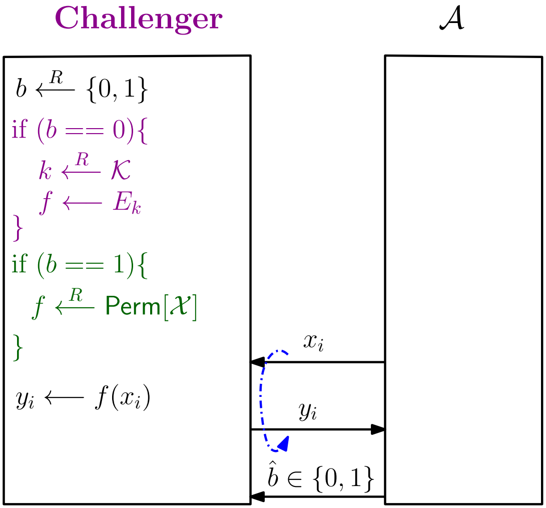

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Theorem

For every PRP E and every PPT adversary $\mathcal{A}$, we have

$$\mathsf{BCadv}[\mathcal{A}, E] = 2 \cdot \mathsf{BCadv}^*[\mathcal{A}, E].$$

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- Although most block ciphers in use today are designed to satisfy the second, stronger requirement, a scheme that can be proven secure based on the former, weaker assumption may be preferable (since the requirements on the block cipher are potentially easier to satisfy).
- Strong pseudorandom permutations are useful in the design and analysis of efficient cryptographic schemes, we will only use pseudorandom permutations(that are not necessarily strong) in the rest of this lecture.

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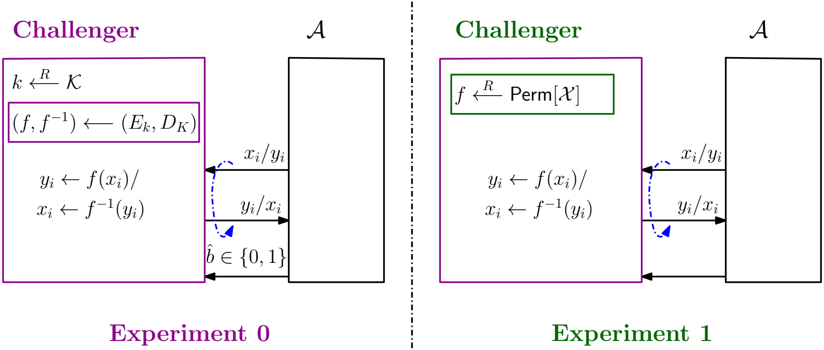

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Let $\mathfrak{E} = (\mathcal{E}, \mathcal{D})$ be a block cipher defined over $(\mathcal{K}, \mathcal{X})$, and let $N := |\mathcal{X}|$. Now suppose that $\mathfrak{E}$ is a secure block cipher; that is, no efficient adversary can effectively distinguish $\mathfrak{E}$ from a random permutation. **Does this imply that $\mathfrak{E}$ is also a secure PRF?**

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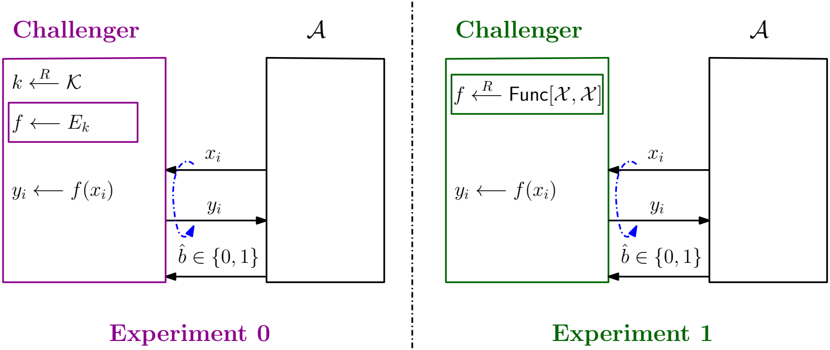

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- PRFadv $= \mid \Pr[W_0] - \Pr[W_1] \mid \geqslant \frac{1}{2}$, not negligible.

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- As $\mathcal{A}$ is efficient PPT adversary, Q must be poly-bounded. Therefore, we can not take $Q = N$.
- $\Pr[W_0] = 0$
-

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&= 0 + \Pr[\text{Collision} \mid f \notin \mathsf{Prem}[\mathcal{X}]] \cdot \Pr[f \notin \mathsf{Prem}[\mathcal{X}]] \\
&= \frac{Q(Q-1)}{4N}\left(1 - \frac{N!}{N^N}\right)
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- Q is poly-bounded and N is superpoly, then $\frac{Q(Q-1)}{4N}$ is negligible.

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- Q is poly-bounded and N is superpoly, then $\frac{Q(Q-1)}{4N}$ is negligible.
- $\left(1 - \frac{N!}{N^N}\right) < 1$.
- $\Pr[W_1] \leqslant$ negligible
- $\mathsf{PRFadv} = |\Pr[W_0] - \Pr[W_1]| \leqslant$ negligible.

PF Indistinguishability Game

For a given finite set $\mathcal{X}$, and for a given adversary $\mathcal{A}$, we define two experiments, Experiment 0 and Experiment 1. For $b = 0, 1$, we define Experiment b as:

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Permutations Vs. Functions

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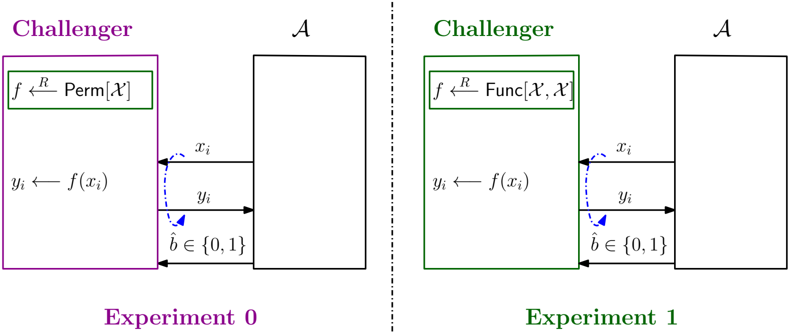
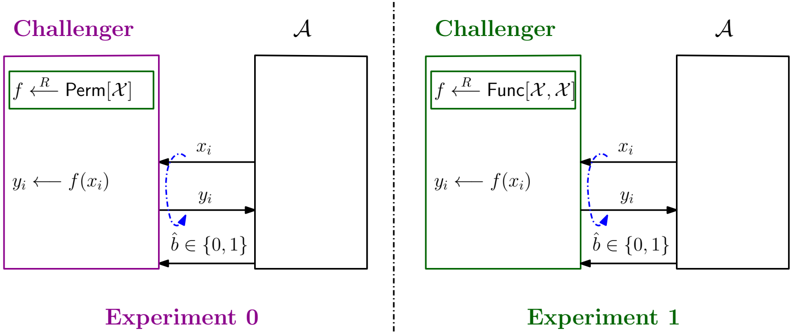
   - The queries are adaptive.
3. The adversary computes and outputs a bit $\hat{b} \in \{0, 1\}$.

Permutations Vs. Functions





PF Advantage

For $b = 0, 1$, let W_b be the event that $\mathcal{A}$ outputs 1 in Experiment b. We define $\mathcal{A}$'s advantage with respect to $\mathcal{X}$ as

$$\mathsf{PFadv}[\mathcal{A},\mathcal{X}] = |\Pr[W_0] - \Pr[W_1]|.$$

We say that $\mathcal{A}$ is a Q-query PF adversary if $\mathcal{A}$ issues at most Q queries.

Permutations Vs. Functions

Theorem

Let $\mathcal{X}$ be a finite set of size N. Let $\mathcal{A}$ be an adversary that makes at most Q queries to its challenger. Then

$$\mathsf{PFadv}[\mathcal{A}, \mathcal{X}] \leqslant \frac{Q^2}{2N}.$$

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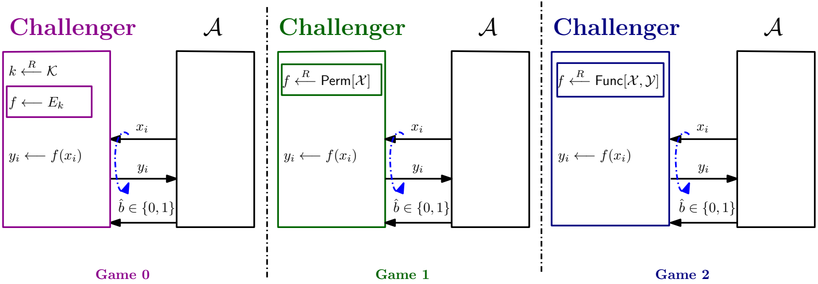
PRF Switching Lemma

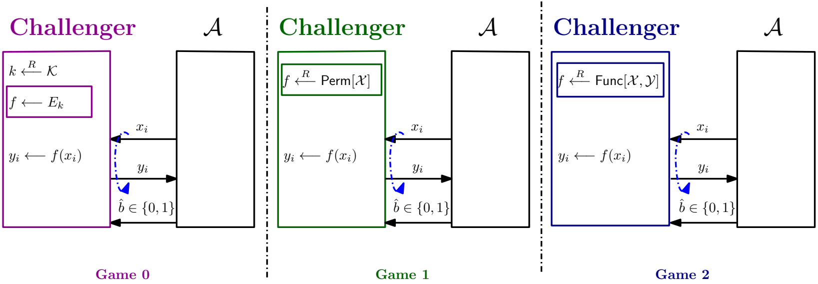
Let $\mathfrak{E} = (\mathcal{E},\mathcal{D})$ be a block cipher defined over $(\mathcal{K},\mathcal{X})$, and let $N := |\mathcal{X}|$. Let $\mathcal{A}$ be an adversary that makes at most Q queries to its challenger. Then

$$|\mathsf{BCadv}[\mathcal{A},\mathfrak{E}] - \mathsf{PRFadv}[\mathcal{A},\mathcal{E}]| \leqslant \frac{Q^2}{2N}.$$

PRF Switching Lemma



PRF Switching Lemma



PF AdvantagePRF Switching Lemma

- $p_0 = \Pr[\mathcal{A}$ outputs 1 in Game 0$]$.
- $p_1 = \Pr[\mathcal{A}$ outputs 1 in Game 1$]$.
- $p_2 = \Pr[\mathcal{A}$ outputs 1 in Game 2$]$.

PRF Switching Lemma

PRF Switching Lemma

- $\mathsf{BCadv}[\mathcal{A},\mathfrak{E}] = \mid p_1 - p_0 \mid$
- $\mathsf{PRFadv}[\mathcal{A},\mathcal{E}] = \mid p_2 - p_0 \mid$

$$
\begin{aligned}
|\mathsf{BCadv}[\mathcal{A},\mathfrak{E}] - \mathsf{PRFadv}[\mathcal{A},\mathcal{E}]| &= \;||\, p_1 - p_0 \mid - \mid p_2 - p_0 \,|| \\
&\leqslant |p_1 - p_0 - p_2 - p_0| \\
&= |p_2 - p_1| \\
&= \mathsf{PFadv}[\mathcal{A},\mathcal{X}] \\
&\leqslant \frac{Q^2}{2N}.
\end{aligned}
$$

Modes of Operation

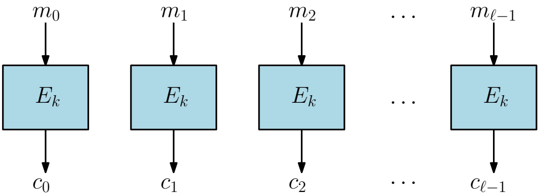
- Essentially, a way of encrypting arbitrary-length messages using a block cipher or PRP.
- Arbitrary-length messages can be unambiguously padded to a total length that is a multiple of any desired block size by appending a 1 followed by sufficiently-many 0s.
- Assume that the length of the plaintext message is an exact multiple of the block size.
- Let data block size of pseudorandom permutation/block cipher = n
- Let $\mathcal{X} = \{0,1\}^n$
- Consider messages consisting of ℓ blocks each of length n.

Modes of Operation

Five most popular modes of operations:

- Electronic CodeBook mode (ECB mode),
- Cipher Block Chaining mode (CBC mode),
- Output FeedBack mode (OFB mode),
- Cipher FeedBack mode (CFB mode), and
- Counter mode (CTR mode).

Electronic codebook mode (ECB mode)



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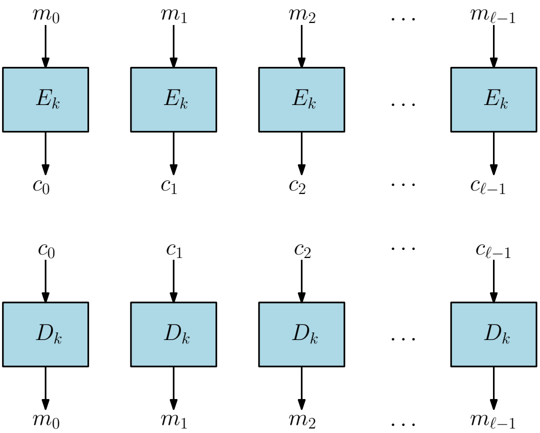

Encryption(m, k)

1. For $i = 0, 1, \ldots, \ell - 1$ do
2. Compute $c_i := E_k(m_i) = E(k, m_i)$
3. End For;
4. Return $c = (c_0, c_1, \ldots, c_{\ell-1})$.

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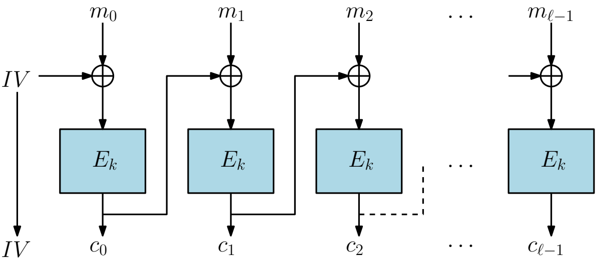
Decryption(c, k)

1. For $i = 0, 1, \ldots, \ell - 1$ do
2. Compute $m_i := E_k^{-1}(m_i) = D(k, c_i)$
3. End For;
4. Return $m = (m_0, m_1, \ldots, m_{\ell-1})$.

Cipher Block Chaining mode (CBC mode)



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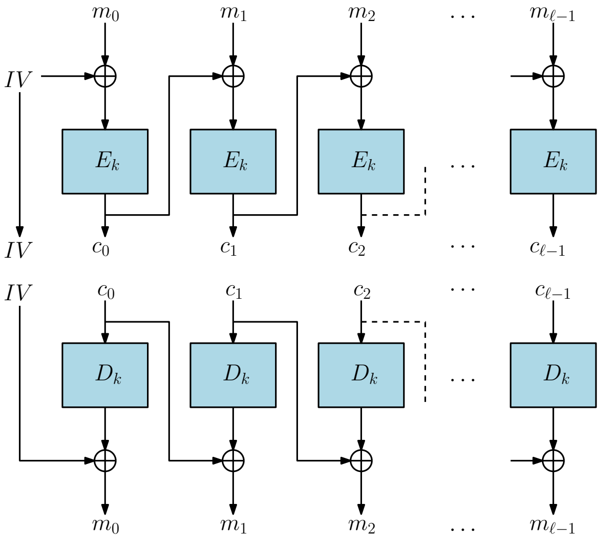

Cipher Block Chaining mode (CBC mode)

Encryption(m, k)

1. Choose a random $IV \xleftarrow{R} \mathcal{X}$
2. Compute $c_0 := E_k(IV \oplus m_0) = E(k, IV \oplus m_0)$
3. For $i = 1, \ldots, \ell - 1$ do
4. Compute $c_i := E_k(m_i \oplus c_{i-1}) = E(k, m_i \oplus c_{i-1})$
5. End For;
6. Return (IV, c), where $c = (c_0, c_1, \ldots, c_{\ell-1})$.

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6. Return (IV, c), where $c = (c_0, c_1, \ldots, c_{\ell-1})$.

Decryption$((IV, c), k)$

1. Compute $m_0 := D_k(c_0) \oplus IV = D(k, c_0) \oplus IV$
2. For $i = 1, \ldots, \ell - 1$ do
3. Compute $m_i := D_k(c_i) \oplus c_{i-1} = D(k, c_i) \oplus c_{i-1}$
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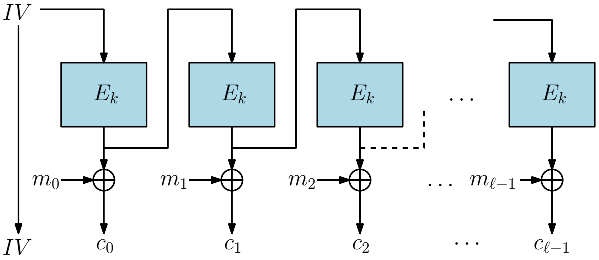
Output FeedBack mode (OFB mode)



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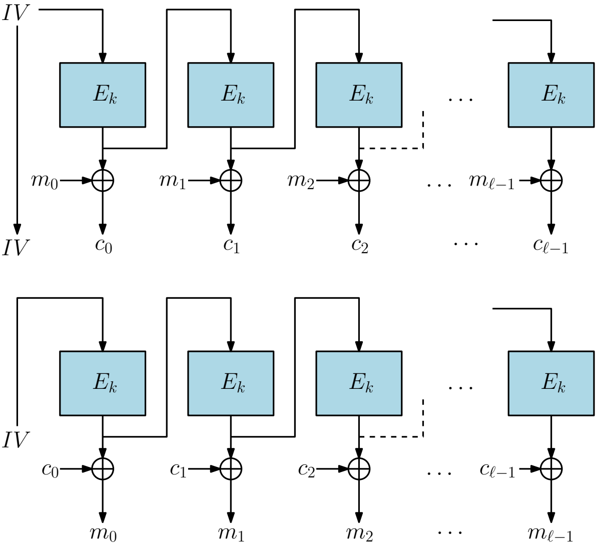

Output FeedBack mode (OFB mode)

Encryption(m, k)

1. Choose a random $IV \xleftarrow{R} \mathcal{X}$
2. $y_0 := E_k(IV) = E(k, IV)$; $c_0 := y_0 \oplus m_0$
3. For $i = 1, \ldots, \ell - 1$ do
4. Compute $y_i := E_k(y_{i-1}) = E(k, y_{i-1})$
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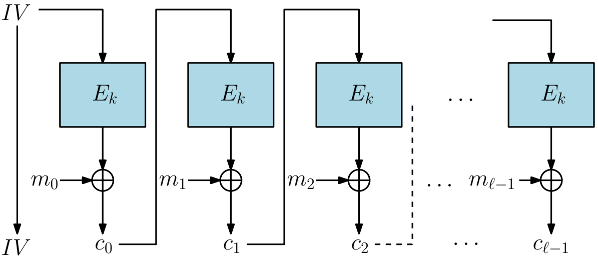
Cipher FeedBack mode (CFB mode)



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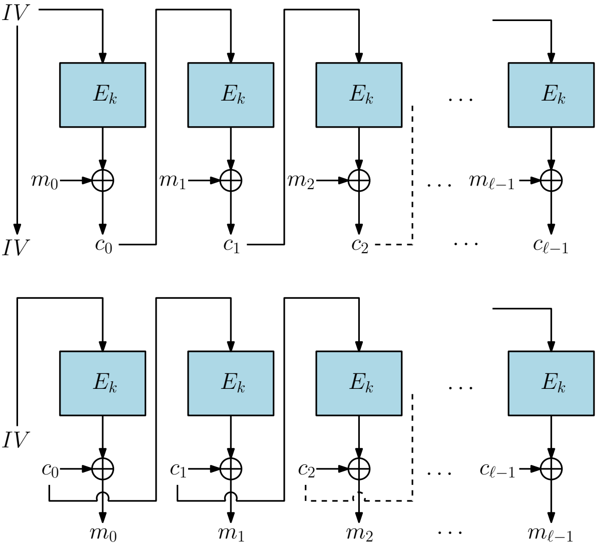

Cipher FeedBack mode (CFB mode)

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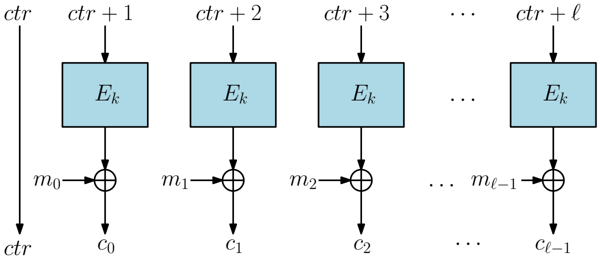

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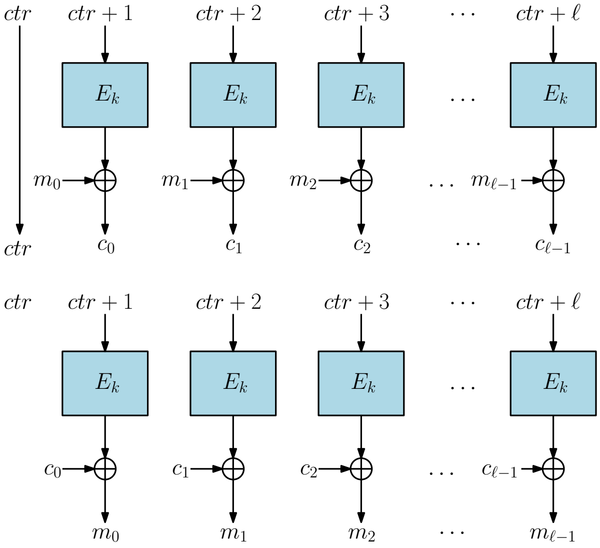

Counter mode (CTR mode)

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1. Choose a random $ctr \xleftarrow{R} \mathcal{X}$
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4. Compute $ctr_i := ctr + i + 1 \pmod{2^n}$
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4. End For;
5. Return $m = (m_0, m_1, \ldots, m_{\ell-1})$.

Theorem

ECB-mode encryption does not have indistinguishable encryptions in the presence of an eavesdropper.

Electronic codebook mode (ECB mode)

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- Not secure.

Electronic codebook mode (ECB mode)

Theorem

Let $\mathfrak{E} = (\mathcal{E}, \mathcal{D})$ be a block cipher. Let $\ell \geqslant 1$ be any poly-bounded value, and let $\mathfrak{E}' = (\mathcal{E}', \mathcal{D}')$ be the ℓ-wise ECB cipher derived from $\mathfrak{E}$, but with the message space restricted to all sequences of at most ℓ distinct data blocks. If $\mathfrak{E}$ is a secure block cipher, then $\mathfrak{E}'$ is a semantically secure cipher.

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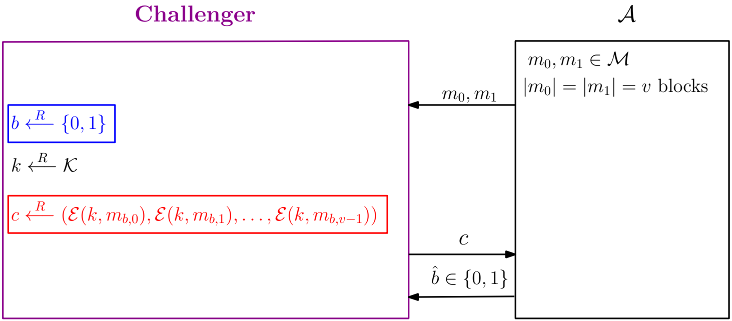
In particular, for every indistinguishability adversary $\mathcal{A}$ that plays symmetric-encryption indistinguishability with respect to $\mathfrak{E}'$, there exists a BC adversary $\mathcal{B}$ that plays PRP indistinguishability with respect to $\mathfrak{E}$, where $\mathcal{B}$ calls $\mathcal{A}$ as subroutine, such that

$$\mathsf{INDadv}[\mathcal{A}, \mathfrak{E}'] = 2 \cdot \mathsf{BCadv}[\mathcal{B}, \mathfrak{E}].$$

Electronic codebook mode (ECB mode)

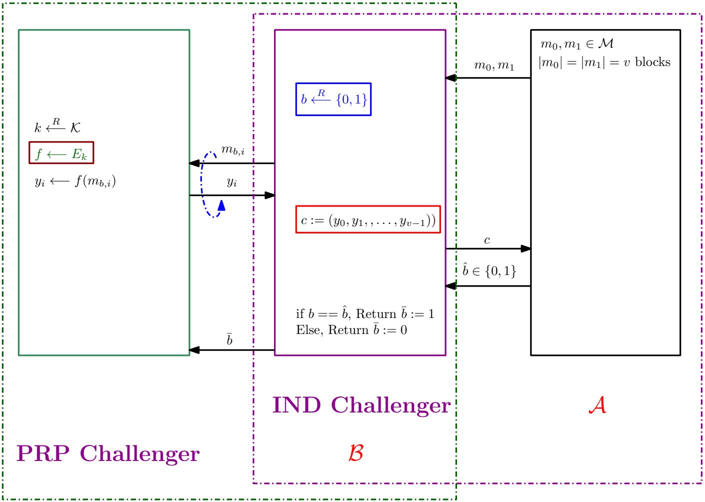
Proof

- If $\mathfrak{E}$ is defined over $(\mathcal{K}, \mathcal{X})$, let $\mathcal{X}_*^{\leqslant \ell}$ denote the set of all sequences of at most ℓ distinct elements of $\mathcal{X}$.

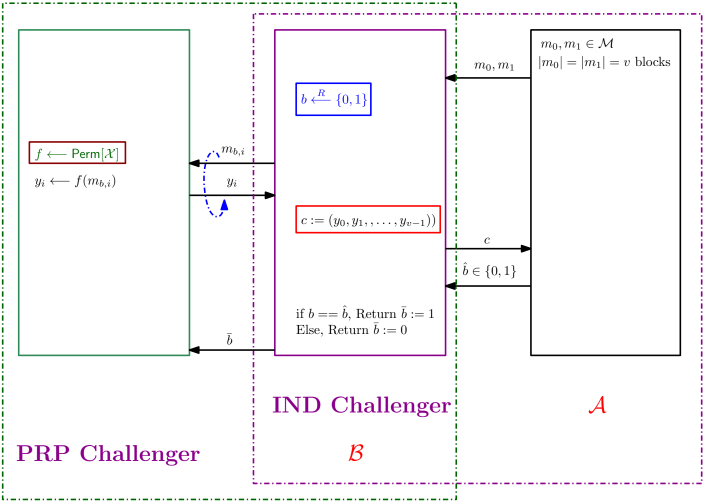


IND Bit-Guessing Experiment

Electronic codebook mode (ECB mode)

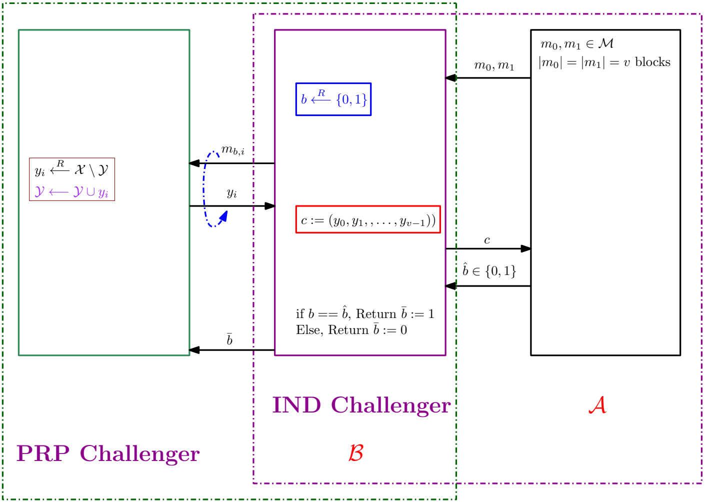


Game 0


$m_0, m_1 \in \mathcal{M}$
$|m_0| = |m_1| = v$ blocks
m_0, m_1
$b \xleftarrow{R} \{0,1\}$
$f \longleftarrow \mathsf{Perm}[\mathcal{X}]$
$y_i \longleftarrow f(m_{b,i})$
$m_{b,i}$
y_i
$c := (y_0, y_1, , \ldots, y_{v-1}))$
c
$\hat{b} \in \{0,1\}$
if $b == \hat{b}$, Return $\bar{b} := 1$
Else, Return $\bar{b} := 0$
$\bar{b}$
IND Challenger
$\mathcal{A}$
PRP Challenger
$\mathcal{B}$

Game 1

Electronic codebook mode (ECB mode)



Game 2

Proof

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Proof

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$$\Pr[W_1] = \Pr[W_2].$$

Electronic codebook mode (ECB mode)

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- As $\mathsf{INDadv}[\mathcal{A}, \mathfrak{E}'] = 2 \cdot \mathsf{INDadv}^*[\mathcal{A}, \mathfrak{E}']$,

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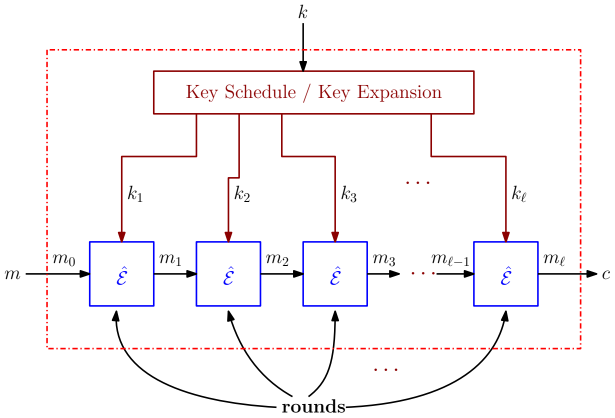
Block Cipher Design Paradigm

Design Paradigm

- Commonly designed as iterated cipher.
- Has a Round Function, say $(\hat{\mathcal{E}}, \hat{\mathcal{D}})$.
- Has a Key Schedule algorithm.
  - $k_1, k_2, \ldots, k_\ell$ are called Key.
- Round function is applied multiple times, say ℓ times.

Block Cipher Design Paradigm

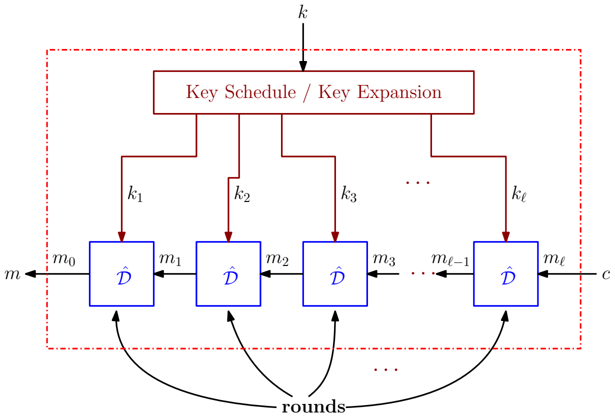


Block Cipher Design Paradigm

$c := \mathcal{E}(k,m)$

$$
\begin{aligned}
m_0 &\longleftarrow m;\\
m_1 &\longleftarrow \hat{\mathcal{E}}(k_1, m_0);\\
m_2 &\longleftarrow \hat{\mathcal{E}}(k_2, m_1);\\
m_3 &\longleftarrow \hat{\mathcal{E}}(k_3, m_2);\\
&\vdots\\
m_\ell &\longleftarrow \hat{\mathcal{E}}(k_\ell, m_{\ell-1});\\
c &\longleftarrow m_\ell;
\end{aligned}
$$

Block Cipher Design Paradigm



Block Cipher Design Paradigm

$m := \mathcal{D}(k, c)$

$$\begin{aligned}
c &\longleftarrow m_\ell; \\
m_{\ell-1} &\longleftarrow \hat{\mathcal{D}}(k_\ell, m_\ell); \\
&\vdots \\
m_2 &\longleftarrow \hat{\mathcal{D}}(k_3, m_3); \\
m_1 &\longleftarrow \hat{\mathcal{D}}(k_2, m_2); \\
m_0 &\longleftarrow \hat{\mathcal{D}}(k_1, m_1); \\
m &\longleftarrow m_0;
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Round Function

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- Let $x \in \{0,1\}^{lm}$, then we can write x as

  $x = (x_{<1>}, x_{<2>}, \ldots, x_{<m>})$, where $x_{<i>} = x_{(i-1)l+1} x_{(i-1)l+2} \cdots x_{(i-1)l+l}$.

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- $\hat{\mathcal{E}}_k(x) = f_1(x_{<1>}) \| f_2(x_{<2>}) \| \cdots f_m(x_{<m>})$.

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- By linearity, we imply

$$S(x \oplus y) = S(x) \oplus S(y), \forall x, y.$$

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    - If $\hat{\mathcal{E}}$ is truly random, it is expected to that the change in one bit of input will affect all the output bits.

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- Goal is to achieve avalanche effect.

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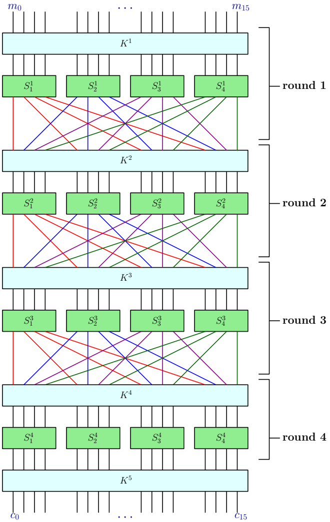
- Small changes to the input have a significant effect on the output.
- Expected result is a pseudorandom permutation.

Substitution-Permutation Network (SPN)

SPN

- Introduced by Feistel in 1973.
- Let l and m be two positive integers.
- Block length = lm
- Has three operations per round:
  - Substitution by S-box,
  - Mixing permutation, and
  - Key Mixing.

Substitution-Permutation Network (SPN)



$\ell = 4$, $l = 4$, and $m = 4$

Substitution-Permutation Network (SPN)

S-Box

$$\pi_S : \{0,1\}^l \longrightarrow \{0,1\}^l.$$

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input	0	1	2	3	4	5	6	7
output	E	4	D	1	2	F	B	8
input	8	9	A	B	C	D	E	F
output	3	A	6	C	5	9	0	7

Mixing Permutation

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input	1	2	3	4	5	6	7	8
output	1	5	9	13	2	6	10	14
input	9	10	11	12	13	14	15	16
output	3	7	11	15	4	8	12	16

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SPN: example one round

- $x = 0010 \;\; 0110 \;\; 1011 \;\; 0111$
- $K^1 = 0011 \;\; 1010 \;\; 1001 \;\; 0100.$

$$\begin{array}{lcl} w0 & = & 0010 \;\; 0110 \;\; 1011 \;\; 0111 \\ K^1 & = & 0011 \;\; 1010 \;\; 1001 \;\; 0100 \end{array}$$

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Substitution-Permutation Network (SPN)

SPN

- Let the input be $x \in \{0,1\}^{lm}$.
- We can write x as $x = x_{<1>} \| x_{<2>} \| \cdots \| x_{<m>}$, where

$$x_{<i>} = x_{(i-1)l+1} x_{(i-1)l+2} \cdots x_{(i-1)l+l}.$$

$w^0 \longleftarrow x$
for $r \longleftarrow 1$ to $\ell - 1$ do
    $u^r \longleftarrow w^{r-1} \oplus K^r$
    for $i \longleftarrow 1$ to m do
        $v^r_{<i>} \longleftarrow \pi_S \left(u^r_{<i>} \right)$
    $v^r := v^r_{<1>} \| v^r_{<2>} \| \cdots \| v^r_{<m>}$
    $w^r \longleftarrow \pi_P \left(v^r \right)$
$u^\ell \longleftarrow w^{\ell-1} \oplus K^\ell$
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$v^\ell := v^\ell_{<1>} \| v^\ell_{<2>} \| \cdots \| v^\ell_{<m>}$
$y \longleftarrow v^\ell \oplus K^{\ell+1}$
Return y

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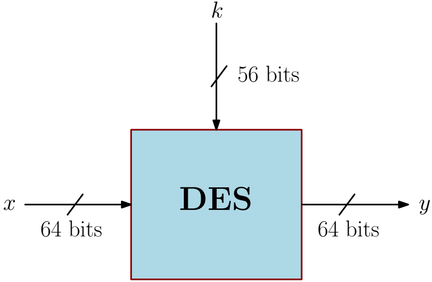
  - Submitted algorithm is based on the cipher Lucifer of 128 data-block with 128-bit key.
  - Lucifer was a family of ciphers developed by Horst Feistel in the late 1960s.

DES

- In 1972, US National Bureau of Standards (NBS), which is now called National Institute of Standards and Technology (NIST), initiated a request for proposals for a standardized cipher in the USA.
- The NBS received the proposal of DES in 1974 from a team of cryptographers working at IBM.
  - Submitted algorithm is based on the cipher Lucifer of 128 data-block with 128-bit key.
  - Lucifer was a family of ciphers developed by Horst Feistel in the late 1960s.
  - DES is a special type of iterated cipher called Feistel Cipher.


k
56 bits
x
64 bits
DES
64 bits
y

DES

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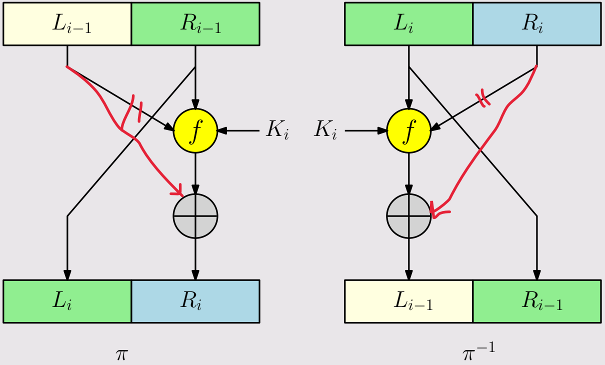
\end{aligned}
$$

Feistel Permutation



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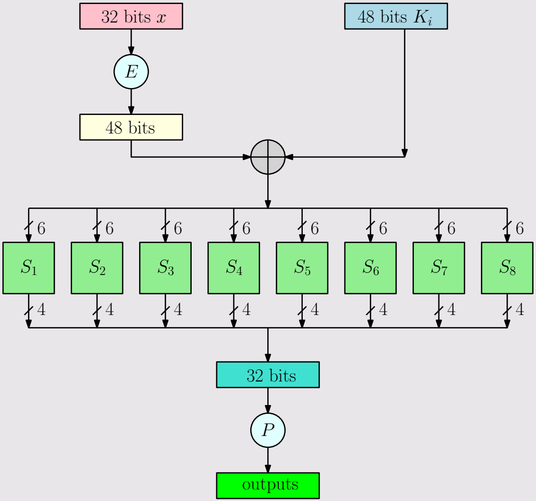
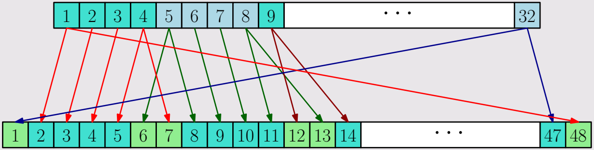
Structure of f



Expansion function E



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  - For S-box S_7, if $b = 110010$, then output is 1111.

16 Column

4 Rows

S_i	0	$\cdots$	8	9	10	$\cdots$	15
0							
1							
2				15			
3							

Exhaustive search on DES

- The adversary is given a small number of plaintext-ciphertext pairs $(x_i, y_i) \in \mathcal{X}^2, 1 \leqslant i \leqslant Q$ using a block cipher key $k \in \mathcal{K}$.

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- For block ciphers like DES and AES-128 three blocks are enough to ensure that with high probability there is a unique key mapping the given plaintext blocks to the given ciphertext blocks.

DES challenges

The DES challenges were set up by RSA data security.

- Rules:
  - n DES outputs $y_1, y_2, \ldots, y_n$ where the first three outputs, y_1, y_2, y_3, were the result of applying DES to the 24-byte plaintext message: (x_1, x_2, x_3)=The unknown message is:
  - The first group to find the corresponding key wins ten thousand US dollars.

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Data Encryption Standard (DES)

Theorem

Let $\mathfrak{E} = (\mathcal{E}, \mathcal{D})$ be a block cipher defined over $(\mathcal{K}, \mathcal{X})$. There is an algorithm $\mathcal{A}_{EX}$ that takes as input Q plaintext/ciphertext pairs $(x_i, y_i) \in \mathcal{X}$ for $i = 1, \ldots, Q$ and outputs a key pair $(k_1, k_2) \in \mathcal{K}^2$ such that

$$\mathcal{E}_2((k_1, k_2), m) := \mathcal{E}(k_2, \mathcal{E}(k_1, m)), \forall i = 1, \ldots, Q.$$

Its running time is dominated by a total of $2Q \cdot |\mathcal{K}|$ evaluations of algorithms $\mathcal{E}$ and $\mathcal{D}$.

Proof

Let $\hat{x} := (x_1, x_2, \ldots, x_Q)$ and $\hat{y} := (y_1, y_2, \ldots, y_Q)$.

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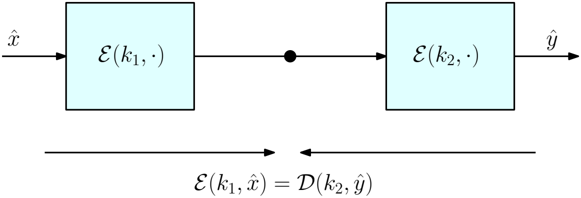

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- Step 1 requires $Q \cdot |\mathcal{K}|$ evaluations of $\mathcal{E}$.
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- Assumption: Insertion in to table T and lookup takes negligible time.

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  - too long to run in practice.

Advanced Encryption Standard (AES)

The AES process

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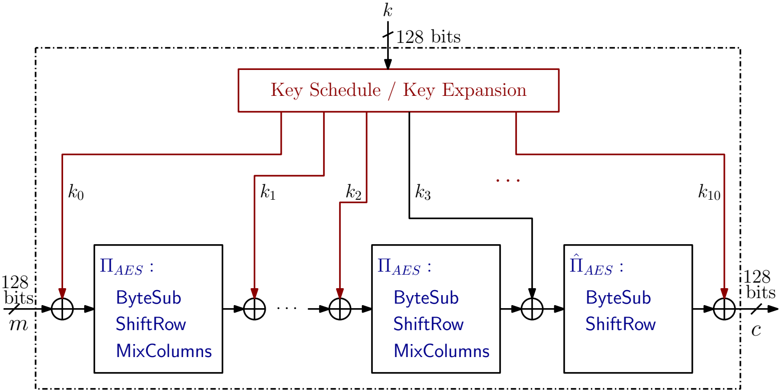
- Rijndael was designed by Belgian cryptographers Joan Daemen and Vincent Rijmen.

Cipher Name	Key-size (bits)	Block Size (bits)	Number of Rounds
AES-128	128	128	10
AES-192	192	128	12
AES-256	256	128	14

Advanced Encryption Standard (AES)



AES 128

AES

- Ciphers that follow the structure shown in Figure are called alternating key ciphers.
- They are also known as iterated Even-Mansour ciphers.

AES round permutation

- The permutation Π_{AES} is made up of a sequence of three invertible operations
  - SubBytes
  - ShiftRows
  - MixColumns

AES round Input

- The 128 bits are organized as a blue 4×4 array of cells, where each cell is made up of eight bits.

$$m = m_0 \| m_1 \| m_2 \| m_3 \| m_4 \| m_5 \| m_6 \| m_7 \| m_8 \| m_9 \| m_{10} \| m_{11} \| m_{12} \| m_{13} \| m_{14} \| m_{15},$$

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AES round operation: SubBytes

- Let $S : \{0,1\}^8 \longrightarrow \{0,1\}^8$ be a fixed permutation (a one-to-one function).
- Applied to each of the 16 cells, one cell at a time.
- The permutation S is specified in the AES standard as a hard-coded table of 256 entries.
- It is designed to have
  - No fixed points, namely $S(x) \neq x$ for all $x \in \{0,1\}^8$.
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$$\begin{pmatrix} S(m_0) & S(m_1) & S(m_2) & S(m_3) \\ S(m_4) & S(m_5) & S(m_6) & S(m_7) \\ S(m_8) & S(m_9) & S(m_{10}) & S(m_{11}) \\ S(m_{12}) & S(m_{13}) & S(m_{14}) & S(m_{15}) \end{pmatrix} = \begin{pmatrix} a_0 & a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 & a_7 \\ a_8 & a_9 & a_{10} & a_{11} \\ a_{12} & a_{13} & a_{14} & a_{15} \end{pmatrix}$$

AES round operation: ShiftRows

- The First row is cyclically shifted zero byte to the left,
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AES round operation: MixColumns

$$\begin{pmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{pmatrix} \times \begin{pmatrix} a_0 & a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 & a_7 \\ a_8 & a_9 & a_{10} & a_{11} \\ a_{12} & a_{13} & a_{14} & a_{15} \end{pmatrix} \longrightarrow \begin{pmatrix} a'_0 & a'_1 & a'_2 & a'_3 \\ a'_5 & a'_6 & a'_7 & a'_4 \\ a'_{10} & a'_{11} & a'_8 & a'_9 \\ a'_{15} & a'_{12} & a'_{13} & a'_{14} \end{pmatrix}$$

AES round operation: MixColumns

- Multiplications are done over the field $GF(2^8)$.
- Irreducible Polynomial: $x^8 + x^4 + x^3 + x + 1$.

End