Tutorial 6

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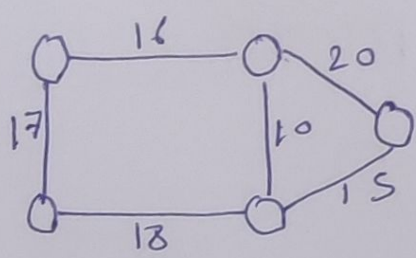
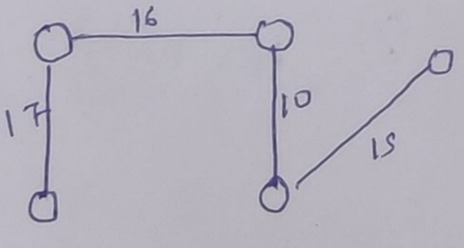
Q.1. Minimum Spanning Tree

A spanning tree of an undirected graph is a subgraph that is a tree & joined by all vertices. One of those tree which has minimum total cost would be its minimum spanning tree.

Eg:



Minimum cost spanning tree



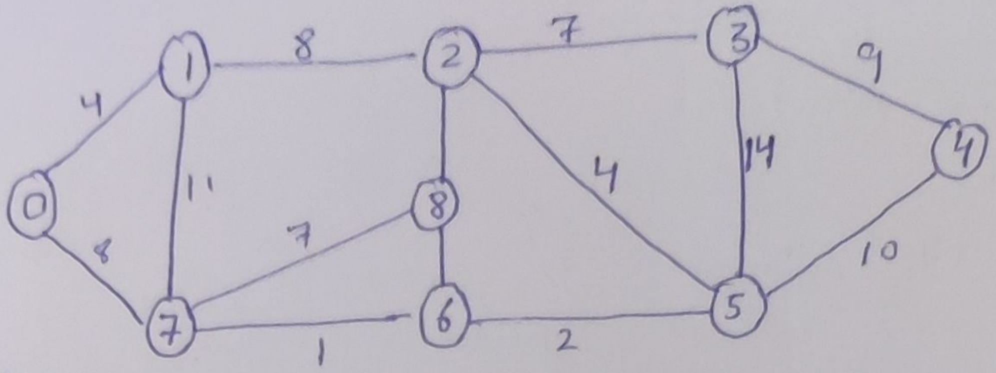
Applications of MST

- It has direct applications in the design of networks including computer networks, telecommunication networks, transportation networks etc.

Q.2.

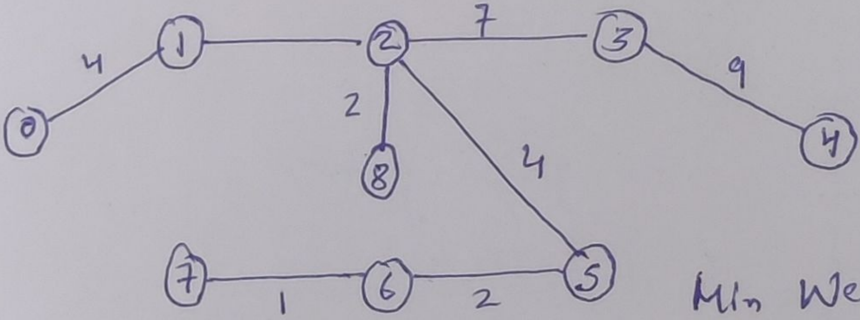
	Prim's Algorithm	Krushkal's A.	Dijkstra's A.	Bellmanford
T.C	$O(V^2)$	$O(E \log V)$	$O(V+E\log V)$	$O(VE)$
S.C.	$O(V+E)$	$O(\|E\|+\|V\|)$	$O(V^2)$	$O(V^2)$

Q.3.



→ Prim's Algo

0	1	2	3	4	5	6	7	8
0	~~∞~~	∞	∞	∞	∞	∞	~~∞~~	∞
	4	8						
			7		$\boxed{4}$		~~8~~	$\boxed{2}$
				~~10~~		~~6~~	~~8~~ 7	
				$\boxed{9}$		$\boxed{2}$	$\boxed{1}$	



Min Weight = 37

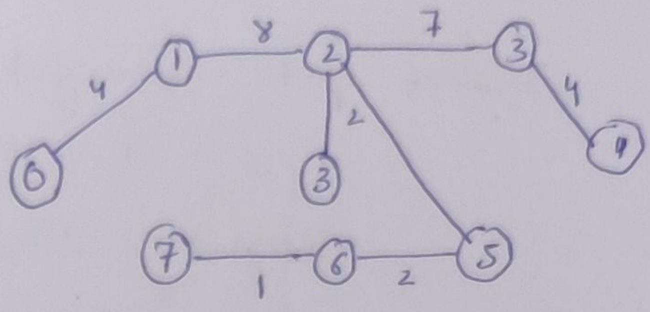
Parent

0	1	2	3	4	5	6	7	8
-1	~~-1~~	~~-1~~	~~-1~~	-1	~~-1~~	-1	~~-1~~	~~-1~~
	0	1	2		2		~~0~~	2
						~~8~~	~~8~~	
				~~8~~		5	6	
				3				

Parent : -1 0 1 2 3 2 5 6 2

→ Krushkal's Algo

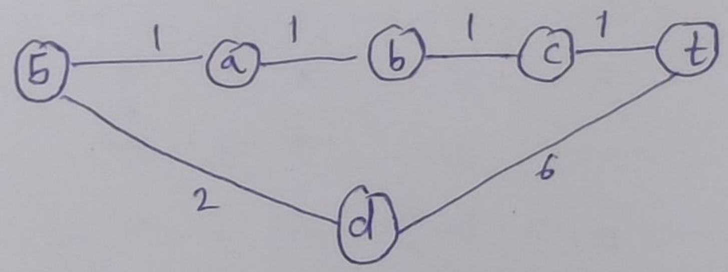
u	v	w	
7	6	1	✓
6	5	2	✓
2	8	2	✓
2	5	4	✓
0	1	4	✓
8	6	6	×
7	8	7	×
2	3	7	✓
1	2	8	✓
0	7	8	×
3	4	9	✓
5	4	10	×
1	7	11	×
3	5	14	×



Weight = 37

Q.4. i) If 10 units is added to each edge, the overall weight of the path may change.

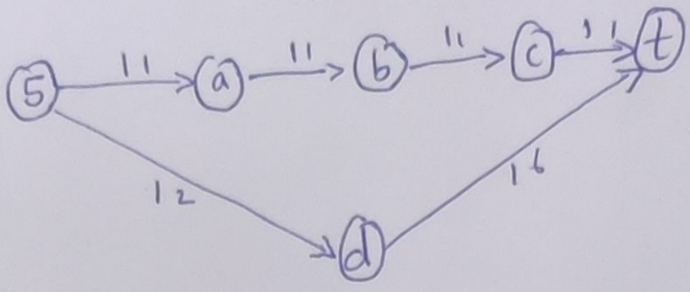
Eg :



Shortest path is 8 ⇉ a → b → c → t

weight 1 + 1 + 1 + 1 = 4

Now if 10 units weight is added to each edge



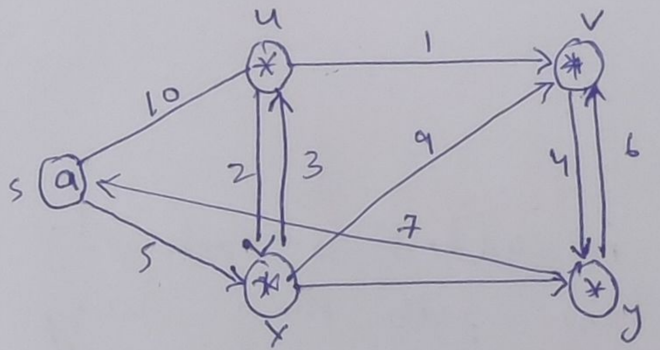
Shortest path changed to

$s \rightarrow d \rightarrow t$

Weight = 28

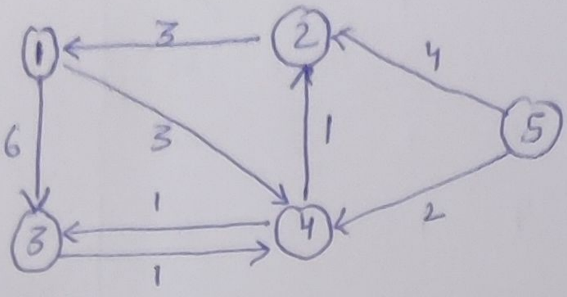
ii) Multiplying the weight of each edge by 10 will have no impact on the shortest path.

Q.5.



s	u	v	x	y
0	∞	∞	∞	∞
0	10	∞	5	∞
0	10	11	5	∞
0	10	11	5	7

All pair shortest path algorithm – Floyd Warshall



$$A^0 = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$$A^1 = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$A^0[2, 3] = \infty$

$A^0[2, 1] + A^0[1, 3] = 3 + 6 = 9$

$9 < \infty$

Similarly $A^0[2, 4] = \infty$

$A^0[2, 1] + A^0[1, 4] = 3 + 3 = 6$

$\Rightarrow 6 < \infty$

$A^0[2, 5] = \infty$

$A^0[2, 1] + A^0[1, 5] = 3 + \infty$

$$A^2 = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array}$$

$A'[1,3] = 6$

$A'[1,2] + A'[2+3] = \infty + 9$

$6 < \infty + 9$

$$A^3 = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array}$$

$$A^4 = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ \infty & 3 & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 3 & 3 & 2 & 0 \end{bmatrix} \end{array}$$

$$A^5 = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ \infty & 3 & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 3 & 3 & 2 & 0 \end{bmatrix} \end{array}$$