Tutorial 4

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Q.1. $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n^2)$

$a \geq 1, b \geq 1$

On comparing

$a = 3, b = 2, f(n) = n^2$

Now, $c = \log_b a = \log_2 3 = 1.584$

$n^2 = n^{1.584} < n^2$

$\because f(n) > n^c$

$\therefore T(n) = O(n^2)$

Q.2. $T(n) = 4T(n/2) + n^2$

$a \geq 1, b > 1$

$a = 4, b = 2$

$c = \log_2 4 = 2$

$n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \theta(n^2 \log_2 n)$

Q.3. $T(n) = T(n/2) + 2^n$

$a = 1, b = 2$

$f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^b = 1$

$f(n) > n^c$

$T(n) = \theta(2^n)$

Q.4. $T(n) = 2^n T(n/2) + n^n$

$a = 2^n$, $b = 2$

$f(n) = n^n$

$c = \log_b a = \log_2 2^n$

$= n$

$n^c \rightarrow n^n$

$f(n) = n^c$

$f(n) = \theta(n^n \log_2 n)$

Q.5. $T(n) = 16\,T(n/4) + n$

$a = 16$, $b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2 \log_4 4$

$= 2$

$n^c = n^2$

$n^c > f(n)$

$\therefore T(n) = \theta(n^2)$

Q.6. $T(n) = 2T(n/2) + n \log n$

$a = 2$, $b = 2$

$f(n) = n \log n$

$c = \log_2 2 = 1$

$n^c = n$

$n \log n > n$

$f(n) > n^c$

$\therefore T(n) = \theta(n \log n)$

Q.7. $T(n) = 2T(n/2) + n/\log n$

$a = 2, b = 2, f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$\frac{n}{\log n} < n$

$f(n) < n^c$

$\therefore T(n) = \theta(n)$

Q.8. $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, b = 4, f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \theta(n^{0.51})$

Q.9. $T(n) = 0.5T(n/2) + 1/n$

$a = 0.5, b = 2$

$a \not\geq 1$ but a is 0.5

So we cannot apply Master's Theorem.

Q.10. $T(n) = 16T(n/4) + n!$

$a = 16, b = 4, f(n) = n!$

$c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

$n! > n^2$

$\therefore T(n) = \theta(n!)$

Q.11. $4T(n/2) + \log n$

$a = 4, \; b = 2, \; f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) < n^c$

$\therefore T(n) = \theta(n^c)$

$= \theta(n^2)$

Q.12. $T(n) = \text{sqrt}(n) + T(n/2) + \log n$

$a = \sqrt{n}, \; b = 2$

$c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2} \log n$

$\therefore \frac{1}{2} \log n < \log n$

$f(n) > n^c$

$T(n) = \theta(\log(n))$

Q.13. $T(n) = 3T(n/2) + n$

$a = 3, \; b = 2, \; f(n) = n$

$c = \log_b a = \log_2 3 = 1.58$

$n^c = n^{1.54}$

$n < n^{1.58}$

$f(n) < n^c$

$T(n) = \theta(n^{1.58})$

Q.14. $T(n) = 3T(n/3) + \text{sqrt}(n)$

$a = 3, \; b = 3$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$\text{sqrt}(n) < n$

$f(n) < n^c$

$T(n) = \theta(n)$

Q.15. $T(n) = 4T(n/2) + n$

$a = 4, \quad b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$n < n^2$ (for any constant)

$f(n) < n^c$

$f(n) = \theta(n^2)$

Q.16. $T(n) = 3T(n/4) + n \log n$

$a = 3, \quad b = 4, \quad f(n) = n \log n$

$c = \log_b a = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$n^{0.792} < n \log n$

$T(n) = \theta(n \log n)$

Q.17. $T(n) = 3T(n/3) + n/2$

$a = 3, \quad b = 3$

$c = \log_b a = \log_3 3 = 1$

$f(n) = n/2$

$n^c = n^1 = n$

$n/2 < n$

$f(n) < n^c$

$T(n) = \theta(n)$

Q.18. $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, \quad b = 3$

$c = \log_b a = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

$n^{1.6309} < n^2 \log n$

$\therefore T(n) = \theta(n^2 \log n)$

Q.19. $T(n) = 4T(n/2) + n/\log n$

$a = 4, \; b = 2, \; f(n) = \frac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\frac{n}{\log n} < n^2$

$T(n) = \theta(n^2)$

Q.20. $T(n) = 64T(n/8) - n^2 \log n$

$a = 64, \; b = 8$

$c = \log_b a = \log_8 64 = \log_8 (8)^2 = 2$

$n^c = n^2$

$n^2 \log n > n^2$

$\therefore T(n) = \theta(n^2 \log n)$

Q.21. $T(n) = 7T(n/3) + n^2$

$a = 7, \; b = 3, \; f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.77}$

$n^{1.77} < n^2$

$T(n) = \theta(n^2)$

Q.22. $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, \; b = 2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$n(2 - \cos n) > n^c$

$\therefore T(n) = \theta(n(2 - \cos n))$