Quantitative Finance

Xi Tan (xtan3.1415926@gmail.com)

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Contents

Preface

This book project, which consists of four subjects: Finance, Mathematics, Statistics, and Computer Science, is tailored specifically to prepare someone for a quant career. It originated from my general belief of the hierarchy of solving a problem — problems are solved at strategic, tactical, and operational levels.

Microeconomics and *Macroeconomics* explain the driving forces of capital markets, from a legislator's perspective. *Accounting* and *Corporate Finance* take a closer and necessary look at these forces, from a different angle. *Stochastic Calculus* and *Asset Pricing* provide with a set of tools and ideas that enables us to **strategically** model one of the central problems in Quantitative Finance.

Generally speaking, there are two paths to solve a quantitative finance problem at the **tactical** level: the mathematical way and the statistical way. There are only two pieces of math we need to know: *Analysis*, in particular measure-theoretical probability and differential equations; and *Linear Algebra*, with functional analysis in mind. Statistics, on the other hand, should start with *Statistical Experiment Design*, from which we learn how to collect data for statistical models. Next, the study of *Random Variables* and *Stochastic Processes* introduce the building blocks of the statistical "pillbox", with *Mathematical Statistics* the "scaffold". Once the "pillbox" is ready, we are equipped to tackle our problems using *Machine Learning*, which is essentially a collection of statistical models and optimization algorithms.

Computer Architecture and *Operating System* are respectively about the "hardware" and "software" of a single computer. The interaction of multiple computers is understood in *Computer Network*. Once we are comfortable with these concepts, we will be able to use *Data Structure and Algorithms* to solve problems at the **operational** level, and use *C++* and/or *Java* to implement our ideas.

I am aware that it can take a while, and even multiple advanced degrees, to finish this curriculum, but let's remember the motto from the Leipzig Gewandhaus Orchestra: "***Res severa est verum gaudium***".

Xi Tan
West Lafayette, IN
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Part I

Background

Chapter 1

Economical Finance

1.1 Capital Market Overview

1.2 Trading and Exchanges

1.3 Macroeconomic Environment

1.4 Classic Finance Theory

1.4.1 Time Value of Money

1.4.2 Capital Asset Pricing Model (CAPM)

Chapter 2

Mathematical Finance

2.1 Probability Theory

2.1.1 Probability Space

2.1.2 Information and σ-Algebra

2.1.3 Conditional Expectation

2.1.4 Martingales

2.1.5 Change of Measure and Girsanov's Theorem

2.2 Brownian Motion and Stochastic Calculus

2.2.1 Stochastic Processes

2.2.2 Brownian Motion

2.2.3 Geometric Brownian Motion

2.2.4 Itô Integral and Itô Process

2.2.5 Function of Itô Processes and Itô's Lemma

2.3 Stochastic Differential Equations

2.3.1 The Feynman–Kac Formula

2.3.2 Kolmogorov Forward and Backward Equations

2.3.3 Explicitly Solvable Stochastic Differential Equations

2.3.4 Backward Induction (BI)

2.3.5 Autonomous System

2.3.6 Milstein Method

2.3.7 Euler–Maruyama Method

2.3.8 Runge–Kutta Method

2.4 Probability Distribution

2.4.1 Poisson Distribution

Chapter 3

Statistical Finance

3.1 Regression

3.2 Classification

3.3 Machine Learning

3.4 Monte Carlo Simulation Methods

3.5 Moment Matching Methods

3.6 Copula Methods

Chapter 4

Computational Finance

4.1 Linear Algebra

4.2 Numerical Analysis

4.3 Interpolation Methods

4.4 Root-finding Methods

4.4.1 The Bisection Method

4.4.2 The Newton–Raphson Method

4.4.3 The Secant Method

4.5 Optimization Methods

4.5.1 Linear Optimization

4.5.2 Non-linear Optimization

4.6 Software Engineering

Part II

Pricing, Investment, and Risk Management

Chapter 5

Financial Modeling

5.1 Overview of (Arbitrage) Pricing Strategies

5.2 The Black–Scholes Model and Its Variants

5.2.1 The Black–Scholes Model

5.2.2 The Back–Scholes Model with Constant Dividend Yield (BS-D)

5.2.3 The Black's Model

5.3 Forward Models

5.3.1 Funding Spread Models

5.3.2 Dividend Models

5.4 Short Rate Models

5.4.1 Overview

5.4.2 Ornstein–Uhlenbeck Process

5.4.3 Square-root Process

5.5 Volatility Models

5.5.1 Implied Volatility Surface

5.5.2 Local Volatility Model

$$C(K,T;S_0) = \tag{5.1}$$

Differentiating this twice with respect to K to obtain

$$
\begin{aligned}
\frac{\partial C^2}{\partial K^2} &= \frac{1}{\partial K^2}\left[\int_K^\infty \mathrm{d}\, S_T \phi(S_T, T; S_0) S_T - K \int_K^\infty \mathrm{d}\, S_T \phi(S_T, T; S_0)\right] && (5.2)\\
&= \frac{1}{\partial K}\left[K\phi(K, T; S_0)K - \int_K^\infty \mathrm{d}\, S_T \phi(S_T, T; S_0) - KK\phi(K, T; S_0)\right] && (5.3)\\
&= -\frac{1}{\partial K}\left[\int_K^\infty \mathrm{d}\, S_T \phi(S_T, T; S_0)\right] && (5.4)\\
&= -K\phi(K, T; S_0) && (5.5)\\
& && (5.6)
\end{aligned}
$$

5.5.3 Stochastic Volatility Models

5.6 Jump Models

Chapter 6

Investment Management

6.1 Asset Classes

6.1.1 Fixed Income

6.1.2 Equities

6.1.3 Foreign Exchange

6.1.4 Credit Derivatives

6.1.5 Interest Rate Derivatives

Chapter 7

Risk Management

7.1 Risk Management Overview

7.1.1 Short-term Risk Management: Politics, Macroeconomics, Fundamental Analysis

7.1.2 Mid-term Risk Management: Technical Analysis, Sensitivity Analysis

7.1.3 Short-term Risk Management

7.2 Option Greeks

7.2.1 Delta

7.2.2 Gamma

7.2.3 Vega

7.2.4 Theta

7.2.5 Hedging

7.3 Correlation and Skew

7.3.1 Implied Volatility Surface

7.3.2 Correlation

7.3.3 Skew

7.4 Term Structure, Duration, and Convexity

7.4.1 Term Structure of Interest Rate

7.4.2 Duration

7.4.3 Convexity

Part III

Miscellaneous

Chapter 8

Study Notes

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Index