品职教育
PZACADEMY.COM

Quantitative Analysis

FRM一级知识框架图

Quantitative Methods

讲师：何 旋

www.pzacademy.com

Framework

Probability & Statistics	1. Probability
	2. Basic Statistics
	3. Distributions
	4. Hypothesis Tests and Confidence Intervals
Linear Regression	5. Linear Regression with One Regressor
	6. Linear Regression with Multiple Regressors
	7. Elements of Forecasting
Simulation, Volatilities and Correlation	8. Simulation Modeling
	9. Estimating Volatilities and Correlations
	10. Correlations and Copulas

Section 1

Probability

Basic Probability Concepts 概念对比、结论

Properties of Probability	✓ $0 \le P(E) \le 1$ ✓ $P(E_1) + P(E_2) + \ldots\ldots + P(E_n) = 1$ • $E_1 \ldots\ldots E_n$: Mutually exclusive and Exhaustive	
Event分类	Mutually exclusive	P(AB)=P(A|B)=P(B|A)=0
	Exhaustive events	include all possible outcomes
	Independence ★★	✓ P(AB)=P(A)×P(B) ✓ If exclusive, must not independence ✓ Independence →ρ=0; 反之不对

Basic Probability Calculation ★★ 概念对比

Joint probability	Multiplication rule： P(AB)=P(A|B)×P(B)= P(B|A)×P(A)
	A and B mutually exclusive events： P(AB)=P(A|B)=P(B|A)=0
Probability that at least one of two events will occur	Addition rule: P(A or B)=P(A)+P(B)-P(AB)
	A and B mutually exclusive events： P(A or B)=P(A)+P(B)

Probability Distribution ★ 概念对比

分类	定义	性质
Discrete	the number of outcomes is **counted**	measurable and positive probability
Continuous	the number of outcomes is **infinite**	P (x)=0 even though x can occur
注意：Cumulative probability function $F(X) = P(X \le x)$		

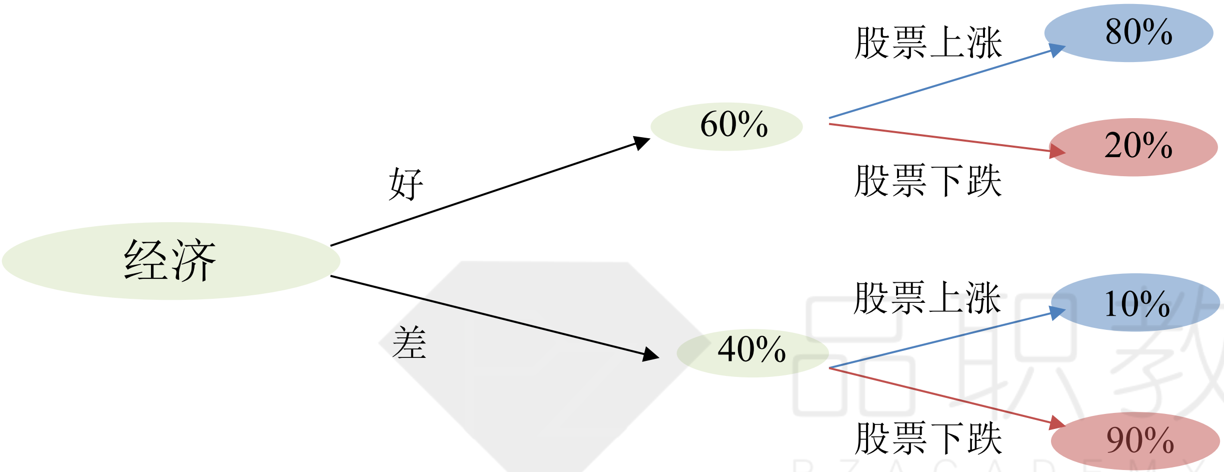
分类	定义	性质
Discrete	the number of outcomes is **counted**	measurable and positive probability
Continuous	the number of outcomes is **infinite**	P (x)=0 even though x can occur
注意：Cumulative probability function $F(X) = P(X \le x)$		

Prob

Probability function	For discrete random variables， $0 \le p(x) \le 1$， $\Sigma p(x)=1$
Probability density function (p.d.f): f(x)	For continuous random variable commonly
Cumulative probability function (c.p.f): F(x)	Discrete: F(x)=P(X<=x)， Continuous: $F(x) = \int_{-\infty}^{x} f(u)du$

类型	性质&计算
Probability Distribution of a Discrete Random Variable	$f(X=x_i)=P(X=x_i), i=1,2,3...$ $f(X=x_i)=0, x\neq x_i$，$0\leq f(x_i)\leq 1$，$\sum_x f(x_i)=1$
Probability Distribution of a Continuous Random Variable	$$P(x_1<X<x_2)=\int_{x_1}^{x_2} f(x)dx$$ 1. The total area under the curve f(x) is 1 2. $P(x_1<X<x_2)$is the area under the curve between x_1 and x_2. 3. $P(x_1\leq X\leq x_2)=P(x_1<X\leq x_2)=P(x_1\leq X<x_2)=P(x_1<X<x_2)$
Cumulative Distribution Function (CDF)	$F(X)=P(X\leq x)$ 1. $F(-\infty)=0$ and $F(\infty)=1$ 2. F(X) is a non-decreasing function such that if $x_2>x_1$ then $F(x_2)\geq F(x_1)$ 3. $P(X\geq k)=1-F(k)$ 4. $P(x_1\leq X\leq x_2)=F(x_2)-F(x_1)$
The bivariate probability distribution	f(X,Y)=P(X=x and Y=y) 1. f(X,Y) ≧0 for all pairs of X and Y. This is so because all probabilities are nonnegative. 2. $\sum\sum f(X,Y)=1$

Bayes' Formula ★★★ 计算，用二叉树图形，不用记公式



注意：把非条件概率画在第一支

$$P(A|B)=\frac{P(B|A)}{P(B)}*P(A)$$

Total Probability Rule:

$$P(R)=P(R|S_1)\times P(S_1)+P(R|S_2)\times P(S_2)+\cdots+\ P(R|S_n)\times P(S_n)$$

$$P(S_i\,|\,R)=\frac{P(R\,|\,S_i)P(S_i)}{P(R)}$$

Section 2

Basic Statistics

Descriptive statistics ★★★ 计算、性质

	计算	性质
Expected Value	$E(X)=\sum P(x_i)x_i$ $=P(x_1)x_1+P(x_2)x_2+\ldots+P(x_n)x_n$	1. If b is a constant, E(b)=b 2. E(X+Y)=E(X)+E(Y) ***3. In general, E(XY)≠E(X)E(Y); If X and Y are independent random variables, then E(XY) =E(X)E(Y)*** 4. $E(X^2) \neq [E(X)]^2$ 5. If a is a constant, E(aX)=aE(X) ***6. If a and b are constants, then E(aX+b)=aE(X)+E(b)=aE(X)+b***
Variance	$\operatorname{var}(X)=\sigma_x^{\ 2}=E(X-\mu_X)^2$ $\operatorname{var}(X)=E(X^2)-[E(X)]^2$	***1. The variance of a constant is zero.*** By definition, a constant has no variability. 2. If X and Y are two ***independent*** random variables, then: var(X+Y)=var(X)+var(Y) and var(X-Y)=var(X)+var(Y) 3. If a,b is a constant, then: $\operatorname{var}(aX+b)=a^2\operatorname{var}(X)$ ***4. If a and b are constant, then: $\operatorname{var}(aX+b)=a^2\operatorname{var}(X)$*** 5. For computational convenience, we can get: $\operatorname{var}(X)=E(X^2)-[E(X)]^2$, that $E(X^2)=\sum_x x^2 p(x)$

Population Mean: $\mu=\frac{\sum_{i=1}^{N} X_i}{N}$; Population Variance: $\sigma^2=\frac{\sum_{i=1}^{N}(X_i-\mu)^2}{N}$. Sample Mean: $\overline{X}=\frac{\sum_{i=1}^{n} X_i}{n}$; Sample Variance: $s^2=\frac{\sum_{i=1}^{n}(X_i-\overline{X})^2}{n-1}$

	计算	性质
Covariance	$COV = E[(X-E(X))(Y-E(Y))]$	✓ how one random variable moves with another random variable ✓ The covariance of X with itself is equal to the variance of X ✓ Covariance ranges from ***negative infinity to positive infinity***
Correlation	$\rho_{XY} = \dfrac{COV(X,Y)}{\sqrt{Var(X)Var(Y)}}$	✓ Measure ***the linear relationship*** between two random variables ✓ ***No units***, ranges from ***–1 to +1***, standardization of covariance ✓ If ***ρ=0***, this ***doesn't indicates independence***. ✓ Variances of correlated Variables: $\operatorname{var}(X \pm Y) = \operatorname{var}(X) + \operatorname{var}(Y) \pm 2\rho\sigma_x\sigma_y$
Coefficient of variation	$CV = \dfrac{S_x}{\overline{X}} \times 100\%$	✓ measures the amount of dispersion in a distribution relative to the distribution's mean. (***relative dispersion***)

Portfolio Expected Return: $E(r_p) = \sum_{i=1}^{n} w_i E(R_i)$; Portfolio Variance: $\sigma^2{}_p = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \operatorname{cov}(R_i, R_j)$

Moment	$m_k = E\left[X^K\right]$	✓ If k=1, then m_1=E[X], it is mean
Central moment	$\mu_K = E\left[(X-\mu)^K\right]$	✓ k=1→0； k=2→variance； k=3→skewness； k=4→kurtosis

Skewness★★★

Positive skewed	✓ Mode<median<mean ✓ right fat tail(The third cross central moment is known as coskewness) ✓ frequent small losses and a few extreme gains (mean＝0时) ✓ 投资者更加prefer positive skewness
Negative skewed	✓ Mode>media>mean ✓ left fat tail(the fourth cross central moment is known as cokurtosis) ✓ frequent small gains and a few extreme losses. (mean＝0时)

kurtosis★★★

Leptokurtic	✓ Sample kurtosis>3； Excess kurtosis>0 ✓ 相同σ，尖峰肥尾(more frequent extremely large deviations from the mean than a normal distribution.)
Platykurtic	Sample kurtosis<3； Excess kurtosis<0
Normal distribution	Sample kurtosis=3； Excess kurtosis=0

t分布：和z分布比，低峰肥尾。所以t分布更加分散一些，σ更大。

Chebyshev's inequality ★★ 计算

计算：$P(|X-\mu| \le k\sigma) \ge 1 - 1/k^2$， $k>1$

1. ***regardless of the shape of the distribution;***
2. the proportion of the values that lie within k standard deviations of the mean is at least $1 - 1/k^2$

Section 3

Distributions

Parametric and Nonparametric Distributions★ 概念对比

Parametric distribution	✓ can be described by using a ***mathematical function***. ✓ also make restrictive assumptions, which are ***not necessarily supported by real-world pattern***
Nonparametric distribution	✓ ***cannot be described by using a mathematical function***.

Discrete Probability Distribution

类型	性质&计算
Discrete uniform	例：X={1,2,3,4,5}, p(x)=0.2
Bernoulli random variable	P(Y=1)=p，P(Y=0)=1-p Expectation=p, Variance=p(1-p)
Binomial random variable★★	$p(x)=P(X=x)={}_nC_x p^x(1-p)^{n-x}$ Expectation=np, Variance=np(1-p)
Poisson Distribution★★★	$p(k)=P(X=k)=\frac{\lambda^k}{k!}e^{-\lambda}$ ，$(\lambda=np)$ E(X)=var(X)= λ ✓ X refers to the number of success per unit ✓ λ indicates the rate of occurrence of the random events

Continuous Probability Distribution

<table>
<tr><th>类型</th><th colspan="2">性质&计算</th></tr>
<tr><td>Uniform Distribution</td><td colspan="2">取值区间：(a, b)
$P(x_1 \le X \le x_2) = (x_2 - x_1)/(b-a)$</td></tr>
<tr><td rowspan="3">Normal distribution
★★★</td><td>Properties</td><td>✓ X~N(μ , σ²); 取值区间：(-∞, +∞)
✓ Symmetrical distribution: skewness=0; kurtosis=3
✓ A linear combination of normally distributed is also normally distributed.
✓ The tails get thin and go to zero but extend infinitely.</td></tr>
<tr><td>confidence intervals
★★</td><td>68% confidence interval is $[\mu-\sigma, \mu+\sigma]$
90% confidence interval is $[\mu-1.65\sigma, \mu+1.65\sigma]$
95% confidence interval is $[\mu-1.96\sigma, \mu+1.96\sigma]$
99% confidence interval is $[\mu-2.58\sigma, \mu+2.58\sigma]$</td></tr>
<tr><td>Standard, Z分布</td><td>$Z = \dfrac{X-\mu}{\sigma}$</td></tr>
<tr><td>Lognormal distribution
★★★</td><td colspan="2">✓ If lnX is normal, then X is lognormal.
✓ Lognormal →the price of asset; normal →the return of asset
✓ Right skewed; Bounded from below by zero (取值不能小于0)</td></tr>
</table>

类型	性质&计算	
Chi-Square Distribution	Probability Distribution	$\chi^2_{n-1} = \dfrac{(n-1)s^2}{\sigma_0^2}$
	Properties	✓ ***only positive value,*** ranges from 0 to infinity ✓ ***positive skewed distribution***, the degree of the skewness depending on the d.f. ✓ ***E(X)= k, var(X)=2k***, where k is the d.f.
Student's T-distribution ★★★	Probability Distribution	$t = \dfrac{\bar{X} - \mu_X}{S_x / \sqrt{n}} \sim t_{(n-1)}$ → $X \sim N(0,1); Y \sim \chi^2(n)$; 且X, Y独立， $t = \dfrac{X}{\sqrt{Y/n}} \sim t(n)$
	Properties	✓ ***Symmetric*** ✓ Degrees of freedom (df): ***n-1*** ✓ Mean=**0**, variance=***n/(n-2)*** ✓ ***flatter*** than the normal distribution, but as ***n increases***, the variance of the t distribution ***approaches*** the variance of the standard normal distribution.
F Distribution	Probability Distribution	$F = \dfrac{S_X^2}{S_Y^2} = \dfrac{\sum (X_i - \bar{X})^2 / (m-1)}{\sum (Y_i - \bar{Y})^2 / (n-1)} \sim F_{(m-1),\ (n-1)}$ → $U \sim \chi^2(n_1), V \sim \chi^2(n_2)$, 且$U,V$独立, $F = \dfrac{U/n_1}{V/n_2} \sim F(n_1, n_2)$
	Properties	***Skewed to the right*** and also ***ranges between 0 and infinity***.

Mixture Distribution

了解、名词掌握

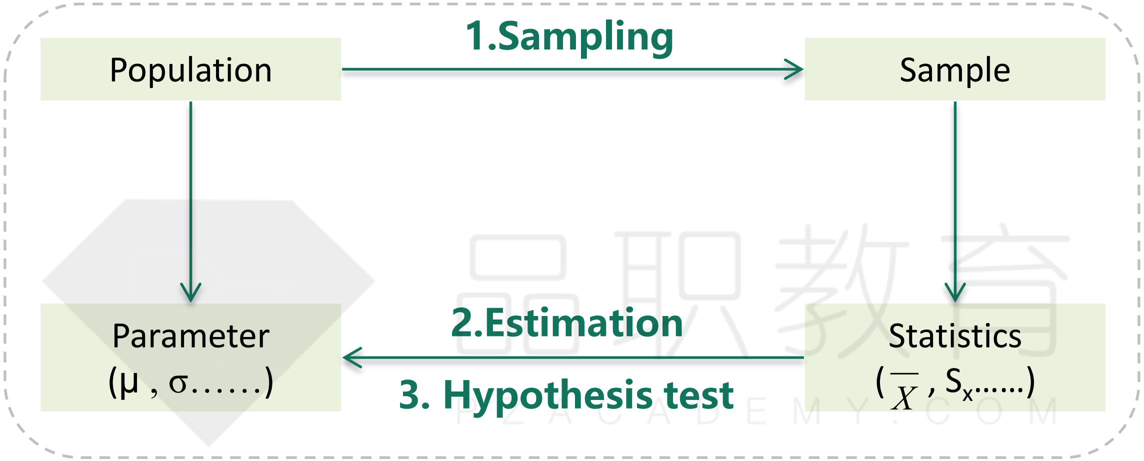
$f(x)=\sum_{i=1}^{n} w_i p_i(x)$, where $p(.)$ are pdf

1. In a typical mixture distribution, the ***component distributions are parametric*** but the ***weights are based on empirical (non-parametric) data***;
2. there is a ***trade-off*** between parametric distributions and non-parametric distributions;
3. By adding more and more component distributions, we can approximate any data set with increasing precision.

Section 4

Hypothesis Tests and Confidence Intervals

Framework



1.Sampling
Population
Sample
Parameter
(μ , σ……)
2.Estimation
3. Hypothesis test
Statistics
($\overline{X}$, S_x……)

1. Sampling

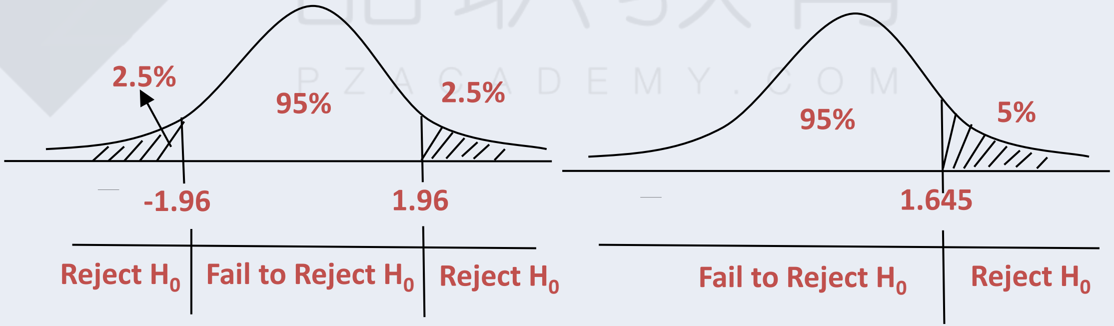
抽样方法	✓ Simple random sampling ✓ Stratified random sampling: to separate the population into smaller groups
Sample statistic 特点★★	✓ ***sampling error*** of the mean= sample mean- population mean ✓ The sample statistic itself is a random variable ✓ ***Central Limit Theory***：n>=30 → sample mean ~$N(\mu\ ,\sigma^2/n)$ ✓ Standard error = $\ /\sqrt{n}$ or $s/\sqrt{n}$

2. Estimation

Desirable properties ★★	✓ ***Unbiasedness:*** expected value of the estimator is equal to parameter that estimate ✓ ***Efficiency:*** variance is smaller ✓ ***Consistency:*** the accuracy increases as n increases.
	Best Linear Unbiased Estimator (BLUE): best available (i.e., has the minimum variance), exhibits linearity, and is unbiased
Estimation ★★★	1. Point estimate 2. Confidence interval estimate:★★ $\bar{x} \pm z_{/2}\frac{\ }{\sqrt{n}}$ or $\bar{x} \pm t_{/2}\frac{s}{\sqrt{n}}$
选择哪一个分布？ ★	✓ 方差已知用z，方差未知用t，非正态总体小样本不可估计； ✓ 如果n≥30，都可以用z.

3. Hypothesis test ★★★

步骤：检验μ

1. 提出假设	• Two-tailed $H_0 : \quad = \ _0 \quad H_a : \quad \ _0$ • One-tailed $H_0 : \mu \geq \mu_0 \ H_a : \mu < \mu_0 \ or, H_0 : \mu \leq \mu_0 \ H_a : \mu > \mu_0$ • H_0 is what we want to reject
2. 计算test statistic	$Test\ Statistic = \dfrac{\overline{X} - \mu_0}{s / \sqrt{n}}$
3. 画分布找到 critical value	
4. 判断	Reject H_0 if |test statistic|>critical value →***** is significantly different from ****** Fail to reject H_0 if |test statistic|<critical value →***is not significantly different from ***

P – value ➡ P – value <α →reject H_0

Type I error and Type II error

Decision	True condition	
	H_0 √	H_0 X
Do not reject H_0	Correct Decision	Incorrect Decision Type Ⅱ error
Reject H_0	Incorrect Decision Significance level =P (Type I error)	Correct Decision Power of test = 1- P (Type Ⅱ error)

结论: 1. Type I error ↑→ Type Ⅱ error ↓; 2. Increase the Sample Size →Type I error & Type Ⅱ error ↓

其他检验

Test type	Assumptions	H_0	Test-statistic	distribution
Mean hypothesis testing	Normally distributed population, known population variance	μ=0	$Z=\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$	N(0,1)
	Normally distributed population, unknown population variance	μ=0	$t=\frac{\bar{x}-\mu_0}{s/\sqrt{n}}$	t(n-1)
Variance hypothesis testing	Normally distributed population	$\sigma^2=\sigma_0^2$	$\chi^2=\frac{(n-1)s^2}{\sigma_0^2}$	$\chi^2(n-1)$
	Two independent normally distributed populations	$\sigma_1^2=\sigma_2^2$	$F=s_1^2/s_2^2$	$F(n_1-1,n_2-1)$

Section 5

Linear Regression with One Regressor

The Basics of Simple Linear Regression

建模 → ANOVA Table分析 → 检验模型 → 预测

1.建模

$$Y_i = b_0 + b_1 X_i + \varepsilon_i, i = 1,\ldots,n$$

- Y_i = ***dependent variable***, explained variable, predicted variable
- X_i = ***independent variable***, explanatory variable, predicting variable.

Assumption ★	① A linear relationship exists between X and Y ② X is uncorrelated with the error term. ③ The expected value of the error term is zero (i.e., $E(\varepsilon_i)=0$) *④ The variance of the error term is constant (homoskedastic)* *⑤ The error term is uncorrelated across observations ($E(\varepsilon_i\varepsilon_j)=0$ for all $i \neq j$)* ⑥ The error term is normally distributed.
Coefficient估计 ★★	解释： ✓ An estimated slope coefficient of 2：Y will change two units for every 1 unit change in X. ✓ Intercept term of 2%：the X is zero, Y is 2%.
	$b_1 = \dfrac{Cov(X,Y)}{Var(X)}$ $b_0 = \bar{Y} - b_1\bar{X}$ 计算

2. ANOVA Table分析 ★★★

	df	SS	MSS
Explained (Regression)	k=1	ESS	MSE=ESS/k
Residual	n-k-1	SSR	MSR=SSR/(n-2)
Total	n-1	SST	-

Coefficient Determination (R^2)	**计算**：$R^2 = \frac{ESS}{TSS} = 1 - \frac{SSR}{TSS} = \frac{\text{explained variation}}{\text{total variation}} = 1 - \frac{\text{unexplained variation}}{\text{total variation}}$
	$R^2 = r^2_{Y\hat{Y}}$ (多元都成立) $R^2 = r^2_{XY}$ (一元)
	解释：R^2 of 0.90 indicates that the variation of the independent variable explains 90% of the variation in the dependent variable.
	multiple R = absolute value of correlation between the actual Y and forecast Y
SEE	**计算**：$SER = \sqrt{\frac{SSR}{n-2}} = \sqrt{\frac{e_i^2}{n-2}}$
	性质： ✓ The SER gauges the “fit” of the regression line. The smaller the standard error, the better the fit. ✓ The SER is the standard deviation of the error terms in the regression.

3. 检验模型：回归分析相当于抽样估计

考试时给定条件

	Coefficient	Standard error	t-statistic	p-value
Intercept	$\hat{b}_0$	$s_{\hat{b}_0}$	?	0.18
Slope	$\hat{b}_1$	$s_{\hat{b}_1}$	?	<0.001

参数估计 ★★★ (confidence interval)	$\hat{b}_1 \pm t_c s_{\hat{b}_1}$ → t_c: 查表 { t分布; Confidence level (置信度) } As SER rises, $s_{\hat{b}_1}$ also increases
假设检验 ★★★ (significance test)	✓ H_0: b_1=0 (没有特殊说明，题目中假设检验都是检验是否为0) $t = \frac{\hat{b}_i \quad 0}{s_{\hat{b}_i}}$ df=n-2 ✓ Decision rule: reject H_0 if $+t_{critical} < t$, or $t < -t_{critical}$ ✓ Rejection of the null means that the slope coefficient is different from zero

4.预测 (Predicted Value of Y)

Point estimate★	$\hat{Y} = \hat{b}_0 + \hat{b}_1 X'$
Confidence interval estimate	$\hat{Y} \pm \left(t_c \quad s_f\right)$ 了解

Section 6

Linear Regression with Multiple Regressors

Multiple Regression & Simple Linear Regression

Difference	要点
Interpreting the coefficient	Each slope coefficient is the estimated change in Y for a one unit change in X_i, ***holding the other independent variables constant.***
Significance test ★★★	**单个检验**(t-test)： H_0: b_j=0 $t = \dfrac{\hat{b}_j}{s_{\hat{b}_j}}$ df = n-k-1
	联合检验(F-test)： The test assesses the effectiveness of the model as a whole in explaining the dependent variable H_0: b_1= b_2= b_3= … = b_k=0 H_a: at least one b_j≠0 (j = 1 to k) reject H_0 : if F (test-statistic) > F_c (critical value) $F = \dfrac{ESS/k}{RSS/(n-k-1)}$ The F-test here is always a ***one-tailed test*** Analysts typically ***do not use ANOVA and F-tests in simple linear regression*** because the ***F-statistic is the square of the t-statistic*** for the slope coefficient.

Difference	要点
R^2 ★★	**解释：** R^2 of 0.90 indicates that the model, ***as a whole***, explains 90% of the variation in the dependent variable.
	缺点： R^2 almost always increases as variables are added to the model, even if the ***marginal contribution*** of the new variables is not statistically significant.
	(计算) **Adjusted R^2:** $R^2 = 1 - \frac{n-1}{n-k-1}\frac{SSR}{TSS}$ ➡ $adjusted\ R^2 = 1 - \left[\left(\frac{n-1}{n-k-1}\right)\times\left(1-R^2\right)\right]$
	$R^2 = r^2_{Y\hat{Y}}$ R^2 r^2_{XY}
Assumptions	**增加**: There is no exact linear relation between any two or more independent variables.
	Gauss-Markov Theorem: the OLS estimators are BLUE (best linear unbiased estimators). • The OLS estimated coefficients have the ***minimum variance*** • The OLS estimated coefficients are based on ***linear*** functions. • The OLS estimated coefficients are ***unbiased***. [i.e., $E(b_0) = B_0$ and $E(b_1) = B_1$. • The OLS estimate of the variance of the errors is ***unbiased*** [i.e., $E(\hat{\sigma}^2) = \sigma^2$].

Dummy Variables

了解、结论

Definition	• To use ***qualitative variables*** as independent variables. (two values, ***0 and 1***) • If we want to distinguish between n categories, we need n−1 dummy variables
Testing and Interpreting The Coefficients	The coefficient on dummy variables indicates the difference 例：$EPS_t = b_0 + b_1Q_{1t} + b_2Q_{2t} + b_3Q_{3t} + \varepsilon_t$ ✓ b_0：average value of EPS for the fourth quarter ✓ Slope coefficient：difference in EPS (on average) between the respective quarter (i.e., quarter 1, 2, or 3) and the omitted quarter. 比如，$b_1 = EPS_1 - EPS_4$

Violations of Regression Assumption ★

概念对比、结论

Multicollinearity

Definition	Two or more independent variables are highly (but not perfectly) correlated with each other.
Consequences	• Estimates become extremely imprecise and unreliable. • Impossible to distinguish the individual impacts of the independent variable • Inflated OLS standard error, t-test have little power
Detection	① ***t-tests*** indicate that none of the individual coefficients is significantly different than zero, while the ***F-test*** indicates overall significance and the ***R^2*** is high ② $\lvert r_{X1X2} \rvert > 0.7$
Correction	Remove one or more independent variables

Violations of Regression Assumption

Heteroskedasticity

Definition

The variance of the residuals is ***not the same*** across all observation

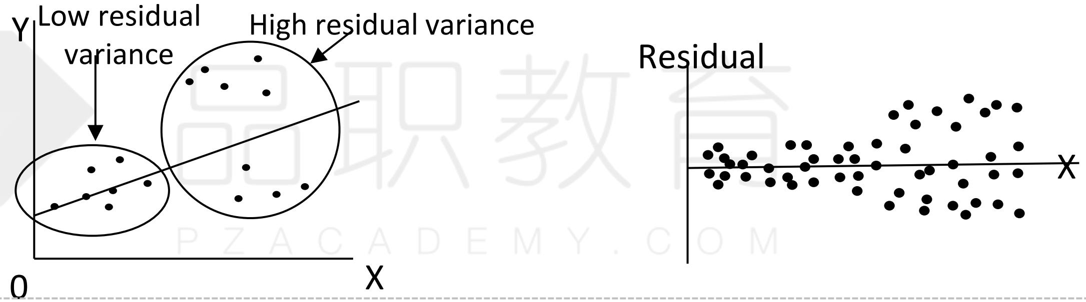
Unconditional heteroskedasticity (***no major problems***)

- the heteroskedasticity is not related to the level of the independent variables

Conditional heteroskedasticity (***create significant problems***)

- Variance of error term is ***related to the level of the independent variables.***

Detecting



Consequences

- The ***coefficient estimates (the b_1) are not affected.***
- The ***standard errors*** are usually ***unreliable*** estimates

Serial Correlation (Autocorrelation)

The error terms are correlated with one another

Positive serial correlation; Negative serial correlation

Omit Variable

Condition	• The omitted variable is correlated with the movement of the independent variable in the model • The omitted variable is a determinant of the dependent variable
Impact	the results will likely lead to incorrect conclusions

Section 7

Elements of Forecasting 了解

Modeling and Forecasting Trend

Modeling Trend ➡
- Simple linear function of time: $T_t = \beta_0 + \beta_1 TIME_t$
- Quadratic trend models: $T_t = \beta_0 + \beta_1 TIME_t + \beta_2 TIME_t^2$
- Exponential trend (log-linear trend): $T_t = \beta_0 e^{\beta_1 TIME_t} \rightarrow \ln(T_t) = \ln(\beta_0) + \beta_1 TIME_t$

Selecting Forecasting Models

Criteria	公式	判别
Akaike information criterion(AIC)	$AIC = e^{\left(\frac{2k}{T}\right)} \frac{\sum_{t=1}^{T} e_t^2}{T} \rightarrow$ MSE	优先考虑AIC最小的模型，即最好地解释数据但包含最少自由参数的模型，尽量避免过度拟合(Overfitting)
Schwarz Criteria	$SIC = T^{\left(\frac{k}{T}\right)} \frac{\sum_{t=1}^{T} e_t^2}{T}$	• The model with the lower value of SIC is the one to be preferred. • The SIC generally penalizes free parameters more strongly than does the Akaike information criterion

Modeling and Forecasting Seasonality

The Nature and Sources of Seasonality ➡
- A seasonal pattern is one that ***repeats itself every year***. The annual repetition can be ***exact***, in which case we speak of deterministic seasonality, or ***approximate***, in which case we speak of stochastic seasonality.

Modeling Seasonality ➡
- remove it and then to model and forecast the ***seasonally adjusted time series***.
- Seasonal dummy variables: $y_t = \beta_1 TIME_t + \sum_{i=1}^{s-1} \gamma_i D_{it} + \varepsilon_t$

Characterizing Cycles

Stationary

- Covariance stationary: mean and covariance structure to be stable. Many series that are ***clearly nonstationary in levels*** appear ***covariance stationary in growth rates.***

White noise

- Such a process, with zero mean, constant variance, and no serial correlation, is called zero-mean white noise, or simply white noise. $\varepsilon_t \text{ or } y_t \sim \mathrm{WN}(0,\sigma^2)$

Wold's Theorem

$$y_t = B(L)\varepsilon_t = \sum_{i=0}^{\infty} b_i \varepsilon_{t-i};\ \varepsilon_t \sim WN(0,\sigma^2)$$

Sample autocorrelation function, or correlogram: Box-Pierce Q-statistic & Ljung-Box Q-statistic

Modeling Cycles

AR Model

$$y_t = \varphi y_{t-1} + \varepsilon_t; \varepsilon_t \sim WN(0, \sigma^2)$$

- The AR(1) model is capable of capturing much more persistent dynamics than is the MA(1). $|\varphi| < 1$ is the condition for covariance stationary in the AR(1).

MA Model

$$y_t = \varepsilon_t + \theta \varepsilon_{t-1} = (1 + \theta L)\varepsilon_t; \varepsilon_t \sim WN(0, \sigma^2)$$

- The current value of the observed series is expressed as a function of current and lagged unobservable shocks.
- MA(1) process with parameter θ = 0.95 varies a bit more than the process with a parameter of θ= 0.4.

ARMA(p,q) model

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \ldots + \varphi_p y_{t-p} + \theta \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} + \varepsilon_t; \varepsilon_t \sim WN(0, \sigma^2)$$

Section 8

Simulation Modeling

Monte Carlo Methods ★★★

Characters	✓ based on their ***assumed distributions***, to produce a distribution of possible security values； ✓ It is fairly ***complex*** and will assume a parameter distribution； ✓ It is ***not an analytic method*** but a ***statistical*** one.
Simulating	$dS_t = \mu_t S_t dt + \sigma_t S_t dz$ 计算
Advantage & Disadvantages	✓ **Advantages:** • Flexibility • build the distribution ✓ **Disadvantages** • High computation costs • Results are imprecise • Results are difficult to replicate • Results are experiment-specific
Reducing Sampling Error	✓ The accuracy of simulations depends on the ***standard deviation and the number of scenarios run*** • Variance reduction techniques: antithetic variates, control variates • ***Speed Versus Accuracy***

Bootstrapping Method ★

与Monte Carlo对比

Definition	✓ draws random return data from a sample of historical data, ***repeatedly draws data*** from a historical data； ✓ uses ***actual historical data*** instead of random data from a probability distribution
Advantage and Limitation	✓ **Advantages:** • ***No assumptions*** are made, can include extreme events • Can include ***fat tails, jumps***, or any departure from the normal distribution; • Account for ***correlations*** across series ✓ **Limitation** • has a methodological problem resulting from ***sampling*** from regressors rather than using a fixed estimate in repeated samples. • Any dependency of variables or ***autocorrelations*** in the original data set will no longer be present, because variables are not drawn in the same sequence as the original data set. • For ***small sample sizes***, may be a poor approximation of the actual one • Results are experiment-specific
Ineffective	✓ outliers exist → have a large number of replications ✓ autocorrelation exists → a moving block bootstrap

Random Number Generation 了解

Inverse Transform Method	X~U[0,1] → cumulative normal distribution → normal distribution
Midsquare technique	将数据的键值中的前几位数取出后平方产生一个新的数值，再从新产生的数值中取出中间某几位数作为数据存储的方法
Congruential Pseudorandom Number Generators	$x_n=g(x_{n-1})$ mod m (modulus 系数) $x_n = Ax_{n-1} - m\left\lfloor \frac{Ax_{n-1}}{m} \right\rfloor$

Section 9

Estimating Volatilities and Correlations

Estimating Volatilities

ARCH Model ★★	$\sigma_n^2 = \gamma V_L + \sum_{i=1}^{m} \alpha_i u_{n-i}^2 \quad (\gamma + \sum_{i=1}^{m} \alpha_i = 1) \xrightarrow{\omega = \gamma V_L} \sigma_n^2 = \omega + \sum_{i=1}^{m} \alpha_i u_{n-i}^2$ ***Mean reversion***
EWMA Model ★★★	$\sigma_n^2 = \lambda \sigma_{n-1}^2 + (1-\lambda) u_{n-1}^2$ ✓ High values of λ will minimize the effect of daily percentage returns ✓ 从现在算起, i日前收益率平方的权重记为α_i，那么:$\alpha_i = (1-\lambda)\lambda^{i-1}$ ✓ Attractive Feature • Relatively ***little data*** needs to be stored. • Need only the ***current estimate*** of the variance rate and the ***most recent observation*** on the value of the market variable. • ***Tracks volatility changes.*** • RiskMetrics uses λ = 0.94 for daily volatility forecasting.
GARCH(1, 1) Model (more appealing) ★★★	$\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2 \xrightarrow{\omega = \gamma V_L} \sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$ ***Mean reversion*** ✓ ***Persistence: 1-γ= (α+β)<1*** • The ***higher the persistence,*** the ***longer it will take to revert to the mean.***

Estimating Correlations ★★★

$$\hat{\rho}_{XY} = \frac{\mathrm{cov}_n}{\sigma_{x,n}\sigma_{y,n}}$$

For **EWMA model**: $\mathrm{cov}_n = \lambda \mathrm{cov}_{n-1} + (1-\lambda) x_{n-1} y_{n-1}$

For **GARCH(1,1) model**: $\mathrm{cov}_n = \omega + \alpha x_{n-1} y_{n-1} + \beta \mathrm{cov}_{n-1}$

Mean Reversion ★★★

- In practice, variance rates do tend to be mean reverting.
- When there is non-zero correlation across days, the two-day variance is

$$V(R_2) = V(R_1) + V(R_1) + 2\rho V(R_1) = 2(1+\rho)V(R_1)$$

- ρ is called the (serial) ***autocorrelation coefficient***
    - ***Positive*** coefficient: ***trend***
    - ***Negative*** coefficient: ***mean reversion***

Section 10

Correlations and Copulas 了解

Correlations and Copulas	✓ **Correlation:** ***linear dependency*** ✓ **Copulas:** $f(x,y) = f(x) \times f(y) \times copulas$ • ***creates a joint probability distribution between two or more variables*** • Copula helps transform variables V_i into new variables U_i that have bivariate normal distributions for which it is easy to define a correlation structure. ✓ **One-factor model:** $U_i = a_i F + \sqrt{1-a_i^2} Z_i$ • correlations between normally distributed variables • each U has a component dependent on a common factor, F and a component that is uncorrelated with the other variables. The Z are uncorrelated with each other and uncorrelated with F. • The correlated between U_i and U_j is $a_i a_j$.
Types of Copulas	✓ Gaussian copula: are mapped into standard normal distributions ✓ Student-t copula: variables are assumed to have a bivariate Student-t distribution ✓ Multivariate copula ✓ One-factor copula
Tail Dependence	✓ greater tail dependence in a bivariate Student's t-distribution ✓ the Student's t-copula is better when assets that historically have extreme outliers
Application to Loan Portfolios: Vasicek's Model	$U_i = \sqrt{\rho} F + \sqrt{1-\rho} Z_i \longrightarrow WCDR(T,X) = N\left(\frac{N^{-1}(\text{PD}) + \sqrt{\rho} N^{-1}(X)}{\sqrt{1-\rho}}\right)$

Thank You!

品职教育
PZACADEMY.COM

Foundations of Risk Management

FRM一级知识框架图

Foundation

讲师：何 旋

www.pzacademy.com

Risk Management ★

1. Risk Management: A Helicopter View
2. Corporate Risk Management
3. Risk-Taking and Risk Management by Banks
4. Risk Management Failures

Portfolio Theory ★★★

5. The Standard Capital Asset Pricing Model
6. Applying the CAPM to Performance Measurement
7. Arbitrage Pricing Theory and Multifactor Models of Risk and Return

Data Quality

8. Principles for Effective Risk Data Aggregation and Risk Reporting

Financial Disasters ★★★

9. Financial Disasters
10. Credit Crunch 2007—2008

GARP Code of Conduct ★★

Section 1

Risk management: A Helicopter View

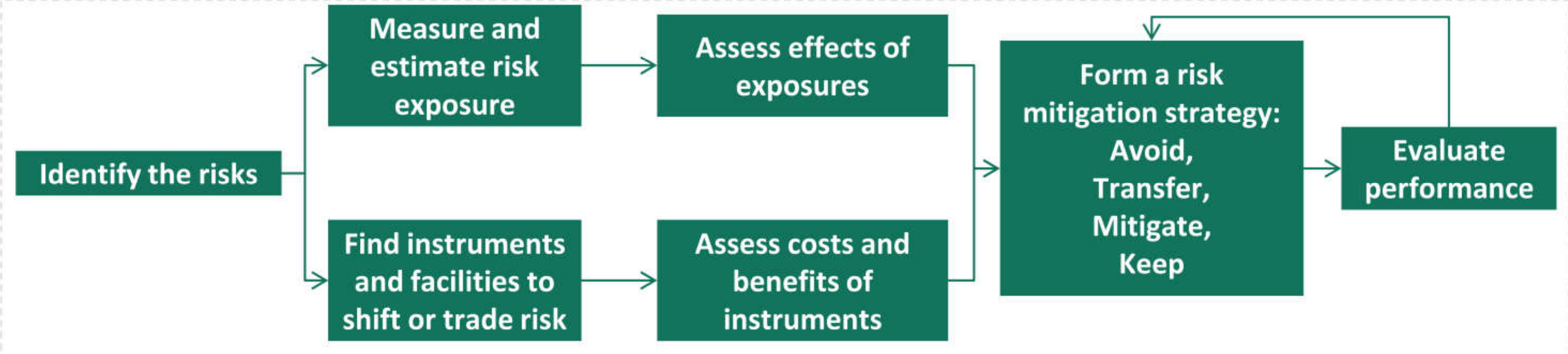
Risk and Risk Management 了解

The Concept of Risk

- **Trade-off between risk and return:**
  - Risk arises from the ***uncertainty*** regarding an entity's future losses as well as future gains.
- **Expected and Unexpected Loss:**
  - Expected loss considers how much an entity expects to lose in the ***normal course of business***. As one of ***the costs of doing business***, and it is ***priced*** into the products and services.
  - Unexpected loss considers how much an entity could lose ***outside of the normal course*** of business.

Risk Management

- Risk management is actually ***broader*** in the sense that it considers how an entity can consciously determine ***how much risk it is willing to take*** to earn future uncertain returns, which involves risk taking.



Risk Classes ★★ 概念对比

- Risk
  - Financial Risks
    - Market Risk
      - Interest rate risk
      - Equity price risk,
      - Foreign exchange risk,
      - commodity price risk.
    - Credit Risk
      - Default risk
      - Bankruptcy risk
      - Downgrade risk
      - Settlement risk
    - Operational Risk
      - inadequate computer systems
      - insufficient internal controls,
      - incompetent management,
      - fraud
      - human error
      - natural disasters.
    - Liquidity Risk
      - Asset liquidity risk
      - Funding liquidity risk
  - Business Risks

Market risk	Definition
Interest rate risk	• If market interest rates rise, ***the value of the bond will decrease.***
Equity price risk	• Two parts: ***(1) general market risk***, which is the sensitivity of the price of a stock to changes in broad market indices, and ***(2) specific risk***, which is the sensitivity of the price of a stock due to unique factors of the entity
Foreign exchange risk	• because of unhedged or not fully hedged foreign currency positions.
Commodity price risk	• Price volatility of commodities. • The resulting lack of trading liquidity tends to increase the amount of price volatility compared to financial securities.
Absolute Risk	Measured in terms of shortfall relative to the initial value of the investment and focuses on the ***volatility of total returns***.
Relative risk	Measured ***in terms of shortfall relative to a benchmark*** (e.g. market index).
Directional risks	Involve exposures to the direction of movements in major financial Market variables. These directional exposures are measured by ***first order*** or linear approximations.
Non-Directional risks	Are risks that have ***non-linear exposures*** or neutral exposures to changes in economic or financial variables
Basis risk	The risk that the price of a hedging instrument and the price of the asset being hedged are nor perfectly correlated.
Volatility risk	Risk of loss from changes in actual or implied volatility of market prices.

Credit risk	Definition
Default risk	• The non-payment of interest and/or principal on a loan by the borrower to the lender. • The period of default past the due date could be at least 30 or 60 days.
Bankruptcy risk	• Take possession of any collateral provided by the defaulting counterparty. The risk is that the liquidation value of the collateral is insufficient to recover the full loss on default.
Downgrade risk	• decreased creditworthiness • A creditor may subsequently charge the downgraded entity a higher lending rate to compensate for the increased risk.
Settlement risk	• Settlement risk could be illustrated using a derivatives transaction between two counterparties. The position that is losing may simply refuse to pay and fulfill its obligations.

Operational Risk

- Operational risk considers a wide range of "non-financial" problems such as ***inadequate computer systems*** (technology risk), ***insufficient internal controls***, ***incompetent management***, ***fraud*** (e.g., losses due to intentional falsification of information), ***human error*** (e.g., losses due to incorrect data entry or accidental deletion of a file), and ***natural disasters***.

Measuring and Managing Risk 了解

Types	Description
Quantitative Measures	• ***Value at risk (VaR)*** states a certain loss amount and its probability of occurring. • ***Economic capital*** refers to holding sufficient liquid reserves to cover a potential loss.
Qualitative Assessment	• ***Scenario analysis*** takes into account potential risk factors with uncertainties that are often non-quantifiable. • ***Stress testing*** is a form of scenario analysis that examines a financial outcome based on a given "stress" on the entity.
Enterprise Risk Management (ERM)	ERM takes an ***integrative approach*** to risk management within an entire entity, dispensing of the traditional approach of independently managing risk within each department or division of an entity.

Section 2

Corporate Risk Management

Corporate Risk Management: A Primer 了解

Hedging Risk Exposures

Disadvantages	Advantages (In Practice)
In Theory • Assume ***no transaction costs or taxes.*** The ***perfect capital markets assumption*** is not realistic in practice. **In Practice** • Distract management from its ***core business***. • A risk management strategy can ***drag a firm down even more quickly*** than the underlying risk. • Even a well-developed risk management strategy has ***compliance costs***	• Possibility of lowering its cost of capital • Reducing the volatility of its earnings/cash flows • Allow management to control its financial performance to meet the requirements of the board of directors. • Result in ***operational improvements*** within a firm. • Derivatives: ***cheaper*** than purchasing an insurance policy • The ***existence of progressive tax rates may result in volatile earnings (with no hedging).***

Hedging Decisions

The Role of the Board of Directors *(risk management goal)*

- ***Management and the board*** need to set and communicate the firm's risk appetite.
- The board may wish to implement definitive and quantitative ***risk limits*** within which management is allowed to transact at its own discretion.

The Process of Mapping Risk *(next step)*

- Mapping risks could be performed for market risk, credit risk, business risk, and operational risk
- Hedging Operational (***income statement*** activities)and Financial Risks (***balance sheet***)

Risk Management Instruments: devised internally & Exchange-traded versus over-the-counter (OTC) instruments

Corporate Governance and Risk Management

主旨：风险管理和公司治理不是分割的

Best-Practice Corporate Governance

- The ***board*** has a duty to ***shareholders*** but must also be sensitive to ***debt holders***
- But ***conflicts of interest*** (or less severely, lack of alignment) can easily happen. The board should maintain its ***independence*** from management.
- Many corporations have created the role of ***chief risk officer (CRO)***. After the financial crisis of 2007–2009, many CROs were given a ***direct reporting line to the board or its risk committees*** in addition to reporting to the ***executive team and CEO.***

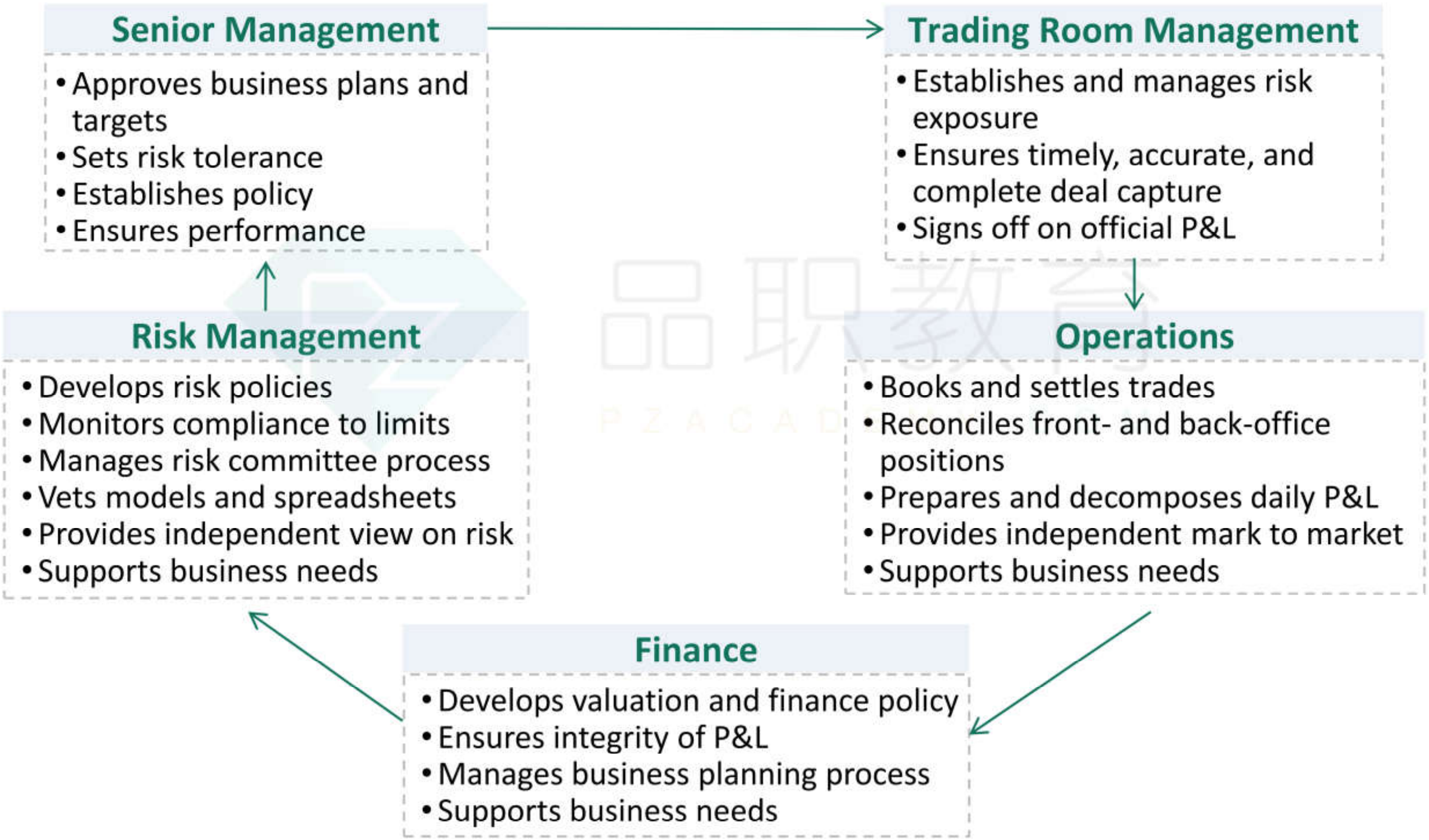
Risk Governance

- **Risk Advisory Director:** board member
- **Risk Management Committee:** ***identifying, measuring, and monitoring*** financial risks
- **Compensation Committee:** ***independent of management***. The committee should ***avoid*** designing compensation plans with ***bonuses based on short-term profits or revenues.***
- **Audit Committee:** responsible for the ***reasonable accuracy of the firm's financial statements and its regulatory reporting requirements.***

Risk Appetite and Business Strategy

- A firm's risk appetite reflects its tolerance (especially willingness) to accept risk.
- The ***board*** needs to develop/approve the firm's risk appetite as well as assist management in developing the firm's overall strategic plan.

Interdependence of Functional Units



What is ERM? ★ 名词掌握

Definitions of ERM

- **ERM** is a ***comprehensive and integrated*** framework for managing key risks
- **Traditional approach** to risk management:
    - Each of these primary risk types was evaluated by a ***specific unit*** within the organization in ***isolation***, independent of the other risk types.
    - Shortcoming: ***ignoring the dynamic nature of risks and their interdependencies.***

ERM Benefits and Costs

- There are three primary motivations
    - ***Integration of Risk Organization***
    - ***Integration of Risk Transfer***
    - ***Integration of Business Processes***
- Implementation of an integrated firm-wide initiative is ***costly*** (both capital and labor intensive) and ***time-consuming***.

ERM Components

1. Corporate governance
Establish top-down risk management

2. Line management
Business strategy alignment

3. Portfolio management
Think and act like a "fund manager"

4. Risk transfer
Transfer out concentrated or inefficient risks

5. Risk analytics
Develop advanced analytical tools

6. Data technology and resources
Integrate data and system capabilities

7. Stakeholder management
Improve risk transparency for key stakeholders

Section 3

Risk-Taking and Risk Management by Banks

了解

Risk-Taking

Optimal Level of Risk

- Targeting a ***certain default probability:*** Aiming for AAA may constrain the bank's risk-taking ability and reduce its returns. Aiming for BBB may result in lost customers due to the perception that the bank is engaging in excessive risk-taking activities.
- The optimal level of risk depends on the ***specific focus of the bank's activities***, so it will differ among banks.
    - deposits, relationship lending customers → ***desire for safety***
    - transactional activities → ***risk higher***

Risk Management

Risk Management Limitations

- Limitations of Hedging
    - ***Risk measurement technology limitations***
    - ***Hedging limitations:*** many risks are nearly or entirely impossible to hedge (e.g., terrorism risk)
    - ***Risk-taker incentive limitations***
- Role of Risk Management: ***independent of the activities of the business lines.***

Bank Risk Profile and Performance

- **Governance**
    - It is ***difficult to demonstrate*** that a bank's governance has a significant impact on its risk profile and performance
- **Incentive structure**
    - ***reward managers*** for taking risks that create value
- **Risk Culture**
    - Companies where managers were perceived as honest and trustworthy were more profitable
    - Shareholder governance improvements would change a firm's culture from focusing on employee integrity and customer service

Section 4

Risk Management Failures: What Are They and When Do They Happen?

了解

The Role of Risk Management

- A large ***loss*** is not necessarily an indication of a risk management ***failure***.
- As long as risk managers ***understood and prepared for the possibility of loss***, then the implemented risk management successful.

Risk Management Failures

1. Mismeasurement of known risks
2. Failure to take risks into account

→ **Measuring risks**

3. Failure in communicating the risks to top management

→ **Communicate**

4. Failure in monitoring risks
5. Failure in managing risks
6. Failure to use appropriate risk metrics

→ **Communicate**

The Role of Risk Metrics

- VAR is a widely used risk metric that is narrow in scope in several ways. Such as:
  - ***VAR does not capture the implications of extremely large losses*** that have a very low probability of occurring.
  - One misuse of VAR is ***choosing a time period*** (e.g., daily or weekly)
  - VAR also ***assumes the distributions of losses are not correlated over time.***

Section 5

The Standard Capital Asset Pricing Model

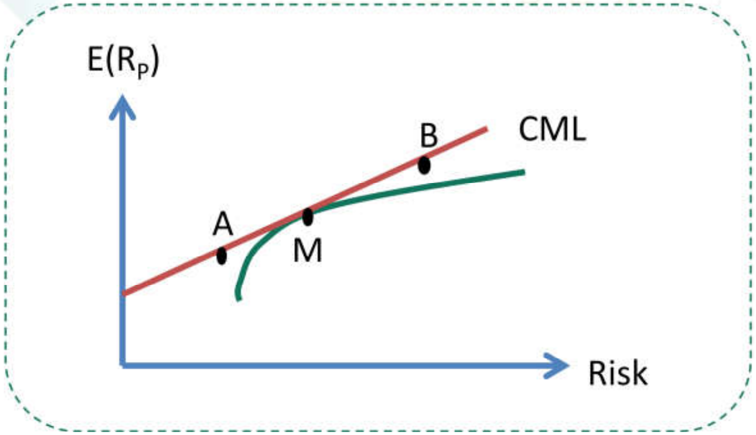
核心思想

Asset allocation: Efficient frontier $\xrightarrow{+\,R_f}$ CAL $\xrightarrow{\text{identical expectations}}$ CML

Pricing (return): CAPM (SML)

<table>
<tr><th colspan="3">Markowitz Modern portfolio theory★★★ 计算、性质</th></tr>
<tr><td>Assumption</td><td colspan="2">✓ Each investment can be measured by expected returns and Risk, which is measured in terms of variance (or standard deviation);
✓ Investors make their decision only based on expected return, standard deviation and covariance;
✓ Utility maximization;
✓ Risk averse.</td></tr>
<tr><td rowspan="3">推导</td><td>Portfolio diversification</td><td>$E(r_p)=\sum_{i=1}^{n} w_i E(R_i)$ $\sigma_P=\sqrt{\sigma_P^2}=\sqrt{\sum_{i=1}^{n} w_i^2\sigma_i^2+\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov_{i,j}}$
特殊的(要求计算)：$\sigma_p^2 = w_1^2\sigma_1^2+w_2^2\sigma_2^2+2w_1w_2\sigma_1\sigma_2\rho_{1,2}$
1. n越大, Correlation越小 → diversification benefit越大
2. $\rho_{1,2}=1$, $\sigma_p= w_1\sigma_1+w_2\sigma_2$, 两个资产组合E(R)和σ在一条直线上, σ_p最大;
$\rho_{1,2}=-1$, $\sigma_p= /w_1\sigma_1 - w_2\sigma_2/$, 两个资产组合E(R)和σ在一条折线上，一定有一个点$\sigma_p=0$</td></tr>
<tr><td></td><td>Equally-weighted portfolio of n stocks: $\sigma^2_p=\frac{1}{n}\bar{\sigma}^2+\frac{n-1}{n}\overline{\text{cov}}=\sigma^2(\frac{1-\rho}{n}+\rho)$</td></tr>
<tr><td>Indifference curve</td><td>Risk averse： accept a riskier investment only if they are compensated in the form of greater expected return (Indifference curve是凸的)</td></tr>
<tr><td>结论</td><td colspan="2">✓ Minimum variance frontier、Global minimum-variance portfolio
✓ Efficient frontier
✓ Optimal portfolio (EF和indifference curve的切点)</td></tr>
</table>

CML★★★ 计算、性质	
公式	$E(R_P)=R_F+\frac{E(R_M)-R_F}{\sigma_M}\sigma_P$
Market portfolio	① tangent point where the CML touches the Markowitz EF. ② Consists of every risky assets ③ Weighting on each asset are equal to the percentage of the market value of the asset to MV $_{\text{Portfolio}}$
特点	① CML上的点：Efficient portfolio ② Passive investment strategy using CML：risk-free asset +M ③ A: lending portfolio B: borrowing portfolio



	CPAM (SML) ★★★ 计算、性质
基础	• Systematic risk : cannot be diversified away. (or market risk) • Interest rate risk、currency risk、macroeconomic risk…… • Unsystematic risk： diversifiable, firm-specific risk
Assumption	• Investors face ***no transaction costs*** when trading assets. • Assets are ***infinitely divisible***. • There are ***no taxes.*** • Investors are ***price takers.*** The market for assets is perfectly competitive. • Investors' utility functions are based solely on ***expected portfolio return and risk.*** • ***Unlimited short-selling*** is allowed. • Investors can ***borrow and lend unlimited*** amounts at the ***risk-free rate.*** • Investors are only concerned about returns and risk over a ***single period***, and the single period is the same for all investors. • All investors have the ***same forecasts*** of expected returns, variances, and covariances. This is known as ***homogeneous expectations.*** • All assets are ***marketable***, including human capital.
公式	$E(R_i)=R_f+\beta_i[E(R_M)-R_f]$ $\beta_i=\frac{Cov_{i,mkt}}{\sigma_{mkt}^2}=(\frac{\sigma_i}{\sigma_{mkt}})\times\rho_{i,mkt}$
应用	Undervalued (buy): market estimated return > Expected return from the SML Overvalued (sell): market estimated return < Expected return from the SML

Difference between SML and CML ★★★		
Difference	**SML**	**CML**
Measure of risk	systematic risk	standard deviation (total risk)
Application	determine the appropriate expected returns for securities	determine the appropriate asset allocation
Definition	Graph of the capital asset pricing model	Graph of the efficient frontier
Slope	Market risk premium	Market portfolio Sharpe ratio

Section 6

Applying the CAPM to Performance Measurement

★★★

	公式	适用范围
Sharpe ratio	$SR=\frac{E(R_P)-R_F}{\sigma(R_P)}$	• ***measuring historical performance*** • measuring portfolio performance that are ***not very diversified*** • evaluating the performance of a portfolio that represents an ***individual's total investment.***
Treynor measure	$TR=\frac{[E(R_P)-R_F]}{\beta_P}$	• comparing ***well-diversified portfolios*** • a ***more forward-looking measure.***
Jensen's alpha	$\alpha_P=E(R_P)-\{R_F+\beta_P[E(R_M)-R_F]\}$	comparing portfolios that have the ***same beta*** ***rank portfolios within peer groups.***
Information ratio	$IR=\frac{\alpha_P}{\sigma_{eP}}=\frac{[E(R_P)-E(R_B)]}{\sigma_{eP}}$	Tracking error: standard deviation of the difference between the portfolio return and the benchmark return.
Sortino ratio	$SOR=\frac{E(R_P)-R_{\min}}{\sqrt{MSD_{\min}}}$	• appropriate for a case where ***returns are not symmetric*** • Less widely used

Section 7

Arbitrage Pricing Theory and Multifactor Models of Risk and Return

Single-Factor Model and Multifactor Model ★★★

	公式	字母含义
single-factor model	$R_i = E(R_i) + \beta_i F + e_i$ • ***a common or macroeconomic factor*** • ***firm-specific events.***	• F is the ***deviation of the common factor from its expected value***; • β_i is the ***sensitivity*** of firm i to that factor; • e_i is the ***firm-specific disturbance***; • $E(R_i)$ is the expected return on stock i.
Multifactor model	$R_i = E(R_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \ldots + \beta_{ik}F_k + e_i$ • ***economy-wide factors*** e.g. business-cycle risk, interest or inflation rate risk, energy price risk	• R_i = return for stock i • $E(R_i)$ =expected return for asset i • F_j= ***deviation of macroeconomic factor j from its expected value*** • β_{ij}= factor beta for stock i • e_i = ***firm-specific surprise***

Factor portfolios

- ***betas equal to one for a single risk factor and betas equal to zero on all remaining factors.***
- **Hedging Exposures to Multiple Factors**:
  - Example: ***GDP beta =0.5,*** Consumer sentiment beta =0.3. Assume the investor wishes to hedge away GDP factor risk, yet maintain the 0.30 exposure to consumer sentiment.
  - The investor should combine the original portfolio with a ***50% short position in the GDP factor portfolio.***

The Single-Factor Security Market Line ★★★

The Single-Factor Security Market Line

- Well-diversified portfolio: $R_p = E(R_p) + \beta_p F$
- The single-factor security market line (SML)

$$E(R_p) = R_F + \beta_p[E(R_M) - R_F]$$

  - M is an observable, well-diversified, market index.
  - β is the beta of any portfolio, P, relative to the market index, M.

The Single-Factor Security Market Line & CAPM

- ***CAPM*** relies on the existence of the mean-variance efficient market portfolio
- ***Single-factor security market line merely relies on the assumptions that security returns be explained by a single-factor model***, and that ***no arbitrage opportunities*** exist.

Arbitrage Pricing Theory ★★★

公式	$E(R_P) = R_F + \beta_{P,1}(\lambda_1) + \beta_{P,2}(\lambda_2) + ... + \beta_{P,k}(\lambda_k)$ **（均衡模型）** 计算 • λ:factor risk premium (or factor price) • β_p: factor sensitivities
Assumption	• A ***factor model*** describes asset returns • Investors can form ***well-diversified portfolios*** that eliminate asset-specific risk • ***No arbitrage opportunities*** exist among well-diversified portfolios
The risk premiums	Calculate risk premiums for each factor portfolio: Factor Portfolio 1 → $E(R_1) - R_F$ $E(R_i) = R_F + \beta_{i1}[E(R_1) - R_F] + \beta_{i2}[E(R_2) - R_F] + ... + \beta_{ik}[E(R_k) - R_F]$
APT vs. CAPM	APT is ***less restrictive*** in its assumptions. The CAPM can be considered a ***"special case"*** of the APT

Section 8

Principles for Effective Risk Data Aggregation and Risk Reporting

了解

Principles for Effective Risk Data Aggregation and Risk Reporting

Definition of risk data aggregation

- ***Defining, gathering and processing*** risk data according to the bank's risk reporting requirements to enable the bank to measure its performance against its risk tolerance/ appetite.

Governance	
i. Overarching governance and infrastructure	• Governance • Data architecture and IT infrastructure
ii. Risk data aggregation capabilities	• Accuracy and Integrity • Completeness • Timeliness • Adaptability
iii. Risk Reporting Practices	• Accuracy • Comprehensiveness • Clarity and usefulness • Frequency • Distribution
iv. Supervisory review, tools, and cooperation	• Review • Remedial actions and supervisory measures • Home/host cooperation

Section 9

Financial Disasters

★★★ 结论

Misleading Reporting Cases

Cases where the "striking feature" is that a firm—or its investors and lenders—have been ***misled with deliberate intent*** about the size and nature of its position(s)

Chase Manhattan Bank and Drysdale Securities	
Incident & Results	• Drysdale Government Securities, obtaining ***unsecured borrowing*** by exploiting a flaw in the market practices for computing the value of U.S. ***government bond*** collateral. • Chase experienced reputational damage and an impact on to their stock price.
Key Factors	• Chase failed to detect the unauthorized positions • Inexperienced managers • Did not correctly interpret borrowing agreements
Lessons	• Need for more precise methods for ***computing the value of collateral***. • Need for ***better process control.***

Kidder Peabody	
Incident & Results	• Joseph Jett, head of the government bond trading desk, entered into a series of trades that were incorrectly reported in the firm's accounting system, ***artificially inflating reported profits***. • Announcement of such a massive misreporting of earnings triggered a substantial loss of confidence
Key Factors	• The ***computer system*** did not account for a forward contract's present value. • Management did not react to visible suspicions
Lessons	• Always investigate a stream of large unexpected profits & Periodically review models and systems

Barings	
Incident & Results	• Junior trader (***Leeson***) took large ***speculative positions*** from the Singapore office. • Losses of roughly 1.25 billion forced ***Barings into bankruptcy***
Strategy	• ***Short Straddles on the Nikkei 225*** • Leeson ***abandoned*** the hedged posture in the ***long-short futures arbitrage*** strategy and initiated a ***speculative long-long futures*** positions on both exchanges
Key Factors	• Allowing Leeson to function ***as a head of trading and the back office*** • The ***management failed to inquire*** how a low-risk trading strategy was supposedly generating such a large profit. • Information provided to the credit risk area was not integrated with information provided to funding and showed no such credit extension. (***poor reporting***)
Lessons	• Absolute necessity of an ***independent trading back office***. • The need to ***make thorough inquiries about unexpected sources of profit (loss)*** and/or cash movements • The need to ***establish information, reporting, and control systems***

Allied Irish Bank	
Incident & Results	• ***John Rusnak***, a ***currency option trader*** entered into massive unauthorized trades. He was disguising large naked position. This resulted in a major blow to AIB's reputation and stock price
Key Factors	• Rusnak was able to ***hide the trading activities*** from management ***by creating imaginary trades*** • The ***back-office failed to confirm all trades*** • Suspicious trades and trading profits were ignored. • ***AIB's management was so inexperienced that failed to figure out*** Rusnak's trading activates.
Lessons	• This case is similar to Barings. But Rusnak had ***not dual role.*** • Strong and enforceable ***back-office controls*** are essential.

Union Bank of Switzerland	
Incident & Results	• UBS lost $700 million due to a large position in Long Term Capital Management (LTCM).
Key Factors	• poor decision-making, merely bad luck, or an improper control structure. • failed to receive adequate scrutiny from the firm's risk controllers • The senior risk manager authority for the department doubled
Lessons	• The incident emphasizes the need for ***independent risk oversight***.

Société Générale	
Incident & Results	• Junior traders, Jérôme Kerviel, was involved in unauthorized trading activity that resulted in losses of $7.1 billion. The incident damaged the reputation.
Strategy	• Kerviel established ***large, unauthorized positions in futures contracts and equity securities***. He ***created fake transactions*** that offset the price movements of the actual positions.
Key Factors	• There was ***no procedure in place that required control functions to confirm information*** • ***Lack of supervision*** • The trading assistant may have been operating in collusion with Kerviel • The ***normal precaution*** of forcing a trader to take ***two consecutive weeks of vacation*** was not followed • There were ***no limits or other monitoring of Kerviel's gross positions*** • Cash and collateral reports and inquiry procedures lacked sufficient granularity • Kerviel was reporting trading gains in excess of levels his authorized position.
Lessons	• Institute systems for monitoring patterns • ***Proper supervision*** • ***Control personnel*** • ***Rules for mandatory time away from work*** • Gross positions must be monitored and highlighted in ***control reports*** • Cash and collateral requirements should be monitored • Any patterns of P& L that are ***unusual*** relative to expectations need to be ***identified*** and ***investigated***

Sumitomo	
Incident & Results	• ***Yasuo Hamanaka,*** the lead ***copper trader*** for Sumitomo, attempted to ***corner the copper*** market
Strategy	• Establish a dominant ***long position in futures contracts*** and simultaneously ***purchased large quantities of physical copper***. • ***Sold put options*** for premium to finance his long copper positions
Key Factors	• Sumitomo's ***lack of supervision*** which allowed him to keep ***two sets of trading books***, one of which reported large profits, and the other reported ***huge losses but secret***.
Lessons	• large transactions should have ***required multiple approvals by senior management***

Daiwa	
Incident & Results	• A Treasury bond trader, ***Toshihide Iguchi***, covered up \$1.1 billion in losses
Key Factors	• Iguchi was able to ***not only hide these losses, but also forge customer trading slips*** • This misleading reporting went undetected due to ***Iguchi's dual role as the head of both trading and the back-office support function***.

Large Market Movement Cases

Cases where the positions were known—or investors and lenders had "reasonable knowledge" of positions--but ***adverse market moves*** were not anticipated

Metallgesellschaft	
Incident & Results	• In 1992, an American subsidiary of MG, Metallgesellachaft Refining and Marketing (MGRM), ***offered customers contracts to buy fixed amounts of heating oil and gasoline at a fixed price over a 5-year or 10-year period.*** And the contracts gave MGRM a ***short position*** in long-term forward contracts • MGRM hedged the exposure with ***long positions in near-term futures*** using a ***stack-and-roll hedging strategy*** (滚动套期策略) • MGRM cashed out its positions and reported losses of approximately $1.9 billion.
Key Factors	• The market ***shifted to contango*** that exploited the ***basis risk***. • Led to ***cash flow (liquidity) problems*** • ***German accounting methods*** required Metallgesellschaft to ***show futures losses (i.e., from hedge) but could not recognize unrealized gains from the forward***
Lessons	• Short-term hedge against long-term contracts requires ***liquidity***. • The ***uncertainty of roll cost*** is a key risk for strategies • A firm running short-term hedges against longer-term risk requires the ***flexibility***

Long-Term Capital Management	
Incident & Results	• Most of LTCM's investments strategies could be classified as ***relative value, credit spreads, and equity volatility.*** • Relative value strategies involved arbitraging price differences, among similar securities and profiting when the prices converged • In August of 1998, Russia unexpectedly defaulted on its debt. The flight to quality ***increased***, rather the decreased, ***credit spreads,*** causing huge losses for LTCM.
The risks	• ***Model Risk:*** LTCM's model ***assumed that historical relationships were useful predictors of future relationships*** • ***Limited Reporting*** Obligation to Regulators
Key Factors	• Failure to supplement VaR with a full set of ***stress test scenarios*** • Failure to account for ***illiquidity*** of positions during stress • ***Leverage too was high*** • Too much faith in models; i.e., ***model risk***
Lessons	• ***Stress scenarios*** including extreme stresses and interaction between market & credit risk. • ***Incorporate liquidity*** • Initial margin needed if counterparty is trader • Greater counterparty ***disclosures***

Customer Conduct Cases

Do not involve any direct financial loss to the firm, but were a matter of reputational risk (or fiduciary exposure) due to the ***conduct of customer business***.

Banker's Trust	
Incident & Results	• Due to losses (e.g., P&G lost >$100 million in 1994), ***customers sued BT***. (Proctor & Gamble (P&G) and Gibson Greetings bought complex derivative products offered by BT)
Key Factors	• Complex derivatives • Evidence of some intent to deceive (Discovery evidence)
Lessons	• This case demonstrated the importance of ***matching trades with a client's needs*** • The importance of exercising caution ***with any form of communication*** that could eventually be made ***public***, as it could ***damage a firm's reputation*** if unethical practices are present.

JPMorgan, Citigroup, and Enron	
Incident & Results	• ***JPMorgan Chase and Citigroup were Enron's principal counterparties*** on these trades • One of the ploys that Enron had used was to ***disguise a borrowing as an oil futures contract.***
Key Factors	• Enron had for years been engaging in dubious accounting practices to ***hide the size of its borrowings from investors and lenders.***
Lessons	• JPMorgan Chase and Citigroup agreed to put new controls in place to ascertain that their clients were accounting for derivative transactions

Section 10

Credit Crunch 2007—2008

结论

Deciphering the Liquidity and Credit Crunch 2007—2008

Key Factors of The Housing Bubble

- ***low interest rate environment***
- banking system: "***originate and distribute***" banking model

Rise in Popularity of Securitized and Structured Products

- Structured financial products can ***cater to the needs of different investor groups***.
- securitization allows certain institutional investors to hold assets (indirectly) that they were previously prevented
- ***regulatory and ratings arbitrage***
- structured products may have ***received more favorable ratings***

Funding Liquidity and Market Liquidity

- ***Funding liquidity*** describes ***the ease with which expert investors can obtain funding from financiers.***
- ***Market liquidity*** is low when it is ***difficult to raise money by selling the asset***

Getting Up to Speed on the Financial Crisis

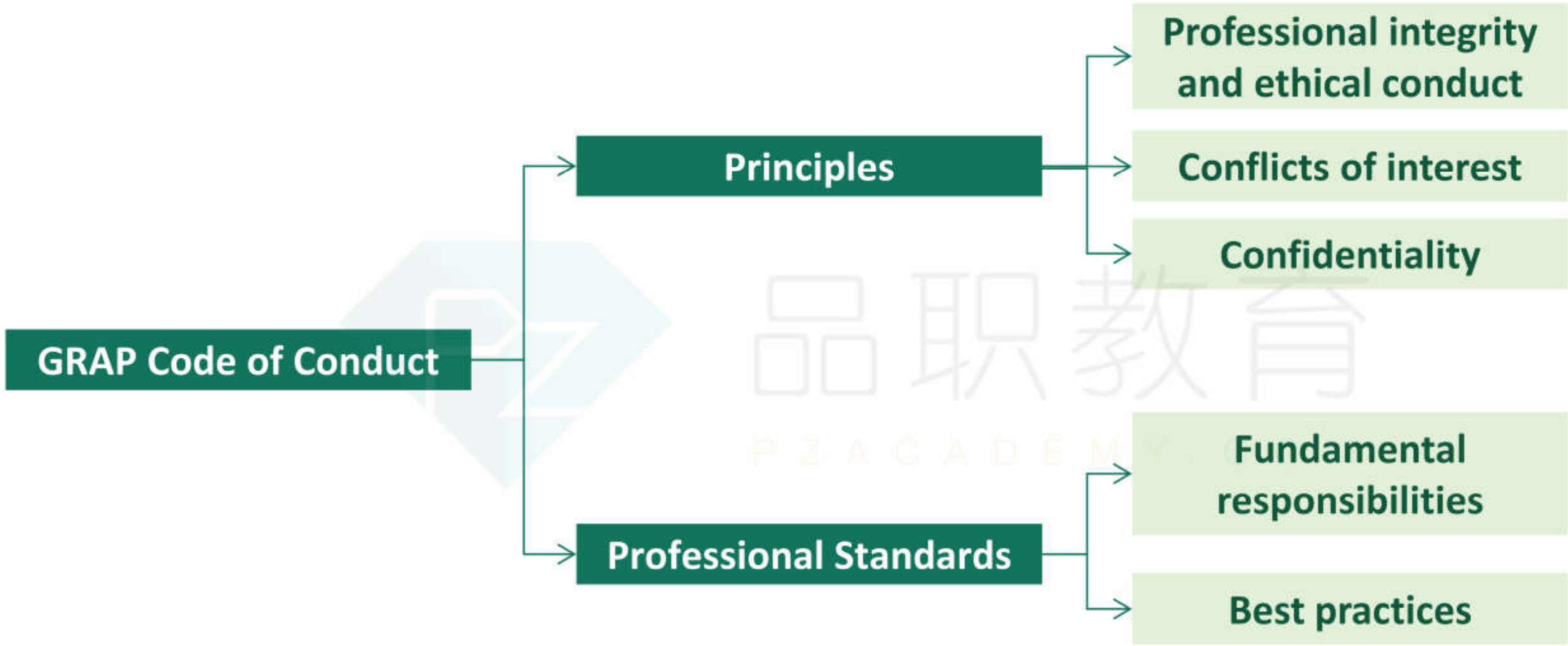
Distinguish between Triggers and Vulnerabilities

- The ***systemic vulnerabilities*** in large part were due to ***changes that had occurred in the financial sector of the economy.*** The financial crisis was a ***bank run***

Section 11

GARP Code of Conduct

★★　概念对比


GRAP Code of Conduct
Principles
Professional Standards
Professional integrity and ethical conduct
Conflicts of interest
Confidentiality
Fundamental responsibilities
Best practices


Thank You!

品职教育
PZACADEMY.COM

Financial Market And Product

FRM一级知识框架图

讲师：李斯克

www.pzacademy.com

Learning Structure

Derivative

Forward Commitment

1. Introduction Of Derivative Contract
2. Mechanics Of Futures Market
3. Hedging Strategies
4. Determination Of Forward And Futures Prices
5. Swaps

Special Topics

13. Commodity Forwards And Futures
14. Foreign Exchange Risk

Option

6. Mechanics Of Options Markets
7. Properties Of Stock Options
8. Trading Strategies Involving Options
9. Exotic Options

Option Valuation

10. Binomial Trees
11. The Black-Scholes-Merton Model
12. The Greek Letters

估值与风险建模部分内容，建议在本科目中学习

Credit Risk In OTC Market

15. Central Counterparties
16. Basic Principles Of Central Clearing
17. Risks Caused By CCPs

Fixed Income

18. Corporate Bonds
19. Mortgages And Mortgage-Backed Securities

建议放在估值与风险建模部分学习

Section 1

INTRODUCTION OF DERIVATIVE CONTRACT

Financial Institutions

Banks

- ***Commercial banking*** → great ***regulation***
- ***Investment Banking*** → ***raising*** debt and equity

Life Insurance

- ***Uses of Life Insurance***
  - hedge against the risk of the ***premature death***
  - ***estate-planning tool***
  - tax-sheltered savings instrument
- ***Types of Life Insurance***
  - ***temporary and permanent***

Annuities

- ***Hedge risk of unknown lifespan***

Funds

- ***Mutual Funds***
  - ***diversification*** opportunities
  - ***three main types*** :
    - ***Bond funds***
    - ***Equity***
    - ***Hybrid funds***
- ***Hedge funds***
  - ***little regulation***.
  - only ***sophisticated individuals and organizations***.
  - ***alternative*** investments

Two key risks of insurance

- Moral hazard
- Adverse Selection

Basic Concept

定义 概念，针对未来交易，回避风险

衍生品分类方法★

根据合约特点分类

- Forward commitment
  - forward
  - futures
  - swap
- Contingent claim
  - option
  - Credit derivatives
  - ABS

根据交易场所分类

- Exchange-traded
  - futures
  - option
- Over-the-counter traded
  - forward
  - swap
  - option

Purpose Of Trading

- Hedging Strategy
- Speculative Strategy
- Arbitrage

Exchange-traded	Over-the-counter
Standardized→ Liquid	Customized→ Specific needs
Regulated	Unregulated
Backed by a clearinghouse (No default)	Trade with counterparty (default risk)
Trade in the a physical exchange	not trade in organized markets → with dealer

Forward And Option

Forward 概念

Definition

分类
- Commodity forward contract
- Financial forward contract

Purposes of trading
- Hedge risk
- Speculation

交割★
- **at expiration**
  - Physical settlement
  - Cash settlement
- **prior to expiration** → Offsetting

Option 概念

概念
- **Option定义**
  - Call option / Put option
  - American option / European option
- **价格**
  - option premium
  - Strike price
- **Moneyness** 定性看long是否赚钱
- **Intrinsic Value** 定量看long赚多少钱
  - C=max[0, S-X]
  - P=max[0, X-S]
- **Option value=intrinsic value + time value**

Section 2

MECHANICS OF FUTURES MARKET

Futures Contract And Basic Characteristics

Forwards	Futures
Private contracts	Exchange-traded
Unique customized contracts	Standardized contracts
Little or no regulation	Regulated
Default risk is present	Guaranteed by clearinghouse
Settlement at maturity	Daily settlement (mark to market)
No margin deposit required	Margin required and adjusted

与Forward区别★★

Open Interest

- Long=short

Futures不会违约原因

Futures contract风险控制方法★★

风控方法	考点
Margin	1. Initial margin 2. Maintenance margin 3. Variation margin 回到IM
Daily Price Limit	Limit move Locked limit
Marking to market	盯市方法→FP converges to SP at termination→Payoff = $S_T(T) - F_0(T) = F_T(T) - F_0(T)$

Basis = spot price - futures price

Characteristics Specified In A Futures Contract

Specified Content	Details
Quality of the underlying asset	the ***quality*** of a good that will be acceptable for settling the contract.
Contract size	specifies the ***quantity*** of the asset that must be delivered
Delivery location	The exchange specifies the ***place*** where delivery will take place.
Delivery time	Futures contracts are referred to by the ***month***
Price quotations and tick size	the ***minimum price fluctuation*** for the contract, (tick size).
Daily price limits	The exchange sets the ***maximum price movement*** for a contract ***during a day***.
Position limits	a ***maximum number of contracts*** that a ***speculator*** may hold Such ***limits do not apply to hedgers***.

Trading Futures Contract

Close a Contract

Four Ways To Terminate Futures Contract

- Delivery
- Cash-settlement Contract
- Make a reverse, or offsetting
- Exchange For Physicals

Trade a Contract

Types Of Orders

- Market orders
    - discretionary order
- Limit orders
- Stop-loss orders
    - Stop-limit orders
    - Market-if-touched orders
- Time-of-day order
- Good-till-canceled (GTC) orders (open orders)
- Fill-or-kill orders

Regulation, Accounting and Taxes

Regulators

- CFTC 主要监管机构
- NFA/SEC/FED/US. Treasury

Accounting

- Recognize Profit & Loss
- ***Hedge***: recognized when asset is disposed
- ***Speculation***: recognized when changes occurred

Taxes

Corporate	capital gains →taxed as ordinary income capital losses are restricted
Non-Corporate	long-term gains subject to maximum rate. capital losses are deductible

60/40 rule for non-corporate taxpayers (not apply to hedging activities)

Section 3

HEDGING STRATEGIES

Hedging Using Futures Contract

Short hedge	Long hedge
short a futures contract	long a futures contract
Against a price decrease	Against a price increase
Appropriate for long positon	Appropriate for short positon

Hedge优缺点★

- Arguments for hedging
  - Reduce price risk, less uncertainty
- Arguments against hedging
  - less profitability.
  - shareholders can easily hedge risk
  - nature of the hedging company's industry.

Perfect Hedge难以实现

Basis Risk ★★

Basis Concept

- Basis=SP-FP
- B=0 at maturity
- spot price increases faster(slower) than the futures price over the hedging horizon, basis increases (decreases).

Three sources of basis risk

- Interruption in the convergence
- Changes in the cost of carry.
- Imperfect matching between the cash asset and the hedge asset.
  - Maturity or duration mismatch.
  - Liquidity mismatch.
  - Credit mismatch.

Hedge Ratio

基本原理★ ★计算

Hedge ratio

$$HR = \rho_{S,F}\frac{\sigma_S}{\sigma_F}$$

一份现货用多少个期货hedge

The effectiveness of the hedge

$$R^2 = \rho_{S,F}{}^2 = \left(\frac{HR \times \sigma_F}{\sigma_S}\right)^2$$

Optimal Number of Futures Contracts

$$N^* = \frac{h^* N_A}{Q_F}$$

一份现货要用多少份期货合约hedge

Hedging With Stock Index Futures ★ ★

$$\text{number of contracts} = (\beta^* - \beta)\frac{P}{A}$$

概念

- Trailing the hedging
- Rollover basis risk

Duration-Based Hedging ★ ★

$$N = -\frac{P \times D_P}{F \times D_F}$$

Duration调成0

应用

Altering Debt and Equity Allocations

Equity ⇄ Cash Beta=0 MD=0 ⇄ Bonds

Interest Rate Derivatives

Interest Rate

Three interest rates

- Treasury rates
- LIBOR
- Repo rates

Discrete vs. continuous compounding

$$FV_1 = A\left(1+\frac{R}{m}\right)^{m\times n} \Longleftrightarrow FV_2 = Ae^{R\times n}$$

Theories Of The Term Structure

- Expectations theory
- The market segmentation theory
- The liquidity preference theory

Bond Pricing

Using Spot Rate

$$B=\left[\frac{c}{2}\times\sum_{j=1}^{N}e^{-\frac{z_j}{2}\times j}\right]+\left[FV\times e^{-\frac{z_N}{2}\times N}\right]$$

Using Bond Yield

YTM等同于Spot Rate的平均值

Bootstrapping Spot Rates

Forward Rates ★

$$R_{Forward}=\frac{R_2T_2\text{-}R_1T_1}{T_2\text{-}T_1}=R_2+(R_2\text{-}R_1)\times\left[\frac{T_1}{T_2\text{-}T_1}\right]$$

FRA ★★

基本概念

LIBOR
EuroDollar
FRA

FRA定义★ → long position→Borrow; Short position→lend

FRA期限

报价★ Example 3×9FRA Payoff计算★★

Interest Rate Risk

一阶导数→Duration ★

Macaulay Duration ➡ $\text{duration} = \sum_{i=1}^{n} t_i \left[\frac{c_i e^{-yt_i}}{B} \right]$

Modified Duration ➡ $\frac{\Delta B}{B} = -\text{duration} \times \Delta y$

DV01

➡ **Macaulay D vs. Modified D**

modified duration = duration/(1+y/m)

二阶导数→Convexity

Disadvantage of duration

- Duration is only good for ***small changes in interest rates***.
- relationship between bond price and yield is ***not linear*** (as assumed by duration) but convex.

➡ ***Convexity effect***

convexity effect = 1/2 x convexity xΔy^2

Quotations For Interest Rate

Day count conventions

- U.S. Treasury bonds use ***actual/actual.***
- U.S. corporate and municipal bonds use ***30/360.***
- Treasury bills use ***actual/360***

报价

Full Price = Clean Price +Accrued Interest

$$\text{accrued interest}=\frac{\#\text{of days from last coupon to the settlement date}}{\#\text{of days in coupon period}}$$

Interest Rate Derivatives

Quotations for T-Bonds

- \$100 par amount in dollars and 32nds.
- cash price = quoted price + accrued interest

T-Bonds Futures★ → CTD & Conversion Factor, 计算

- cash received = (QFP x CF) + AI
- The CTD bond minimizes the following: quoted bond price - (Q_{FP} x CF)

Quotations for T-Bills

- actual/360 day count.
- Quotation=100-Annual Discount Rate

Eurodollar Futures → Convexity Adjustment ★

- one tick=\$25
- can be used to extend the LIBOR zero curve
- actual forward rate = forward rate implied by futures — (1/2 x σ^2 x T1 x T2)

Section 4

DETERMINATION OF FORWARD AND FUTURES PRICES

Pricing and Valuation

Asset
- Investment Asset
- Consumption Asset

price $\xrightarrow{\text{原理}}$ **no-arbitrage principle** →
- Cash-and-Carry Arbitrage ★
- Reverse Cash-and-Carry Arbitrage ★

Limits to Arbitrage

Investment Assets

Contract	T=0→Price	T=t→ Value
T-bill forwards	$F_0 = S_0e^{rT}$	$S_t - Ke^{-R(T-t)}$
dividend-paying stock (Coupon Bond)	$F_0 = (S_0-I)e^{rT}$	$S_t - I - Ke^{-r(T-t)}$
Index	$F_0 = S_0e^{(r-q)T}$	$S_0e^{-qT} - Ke^{-rT}$
Currency	$F_0 = S_0e^{(RA-RB)T}$	$S_te^{-RB(T-t)}-Ke^{-RA(T-t)}$

Futures与Forward估值区别★

Investors will...
Futures price > forward contract.
Futures price = forward contract.
Futures price < forward contract.

Consumption Asset

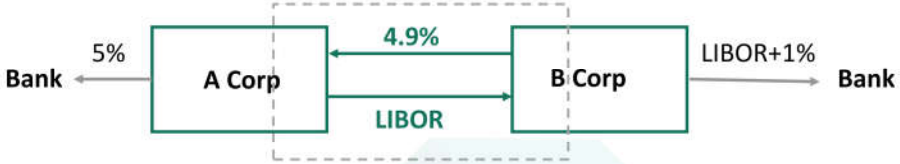
- **Income and Storage Costs** $F_0 = (S_0 + U)e^{rT}$
- **Convenience Yield** $F_0 = S_0e^{(r+u-y)T}$

Section 5

SWAPS

Swap

与Forward相似点 → 代表 Interest Rate Swaps



Interest Rate Swap

Contract	t=t	t=T
Interest Rate Swap	High	Low
Equity Swap	High	Low
Currency Swap	High	higher

interest rate swaps → 估值★★

- **Price→** $C=\dfrac{1-B_n}{B_1+B_2+\cdots+B_n}$ 求Swap rate，discount factor会算
- **Value→** $V_{swap}(X)=B_{flt}-B_{fix}$ 考法很常规，多做例题即可

Currency swaps → 估值★★

- **Price→** 各自币种按照利率互换方法求Swap rate，只对固定利率一方定价
- **Value→** 练习固定换固定，浮动换固定，及浮动换浮动即可

Other Swap →

- Equity swap
- Swaption
- Commodity swap
- Volatility swap

Section 6-7

MECHANICS OF OPTIONS MARKETS
PROPERTIES OF STOCK OPTIONS

Option Concepts

Option

概念

- **Option定义**
  - Call option
  - Put option
  - American option
  - European option
- **价格**
  - option premium
  - Strike price
- **Moneyness** 定性看long是否赚钱
- **Intrinsic Value** 定量看long赚多少钱
  - C=max[0, S-X]
  - P=max[0, X-S]
- **Option value=intrinsic value + time value**

Positions	Right	Obligation
Buyer of a call	Right to buy	
Seller of a call		Obligation to sell
Buyer of a put	Right to sell	
Seller of a put		Obligation to buy

Moneyness	Call Option	Put Option
In-the-money	$S>X$	$S<X$
At-the-money	$S = X$	$S = X$
Out-the-money	$S<X$	$S>X$

Underlying Assets

- Stock options
- Currency options.
- Index options
- Futures options

Other Option-Like Securities

- Warrants
- employee stock options
- Convertible bonds

potential for dilution

Properties Of Stock Options

Six Factors That Affect Option Prices ★★★

Factor	European Call	European Put	American Call	American Put
S	+	-	+	-
X	-	+	-	+
T	?	?	+	+
σ	+	+	+	+
R	+	-	+	-
D	-	+	-	+

Upper And Lower Pricing Bonds ★★★

Option	Minimum Value	Maximum Value
European call	$c \geq \max(S_0 - Xe^{-rT}, 0)$	S_0
American call	$C \geq \max(S_0 - Xe^{-rT}, 0)$	S_0
European put	$p \geq \max(Xe^{-rT} - S_0, 0)$	Xe^{-rT}
American put	$P \geq \max(X - S_0, 0)$	X

Put-Call Parity ★★★

- ***European***

$$c + Xe^{-rT} = S + p$$

fiduciary call=Protective Put

- ***American***

$$S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$$

The Impact Of Dividends ★★

•用St-PVDt代替公式中的St

•美式期权平价公式特殊：

•$S_0 - X - D \leq C - P \leq S_0 - Xe^{-rT}$

Section 8

TRADING STRATEGIES INVOLVING OPTIONS

Option Strategies for Equity Portfolios ★★

考法 → 构成
Max Profit & Loss
Breakeven point

Covered Call & Protective Put

- **Covered call** = short call + long stock → Profit = $(S_T - S_0) - [\max\{0, (S_T - X)\} - C]$
- **Protective Put** = long stock + long put → Profit = $(S_T - S_0) + [\max\{0, (X - S_T)\} - P]$

Spread

一个strategy只用call或者put，而不是二者混用

Bull spread	Call	Bull Call Spread = long call at X_L + short call at X_H → Profit = $[\max\{0, (S_T - X_L)\} - C_L] - [\max\{0, (S_T - X_H)\} - C_H]$
	Put	Bull Put Spread = long put at X_L + short put at X_H →Profit = $[\max\{0, (X_L - S_T)\} - P_L] - [\max\{0, (X_H - S_T)\} - P_H]$
Bear spread	Call	Bear Call Spread = short call at X_L + long call at X_H →Profit = $- [\max\{0, (S_T - X_L)\} - C_L] + [\max\{0, (S_T - X_H)\} - C_H]$
	Put	Bear Put Spread = short put at X_L + long put at X_H → Profit = $- [\max\{0, (X_L - S_T)\} - P_L] + [\max\{0, (X_H - S_T)\} - P_H]$
Butterfly Spreads	Call	Butterfly Spread Using Calls = long call at X_L + long call at X_H +short two calls at X_M →Profit = $[\max\{0, (S_T - X_L)\} - C_L] + [\max\{0, (S_T - X_H)\} - C_H] - 2[\max\{0, (S_T - X_M)\} - C_M]$
	Put	Butterfly Spread Using Puts = long put at X_L + long put at X_H +short two puts at X_M →Profit = $[\max\{0, (X_L - S_T)\} - P_L] + [\max\{0, (X_H - S_T)\} - P_H] - 2[\max\{0, (X_M - S_T)\} - P_M]$

Calendar	Long long-dated option + Short short-dated option (same strike price)
	Diagonal spreads: different strike price

Combinations of Calls and Puts

Straddle	long straddle = long call + long put • This strategy is profitable when the stock price moves strongly in either direction. This strategy bets on volatility.
	short straddle = short call + short put • bets on little movement in the stock
Strangle	• Different strike price (compared with straddle)
Strips	• Long 2 Put+Long 1 Call
Straps	• Long 2 Call+Long 1 Put
Collar	Collar = protective put + covered call = long stock + short call + long put • If the premium of the two are equal, it is called a zero-cost collar.
Box Spread	Box Spread = bull call spread + bear put spread • If the options are priced correctly, the payoff must be the risk-free.

Interest rate Option ★

Call/Put	Call: floating-rate borrower Put: floating-rate investor
	计算payoff • ***Call***: P=max (0, NP * (LIBOR - cap strike) * actual days /360) • ***Put***: P=max (0, NP * (floor strike - LIBOR) * actual days /360)

Options on Rate vs. Options on Prices

Option	If Rates Increase and Bond Prices Decrease	If Rates Decrease and Bond Prices Increase
Value of call on LIBOR	Increases	Decreases
Value of call on bond price	Decreases	Increases
Value of put on LIBOR	Decreases	Increases
Value of put on bond price	Increases	Decreases

Section 9

EXOTIC OPTIONS

Exotic Options

Payoff for a gap option ★

- has ***two strike prices***, X_1 and X_2. (X_2 is sometimes referred to as the ***trigger price***.)

Payoff for a gap call option			
Condition	$S_T > X_2 > X_1$	$S_T \le X_2$	$X_2 < X_1$, $X_2 < S_T$
Pay off	$S_T - X_1$	0	< 0

Payoff for a gap put option			
Condition	$S_T < X_2 < X_1$	$S_T \ge X_2$	$X_2 > X_1$, $X_2 > S_T$
Pay off	$X_1 - S_T$	0	< 0

Binary Options ★

Cash-or-nothing call → $Qe^{-rT}N(d_2)$

Asset-or-nothing call → $S_0e^{-qT}N(d_1)$

Lookback Options ★

➢ **A fix lookback options**

- Call: Max ($S_{MAX} - K$, 0).
- Put: Max ($K - S_{MIN}$, 0).

➢ **A float lookback options**

- Call: Max ($S_T - S_{MIN}$, 0).
- Put: Max ($S_{MAX} - S_T$, 0).

Barrier Options ★

- Down-and-out call (put)
- Down-and-in call (put)
- Up-and-out call (put)
- Up-and-in call (put)

Vega may be ***negative for a barrier option***

比一般期权便宜

Option	Details
Forward Start Options	options that begin their ***existence at some time in the future (eg. ESOP)***
Compound Options	• Call on call • Call on put • Put on call • Put on put
Chooser Options	• max (c, p) (比一般期权贵)
Shout Options	• the owner receives the ***maximum of the shout intrinsic value*** or the option ***expiration intrinsic value*** • *比一般期权贵*
Asian Options	• $c_T = Max(S_{AVE}(t, T) - K, 0)$ • 比一般期权便宜
Exchange Options	• exchange one asset for another
Basket options	• simply options to purchase or sell ***baskets of securities (rainbow options)***
A volatility swap	• the exchange of ***volatility*** based on a notional principal
A variance swap	• exchanging a pre-specified fixed ***variance*** rate for a realized variance rate • ***easier to price and hedge***

Section 10-12

BINOMIAL TREES
THE BLACK-SCHOLES-MERTON MODEL
THE GREEK LETTERS

Binomial Trees

Option到期是否行权 —须知S_T→ **一期二叉树**

Method 1: Synthetic Call Replication

call price = hedge ratio x [stock price - PV(borrowing)]

$$HR = \frac{c_U - c_D}{S_U - S_D}$$

Method 2: Risk-Neutral Valuation ★★★

$$Se^{r\Delta t} = pSu + (1-p)Sd \quad p = \frac{e^{r\Delta t} - d}{u - d} \rightarrow u = e^{\sigma\sqrt{\Delta t}} \quad d = e^{-\sigma\sqrt{\Delta t}}$$

↓

二期二叉树

- ***European Option***→ T时刻将IV往0时刻折现
- ***American Option***→ 每个节点判断是否行权，挑大的值往前折

↓

无穷多期二叉树

BSM模型

BSM Model

BSM ★★

$$C_0 = [S_0 \times N(d_1)] - [X \times e^{-R_f^c \times T} \times N(d_2)]$$ 计算

$$d_1 = \frac{\ln(\frac{S_0}{X}) + \left[R_f^c + \left(0.5 \times \sigma^2\right)\right] \times T}{\sigma \times \sqrt{T}}; \quad d_2 = d_1 - \sigma\sqrt{T}$$

effect of dividend

$$S \rightarrow Se^{-qT}$$

assumptions ★★

- Price of the underlying asset → lognormal distribution.
- The (continuous) risk-free rate is known and constant.
- Volatility of the underlying asset is known and constant.
- The markets are frictionless.
- There are no cash flows on the underlying asset.
- The options valued are European options.

Lognormal stock price→ $E(S_T) = S_0 e^{\mu T}$

Other

Early Exercise

American call $D_n > X\left(1 - e^{-r(T-t_n)}\right)$

American put $D_n < X\left(1 - e^{-r(T-t_n)}\right)$

Valuation Of Warrants

$\frac{N}{N+M} \times$ value of regular call option

Implied Volatility

$$R_i = \ln(\frac{S_i}{S_{i-1}}) \quad \sigma^2 = \frac{\sum_{i=1}^{N}\left(R_i^c - \overline{R_i^c}\right)^2}{N-1}$$

Delta Hedge

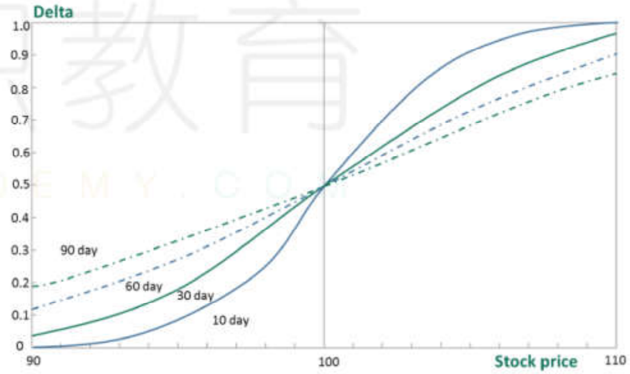
Greek → BSM输入变量对期权价格影响

- S→Delta ★ →Gamma ★
- X
- σ→Vega
- Rf→Rho
- T→Theta

Delta ★ ★

Delta介绍 计算，概念，d1要自己求

$$\Delta=\frac{\partial c}{\partial S}=N(d_1)\sim(0,1);\Delta=\frac{\partial p}{\partial S}=N(d_1)-1\sim(-1,0)$$



- When t→T, delta is ***unstable***
- When S<K, ***greater*** Δ with ***longer*** T; when S>K, ***greater*** Δ with ***shorter*** T
- For a call option ***at the money***, Δ =***0.5***
- ***Forward Delta=1 (Futures不等于1)***

Delta Hedge ★ ★

$$\text{number of options need to hedge}=\frac{\text{number of shares hedged}}{\text{delta of call option}}$$

已有股票，用期权Hedge。前面求HR是已有期权，用股票hedge。本质相同

Maintain The Hedge

only holds for very ***small changes*** in stock.

Dynamic Hedge

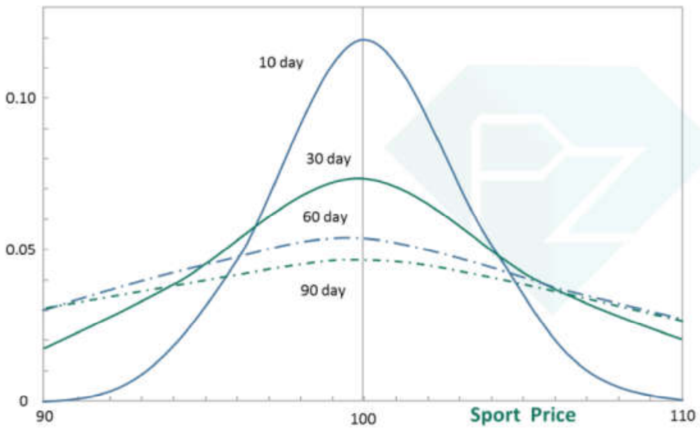
计算,求如何实现动态对冲

$$\text{portfolio delta}=\Delta_p=\sum_{i=1}^{n}w_i\Delta_i$$

Other Greek Letters

Gamma ★

计算，求Gamma Hedge，注意对冲Gamma的Option会产生新的Delta



Key Point
Gamma is largest when an option is at-the-money
When gamma is large, delta will be changing rapidly
Can be used to minimize the hedging error associated with a linear relationship (delta) to represent the curvature of the call-price function
Gamma is the same for call and put options.
Delta neutral positions: Hedge against small changes in stock price. Gamma neutral positions: Hedge against larger changes in stock price.

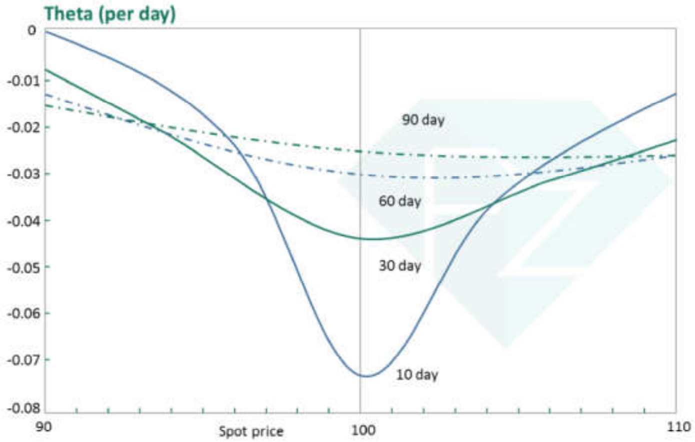
$$S >> K,\ \Delta_C \to 1,\ \Delta_P \to 0,\ \Gamma \to 0$$

$$S \approx K,\ \Delta_C \to 1/2,\ \Delta_P \to -1/2,\ \Gamma \to Max$$

$$S << K,\ \Delta_C \to 0,\ \Delta_P \to -1,\ \Gamma \to 0$$

Theta

$$\Theta = \frac{\partial c}{\partial t}$$

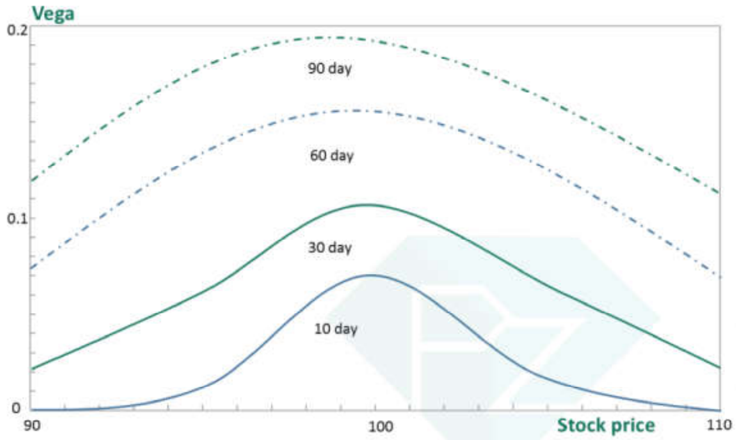


Theta (time decay)
Affect the value of call and put in a similar way.
Vary with changes in stock prices and as time passes.
Most pronounced when the option is at the money, especially near expiration.
Usually negative.
Usually increase in absolute value as expiration approaches.
European in the money put option is possible to have a positive theta.

> **Theta**

- For ***long*** position, theta < 0, means option lose value as time goes by;
- For ***American*** option, theta is ***always negative***;
- ***Short*** term at the money option has a greatest ***negative theta***.

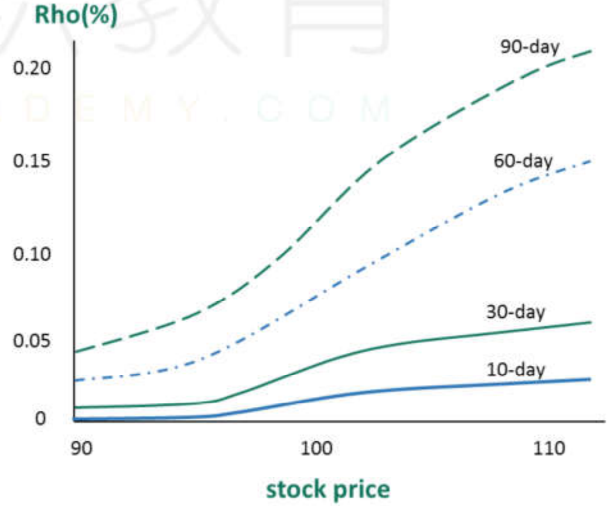
Vega



Vega
Vega of a call is equal to the Vega of a put.
Most sensitive to changes in volatility when they are at the money.

Rho

- The equity options are ***not*** as other variables.
- ***In the money calls and puts*** are ***more sensitive*** to changes in rates than ***out-of-the-money*** options.
- ***Increase*** in rates cause larger ***increases*** for in-the-money ***call*** prices and larger ***decreases*** for in-the-money ***puts***.



Section 13

COMMODITY FORWARDS AND FUTURES

Commodity Forwards And Futures

持有与出租互斥，不会同时出现

期货定价

Commodity → 现货 → 持有 / 出租

- 持有
  - 成本: Storage Cost → $F_{0,T} = S_0 e^{(r+\lambda)T}$
  - 收益: Convenience Yield → $F_{0,T} = S_0 e^{(r+\lambda-c)T}$
- 出租
  - 收益: Lease Rate → $F_{0,T} = S_0 e^{(r-\delta_1)T}$

→ Contango→FP>SP

Backwardation→SP>FP

Commodity	Details
Gold	can earn a return by being loaned out. influenced by the cost of production.
Corn	a commodity with seasonal production and a constant demand. the forward curve is increasing until harvest time, and it drops sharply
Electricity	***not a storable commodity***. Futures prices on electricity will ***vary much***
Natural Gas	a commodity with ***constant production*** but ***seasonal demand***. ***rises steadily in the fall***.
Oil	***easier to transport than natural gas***. Therefore, the price of oil is ***comparable worldwide***

Other Issue

Commodity Spread ★

- ***crush spread:*** 物理变化
- ***crack spread*** (eg. 7-4-3): 化学变化

Basis Risk

- $\Delta SP \neq \Delta FP$
- Roll Over Risk

Cross Hedging 原因★

- The ***liquidity*** of the futures contract (since delivery may not be an option).
- The ***correlation*** between the ***underlying*** for the futures contract and the asset(s) being hedged.
- The ***maturity*** of the futures contract.

Strip Hedge VS. Stack Hedge ★

- **a strip hedge**
  - by buying futures contracts that match the maturity and quantity for every month of the obligation.
- **a stack hedge.**
  - A strategy of continually rolling into the next near-term contract is referred to as stack and roll.

Advantages of stack hedge ★

- when near-term contracts are ***more readily available*** due to ***heavier volume*** and more ***liquidity***.
- ***distant futures*** on commodities often have wider ***bid-ask spreads*** and therefore ***larger transaction costs*** compared to near term contracts
- an ***oil producer*** may prefer a ***stack hedge*** in order to ***speculate*** on the shape of the forward curve

Section 14

FOREIGN EXCHANGE RISK

Foreign Exchange Risk

Net positon exposure
- Positive Net Position→Long Currency
- Negative Net Position→Short Currency

获得头寸方法
- business transactions
- foreign investments.
- hedging purposes.
- Speculating

net EUR exposure = (EUR assets - EUR liabilities) + (EUR bought - EUR sold)

额外风险，**Hedge**

- On-Balance-Sheet Hedging
- Off-Balance-Sheet Hedging

计算，掌握例题即可

Forward Price定价→IRP

离散复利

$$\text{fowrad} = \text{spot}\left[\frac{(1+r_{DC})}{(1+r_{FC})}\right]^{T}$$

连续复利

$$\text{fowrad} = \text{spot} \times e^{(r_{DC}-r_{FC})T}$$

Section 15-17

CENTRAL COUNTERPARTIES

Developing Of CCP

OTC向Exchange Traded学习

	Exchange-traded Derivatives	OTC Derivatives
Terms	Standardized	Custom ,negotiable
Maturity	Standardized	Negotiable, non-standard
Liquidity	Strong	Weak
Credit risk	Little(CCP guarantee)	High(bilateral)

OTC违约，引入CCP

CCP以外OTC Risk Mitigation方法

- ***capital*** requirements, ***regulation***, ***netting***, and ***margining***
- (***SPVs***), derivatives product companies (***DPCs***), ***monolines***, and ***credit derivative*** product companies (***CDPCs***).

CCP作用★

- set standards
- ensure netting of trades (Novation)
- maintain margin
- loss mutualization (i.e., risk sharing)
- make margin calls,
- carry out trade settlement

CCP优点★

- Netting: Multilateral netting
- Margin
- Loss mutualization
- Liquidity

CCP缺点★

- Failure of a CCP
- Moral hazard
- Costs
- Adverse selection

CCP交易机制

conditions needed to be centrally cleared

- Standardization
- Complexity
- Liquidity

CCP交易会员准入

- Admission criteria
- Financial commitment
- Operational criteria

*CCP数量→*最优是1个，但是难实现

- Regional differences
- product types
- Regulatory reasons

Types of CCPs

- A utility-driven CCP
- A profit-driven CCP

- **profit-driven CCPs效果更好**

Failure of a CCP

While it is an ***infrequent event***, CCPs do ***fail***. The financial trouble of a CCP may ultimately result in ***liquidity support from a central bank***

Risk Faced by Central Counterparties ★

- Default Risk
- Model Risk
- Liquidity Risk
- Operational Risk
- Legal Risk
- Investment risk
- Settlement and payment risk
- Foreign exchange risk
- Custody risk
- Concentration
- Sovereign risk
- Wrong-way risk


Thank You!

品职教育
PZACADEMY.COM

Valuation and Risk Models

FRM一级培训项目

Risk model

讲师：何　旋

www.pzacademy.com

Learning Structure

Fixed Income Valuation

- 1. Corporate Bonds
- 2. The Rating Agencies（删）
- 5. Mortgages and Mortgage-Backed Securities

金融市场与产品部分内容，建议在本科目中学习

- 3. Bond Price and Yield Measures
- 4. Risk Metrics and Hedges

Option Valuation 建议放在金融市场与产品部分学习

- Binomial Trees
- The Black-Scholes-Merton Model
- The Greek Letters

Risk Models

- 6. Market Risk Measurement and Management
- 7. Credit Risk Measurement and Management
- 8. Operational Risk Measurement and Management

Section 1

Corporate Bonds

Financial Statement Elements 了解

Basic Features of A Bond
Par value/face value/maturity value, Coupon rate, Maturity, Issuer/borrower & Bondholder
Role of corporate trustee: interpret legal language in bond indenture and represent the interests of the bondholders.

Bond Types ★★ 概念对比

Interest Payment Classifications

- Straight-coupon bonds. Variations:
  - ***Participating bonds*** pay at least the specified interest rate but may pay ***more*** if company's profits increase.
  - ***Income bonds*** pay at most the specified interest, but they may pay ***less*** if the company's income is not sufficient.
- Floating-rate bonds: Coupon rate = reference rate + quoted margin
- Zero-coupon bonds. Variations of the zero-coupon bond
  - The ***deferred-interest bond (DIB)***
  - ***Payment-in-kind (PIK) bonds***: pay ***interest with additional bonds*** for the initial period, and then cash interest after that period ends
  - A zero-coupon bond's interest rate is determined by the original-issue discount (OID): ***original-issue discount (OID) = face value — offering price***
  - One advantage of zero-coupon bonds is ***zero reinvestment risk***. A disadvantage is that the bondholder must ***pay taxes each year on the accrued interest***

Types of Collateral Backing

- Mortgage bonds: Corporate bonds can have a security, such as ***real property***, underlying the issue. It can be ***issued in a series*** in a blanket arrangement.
- Collateral trust bonds are ***backed by stocks, notes, bonds, or other similar obligations***.
- Equipment trust certificates (ETCs): a ***particular piece of equipment*** underlies the bond.
- Debentures are unsecured bonds
- Subordinated debenture bonds have a claim that is at ***the bottom of the list of creditors if the issuer goes into default.***
- Convertible debentures give the bondholder the right to convert the bond into common stock.
- Guaranteed bonds. Bonds issued by one company may also ***be guaranteed by other companies***.

Retiring Debt Guaranteed Bonds

- Call and refunding provisions ***give the issuer the right to purchase at a fixed price*** prior to maturity.
  - ***Fixed-price call.*** The firm can ***call back the bonds at specific prices that vary over the life*** of the bonds.
  - ***Make-whole call.*** In this case, market rates determine the call price, which is the ***present value of the bond's remaining cash flows*** subject to a floor price equal to par value.
- A sinking fund provision generally means the issuing firm retires a specified portion of the debt each year as outlined in the indenture.
  - An ***accelerated sinking- fund provision,*** which allows the firm to ***call back more bonds in early years***, which the firm would do if ***interest rates fall*** in those early years.
- A maintenance and replacement fund (M&R) ***maintain the credibility of the property*** backing the bonds.
- Tender offers are usually ***a means for retiring debt for most firms***.

Risks of Corporate Bond

了解、结论

Credit Risk

Types	Description
Credit default risk	The ***uncertainty concerning the issuer making timely payments of interest and principal*** as prescribed by the bond’s indenture.
Credit spread risk	Focuse on the ***difference between a corporate bond’s yield and the yield on a comparable-maturity benchmark Treasury*** security.

衡量方法

- ***Spread duration:*** the approximate percentage change in a bond's price for a 100 basis point change in the credit spread assuming that the Treasury rate is constant.
- ***Default Rate & Recovery Rate***
  - ***The issuer default rate*** is the number of issuers that defaulted over a year divided by the total number of issuers at the beginning of the year.
  - ***The dollar default rate*** is the par value of all bonds that defaulted in a given calendar year divided by the total par value of all bonds outstanding during the year.
  - ***The recovery rate*** is the amount received as a proportion of the total obligation after a bond defaults.

Event Risk

- Address the ***adverse consequences from possible events*** such as mergers, recapitalizations, restructurings, acquisitions, leveraged buyouts, and share repurchases.

High-yield Bonds

了解

- High-yield bonds (a.k.a. junk bonds) are those bonds rated ***below investment grade*** by ratings agencies.
- Over ***long*** periods of time, high-yield bonds should offer ***higher average returns.*** However, over ***shorter*** periods, the returns will be ***volatile where large losses are possible.***

Types of high-yield bonds	
Young and growing companies	Do ***not have strong financial statements***
Established firms	deteriorating financial situation
Fallen angels	They are bonds that were ***issued with an investment-grade rating, but then events led to the ratings agencies lowering the rating*** to below investment grade.
Restructurings and leveraged buyouts	May ***increase the credit risk*** of a company to the point where the bonds become non-investment grade. In this process, the firm may issue non-investment grade debt to pay off the bridge loans taken to finance the acquisition.

Section 2

The Rating Agencies

The Rating Agencies ★ 名词掌握

Rating Scales		
Investment Grade	S&P	Moody's
Highest quality	AAA	Aaa
High quality	AA	Aa
Strong capacity for repayment	A	A
Adequate capacity for repayment	BBB	Baa
Speculative Grade	S&P	Moody's
Likely to meet obligations with uncertainty	BB	Ba
High-risk obligations	B	B
Currently vulnerable to default	CCC	Caa
	CC	Ca
Lowest quality	C	C
In default	D	

- In a ***subscriber-pay model***, investors subscribed to the ratings agencies and paid for their services.
- The ***issuer-pay ("pay-for-rating") model*** is sometimes questioned as having the potential to distort the independence of the rating process.

Characteristics of ratings performance for corporate bonds are as follows:

- Ratings and corporate ***default rates*** are inversely related.
- ***Yield spreads*** over treasury bonds ***correlate highly with ratings***.
- ***Default rates and ratings remain inversely*** related throughout the business cycle.
- Default rates for ***investment grade*** issues are substantially ***lower*** than default rates for ***speculative grade*** issues.

Section 3

Bond Price and Yield Measures

★★★ 整章都很重要，以计算为主

Bond Price

计算

Bond Price Quotations:

- ***Treasury*** notes and bonds use a ***"32nds"*** convention. A bond quoted as 97-6 (or 97:06 or 97.6) is interpreted as 97 6/32% of par value. ***Corporate and municipal*** bonds are quoted ***in eighths*** (e.g., 102 1/8 indicates the price of the bond is 102.125% of par).
- A ***"+"*** in the quote indicates a ***half tick***. If the price is quoted as 101-12+, the bond would sell at 101 + 12.5/32.

Valuation with A Single Yield

计算

$$bond\ price = \frac{CPN_1}{(1+YTM/2)} + \frac{CPN_2}{(1+YTM/2)^2} + \ldots + \frac{CPN_{2N}+Par}{(1+YTM/2)^{2N}}$$

$$\text{Price of a perpetuity} = \frac{c}{y}F$$

Price & Time

定性

① At maturity date， price＝par

② Discount： 随着到期日临近， price上升

③ Premium： 随着到期日临近， price下降

The relationship between YTM, coupon, rate, and price

If coupon rate > YTM	at a premium.
If coupon rate = YTM	par value.
If coupon rate < YTM	at a discount.

结论

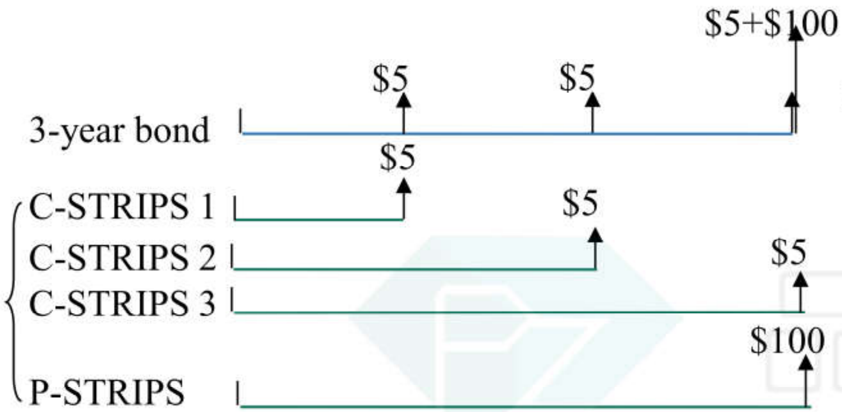
Relationships Between Price and Yield:

- A bond's ***price and YTM are inversely related***.
- A bond will be priced at a ***discount (premium)*** to par value if ***coupon rate is less (more) than its YTM.***
- For a given change in yield, the ***percentage*** price ***increase*** is ***greater*** than the percentage price ***decrease.***

Coupon effect:

- The lower(higher) the coupon rate, the greater(lower) the interest-rate risk.

Identifying Arbitrage Opportunities 计算



$$\text{no-arbitrage price}=\frac{CPN_1}{(1+S_1)}+\frac{CPN_2}{(1+S_2)^2}+\cdots\cdots+\frac{CPN_N+Par}{(1+S_N)^N}$$

Constructing A Replicating Portfolio 基础班例题

	Coupon	Face Amount	CF(t=0.5)	CF(t=1)	CF(t=1.5)	CF(t=2)
Bond2	7%	1.73%	1.7906			
Bond3	12%	1.79%	0.1074	1.8974		
Bond4	5%	1.89%	0.0473	0.0473	1.9373	
Bond5	6%	101.94%	3.0582	3.0582	3.0582	101.94%*103
Total CFs			5	5	5	105
Bond 1 CFs			5	5	5	105

Computing Price Between Coupon Dates

计算

Coupon payment dates — 06/30

Settlement date — 09/30

Coupon payment dates — 12/31

Full price:
- Seller's interest → Accrued Interest → Seller's interest
- Agreed-upon Bond Price → Clean price → Buyer's interest

Total interest that buyer will receive: Seller's interest + Buyer's interest

① Full Price = Clean Price +Accrued Interest

② Full price：折现求和得到的价格

Spot, Forward, and Par Rates

计算、结论

The Spot Rate Curve & Discount Factors

The Spot Rate: yield to maturity on a zero-coupon bond

$$\left(1+\frac{z(t)}{2}\right)^{2t}=\frac{1}{d(t)}$$

Forward Rate

$$1\cdot(1+R_1)^{T_1}\cdot(1+F_{1,2})^{(T_2-T_1)}=1\cdot(1+R_2)^{T_2}$$

$$e^{R_1T_1}\times e^{F_{1,2}(T_2-T_1)_1}=e^{R_2T_2}$$

Par Rates

the rate at which the present value of a bond equals its par value.

$$\frac{\text{par rate}}{2}\times\sum_{t=1}^{2T}d\left(\frac{t}{2}\right)+d(T)=1$$

- Bond ***prices*** will tend to ***increase*** with maturity when ***coupon rates are above the relevant forward rates.***
- When ***short-term rates are above the forward rates*** utilized by bond prices, the investors who ***rolls over shorter-term investments will tend to outperform*** investors who invest in longer-term investments.

Bond Yield

计算、结论

Yield to Maturity

$$bond\ \ price = \frac{CPN_1}{(1+YTM/2)} + \frac{CPN_2}{(1+YTM/2)^2} + \ldots + \frac{CPN_{2N} + Par}{(1+YTM/2)^{2N}}$$

Critical Assumptions

- hold the bond ***until maturity***
- ***full, timely coupon, principal payments*** (no default)
- coupons are ***reinvested at original YTM***

limitations of YTM

- The yield to maturity ***assumes cash flows will be reinvested at the YTM*** and assumes that the bond will be ***held until maturity***
- If the ***"average" reinvestment rate is below the YTM, the realized yield will be below the YTM***

Sport Rates and YTM

- When the term structure of spot rates is ***upward - sloping***, the ***YTM*** will be ***below spot rate.***
- When the term structure of spot rates is ***downward - sloping***, the ***YTM*** will be ***above spot rate.***

Bond Equivalent Yield (BEY)

- BEY=2 x the semiannual discount rate
- BEY is ***semiannual YTM*** or semiannual-pay YTM.

Bond Equivalent Yield (BEY)

- The calculation of the yield to call is the same as the calculation of yield to maturity except that ***the call price is substituted for the par value in FV and the number of semiannual periods until the call date is substituted for periods to maturity, N.***

Realized Return: YTM Assumption

计算、结论

The coupon payments can be reinvested at an interest rate equal to the yield to maturity

Reinvestment risk exist. Future interest rates can be less than the yield to maturity at the time bond is purchased.

The bond is held to maturity

If the bond is not held to maturity, the investor faces the risk that he may have to sell for less than the purchase price, resulting a return that is less than the yield to maturity, known as **interest rate risk**.

Realized Return

$$R_{t-1,t} = \frac{BV_t + C_t - BV_{t-1}}{B_{Vt-1}}$$

Bond Spread ⟶ $$bond\ price = \frac{C}{\left[1+f(1.0)+s\right]} + \frac{C+P}{\left[1+f(1.0)+s\right]\times\left[1+f(2.0)+s\right]}$$

Yield Curves Shapes		
Parallel Shift		
Non-parallel Shift	Yield curve twists	Flattening
		Steepening
	Yield curve butterfly shifts	A positive butterfly
		A negative butterfly

Return Decomposition 结论

total price apprciation $= BV_t(R_t, s_t) - BV_{t-1}(R_{t-1}, s_{t-1})$

The carry-roll-down component	carry - roll - down $= BV_t(R'_t, s_{t-1}) - BV_{t-1}(R_{t-1}, s_{t-1})$ R'_t: expected return term structure
The rate changes component	rate changes $= BV_t(R_t, s_{t-1}) - BV_t(R'_t, s_{t-1})$
The spread change component	rate changes $= BV_t(R_t, s_t) - BV_t(R_t, s_{t-1})$

Section 4

Risk Metrics and Hedges

★★★ 整章都很重要

Summary: Interest Rate Factors 概念对比

Interest Rate Factors	
One-factor approach	Price sensitivity based on ***parallel shifts*** in the yield curve. Assumes that ***a change in one rate*** (e.g., 20-year rate) ***will impact all other rates along the curve in a similar fashion.***
Multi-factor approach	Rates in different regions of the term structure are ***not always correlated.*** The risk that rates along the term structure ***move differently*** (i.e., ***nonparallel shifts***) is called ***yield curve risk.***

One-factor Risk Metrics and Hedges

Bond Duration 计算

Macaulay, Modified and Effective Duration

$$Mac.Dur. = \sum_{t=1}^{T}\left[\frac{PV(C_t)}{P} \times t\right] = \sum_{t=1}^{T}(\omega_t \times t)$$

$$\text{Modified duration} = \frac{\text{Macaulay duration}}{\text{1+periodic market yield}} = \frac{1}{\text{BV}}\frac{\Delta\text{BV}}{\Delta\text{y}}$$

$$Effective\ duration = \frac{BV_{-\Delta y}\text{-}BV_{+\Delta y}}{2 \times BV_0 \times \Delta y}$$

DV01 Application to Hedging

$$\text{DV01} = \frac{\Delta\text{BV}}{10,000 \times \Delta\text{y}}$$

DV01=Duration × Bond Value ×0.01%

$$HR = \frac{DV01(\text{per \$100 of initial position})}{DV01(\text{per \$100 of hedging instrument})}$$

Bond Duration 结论

Properties of Bond Duration

- A ***higher coupon rate or a higher yield-to-maturity reduces the duration*** measures.
- A ***longer time-to-maturity usually leads to a higher duration.***
  - It always does so for a bond priced at a ***premium or at par value***.
  - But if the bond is priced at a ***discount***, a longer time-to-maturity ***might lead to a lower duration.***
- Bond with embedded options (callable bond & putable bond) has lower duration.
- The ***greater the yield to maturity***, all else equal, the ***lower the bond's duration***

Bond Convexity

Duration & Convexity

- ***Duration*** is a good approximation of price changes for relatively ***small changes in interest rates.*** As rate changes grow ***larger***, the ***curvature*** of the bond price-yield relationship becomes more important

计算

$$\text{convexity} = \frac{1}{BV}\frac{d^2BV}{dy^2} \qquad convexity = \frac{BV_{-\Delta y} + BV_{+\Delta y} - 2\times BV_0}{BV_0 \times \Delta y^2}$$

The ***first derivative measures how price changes with yields (i.e., duration)***, while the ***second derivative measures how the first derivative changes with yields (i.e., convexity)***.

Properties of Bond Convexity 结论

- The higher the convexity number, the higher the ***price volatility.***
- Convexity is also a measure of ***dispersion of cash flows.***
- ***Convexity increases*** at an increasing rate as ***duration increases***.

Bond Convexity

Callable bond: Negative convexity 结论

- With ***callable debt***, the ***upside price appreciation*** in response to decreasing yields is ***limited***
- It is only at ***lower yields*** that the callable bond will exhibit ***negative convexity***.

Price Change Using Both Duration and Convexity 计算

$$\Delta P = -\underbrace{\boxed{D^* \cdot P}}_{\text{dollar duration}} \cdot \Delta y + \frac{1}{2}\underbrace{\boxed{C \cdot P}}_{\text{dollar convexity}} \cdot (\Delta y)^2 + \cdots$$

The actual, exact price $P = f(y_0 + \Delta y)$

The duration estimate $P = P_0 - D^* P_0 \Delta y$

The duration and convexity estimate $P = P_0 - D^* P_0 \Delta y + \frac{1}{2} C P_0 (\Delta y)^2$

Portfolio Duration and Convexity

计算

$$D_{port} = \sum_{j=1}^{K} w_j \times D_j$$

$$C_{port} = \sum_{j=1}^{K} w_j \times C_j$$

Barbell & Bullet Strategy 概念对比

- A barbell strategy is typically used when an investment manager uses bonds with ***short and long maturities, thus forgoing any intermediate-term bonds. Barbell*** benefits more from ***interest rate volatility.***
- A bullet strategy is used when an investment manager buy bonds ***concentrated in the intermediate*** maturity range. ***Bullet*** performs well ***if yields remain at current levels***.

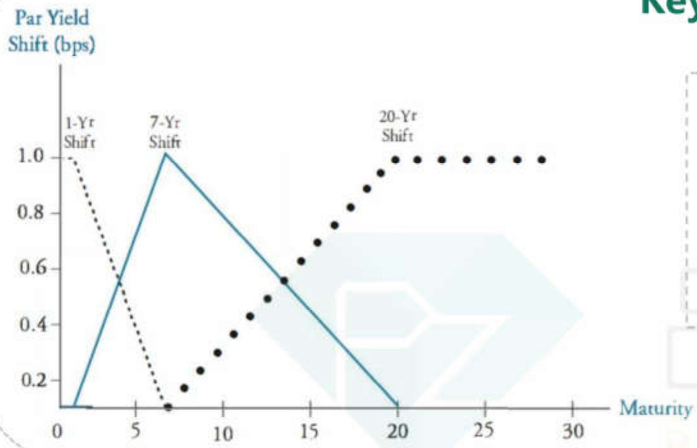
Multi-factor Risk Metrics and Hedges 概念对比、结论

- Weaknesses of Single-factor Approaches: A single-factor approach assumes that ***all rate changes are driven by a single factor*** (***parallel*** fashion).
- Yield Curve Risk is the risk that rates along the term structure ***move differently*** (i.e., nonparallel shifts).

Key Rate Approach	
Key rate exposures (下页补充)	• describe how the risk of a bond portfolio is distributed along the term structure • using ***rates from the most liquid bonds available*** • The most common key rates used for the U.S. Treasury and related markets are par yield bonds——2-, 5-,10- and 30-year par yields (par rate). If ***one of these key rates shifts by one basis point, it is called a key rate shift.***
Partial '01s	• Measuring and hedging risk in ***swap portfolios***. • These approaches are similar to the key rate approach, but instead divide the term structure into ***more parts***. Risk along the yield curve is thus measured ***more frequently*** • Swap market participants fit a ***par rate curve (i.e., swap rate curve)*** daily or even more frequently
Forward-bucket '01s	• Forward-bucket '01s are computed by ***shifting the forward rate*** over several regions of the term structure, one region at a time, after the term structure is divided into various buckets.

Key Rate Exposures 概念对比、结论

Key Rate Shifts



key rates are mostly affected by the ***closest key rate***

- 1-year key rate will affect all rates from 0 to 7 years; the 7-year key rate affects all rates from 1 year to 20 years; and the 20-year key rate affects all rates from 7 years to the end of the curve.

Key Rate '01s and Durations

- For every basis point shift in a key rate, the corresponding ***key rate '01 provides the dollar change in the value of the bond. key rate duration provides the approximate percentage change*** in the value of the bond.

Advantages and disadvantages

- key rate exposure analysis is a ***useful tool for measuring bond price sensitivity;***
- It makes very strong ***assumptions*** about how the term structure behaves.

Key Rate Exposures

- These multi-factor approaches work well ***not only in estimating changes in the level of the portfolio, but also in the estimation of portfolio volatility*** because it ***incorporates correlation effects*** between various interest rate assumptions.

Section 5

Mortgages and Mortgage-Backed Securities

Residential Mortgage

了解

Lien Status

- A ***first lien*** would give the lender the first right to receive proceeds on liquidation

Interest Rate Type

Fixed-rate mortgages
Adjustable-rate mortgages (ARMs): ***default risk is high***

Credit Classification

Prime (A-grade) loans
Subprime (B-grade) loans
Alternative-A loans: are the loans in between prime and subprime.

Prepayments

Prepayments reduce the mortgage balance and amortization period. They can occur because of the following reasons:

- ***Home is sold***
- ***Refinancing*** due to ***lower rates***
- ***Partial prepayments***

Credit Guarantees

- With ***agency MBSs***, the investor bears ***no credit risk*** because the GSEs have been paid a fee to guarantee the underlying mortgages.
- With a ***non-agency MBS***, there is ***some credit risk*** but that is mitigated through the process of subordination.

Fixed-Rated, Level-Payment Mortgages

- The amount of the ***principal*** payment ***increases*** as time passes.
- The amount of ***interest decreases*** as time passes. Interest is calculated on a declining principal balance
- The ability of the borrower to repay results in ***prepayment risk***.

Prepayment ★★

Basic Characteristics 结论

- The option to prepay a mortgage is essentially a ***call option*** for the borrower.
- With agency MBSs, ***prepayments and defaults have the same impact*** on investors.

Factors That Influence Prepayments	
Seasonality	The ***summertime*** is a popular time for individuals to ***move*** (and mortgages must be paid out prior to the sale of a home)
Age of mortgage pool	the lower the age of the mortgage pool, the less likely the risk of prepayment.
Personal	Marital ***breakdown***, ***loss of employment***, family emergencies, and destruction of property
Housing prices	***Property value increases*** may spur an increase in prepayments
Refinancing burnout	***To the extent that there has been a significant amount of prepayment*** or refinancing activity in the mortgage pool in the past, ***the risk of prepayment in the future decreases.***

Prepayment Modeling	
Refinancing a mortgage	The ***path*** that mortgage rates; the ***term structure*** of mortgage rates
Extracting home equity	***cash-out*** refinancing
Due on sale	housing turnover; seasonality
borrower defaults	mortgage guarantors pay the interest and principal outstanding.
Partial payments	tend to occur ***when a mortgage is older*** or has a relatively low balance.

Measure of Prepayment Rate 计算

Single monthly mortality(SMM)
Conditional prepayment rate(CPR)

$$(1-SMM)^{12}=(1-\overline{CPR})$$

$$PRE=SMM_{m}\times(Mort_{begin\ at\ m}-Scheduled\ principal\ pay\ for\ m)$$

Public Securities Association (PSA). 100%PSA:

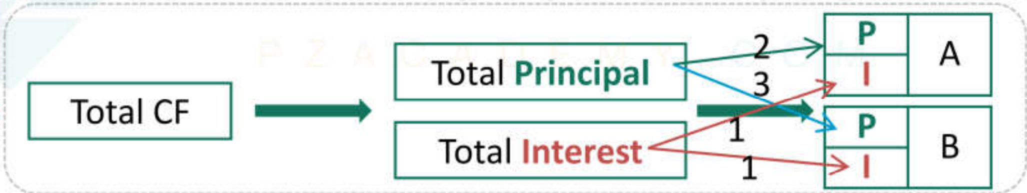
- CPR=0.2% for the first month after origination, increasing by 0.2% per month up to 30 months.
- CPR=6% for months 30 to 360
- After 30 months, no prepayment rate is added.

Securitization 概念对比

Pass-through Securities ➡ Monthly cash flow (MPS)=Monthly cash flow of the underlying mortgages - servicing fees

CMOs ➡

Sequential Pay tranches



PAC & Support tranches

- PAC: low prepayment risk; support: high prepayment risk
- ***broken or busted PAC***

PO & IO

- The underlying ***pass-through security*** exhibits significant ***negative convexity***. The ***PO*** exhibits some ***negative convexity at low rates.***
- Since ***prepayment rates increase as mortgage rates decline, PO prices increase when interest rates fall. IOs' price*** move in the ***same direction as market rates.***
- The ***PO and IO prices are more volatile*** than the underlying pass-through.

MBS Valuation ★★ 结论

总结：bond定价方法

$$P_0=\sum_{i=i}^{N}\frac{CF_i}{(1+r)^i}$$ ➡

	Option-free bond	Bond with embedded option
CF	时间、金额确定	不确定(取决于将来r的变化)
r	Single yield (YTM)：$bond\ \ price=\dfrac{CPN_1}{(1+YTM)}+\dfrac{CPN_2}{(1+YTM)^2}+\cdots+\dfrac{CPN_N+Par}{(1+YTM)^N}$	
	Arbitrage-free: no-arbitrage price$=\dfrac{\text{CPN}_1}{(1+\text{S}_1)}+\dfrac{\text{CPN}_2}{(1+\text{S}_2)^2}+\cdots\cdots+\dfrac{\text{CPN}_\text{N}+\text{Par}}{(1+\text{S}_\text{N})^\text{N}}$	
	Binomial tree: backward induction (考虑将来r变化) → callable, putable	
	Monte Carlo Simulation: 解决*path dependency* → MBS	

Option-Adjusted Spread (OAS)

- 特点: adjusting for ***prepayment risk***
- 计算: OAS is added to all the spot rates of all the interest rate paths, will make the average present value of the paths equal to the actual observed market price plus accrued interest.
- 对比z-spread
  - ***option cost = zero-volatility spread — OAS***
  - OAS ***increases*** as ***volatility declines***, all other things equal.

Section 6

Market Risk Measurement and Management

Measures of Financial Risk ★★★ 概念对比

Measures	Features	优缺点	Coherent
Mean-Variance Framework	standard deviation or variance	• It assume that ***return distributions for portfolios are elliptical distributions. (normal distribution)*** • The use of the standard deviation is ***not appropriate for non-normal distributions.***	Violates monotonicity, Translation invariance
VaR at Risk (VaR)	The ***worst possible loss*** • VaR ***increases*** when ***confidence level*** increases. • VaR will ***increase*** with increases in the ***holding period.***	• VaR measurements work well with ***elliptical return distributions***, such as the normal distribution. • VaR are subject to both model and implementation risk. • VaR does not tell the investor the magnitude of the actual loss. • ***risk-seekers*** investors	Violates subadditivity
Expected Shortfall (Conditional VaR)	the ***expected loss*** given that the portfolio return already lies ***below the pre-specified worst case*** quantile return	• ES gives an estimate of the ***magnitude of a loss for unfavorable events***. • The portfolio risk surface for ES is convex. • ES has ***less restrictive assumptions*** • ***risk-neutral*** investors	Yes
Spectral Risk Measures	the ***weighted averages of the return quantiles*** from the loss distributions.	• ***ES is a special case: tail losses*** have an ***equal weight***, and ***all other quantiles have a weight of zero.*** • The VaR is a special case where ***only a single quantile*** is measured and ***places no weight on tail losses.***	Yes

Types	Definition
Monotonicity	portfolio with greater future returns will likely have less risk $R_1 \geq R_2, then \ \ \rho(R_1) \leq \rho(R_2)$
Subadditivity	the risk of a portfolio is at most equal to the risk of the assets within the portfolio $\rho(R_1+R_2) \leq \rho(R_1)+\rho(R_2)$
Positive homogeneity	the size of a portfolio, β, will impact the size of its risk $\beta>0, \ \rho(\beta R)=\beta\rho(R)$
Translation invariance	the risk of a portfolio is dependent on the assets within the portfolio for constant c, $\rho(R+c)=\rho(R)-c$

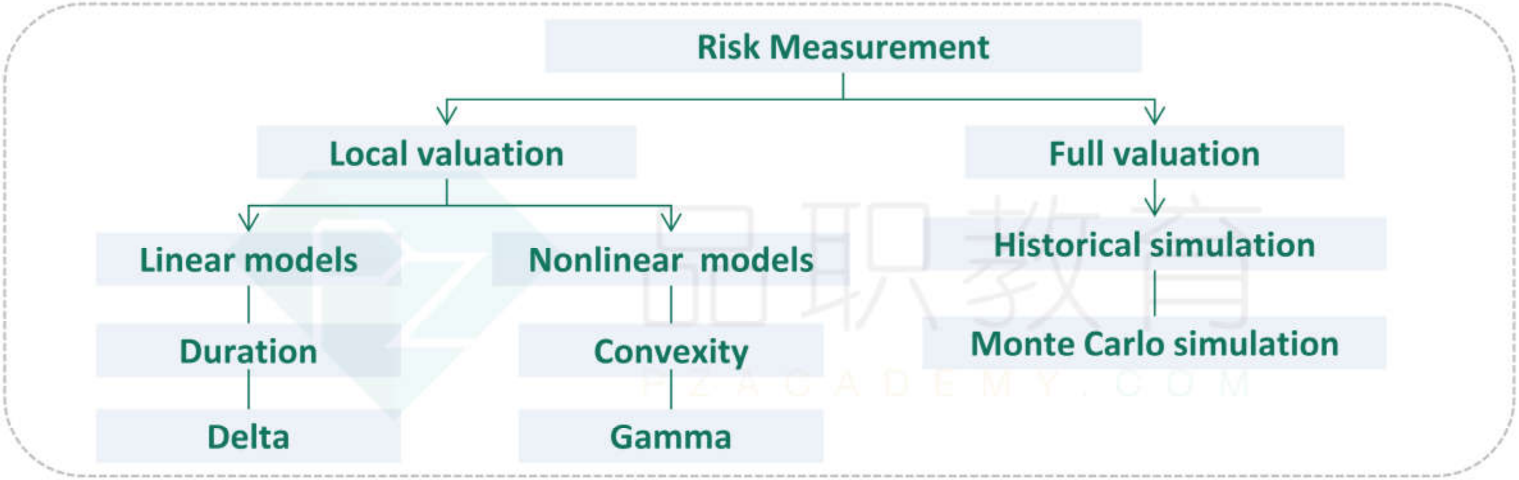
Defining and Calculating VaR ★★★ 计算

Calculating VaR ➡

- **Delta-normal VaR %**
  - assuming that asset returns conform to a ***standard normal distribution***
  - E(R) = 0, $\text{VaR}(X\%) = z_{X\%}\sigma$
  - E(R) ≠ 0, $\text{VaR} = |\text{E(R)} - z\sigma|$
- **dollar VaR** → $\text{VaR}(X\%)_{\text{dollar basis}} = \text{VaR}(X\%)_{\text{decimal basis}} \times \text{asset value} = z_{X\%}\sigma \times \text{asset value}$
- **VaR Conversions**
  - time period: $\text{VaR}(X\%)_{\text{J-days}} = \text{VaR}(X\%)_{\text{1-days}} \sqrt{\text{J}}$
  - confidence levels

VaR at Risk (VaR)

The VaR Methods ★★★



- ***Local valuation:*** Replace portfolio positions with linear exposures on the appropriate risk factor.
- ***Full valuation methods:*** fully reprice the portfolio for each scenario.
  - Full valuation methods is more complex than linear methods, but can be ***accurate*** because they ***include all nonlinear relationships*** and other potential correlations that may not be included in the linear valuation models.

Local Valuation

计算

- **The delta-normal approach:** $\text{VaR} = |\Delta_0| \times (z\sigma S_0)$
- **Delta-gamma:**

$$VaR(dP) = |-D^{*}P| \times VaR(dy) - \frac{1}{2}(C \times P) \times VaR(dy)^2$$

$$VaR(df) = |\Delta| \times VaR(dS) - \frac{1}{2}\Gamma \times VaR(dS)^2$$

结论

Advantages & disadvantages	
Advantages	disadvantage
• ***Easy*** to implement. • ***Quick*** calculation. • ***Conducive to analysis***. • Delta-gamma: consider ***Nonlinear relationships of option-like positions***	• The need to ***assume a normal distribution.*** • The method is ***unable*** to properly account for distributions with ***fat tails***.

Full Valuation

概念对比

Historical Simulation Method	• ***accumulate a number of past daily returns***, rank the returns from highest to lowest, and identify the lowest 5% of returns. • Each observation can be viewed as having a probability distribution with ***50% to the left and 50% to the right***. When considering the previous example, 5% VaR with 100 observations would take the average of the fifth and sixth observations
Monte Carlo Simulation Method	• ***computer software*** that generates hundreds, thousands, or even millions of possible outcomes from the distributions of inputs specified by the user. • The several thousand weighted average portfolio returns will naturally form a distribution, which will approximate the normal distribution. VaR is calculated in the same way as with the delta-normal method.

Advantages & disadvantages of the Historical Simulation Method	
Advantages	disadvantage
Easy to implement	It may ***not be enough historical data*** for al assets.
Calculations are ***simple***	Only ***one path of events is used*** (the actual history).
Horizon is based on the intervals of historical data.	Time variation of risk in the past may ***not represent future***.
Full valuation is based on ***actual prices***.	The model may ***not recognize changes*** in volatility and correlations.
It is ***not exposed to model risk***.	It is slow to ***adapt to new volatilities*** and correlations.
It ***includes all correlations*** as embedded in market price.	A small number of actual observations may lead to ***insufficiently*** defined distribution tails.

Advantages & disadvantages of the Monte Carlo Simulation Method	
Advantages	disadvantage
• It is the ***most powerful*** model. • It can account for both ***linear*** and ***nonlinear*** risks. • It can include ***time variation*** in ***risk*** and ***correlations*** by aging positions over chosen horizons. • It is extremely ***flexible*** and can incorporate additional risk factors easily. • Nearly ***unlimited*** numbers of ***scenarios*** can produce well-described distributions.	• It is subject to ***model risk*** of the stochastic processes chosen. • It is ***expensive*** because of the intellectual and technological skills required. • There is a ***lengthy computation time*** as the number of valuations escalates quickly. • It is subject to ***sampling variation*** at ***lower numbers*** of simulations.

Comparing The Methods		
Method	Appropriate for	Disadvantage
The delta-normal method	Large portfolios ***without significant option-like*** exposures.	***less accurately***
Historical data or Monte Carlo simulations.	large portfolios with ***substantial option-like exposures***, a wider range of risk factors, or a longer-term horizon.	***more time consuming*** and ***costly***.

Deviations from Normality 结论

- ***Fat-tailed:*** distribution with a ***higher probability of observations occurring in the tails*** relative to the normal distribution.
- ***Skewed:*** A distribution is skewed when the distribution is ***not symmetrical***.
- ***Unstable:*** If the parameters of the model are unstable, they are ***not constant*** but vary over time.

Reasons for "Fat Tails"

- Unconditional Distribution: the mean and standard deviation are the ***same for asset returns for any given day***
- Conditional distribution: ***different market or economic conditions*** may cause the mean and variance to ***change*** over time.
- Volatility is time-varying (Volatility Clustering)

A Regime-switching Volatility Model

- assumes ***different market regimes*** exist with ***high or low volatility***.
- The ***regime-switching model captures the conditional normality and may resolve the fat-tail problem*** and other deviations from normality.
- VaR assumes asset returns follow a normal distribution, but asset return distributions tend to exhibit fat tails.

Various Approaches for Estimating VaR ★★★ 概念对比

Historical-based approaches

The parametric approach

- Assume asset returns are ***normally*** or ***lognormally*** distributed with time-varying volatility.

The nonparametric approach

- There are ***no underlying assumptions*** of asset returns distribution. The most common model: historical simulation method

The hybrid approach

- Combine techniques of both parametric and nonparametric methods

The implied-volatility-based approach

- uses ***derivative pricing models such as the Black- Scholes-Merton option pricing model*** to estimate an ***implied volatility based on current market data*** rather than historical data.

Historical-based Approaches

Parametric Approaches for VaR

Historical variance	Future volatility is based on calculating ***historical variance***: $\sigma_t^2 = (r^2_{t\text{-}K,t\text{-}K+1} + \cdots + r^2_{t\text{-}3,t\text{-}2} + r^2_{t\text{-}2,t\text{-}1} + r^2_{t\text{-}1,t})/K$
The RiskMetrics® and GARCH approaches	Exponential smoothing weighting methods • The ***historical standard deviation approach assumes all K returns*** are ***equally weighted***. • The ***exponential smoothing methods*** place a ***higher weight on more recent data***, and the ***weights decline exponentially to zero*** as returns become older. $\sigma_t^2 = (1-\lambda)(\lambda^0 r^2_{t\text{-}1,t} + \lambda^1 r^2_{t\text{-}2,t\text{-}1} + \lambda^2 r^2_{t\text{-}3,t\text{-}2} + \cdots + \lambda^N r^2_{t\text{-}N\text{-}1,t\text{-}N})$ $\sigma_t^2 = a + bt^2_{t-1,t} + c\sigma^2_{t-1}$

Nonparametric Methods

Historical simulation	***All returns are weighted equally*** based on the number of observations ***(1/K).***
Hybrid approach	Step 1: Assign ***weights*** for historical realized returns to the most recent K returns ***using an exponential smoothing*** process as follows: $[(1-\lambda)/(1-\lambda^K)],[(1-\lambda)/(1-\lambda^K)]\lambda^1,\ldots,[(1-\lambda)/(1-\lambda^K)]\lambda^{K-1}$ Step 2: ***Order the returns*** Step 3: Determine the VaR by starting with the lowest return and ***accumulating the weights until x percentage*** is reached.
Multivariate density estimation (MDE)	• Conditional volatility for each market state or regime: $\sigma_t^2 = \sum_{i=1}^{K} \omega(x_{t-i}) r^2_{t-i}$ • The MDE model is very flexible in identifying dependence on state variables

Advantages & Disadvantage of Nonparametric Methods Compared To Parametric Methods	
Advantage	• Nonparametric models do ***not require assumptions regarding the entire distribution*** of returns to estimate VaR. • ***Fat tails, skewness, and other deviations from some assumed distribution*** are no longer a concern in the estimation process for nonparametric methods. • ***Multivariate density estimation (MDE)*** allows for ***weights*** to vary ***based*** on how relevant the data is to the ***current market environment***, regardless of the timing of the most relevant data. • MDE is ***very flexible*** in introducing dependence on economic variables (called state variables or conditioning variables). • ***Hybrid approach*** does ***not require distribution assumptions*** because it uses a historical simulation approach with an exponential weighting scheme.
Disadvantage	• Data is used more efficiently with parametric methods than nonparametric methods. Therefore, ***large sample sizes are required*** to precisely estimate volatility using historical simulation. • Separating the full sample of data into different market regimes ***reduces the amount of usable data*** for historical simulations. • MDE may lead to data snooping or ***over-fitting*** in identifying required assumptions regarding the weighting scheme identification of ***relevant conditioning variables*** and the number of observations used to estimate volatility. • MDE requires a ***large amount of data*** that is directly related to the number of conditioning variables used in the model.

Return Aggregation and VaR	
Parametric Approaches	A single VaR can be estimated by assuming asset returns are all ***normally distributed***. The covariance matrix of asset returns is used to calculate portfolio volatility and VaR.
The historical simulation approach	• Aggregate each period's historical returns weighted. • A major advantage is that ***no parameter estimates are required.***
Strong law of large numbers	• Estimate the volatility of the vector of aggregated returns and assumes normality based on the ***strong law of large numbers.*** • The strong law of large numbers states that an average of a very large number of random variables will end up converging to a normal random variable.

Mean Reversion and Long Time Horizons ★★★ 概念对比

Time series model: $X_i = a + b \times X_{i-1}$

The key parameter in this long-run mean equation	
b = 1	the long-run mean is infinite → ***random walk***
b < 1	the process is mean reverting

结论:

- The long- run mean of this model is evaluated as [a/(1— b)]
- The ***single-period conditional variance of the rate of change is σ^2 and that the two-period variance is $(1 + b^2)\sigma^2$.***
- If ***mean reversion*** exists, the ***long horizon risk (and the resulting VaR calculation) is smaller than square root volatility.***

Stress Testing 了解

Stress testing and VaR ➡

- VaR is useful for ***normal market conditions***. VaR ***cannot*** make predictions about the ***magnitude of the losses*** beyond the threshold
- Stress testing should be used as a ***complement to VaR measures, rather than as a substitute.***

Governance over Stress Testing

Governance Structure

- Board of directors: ultimately responsible
- Senior management: execute

Policies, Procedures and Documentation

Validation and Independent Review

Internal Audit

- provide independent evaluation

Other Key Aspects of Stress-testing Governance

- Stress-testing coverage
- Stress-testing types and approaches
- Capital and liquidity stress testing

Stress Testing and Other Risk-Management Tools

The loss estimates are differently between stress tests and EC:

- Enterprise-wide stress tests have examined a long period
- The role of probabilities: For many stress tests conducted ordinal rank assignments
- the approach to scenarios: Stress-test scenarios are often ad hoc and conditional

Use of VAR Models in Stress Tests

Stressed Calibration of Value at Risk Measures

Principles for Sound Stress Testing Practices and Supervision

- Stress Testing and Risk Governance
- Stress Testing Methodologies
- Stress Testing Scenarios
- Stress Test Handling

Section 7

Credit Risk Measurement and Management

External and Internal Ratings

Credit Transition Matrices ★★★ 计算

State Starting	Ending A	B	C	D	Total Prob.
A	0.97	0.03	0.00	0.00	1.00
B	0.02	0.93	0.02	0.03	1.00
C	0.01	0.12	0.64	0.23	1.00
D	0	0	0	1.00	1.00

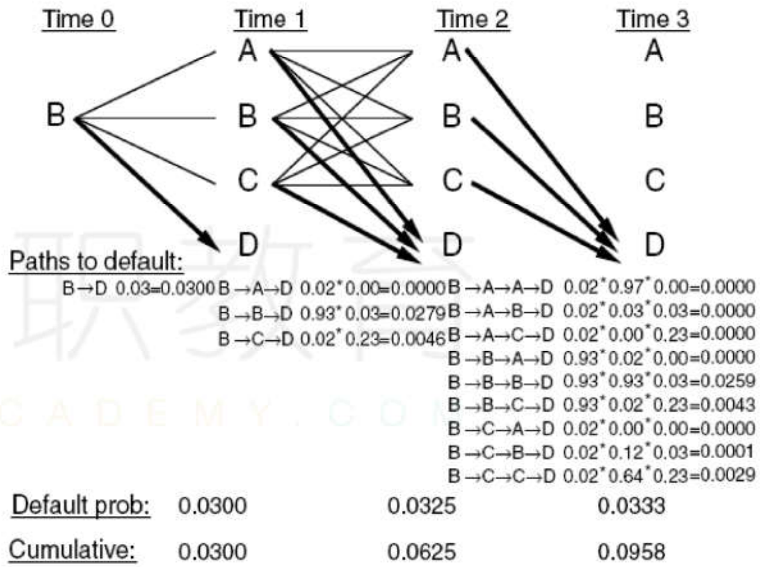
- In the first year, the probability of default

  $P(D_1 \mid B_0) = 3\%$

- In the second year, the probability of default

  $P(D_2 \mid A_1)P(A_1) + P(D_2 \mid B_1)P(B_1) + P(D_2 \mid C_1)P(C_1)$

  $= 0.00 \times 0.02 + 0.03 \times 0.93 + 0.23 \times 0.02 = 3.25\%.$



The Impacted Factors on External Raring	
Time horizon	The probability of default given any rating at the beginning of a cycle increases with the horizon.
Economic and industrial cycles	• The ratings is relatively ***stable*** • The default rate of lower-grade bonds is correlated with the economic cycle
Geographic location	For a ***given rating category, default rates can vary from industry to industry***

Internal Credit Rating

★ 名词掌握

At-the-point approach (short horizon) & Through-the-cycle approach (long horizon)

- Through-the-cycle approaches: ***high-rated firms may be underrated during growth*** periods and overrated during the decline of a cycle.
- The use of ***at-the-point*** approaches may be ***procyclical***. The changes in ratings and lending policies can ***lag*** the economic cycle.
- Through-the-cycle approach and at-the-point approach should not be mixed.

The Biases Affecting A Rating System

- ***Time horizon bias:*** mixing ratings from different approaches to score a company
- ***Homogeneity bias:*** inability to maintain consistent ratings methods
- ***Principal agent bias:*** moral hazard could result if bank employees do not act in the interest of management.
- ***Information bias:*** ratings assigned based on insufficient information
- ***Criteria bias:*** allocation of rating is based on unstable criteria
- ***Scale bias:*** ratings may be unstable over time
- ***Backtesting bias:*** incorrectly linking rating system to default rates
- ***Distribution bias:*** using an incorrect distribution to model probability of default.

Capital Structure in Banks ★★★ 计算、概念

Credit Risk Factors: EL & UL

expected loss = $\boxed{\text{adjusted exposure}}$ × LGD × PD

adjusted exposure = $OS + \alpha \times COM_U$

- OS: outstanding
- α: the fraction of committed funds drawn given default
- COM_U: unused portion of commitments

$$UL = \boxed{EA} \times \sqrt{PD \times \sigma_{LR}^2 + LR^2 \times \boxed{\sigma_{PD}^2}}$$

EA term explicitly recognizes that only the risky portion of the asset is subject to default.

$$\boxed{\sigma_{PD}^2} = PD \times (1 - PD)$$

σ_{PD}^2 is simply the variance of a binomial random

Portfolio Expected and Unexpected Loss

$$EL_P = \sum_i EL_i = \sum_i AE_i \times LGD_i \times EDF_i;\ UL_P = \sqrt{\sum_i \sum_j \rho_{ij} UL_i UL_j} \le \sum_i UL_i$$

For a two-asset portfolio, $UL_P = \sqrt{UL_1^2 + UL_2^2 + 2\rho_{12} UL_1 UL_2}$

The risk contribution (RC) of each asset

$$RC_i = UL_i \frac{\partial UL_p}{\partial UL_i} = \frac{UL_i \sum_j UL_j \rho_{ij}}{UL_p} \text{ and } \sum_i RC_i = UL_P$$

$$RC_1 = UL_1 \frac{\partial UL_p}{\partial UL_1} = \frac{UL_1^2 + \rho_{12} UL_1 UL_2}{UL_p} \text{ and } RC_2 = UL_2 \frac{\partial UL_p}{\partial UL_2} = \frac{UL_2^2 + \rho_{12} UL_1 UL_2}{UL_p}$$

结论:

- This **expected loss** equation describes the ***average behavior of a risky asset.*** The EL measure does not capture the variation in the risky asset’s value.
- This variation is referred to as **unexpected loss.**
- **Diversifiable and Undiversifiable Risk:** In a portfolio context, the ***diversifiable risk is reduced effectively to zero. undiversifiable risk is the residual credit risk*** (i.e., risk contribution) that cannot be diversified away.
- **The Effect of Correlation:** As the ***correlation between assets increases***, the bank suffers from ***concentration risk***.

Economic Capital

$\text{economic capital}_p = UL_p \times \text{capital multiplier (CM)}$	
Economic capital	absorb credit losses
Regulatory capital	Basel Accord; minimum capital

Modeling Credit Risk

- Credit risks are ***not normally distributed*** and tend to be ***highly skewed***, because ***maximum gains are limited*** to receiving promised payments while extreme losses are very ***rare events***.

Country risk

了解

Sources of Country Risk

- Economic growth life cycle
- Political risks
- The legal systems of countries
- The disproportionate reliance of a country on one commodity or service.

Country Risk Exposure

- Economic Growth Life Cycle
- Political risk: democracy or a dictatorship
- Legal Risk: The protection of property rights and the speed with which disputes are settled
- Economic Structure: A disproportionate reliance

Sovereign Defaults

分类

- **Foreign Currency Defaults:** Countries are unable to print money in a foreign currency to repay the debt.
- **Local Currency Defaults:** A study by Moody's finds that countries are increasingly defaulting on both foreign and local currency debts concurrently

Consequences of Sovereign Default

- **GDP growth.** decrease
- **Sovereign ratings and borrowing costs.** Borrowing costs are higher.
- **Trade retaliation.**
- **Fragile banking systems.**
- **Political change.**

Factors Influencing Sovereign Default Risk

- The country's level of indebtedness.
- Pension funds and social services.
- Tax receipts.
- Stability of tax receipts.
- Political risk.
- Backing from other countries/entities.

Measurement of default risk

Rating Agencies

- ***The ratings process*** includes: Evaluating factors that may contribute to default; Ratings recommendation; Foreign currency versus local currency ratings (Notch-up & Notch-down); Ratings review process
- ***Rating agencies have been criticized***
  - Ratings are biased upward
  - Herd behavior
  - Not timely enough
  - Overreaction leads to a vicious cycle
  - Ratings failures: Bad information; Overburdened analysts; Conflicts of interest; off-shoot businesses

The Sovereign Default Spread

- ***Sovereign bond yield*** compared to a ***riskless investment*** in the same currency and maturity.
- **Advantages** of Default Risk Spreads
  - Changes occur in real time.
  - Granularity.
  - Adjust quickly to new information.
- **Disadvantages** of Default Risk Spreads
  - Need for a risk-free security.
  - Cannot compare local currency bonds.
  - Greater volatility

Section 8

Operational Risk Measurement and Management

Defining Operational Risk ★★ 名词掌握

Basel Definition ➡

Definition approach	Cause impact
What cause operational risk	People (human factor)
	Internal processes
	Systems
	External events.
Excluding	Strategic risk and Reputational risk
Including	Legal risk

Basel Loss Event Types

- Clients, products, and business practices
- Internal fraud
- External fraud
- Damage to physical assets
- Execution, delivery, and process management
- Business disruption and system failures
- Employment practices and workplace safety

Operational Risk Events

- ***High Frequency/Low Severity(HFLS)***: occur regularly, but low-level losses
- ***Low Frequency/High Severity(LFHS)***: rare but devastating

Assessing operational risk ★★★ 结论、计算

The Basel Operational Risk Charge	
Basic Indicator Approach	$K_{BIA} = \alpha * GI,\ \alpha = 15\%$ NOTE: If ***negative gross revenues*** are experienced in a previous year, that year is not counted
Standardized Approach	Bank's activities are divided into 8 business lines $K_{TSA} = \sum_{1}^{8} \beta_i \times GI_i$
The advanced measurement approach (AMA)	The operational risk ***capital requirement is equal to the unexpected loss*** in a total loss distribution that corresponds to a confidence level of ***99.9% over a 1-year time horizon.***

Business Line	Beta Factor
corporate finance	18%
trading and sales	18%
retail banking	12%
commercial banking	15%
settlement and payment activities	18%
agency and custody services	15%
asset management	12%
retail brokerage	12%

Loss Distribution Approach

Historical-based Loss Distribution

Parametric-based LDA ➡

- Loss Frequency: is often modeled with a ***Poisson distribution***
- Loss Severity: is often modeled with a ***lognormal distribution*** (asymmetrical, fat-tailed)

➡

Convolution

- using a ***Monte Carlo simulation*** process.

Data Limitations	
Internal Data	The ***historical record of operational risk loss data is currently inadequate***.
External Data	• Two data sources: ***sharing agreements with other banks and public data.*** • Use a ***scale adjustment***
Scenario Analysis	This approach allows management to incorporate events that have not yet occurred.
Forward-Looking Approaches	Causal relationships, Risk and control self assessment (RCSA) program, Key risk indicators (KRIs)

The Power Law: $P(V > X) = K \times X^{-\alpha}$
useful in ***extreme value theory (EVT)*** when we evaluate the ***nature of the tails***

Managing operational risk ★ 结论

Expected loss (EL): high-frequency, low-severity events.	absorbed as an ***ongoing cost***
Unexpected loss (UL): deviation between the quantile loss at some confidence level and the expected loss. lower-frequency, higher-severity events	***capital***
The stress loss: loss in excess of the unexpected loss.	***Insurance:*** moral hazard; Adverse selection


Thank You!