Abstract Algebra: Theory and Applications

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Preface

TBD

Part I

Integers and Equivalence Relations

Chapter 1

Preliminaries

Theorem 1.0.1 Well Ordering Principle. Every nonempty set of positive integers contains a smallest member.

Theorem 1.0.2 Division Algorithm. Let a and b be integers with $b > 0$. Then there exist unique integers q and r with the property that $a = bq + r$, where $0 \leq r < b$. (Note: a and q could be negative.)

Theorem 1.0.3 GCD (Greatest Common Divisor) is a Linear Combination. For any nonzero integers a and b, there exist integers s and t such that $gcd(a, b) = as + bt$. Moreover, $gcd(a, d)$ is the smallest positive integer of the form $as + bt$.

Corollary 1.0.1 If a and b are relatively prime, then there exist integers s and t such that $as + bt = 1$.

Theorem 1.0.4 If $a \mod n = a'$ and $b \mod n = b'$, then $(a + b) \mod n = (a' + b') \mod n$ and $(ab) \mod n = (a'b') \mod n$.

Bibliography

[1] Joseph A. Gallian *Contemporary Abstract Algebra (7th Edition)*. Cengage Learning, 2010.