

Qualifying Exam Preparation III
Theory of Statistics

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Preface

STAT 528: References: Bickel & Doksum (Mathematical Statistics: Basic Ideas and Selected Topics, 2nd ed.); Casella & Berger (Statistical Inference, 2nd ed.).

STAT 553: References; Seber and Lee (Linear Regression Analysis, 2nd ed.) Stapleton (Linear Statistical Models), Christensen (Plane Answers to Complex Questions), Schervish (Theory of Statistics).

TBD

Part I

STAT528 Topics

Chapter 1

Data, Models, Statistics, Parameters

Definition 1.0.1 The set, S , of all possible outcomes of a particular experiment is called the sample space for the experiment.

Definition 1.0.2 An event is any collection of possible outcomes of an experiment, that is, any subset of S (including S itself).

Definition 1.0.3 A random variable is a function from a sample space S into the real numbers.

1.1 Distributions of Functions of a Random Variable

Theorem 1.1.1 From Casella & Berger Theorem 2.1.5) Let X have pdf $f_X(x)$ and let $Y = g(X)$, where g is a monotone function. Suppose that $f_X(x)$ is continuous and that $g^{-1}(y)$ has a continuous derivative. Then the pdf of Y is given by

$$f_Y(y) = f_X(g^{-1}(y)) \left| \frac{d}{dy} g^{-1}(y) \right| \quad (1.1)$$

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Chapter 2

Decision Theory (Bayes and Minimax Criteria, Risk Functions, Estimation and Testing in Terms of the Decision Theoretic Framework

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Bayesian Models, Conjugate (and Other) Prior Distributions

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Prediction (Optimal MSPE and Optimal Linear MSPE)

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Sufficiency (Factorization Theorem)

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Natural Sufficient Statistics

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Minimal Sufficiency

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Estimation (Least Squares,
MLE, Frequency Plug-in,
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Chapter 9

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**Asymptotic Approximation
/ Large Sample Theory
(Consistency, Delta
Method, Asymptotic
Normality of MLE,
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Chi-Square)**

Part II

Stat 553 Topics

Chapter 13

**Linear Models, Estimable
Functions, Least Squares
Estimates=LSE, Normal
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Gauss Markov Theorem,
BLUE**

Chapter 14

Multivariate Normal Distribution and Distribution of Linear and Quadratic Forms

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Properties of LSE and Generalized LSE

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Chapter 22

Hierarchical Bayes Analysis
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