Mathematics Notes

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Contents

Chapter 1

Introduction to $\mathbb{R}^n$

1.1 $\liminf$ and $\limsup$

Definition 1.1.1

$$\liminf_n A_n = \cup_{n=1}^{\infty} \cap_{i=n}^{\infty} A_i = \{x \mid x \in A_i \text{ eventually}\} \tag{1.1}$$

$$\limsup_n A_n = \cap_{n=1}^{\infty} \cup_{i=n}^{\infty} A_i = \{x \mid x \in A_i \text{ for infinitely many } i\} \tag{1.2}$$

The meaning of $\liminf$ can be seen by re-writing the above definition as: $x \in \liminf_n A_n$ if $\exists n \in \mathbb{N}$, s.t. $\forall i \geq n$ and $i \in \mathbb{N}$, $x \in A_i$. Hence the elements in $\liminf_n A_n$ are in all but (the first) finitely many sets, though the "first finitely many sets" may be different for different elements in $\liminf$. $\limsup$ can be best seen by examining its complement, according to the De Morgan's law.

Proposition 1.1.1

$$(\limsup_n A_n)^c = \liminf_n A_n^c \tag{1.3}$$

$$\liminf A_k \subset \limsup A_k \tag{1.4}$$

$$\limsup(A_k \cup B_k) = \limsup A_k \cup \limsup B_k \tag{1.5}$$

$$\liminf(A_k \cap B_k) = \liminf A_k \cap \liminf B_k \tag{1.6}$$