PRML Problems

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Problem 2.1 Prove that

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix}^{-1} = \begin{pmatrix} \mathbf{M} & -\mathbf{M}\mathbf{B}\mathbf{D}^{-1} \\ -\mathbf{D}^{-1}\mathbf{C}\mathbf{M} & \mathbf{D}^{-1} + \mathbf{D}^{-1}\mathbf{C}\mathbf{M}\mathbf{B}\mathbf{D}^{-1} \end{pmatrix} \tag{1}$$

where $\mathbf{M} = (\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C})^{-1}$.

Solution 2.1 We apply Gaussian Elimianation in the following way:

$$\left[\begin{pmatrix} \mathbf{I} & \mathbf{O} \\ -\mathbf{D}^{-1}\mathbf{C} & \mathbf{I} \end{pmatrix} \begin{pmatrix} (\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}C)^{-1} & \mathbf{O} \\ \mathbf{O} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{I} & -\mathbf{B} \\ \mathbf{O} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{O} & \mathbf{D}^{-1} \end{pmatrix} \right] \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} = \begin{pmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{O} & \mathbf{I} \end{pmatrix} \tag{2}$$

and we have

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix}^{-1} = \begin{pmatrix} \mathbf{I} & \mathbf{O} \\ -\mathbf{D}^{-1}\mathbf{C} & \mathbf{I} \end{pmatrix} \begin{pmatrix} (\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}C)^{-1} & \mathbf{O} \\ \mathbf{O} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{I} & -\mathbf{B} \\ \mathbf{O} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{O} & \mathbf{D}^{-1} \end{pmatrix} \tag{3}$$

$$= \begin{pmatrix} (\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C})^{-1} & -(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C})^{-1}\mathbf{B}\mathbf{D}^{-1} \\ -\mathbf{D}^{-1}\mathbf{C}(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C})^{-1} & \mathbf{D}^{-1} + \mathbf{D}^{-1}\mathbf{C}(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C})^{-1}\mathbf{B}\mathbf{D}^{-1} \end{pmatrix} \tag{4}$$

Problem 2.2 Show that the inverse of a symmetric matrix is itself symmetric.

Solution 2.2 Suppose $\mathbf{A} = \mathbf{A}^T$ is a symmetric matrix, and invertible. We have

$$(\mathbf{A}^{-1})^T = (\mathbf{A}^{-1})^T \mathbf{A} \mathbf{A}^{-1} = (\mathbf{A}^{-1})^T \mathbf{A}^T \mathbf{A}^{-1} = (\mathbf{A}\mathbf{A}^{-1})^T \mathbf{A}^{-1} = \mathbf{A}^{-1} \tag{5}$$