Quant Interview Guide

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April 12, 2021

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Part I

Brainteaser

Chapter 1

Logic Brainteaser

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1.1 Games

Question 1: *Poker Game* (Easy) (Tags: #1002.md; Brainteaser, Logic Brainteaser; Baidu;)

Two people play a deck of 52 cards. Each person in turn draws 1 to 4 cards and compare the number. What is your strategy if you are to draw first?

(Answer on page 8)

1.2 Chapter Answers

Answer 1: *Poker Game* (Question on page 7)

Chapter 2

Math Brainteaser

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2.1 Games

Question 2: *Two Strings* (Easy) (Tags: #1101.md; Brainteaser, Math Brainteaser; Microsoft;)

You have two strings. Each string takes exactly one hour to burn. The rate at which the strings are burnt is completely random and the two strings are different. How do you measure 45 minutes with these two strings?

(Answer on page 10)

Question 3: *Coin Game* (Easy) (Tags: #1102.md; Brainteaser, Math Brainteaser; Baidu;)

Two people take 1-5 coins in turn from a pile of 100 coins. Is there a strategy to guarantee that the first person to pick always get the last coin?

(Answer on page 20)

2.2 Chapter Answers

Answer 2: *Two Strings* (Question on page 9)

$45 = 30 + 15$. Burn one string (pick any one of the two) at both ends, and only one end of the other string. As soon as the first string is finished (which takes 30 minutes), burn the other end of the second string. It will take additional 15 minutes for the second string to finish. The total time is 45 minutes.

Answer 14: *Coin Game* (Question on page 18)

Part II

Mathematics

Chapter 3

Calculus

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3.1 Comparison

Question 4: *Compare cos and sin* (Easy) (Tags: #2005.md; Calculus; NA;)

Compare $\cos(\sin x)$ and $\sin(\cos x)$.

(Answer on page 15)

Question 5: *Compare e and pi* (Easy) (Tags: #2008.md; Calculus; 150 Book;)

Which number is larger, π^e or e^π?

(Answer on page 15)

3.2 Derivative

Question 6: *Calculate Integral* (Easy) (Tags: #2006.md; Calculus; Green Book;)

What is the integral of $\sec(x)$ from 0 to $\pi/6$?

(Answer on page 15)

Question 7: *Evaluate Polynomial* (Easy) (Tags: #2007.md; Calculus; World Quant;)

Consider a polynomial $f(x)$ and its derivative $f'(x)$ that are related according to

$$f(x) - f'(x) = x^3 + 3x^2 + 3x + 1.$$

What is $f(9)$?

(Answer on page 15)

3.3 Chapter Answers

Answer 4: *Compare cos and sin* (Question on page 13)

$$\begin{aligned}
f(x) &\equiv \cos\sin x - \sin\cos x \\
&= \cos\sin x - \cos\left(\frac{\pi}{2} - \cos x\right) \\
&= -2\sin\left(\frac{\sin x + (\frac{\pi}{2} - \cos x)}{2}\right)\sin\left(\frac{\sin x - (\frac{\pi}{2} - \cos x)}{2}\right) \\
&= -2\sin\left(\frac{\frac{\pi}{2} + \sqrt{2}\sin(x - \pi/4)}{2}\right)\sin\left(\frac{-\frac{\pi}{2} + \sqrt{2}\sin(x + \pi/4)}{2}\right)
\end{aligned}$$

Since $-1 \le \sin(x \pm \pi/4) \le 1$ and $2\sqrt{2} < \pi$, we have,

$$\begin{aligned}
0 < \frac{\frac{\pi}{2} + \sqrt{2}\sin(x - \pi/4)}{2} < \frac{\pi}{2} \\
-\frac{\pi}{2} < \frac{-\frac{\pi}{2} + \sqrt{2}\sin(x + \pi/4)}{2} < 0
\end{aligned}$$

And finally,

$$f(x) \equiv \cos\sin x - \sin\cos x > 0$$

Note that, the equality is not attained.

Answer 6: *Calculate Integral* (Question on page 13)

NA

Answer 7: *Evaluate Polynomial* (Question on page 14)

Since $f(x)$ is a polynomial, so we may write

$$\begin{aligned}
f(x) &= ax^3 + bx^2 + cx + d \\
f'(x) &= 3ax^2 + 2bx + c \\
f(x) - f'(x) &= ax^3 + (b - 3a)x^2 + (c - 2b)x + (d - c)
\end{aligned}$$

which implies $a = 1, b = 6, c = 15, d = 16$. Hence,

$$f(x) = x^3 + 6x^2 + 15x + 16.$$

Answer 5: *Compare e and pi* (Question on page 13)

Take log on both sides we have

$$\frac{\log\pi}{\pi} < \frac{\log e}{e},$$

because

$$\left(\frac{\log x}{x}\right)' = -\frac{\log x + 1}{x^2} < 0$$

for $x > e$. Therefore, $e^\pi > \pi^e$.

Chapter 4

Combinatorics and Probability

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4.1 Expectation

Question 8: *All Six Sides of A Die* (Easy) (Tags: #2101.md; Expectation; NA;)

What is the expected number of tosses you need to see all six sides of a die?

(Answer on page 20)

Question 9: *First Ace* (Easy) (Tags: #2102.md; Expectation; Green Book, page 95;)

You turn over a card one by one from a deck of 52 cards. What is the expected number of cards that you need to flip before you see the first ace?

(Answer on page 20)

Question 10: *Hand Shakes* (Easy) (Tags: #2112.md; Indicator Variables; Linearity of Expectations; NA;)

There are x people from country A and y people from country B. They sit around a table and shake hands with people on their left and right, but they only shake hands if they're from the same country. We want to know the expected number of handshake made.

(Answer on page 21)

Question 11: *Hunters and Ducks* (Easy) (Tags: #2113.md; Indicator Variables; Linearity of Expectations; Morgan Stanley;)

There are 10 hunters and 10 ducks flying over. Each hunter aims at one duck and there is a probability of 0.5 that a hunter kills the duck he aims at. What is the expected number of ducks that have survived?

(Answer on page 21)

4.2 Conditional Probability

Question 12: *Bayesian Child* (Easy) (Tags: #2103.md; Bayes Theorem; Conditional Probability; SIG;)

In a hospital there were 3 boys and some girls. A woman gave birth to a child in the hospital. A nurse picked up a child at random and was a boy. What is the probability that that woman gave birth to a boy?

(Answer on page 20)

4.3 Uniform Random Variables

Question 13: *Uniform Random Variables* (Easy) (Tags: #2104.md; Uniform Random Variables; NA;)

X, Y, Z are i.i.d. $U[0,1]$. What is the distribution of $(XY)^Z$?

(Answer on page 20)

4.4 Coin Games

Question 14: *Coin Game* (Easy) (Tags: #2111.md; Coin Games; SIG;)

You and I each flip 3 fair coins, if we got same heads I pay you $2, if different you pay me $1. Will you play this game?

(Answer on page 20)

4.5 Deterministic Games

Question 15: *Robot Flipping Coins* (Easy) (Tags: #2114.md; Deterministic Games; Markov Chain; Optiver;)

In a room there are half H and half T. A robot comes into the room and flips the Hs and tosses the Ts, repeatedly. What is the stationary distribution of the Hs and Ts in the room?

(Answer on page 22)

Question 16: *Coupon Collection* (Easy) (Tags: #2115.md; Dice Games; Deterministic Games; Green Book; Black Book; Jump;;)

Expected number of die rolls until all numbers are rolled? If I have already made N rolls, what is the expected number of distinctive numbers?

(Answer on page 22)

Question 17: *Two Heads* (Easy) (Tags: #2116.md; Think Backwards; Symmetry; Coin Games; NA;)

The expected number of flips to see two heads from a series of fair coin tosses.

(Answer on page 23)

Question 18: *One More Coin* (Easy) (Tags: #2117.md; Think Backwards; Symmetry; Coin Games; Green Book; Blue Book; HRT;;)

A tosses a fair coin $n + 1$ times; B tosses a fair coin n times. What is the probability that A has strictly more heads than B?

(Answer on page 23)

4.6 Chapter Answers

Answer 8: *All Six Sides of A Die* (Question on page 17)

The answer is $6 \cdot \sum_{i=1}^{6} 1/i$. Let X_i denote the random variable that tells us the number of trials it takes for us to see the i-th distinct side on a die. Obviously, X_1 is always 1, since the first roll gives us the first observed side. X_2 follows a geometric distribution with success p = 5/6. Similarly X_i follows a geometric distribution with success p = (7-i)/6. So, the total expected number of rolls is E[X1 + ... + X6] = E[X1] + ... E[X6] = 6 * (1 + 1/2 + ... + 1/6).

Answer 9: *First Ace* (Question on page 17)

The answer is 10.6. Setup an indicator function X_i = all aces are behind card i. Then total number of draws is $N = 1 + \sum_{i=1}^{4} 8X_i$, where $p(X_i) = 1/5$. The 1/5 comes from the fact that each non-ace card has a probability of 1/5 being appear before all aces.

Answer 12: *Bayesian Child* (Question on page 18)

$$
\begin{aligned}
P(\text{Boy} \mid \text{Picked a Boy}) &= \frac{P(\text{Picked a Boy} \mid \text{Boy})P(\text{Boy})}{P(\text{Picked a Boy})} && (4.1)\\
&= \frac{P(\text{Picked a Boy} \mid \text{Boy})P(\text{Boy})}{P(\text{Picked a Boy} \mid \text{Boy})P(\text{Boy}) + P(\text{Picked a Boy} \mid \text{Girl})P(\text{Girl})} && (4.2)\\
&= \frac{\frac{4}{3+N+1} \cdot \frac{1}{2}}{\frac{4}{3+N+1} \cdot \frac{1}{2} + \frac{3}{3+N+1} \cdot \frac{1}{2}} && (4.3)\\
&= \frac{4}{4+3} = \frac{4}{7} && (4.4)
\end{aligned}
$$

Answer 13: *Uniform Random Variables* (Question on page 18)

Answer 14: *Coin Game* (Question on page 18)

Naive Way:

$$P(A = B = 0) = \left[\binom{3}{0}\left(\frac{1}{2}\right)^3\right]^2 = \frac{1}{64} \tag{4.5}$$

$$P(A = B = 1) = \left[\binom{3}{1}\left(\frac{1}{2}\right)^3\right]^2 = \frac{9}{64} \tag{4.6}$$

$$P(A=B=2) = \left[\binom{3}{2}\left(\frac{1}{2}\right)^3\right]^2 = \frac{9}{64} \tag{4.7}$$

$$P(A=B=3) = \left[\binom{3}{3}\left(\frac{1}{2}\right)^3\right]^2 = \frac{1}{64} \tag{4.8}$$

So

$$P(A=B) = \frac{1}{64} + \frac{9}{64} + \frac{9}{64} + \frac{1}{64} = \frac{5}{16} \tag{4.9}$$

Don't forget the case when both had no head (A=B=0).

Clever Way: Let both do their tossing and then let B turn over each of her coins. Then the event you are looking for is that exactly three out of six coins show heads. Since B's "trick" doesn't destroy any randomness or independency, the answer is

$$P(A=B) = \binom{6}{3}\left(\frac{1}{2}\right)^3 = \frac{5}{16} \tag{4.10}$$

Answer 10: *Hand Shakes* (Question on page 17)

For each seat, the probability the person sitting there shakes the hand to the left is

$$\frac{x}{x+y} \cdot \frac{x-1}{x+y-1} + \frac{y}{x+y} \cdot \frac{y-1}{x+y-1} = \frac{x(x-1)+y(y-1)}{(x+y)(x+y-1)} \tag{4.11}$$

Hence

$$E\left(\sum_i Z_i\right) = \sum_i Z_i = (x+y) \cdot \frac{x(x-1)+y(y-1)}{(x+y)(x+y-1)} = \frac{x(x-1)+y(y-1)}{x+y-1} \tag{4.12}$$

Answer 11: *Hunters and Ducks* (Question on page 18)

The answer is 6 (but mine is 5?). Define a set of indicator variables:

$$Z_i = \begin{cases} 0, & \text{if the ith duck has survived} \\ 1, & \text{if the ith duck has died} \end{cases}$$

where $i = 1, \dots, 10$. Additionally, we define $Pr(Y_j = i)$ to be the probability that the jth hunter aims at the ith duck. Now,

$$\mathbb{E}\left[\sum_{i=1}^{10} Z_i\right] = \sum_{i=1}^{10} \mathbb{E}[Z_i] = \sum_{i=1}^{10} Pr(Z_i = 1)$$

$$
\begin{aligned}
&= \sum_{i=1}^{10}\sum_{j=1}^{10} Pr(Z_i = 1 | Y_j = i) Pr(Y_j = i) \\
&= \sum_{i=1}^{10}\sum_{j=1}^{10} (1/2)(1/10) \\
&= 5
\end{aligned}
$$

Answer 15: *Robot Flipping Coins* (Question on page 19)

The answer is 1/3 for H and 2/3 for T. We model this as a Markov Chain, which has a transition matrix

$$M = \begin{pmatrix} 0 & 1 \\ 0.5 & 0.5 \end{pmatrix}$$

Suppose in equilibrium, the probability of H is p and the probability of T is $1 - p$, then

$$(p, 1-p) \begin{pmatrix} 0 & 1 \\ 0.5 & 0.5 \end{pmatrix} = (p, 1-p)$$

This gives us $p = 1/3$. Alternatively, using eigendecomposition ($M = Q\Lambda Q^{-1}$), we find:

$$M = \begin{pmatrix} 1 & 1 \\ 1 & -1/2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -0.5 \end{pmatrix} \begin{pmatrix} 1/3 & 2/3 \\ 2/3 & -2/3 \end{pmatrix}$$

and

$$M^\infty = \begin{pmatrix} 1 & 1 \\ 1 & -1/2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1/3 & 2/3 \\ 2/3 & -2/3 \end{pmatrix}$$

This gives us the same result: $(0.5, 0.5)M^\infty = (1/3, 2/3)$.

Answer 16: *Coupon Collection* (Question on page 19)

This is the famous "Coupon Collection" problem. Define a set of variables $X_1, \ldots, X_6$ where X_i denotes the number of additional rolls need to obtain the i^{th} new number, then the answer reduces to computing $\mathbb{E}[\sum_{i=1}^6 X_i]$. However, each X_i follows a Geometric distribution where $p_i = (6 - (i-1))/6$ and $\mathbb{E}[X_i] = 6/(6 - (i-1))$. Therefore,

$$\mathbb{E}\left[\sum_{i=1}^{6} X_i\right] = \sum_{i=1}^{6} \frac{6}{6 - (i-1)} = 6(1/6 + 1/5 + 1/4 + 1/3 + 1/2 + 1) = 14.7$$

In general, the answer for the coupon collection problem is:

$$\mathbb{E} = N \sum_{i=1}^{N} \frac{1}{i}$$

For the second part, introduce the indicator variables Y_i to denote that if number i is in the rolls already made. Then

$$\mathbb{E}\left[\sum_{i=1}^{6} Y_i\right] = \sum_{i=1}^{6} \mathbb{E}[Y_i] = \sum_{i=1}^{6} Pr(Y_i = 1) = \sum_{i=1}^{6} 1 - \left(\frac{5}{6}\right)^N = 6\left[1 - \left(\frac{5}{6}\right)^N\right]$$

Answer 17: *Two Heads* (Question on page 19)

Answer is 4 for to see two heads, and 6 to see two consecutive heads. Expected # of flips to gets 1 H is just, E[X] = 1 + (1-p)(E[X]), where p is prob. of getting H, in the case of a fair coin this is 0.5, therefore rearranging you get E[X] = 1/p, meaning expected number of flips for 1 head is 1/p, for fair coin this is equal to 2. Since coin flipping is memoryless, expected number of flips to get k heads is just 1/p + 1/p + ... = k/p. Therefore 2/0.4 = 4, when k=2.

Answer 18: *One More Coin* (Question on page 19)

The answer is 1/2. First observe that if A were to toss only n times, then there are only three events:

1. With probability P_1, A has more heads than B does.
2. With Probability P_2, A has the same number of heads compared to B.
3. With probability P_3, A has fewer heads than B does.

By symmetry, $P_1 = P_3$ and $P_1 + P_2 + P_3 = 1 \implies 2P_1 + P_2 = 1$. The additional coin toss A has increases the probability of A having more heads by just $0.5P_3$, hence the probability in question is $P_1 + 0.5P_2 = 0.5$.

Chapter 5

Linear Algebra

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5.4 Matrix Decomposition 26
5.5 Chapter Answers 27

5.1 Positive Definiteness and Covariance Matrices

Question 19: *How to check positive definiteness of a matrix?* (Easy) (Tags: #2002.md; Math, Matrix, Linear Algebra, Positive Definite; NA;)

How to check if a matrix is positive definite? How about positive semi-definite?

(Answer on page 27)

5.2 Trace

Question 20: *Trace of Matrix Sum* (Easy) (Tags: #2004.md; Trace; Eigenvector; Eigenvalue;; NA;)

Given a 3×3 matrix $\mathbf{M}$, what is $\operatorname{tr}\left(\sum_{k=0}^{\infty} \mathbf{M}^k\right)$?

(Answer on page 27)

5.3 Determinant

Question 21: *Sherman-Morrison Formula* (Easy) (Tags: #2009.md; Calculus; Sherman-Morrison Formula; World Quant;)

Prove the Sherman-Morrison formula:

$$(\mathbf{I} + \mathbf{u}\mathbf{v}^T)^{-1} = \mathbf{I} - \frac{\mathbf{u}\mathbf{v}^T}{1 + \mathbf{u}\mathbf{v}^T}$$

and the Matrix Determinant Lemma:

$$\det(\mathbf{I} + \mathbf{u}\mathbf{v}^T) = 1 + \mathbf{v}^T\mathbf{u}.$$

(Answer on page 27)

5.4 Matrix Decomposition

Question 22: *QR Decomposition* (Easy) (Tags: #2010.md; Trace; QR Decomposition; NA;)

How do you use QR decomposition in linear regression?

(Answer on page 28)

5.5 Chapter Answers

Answer 19: *How to check positive definiteness of a matrix?* (Question on page 25)

Need to check if the matrix is square first.

Answer 20: *Trace of Matrix Sum* (Question on page 25)

The answer is

$$\operatorname{tr}\left(\sum_{k=0}^{\infty}\mathbf{M}^k\right) = \sum_{i=1}^{3}\frac{1}{1-\lambda_i}$$

where λ_i are the eigenvalues of $\mathbf{M}$. The key is that trace is the sum of all eigenvalues, and if λ is an eigenvalue of $\mathbf{M}$ then λ^k is an eigenvalue of $\mathbf{M}^k$. This is easily seen by noticing that

$$\mathbf{Mx} = \lambda\mathbf{x} \implies \mathbf{M}^2\mathbf{x} = \lambda(\mathbf{Mx}) = \lambda^2\mathbf{x}$$

Now,

$$\operatorname{tr}\left(\sum_{k=0}^{\infty}\mathbf{M}^k\right) = \sum_{k=0}^{\infty}\operatorname{tr}(\mathbf{M}^k) = \sum_{k=0}^{\infty}\sum_{i=1}^{n}\lambda_i^k = \sum_{i=1}^{n}\sum_{k=0}^{\infty}\lambda_i^k = \sum_{i=1}^{n}\frac{1}{1-\lambda_i}$$

The answer is given by $n = 3$.

Answer 21: *Sherman-Morrison Formula* (Question on page 26)

We guess the inverse has the similar form $(\mathbf{I}+\mathbf{uv}^T)^{-1} = \mathbf{I}+\alpha\mathbf{v}^T\mathbf{u}$, thus

$$(\mathbf{I}+\mathbf{uv}^T)(\mathbf{I}+\alpha\mathbf{uv}^T) = \mathbf{I}+a\mathbf{uv}^T+\mathbf{uv}^T+a\mathbf{uv}^T\mathbf{uv}^T = \mathbf{I}+\mathbf{u}(a+1+a\mathbf{v}^T\mathbf{u})\mathbf{v}^T$$

$$\implies a = -\frac{\mathbf{uv}^T}{1+\mathbf{u}^T\mathbf{v}}$$

and hence

$$(\mathbf{I}+\mathbf{uv}^T)^{-1} = \mathbf{I}-\frac{\mathbf{uv}^T}{1+\mathbf{u}^T\mathbf{v}}.$$

If we look at the eigenvalues,

$$(\mathbf{I}+\mathbf{uv}^T)\mathbf{u} = \mathbf{u}+\mathbf{uv}^T\mathbf{u} = (1+\mathbf{v}^T\mathbf{u})\mathbf{u}$$

We see that $\mathbf{u}$ is an eigenvector and $1+\mathbf{v}^T\mathbf{u}$ is its eigenvalue. Moreover, there are $n-1$ orthogonal vectors (suppose $\mathbf{I}$ is of dimension n) $\mathbf{b}$ such that $\mathbf{v}^T\mathbf{b} = 0$, and for each of them we have $(\mathbf{I}+\mathbf{uv}^T)\mathbf{b} = \mathbf{b}$, which suggests that 1 is the other eigenvalue with multiplicity $n-1$. Therefore, the determinant of $\mathbf{I}+\mathbf{uv}^T$ is the product of its eigenvalues, which is $1+\mathbf{u}^T\mathbf{v}$. That is

$$\det(\mathbf{I}+\mathbf{uv}^T) = 1+\mathbf{u}^T\mathbf{v}$$

This proves the Matrix Determinant Lemma.

Answer 22: *QR Decomposition* (Question on page 26)

Recall that the normal equation

$$\boldsymbol{\beta} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$$

involves the inverse of $\mathbf{X}^T\mathbf{X}$, which is numerically unstable when the matrix is near singular and ill-conditioned, i.e., the condition number

$$\kappa(\mathbf{A}) = \frac{\sigma_{\max}(\mathbf{A})}{\sigma_{\min}(\mathbf{A})} = ||\mathbf{A}^{-1}| \cdot |||\mathbf{A}||$$

is large. Now, suppose we have a *thin* QR factorization of the design matrix (usually $n \geq p$)

$$\mathbf{X}_{n\times p} = \mathbf{QR} = [\mathbf{Q}_1, \mathbf{Q}_2]\begin{pmatrix}\mathbf{R}_1 \\ \mathbf{0}\end{pmatrix} = \mathbf{Q}_1\mathbf{R}_1,$$

where Q_1, Q_2 both have orthonormal columns but with sizes $n\times p$ and $n\times(n-p)$, respectively. $\mathbf{R}_1$ is an upper triangular matrix with size $p \times p$. Then since orthogonal matrices preserve the 2-norm, we have:

$$\begin{aligned}
&\min_{\boldsymbol{\beta}} \frac{1}{2}||\mathbf{X}\boldsymbol{\beta} - \mathbf{y}||_2^2 \\
\implies &\min_{\boldsymbol{\beta}} \frac{1}{2}||\mathbf{Q}_1\mathbf{R}_1\boldsymbol{\beta} - \mathbf{y}||_2^2 \\
\implies &\min_{\boldsymbol{\beta}} \frac{1}{2}||\mathbf{Q}_1^T(\mathbf{Q}_1\mathbf{R}_1\boldsymbol{\beta} - \mathbf{y})||_2^2 \\
\implies &\min_{\boldsymbol{\beta}} \frac{1}{2}||\mathbf{R}_1\boldsymbol{\beta} - \mathbf{Q}_1^T\mathbf{y}||_2^2
\end{aligned}$$

Since $\mathbf{R}_1$ is upper triangular, we only need one backward sweep and one forward sweep to solve $\boldsymbol{\beta}$.

Part III

Finance

Chapter 6

Quant Finance

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6.1 Stochastic Calculus **31**
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6.1 Stochastic Calculus

Question 23: *NA* (Easy) (Tags: #3001.md; Stochastic Calculus; NA;)

NA

(Answer on page 54)

6.2 Chapter Answers

Answer 48: *NA* (Question on page 53)

NA

Part IV

Computer Science

Chapter 7

Compiler

Contents

7.1 Compiling Process **35**
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7.1 Compiling Process

Question 24: *Compiling Process* (Easy) (Tags: #4101.md; Compiler; NA;)

What are the stages of compiling?

(Answer on page 36)

7.2 Chapter Answers

Answer 24: *Compiling Process* (Question on page 35)

- Macro Substitution - Syntactic Analysis - Grammatical Analysis - Quadruplet Generation - Table of Variable Maintenance

Chapter 8

Database

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8.1 Database

Question 25: *B Tree vs B+ Tree* (Easy) (Tags: #4201.md; Database; NA;)

What is the difference between **B Tree** and **B+ Tree**? WHy do we need B+ tree?

(Answer on page 38)

8.2 Chapter Answers

Answer 25: *B Tree vs B+ Tree* (Question on page 37)

Chapter 9

Network

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9.1 DNS

Question 26: *DNS* (Easy) (Tags: #4301.md; Network; DNS; NA;)

Explain DNS.

(Answer on page 40)

9.2 Chapter Answers

Answer 26: *DNS* (Question on page 39)

Chapter 10

Operating System

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10.1 Thread

Question 27: *Process vs Thread* (Easy) (Tags: #4401.md; Operating System; NA;)

What is the difference between **process** and **thread**?

(Answer on page 42)

10.2 Chapter Answers

Answer 27: *Process vs Thread* (Question on page 41)

Chapter 11

Programming Languages

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11.1 C++

Question 28: *Code Debug* (Easy) (Tags: #4001.md; C++, Debug; NA;)

What's wrong with the following code?

```
class A {
private:
    int value;
public:
    A(int n) { value = n; }
    A(A other) { value = other.value; }

    void Print() {std::cout << value << std::endl; }
};

int _tmain(int argc, _TCHAR* argv[]) {
    A a = 10;
    A b = a;
    b.Print();

    return 0;
}
```

(Answer on page 47)

Question 29: *C++ Cast* (Easy) (Tags: #4002.md; Casting; NA;)

Describe all the C++ casts and their properties.

(Answer on page 47)

Question 30: *Shallow Copy vs Deep Copy* (Easy) (Tags: #4003.md; Deep Copy, Shallow Copy; NA;)

What is **shallow copy** and **deep copy**? What member function needs to be implemented in order to implement **deep copy**?

(Answer on page 47)

Question 31: *Empty Class* (Easy) (Tags: #4004.md; Computer Science, Empty Class; Jian Zhi Offer;)

What is the **sizeof** of an empty class? Follow-up: 1) why not zero? 2) what if I add a constructor and a destructor? 3) what if the destructor is virtual? 4) what if I add an int and a double? 5) what if I add an int and a double?

(Answer on page 47)

Question 32: *Private Member Variable* (Easy) (Tags: #4005.md; Private Member Variable; NA;)

You can only read the source code of a class, which has a private member variable. How can you get the access of the private member variable?

(Answer on page 47)

Question 33: *Value Type and Reference Type* (Easy) (Tags: #4006.md; Value Type and Reference Type; Jian Zhi Offer;)

Explain **value type** and **reference type**.

(Answer on page 47)

Question 34: *Vector vs List* (Easy) (Tags: #4007.md; STL; NA;)

What is the difference between **vector** and **list**? How does **vector** expand in size, why not use 2x?

(Answer on page 47)

Question 35: *Empty Class* (Easy) (Tags: #4008.md; Class; NA;)

What will be implemented for an empty class?

(Answer on page 47)

Question 36: *Map vs Unorder Map* (Easy) (Tags: #4009.md; STL; NA;)

What is the difference between **map** and **unordered map**?

(Answer on page 48)

Question 37: *Select, Poll and Epoll* (Easy) (Tags: #4010.md; Multithreading; NA;)

What is the difference between **select**, **poll** and **epoll**?

(Answer on page 48)

Question 38: *New and Malloc* (Easy) (Tags: #4011.md; C++, Memory; NA;)

What is the difference between **new** and **malloc**?

(Answer on page 48)

Question 39: *Modify Virtual Function of Parent Class* (Easy) (Tags: #4012.md; C++, Virtual Function; Tencent;)

Can a derived class modify the default parameters of a virtual function of a parent class? If so, at compile time or run time?

(Answer on page 48)

Question 40: *Pointers and References* (Easy) (Tags: #4013.md; C++, Pointers and References; NA;)

What is the difference between pointers and references? Does reference take memory space?

(Answer on page 48)

Question 41: *Smart Pointers* (Easy) (Tags: #4014.md; C++, Smart Pointers; NA;)

Explain smart pointers.

(Answer on page 48)

Question 42: *Six Key Components of STL* (Easy) (Tags: #4015.md; C++, STL; NA;)

What are the six key components of STL? Explain.

(Answer on page 48)

Question 43: *STL Map* (Easy) (Tags: #4016.md; C++, STL; NA;)

Are data in STL map ordered?

(Answer on page 48)

Question 44: *Map vs Set* (Easy) (Tags: #4017.md; C++, STL; NA;)

What is the difference between **map** and **set**?

(Answer on page 48)

Question 45: *When to use a virtual destructor?* (Easy) (Tags: #4018.md; Virtual Destructor; Ubiquant;)

When to use a virtual destructor?

(Answer on page 48)

11.2 Python

Question 46: *Python* (Easy) (Tags: #4019.md; C++, Debug; NA;)

Explain "if __name__ == __main__".

(Answer on page 48)

11.3 Chapter Answers

Answer 28: *Code Debug* (Question on page 43)

Should be

```
A(const A& other)
```

Answer 29: *C++ Cast* (Question on page 44)

Answer 30: *Shallow Copy vs Deep Copy* (Question on page 44)

Answer 35: *Empty Class* (Question on page 44)

(Usually) 1 Byte.

An object has to occupy some physical memory space, otherwise when you instantiate it you wouldn't be able to use it. The size of an empty class is usually 1 byte, but it depends on the compiler and system.

If we add a constructor and a destructor, the **sizeof** will still be one byte. If the destructor is virtual, the **sizeof** will be 8 bytes (in a x386-64 machine, which is 64-bit), since a **vpointer** will be added to point to a **vtable**.

If we add an int, the size will be 4 (in a x386-64 machine), but if we add one more double, the size will be 16, although a double takes 8 bytes and an int take 4 bytes. This is because of **padding**, i.e., shorter types will be padded to align with the longest type.

Answer 32: *Private Member Variable* (Question on page 44)

Answer 33: *Value Type and Reference Type* (Question on page 44)

Answer 34: *Vector vs List* (Question on page 44)

Answer 35: *Empty Class* (Question on page 44)

Answer 36: *Map vs Unorder Map* (Question on page 45)

unordered map is faster than **map**, as the former is implemented via a hash function while the latter is implemented by keeping the keys always sorted using a self-balancing binary tree.

Answer 37: *Select, Poll and Epoll* (Question on page 45)

Answer 38: *New and Malloc* (Question on page 45)

Answer 39: *Modify Virtual Function of Parent Class* (Question on page 45)

Answer 40: *Pointers and References* (Question on page 45)

Answer 41: *Smart Pointers* (Question on page 45)

Answer 42: *Six Key Components of STL* (Question on page 45)

- containers - algorithm - iterator - adapter - allocator - functor

Answer 43: *STL Map* (Question on page 46)

Answer 44: *Map vs Set* (Question on page 46)

Answer 45: *When to use a virtual destructor?* (Question on page 46)

Need to check if the matrix is square first.

Answer 46: *Python* (Question on page 46)

Checking if the current module (== file) contains a main function, and is not imported.

Part V

Algorithm and Optimization

Chapter 12

Optimization

Contents

12.1 Root Finding **51**
12.2 Chapter Answers **52**

12.1 Root Finding

Question 47: *Newton's Method* (Easy) (Tags: #5001.md; Newton's Method; NA;)

Explain Newton's method.

(Answer on page 52)

12.2 Chapter Answers

Answer 47: *Newton's Method* (Question on page 51)

Suppose we have a function $f(x) = 0$ and we want to find its root. We start with x_0 close to its root, and update as

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{12.1}$$

Chapter 13

Algorithm

Contents

13.1 Dynamic Programming **53**
13.2 Chapter Answers **54**

13.1 Dynamic Programming

Question 48: *NA* (Easy) (Tags: #5101.md; Data Science; NA;)

NA

(Answer on page 54)

13.2 Chapter Answers

Answer 48: *NA* (Question on page 53)

NA

Part VI

Data Science

Chapter 14

Data Science

Contents

14.1 Data Science **57**
14.2 Chapter Answers **58**

14.1 Data Science

Question 49: *Naive Bayes vs Logistic Regression* (Easy) (Tags: #6001.md; Naive Bayes; Logistic Regression; NA;)

Compare **Naive Bayes** and **Logistic Regression**.

(Answer on page 58)

14.2 Chapter Answers

Answer 49: *Naive Bayes vs Logistic Regression* (Question on page 57)

NB has high bias as it focuses on a smaller set of models, converges faster but error rate may be high if assumptions are incorrect. Logistic Regression has lower bias as it allows a larger model space, converges to solutions slower but error rate can be lower due to low bias. Logistic Regression tends to be used a LOT more in industry but if your prior information is accurate (data is generated from a well known process), NB may be a better choice.

NB assumes that the input features are conditional independent. NB also requires a prior distribution. Simple LR cannot do nonlinear classification.