The Underlying Math Behind Smooth Grayscale Superimposing

This simple explanation will help you understand the mathematics behind this algorithm that mitigates the occurrence of ghost images in superimposed grayscale pictures.

First, the resulting image should produce two rendering results, one overlaying on white background, and the other overlaying on black background. Consider the grayscale of each pixel in the two **rendering results** as $W_{i,j}$ for white background and $K_{i,j}$ with black background; and such in the original input as $w_{i,j}$ and $k_{i,j}$; where the subscripts i, j denote the x and y coordinates correspondingly. The pixels in the actual result image will be denoted as $B_{i,j}$ and $A_{i,j}$ for brightness and alpha accordingly. WLOG, all the variables mentioned above are constrained between [0, 1], with the value of 1 stands for pure white/completly opaque, and the value of 0 stands for pure black/completly transparent.

By simply applying the linear alpha blending algorithm, one may easily derive the following set of equations:

$$\begin{cases} W_{i,j} = A_{i,j}B_{i,j} + \left(1 - A_{i,j}\right) \\ K_{i,j} = A_{i,j}B_{i,j} \end{cases}$$

The unknown variables then can be represented with $W_{i,j}$ and $K_{i,j}$:

$$\begin{cases} A_{i,j} = 1 + K_{i,j} - W_{i,j} \\ B_{i,j} = \dfrac{K_{i,j}}{1 + K_{i,j} - W_{i,j}} \end{cases}$$

Ideally, $W_{i,j}$ should equal to $w_{i,j}$ and so does $K_{i,j}$. However, with the constrain of all variables may only sit between [0, 1], the following inequality must holds.

$$W_{i,j} > K_{i,j} > W_{i,j} - 1$$

In order to comply with such limitation, we may enforce the following inequalities:

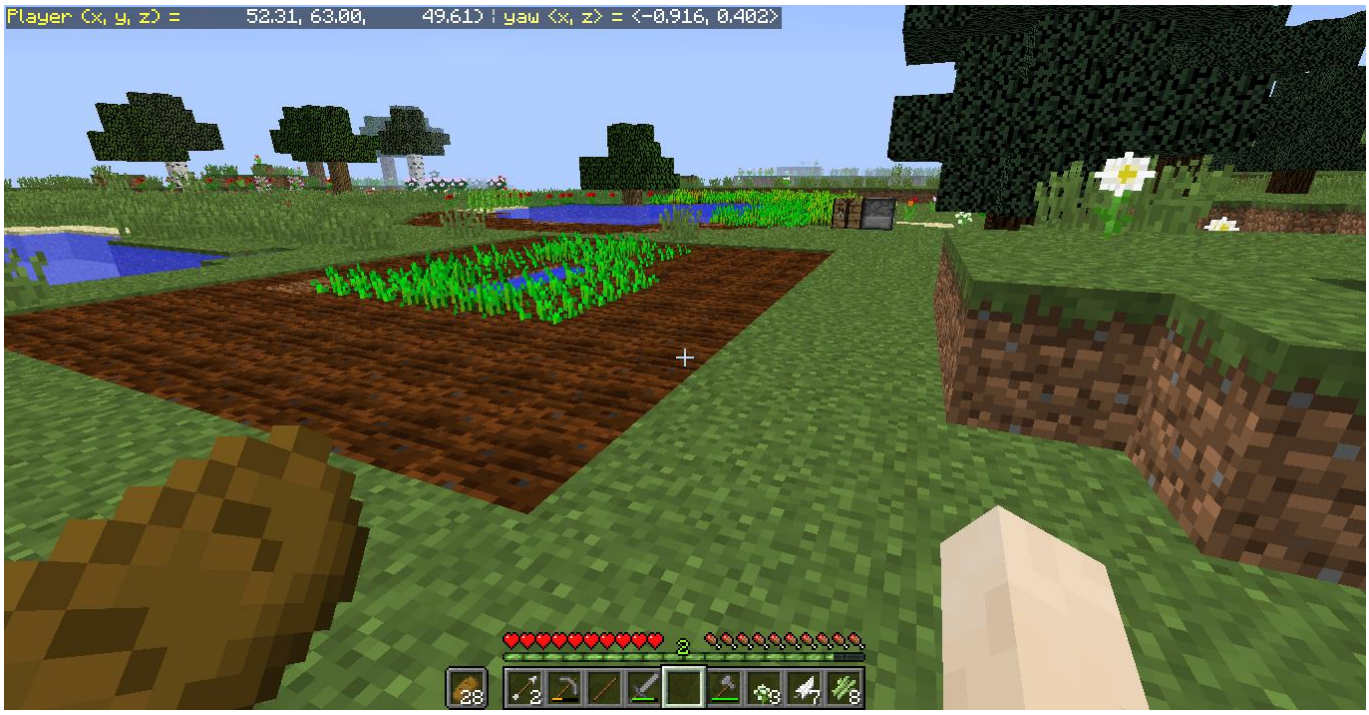
$$\begin{cases} W_{i,j} > 0.5 \\ K_{i,j} \leq 0.5 \end{cases},$$

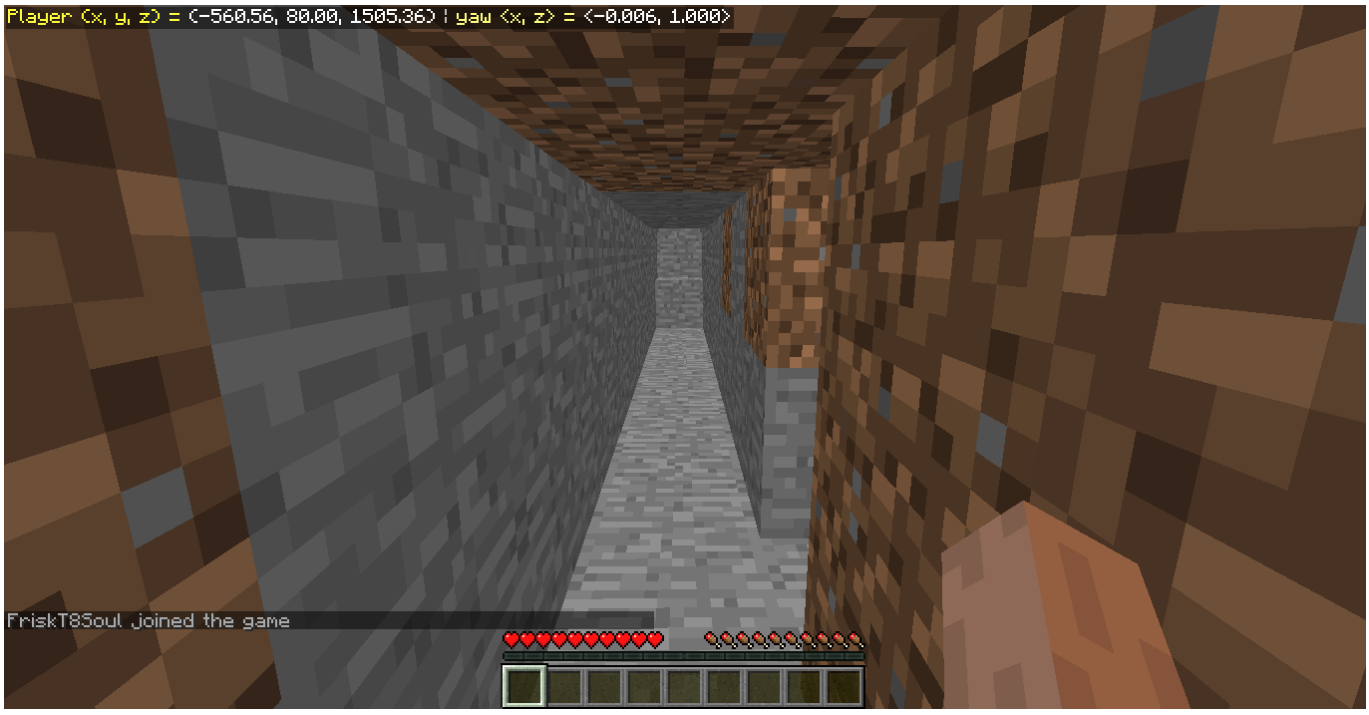
which can be simply implemented by using:

$$\begin{cases} W_{i,j} = \dfrac{w_{i,j} + 1}{2} \\ K_{i,j} = \dfrac{k_{i,j}}{2} \end{cases}.$$

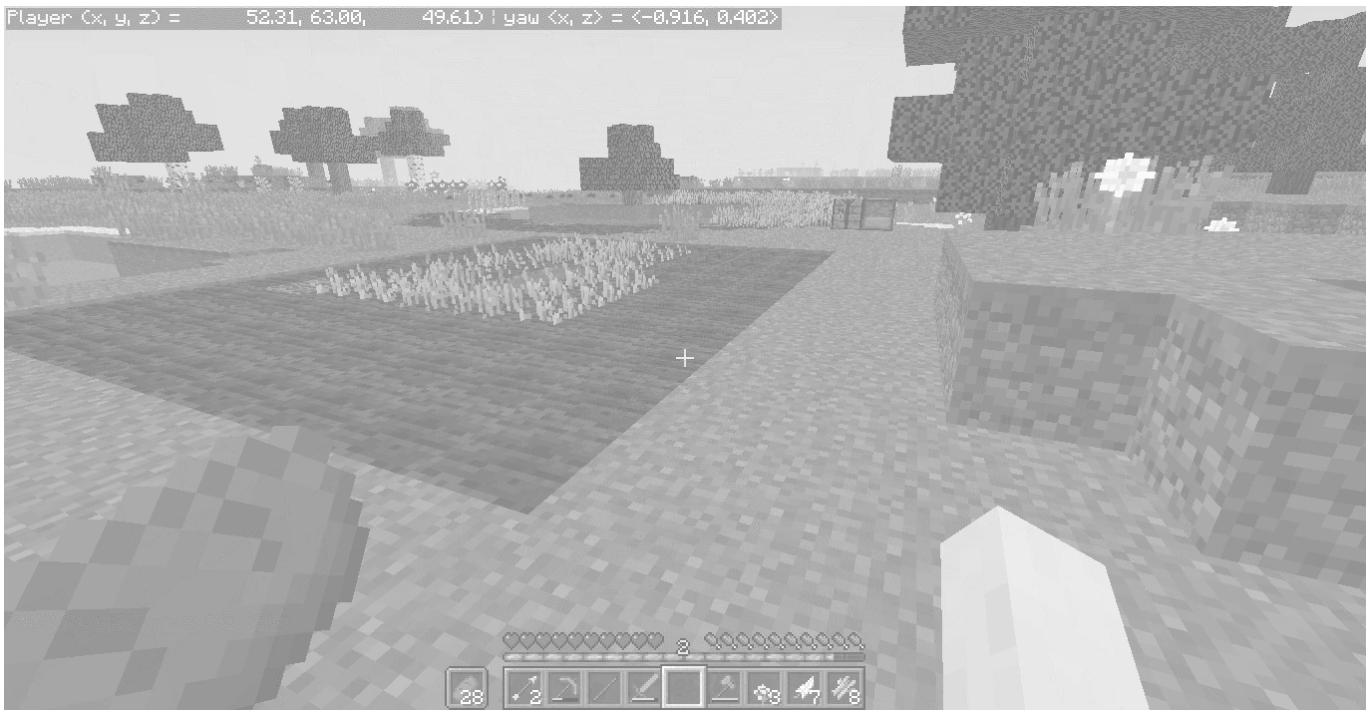
Samples:

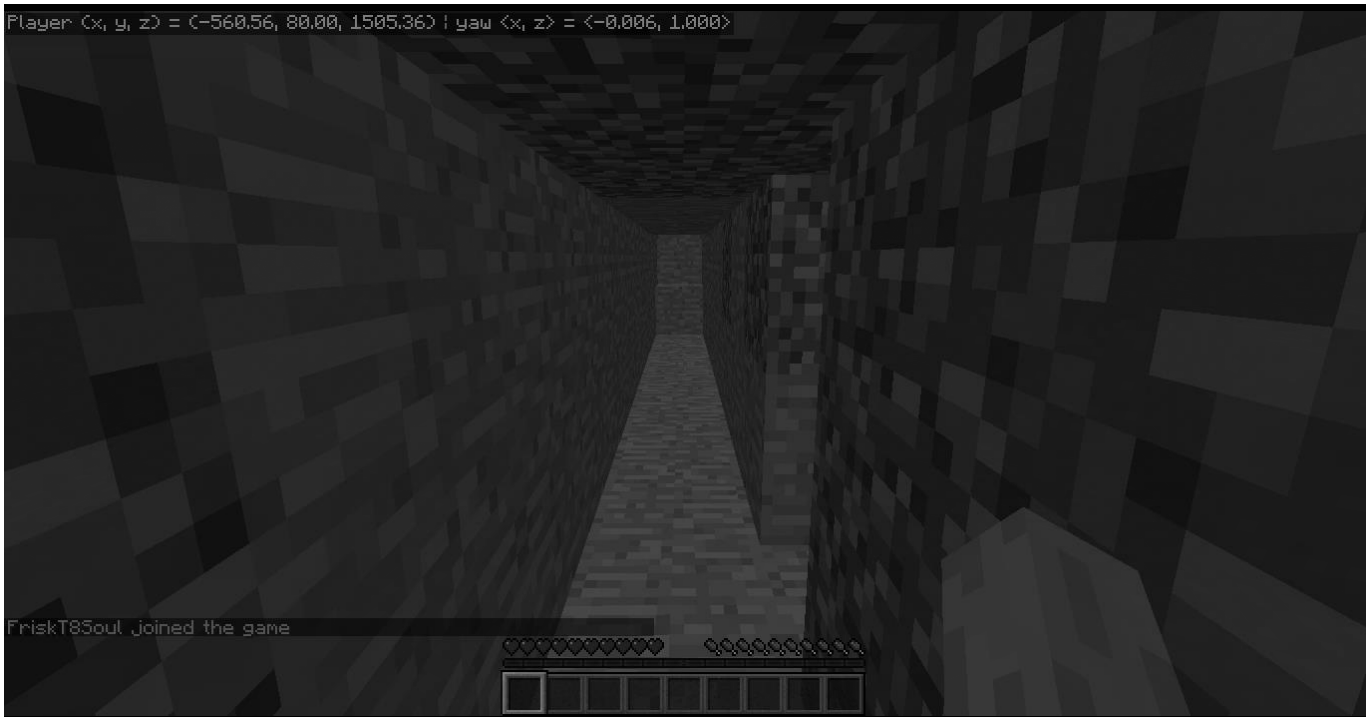
Input files:





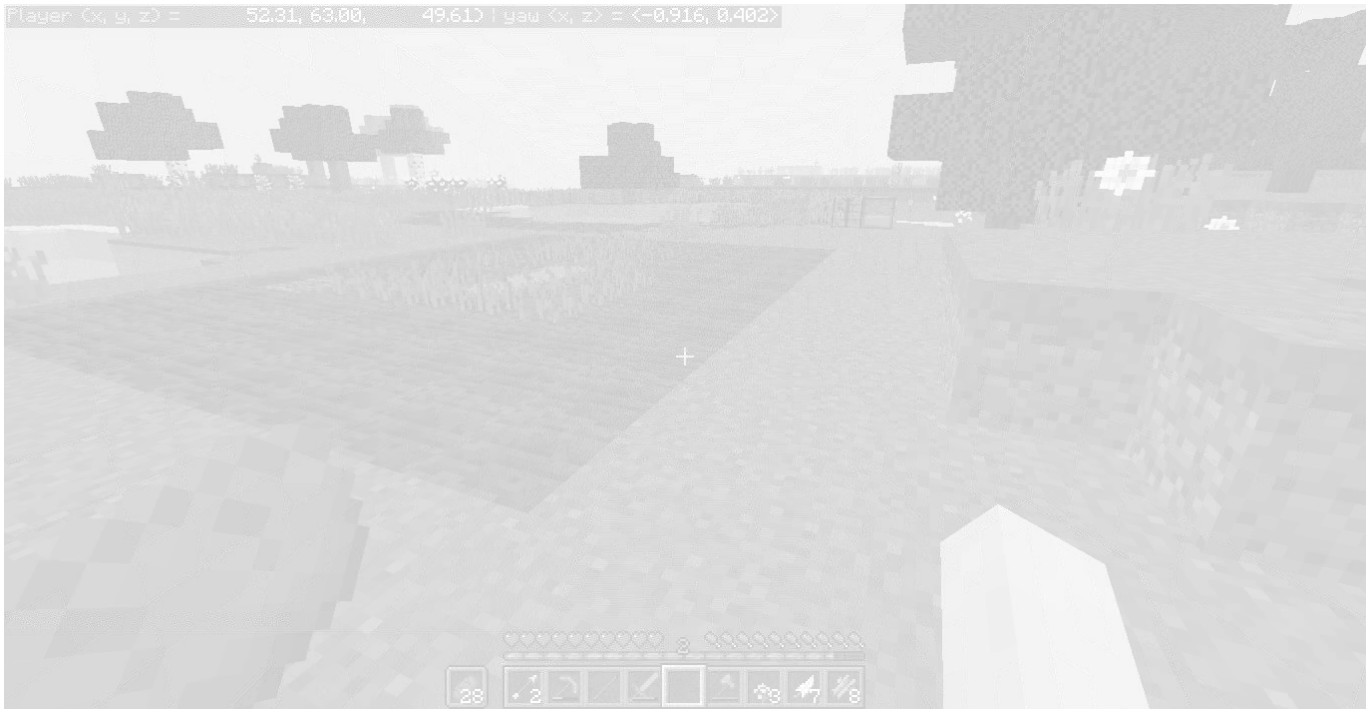
Results generated using mentioned algorithm:

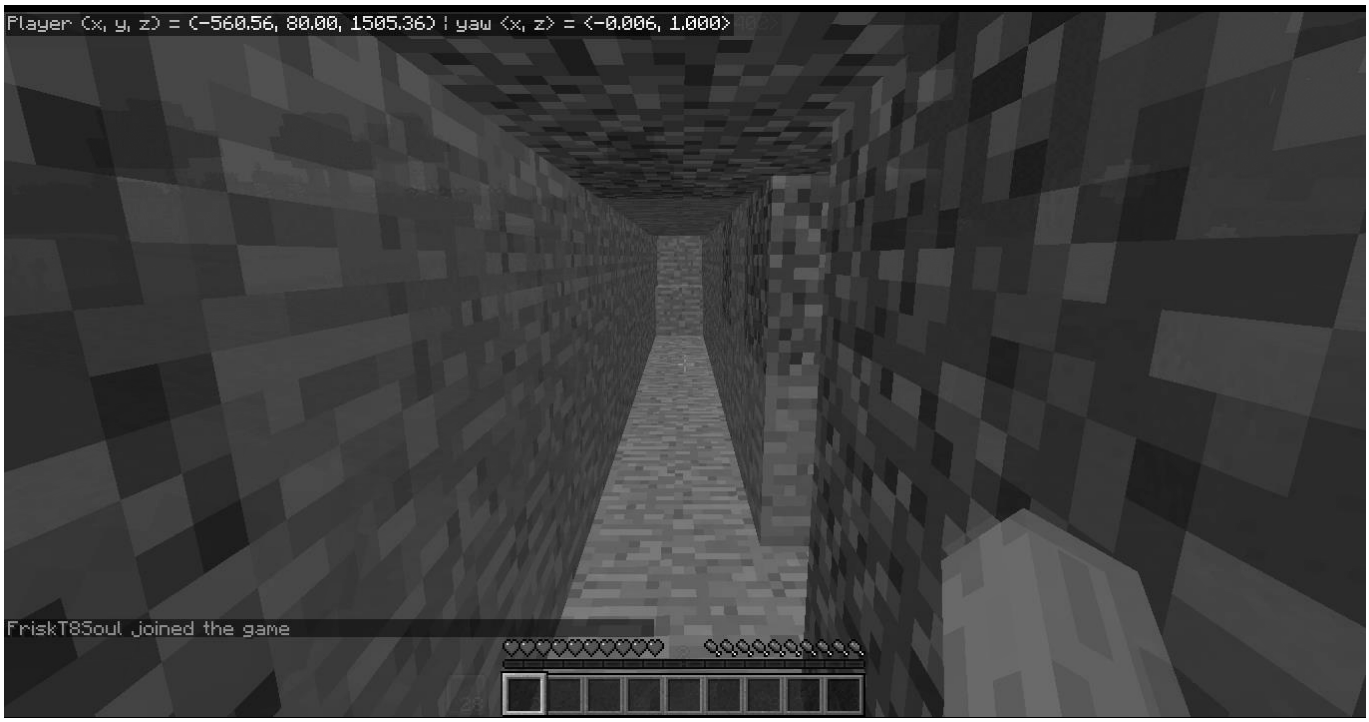




Results generated by using hbl917070's GitHub repository:

(https://github.com/hbl917070/Invisible_image)





With black background, the other image is still visible in hbl917070's implementation while being completely absence in our result.