4.1 page (178)

23) $f_1 = e^{-3x}$, $f_2 = e^{4x}$

$$W(e^{-3x}, e^{4x}) = \begin{vmatrix} f_1 & f_2 \\ f_1' & f_2' \end{vmatrix}$$

$$= \begin{vmatrix} e^{-3x} & e^{4x} \\ -3e^{-3x} & 4e^{4x} \end{vmatrix}$$

$$= 4e^{4x} e^{-3x} + 3e^{-3x} e^{4x}$$

$$= 4e^{x} + 3e^{x}$$

$$= 7e^{x}$$

$\therefore W(e^{-3x}, e^{4x}) = 7e^{x} \neq 0$

~~for $(-\infty < x < \infty)$~~

$\therefore f_1 = e^{-3x}$ and $f_2 = e^{4x}$ are set of Solution of the given function

for $(-\infty < x < \infty)$ The general solution is

$$y = c_1 e^{-3x} + c_2 e^{4x}.$$

25. $f_1 = e^x \cos 2x, \quad f_2 = e^x \sin 2x$

$$W(f_1, f_2) = \begin{vmatrix} e^x \cos 2x & e^x \sin 2x \\ \frac{1}{2} e^x \sin 2x + e^x \cos 2x & \frac{1}{2} e^x \cos 2x + e^x \sin 2x \end{vmatrix}$$

$$= \frac{1}{2} e^{2x} \cos^2 2x + e^{2x} \sin 2x \cos 2x + \frac{1}{2} e^{2x} \sin^2 2x + e^{2x} \cos 2x \sin 2x$$

$$= \frac{1}{2} e^{2x} (\cos^2 2x + \sin^2 2x) + 2 e^{2x} \sin 2x \cos 2x$$

$$= \frac{1}{2} e^{2x} + 2 e^{2x} \sin 2x \cos 2x$$

25] $f_1 = e^x \cos 2x$

$f_1' = 2e^x \sin 2x + e^x \cos 2x$

$= e^x (\cos 2x - 2 \sin 2x)$

$f_2 = e^x \sin 2x$

$= 2e^x \cos 2x + e^x \sin 2x$

$= e^x (2 \cos 2x + \sin 2x)$

$$W = \begin{vmatrix} f_1 & f_2 \\ f_1' & f_2' \end{vmatrix} = \begin{vmatrix} e^x \cos 2x & e^x \sin 2x \\ e^x (\cos 2x - 2 \sin 2x) & e^x (2 \cos 2x + \sin 2x) \end{vmatrix}$$

$$= 2e^{2x} \cos^2 2x + e^{2x} \cos 2x \sin 2x - 2e^{2x} \sin 2x \cos 2x + 2e^{2x} \sin^2 2x$$

$$= 2e^{2x} (\cos^2 2x + \sin^2 2x)$$

$$= 2e^{2x} \cdot 1 = 2e^{2x}$$

$W = 2e^{2x} \neq 0$

$\therefore f_1$ and f_2 are solutions of the given function

26 $y_1 = e^{x/2}$, $y_2 = x e^{x/2}$

$y_1' = \frac{1}{2} e^{x/2}$ $\quad y_2' = \frac{1}{2} x e^{x/2} + e^{x/2}$

$$W = \begin{vmatrix} e^{x/2} & x e^{x/2} \\ \frac{1}{2} e^{x/2} & \frac{1}{2} x e^{x/2} + e^{x/2} \end{vmatrix}$$

$$= \frac{1}{2} x e^{x} + e^{x} - \frac{1}{2} x e^{x}$$

$$= e^{x}$$

$$= \frac{1}{2} x e^{x} + \frac{1}{2} e^{x}$$

$$= \frac{1}{2} e^{x} (x+1)$$

$\therefore W \neq 0$.

$y = c_1 e^{x/2} + c_2 x e^{x/2}$

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$$W = \begin{vmatrix} x^3 & x^4 \\ 3x^2 & 4x^3 \end{vmatrix}$$

$= 4x^6 - 3x^6$

$= x^6$

$\therefore W = x^6 \neq 0$

$y = c_1 x^3 + c_2 x^4$