COL226: Programming Languages

Mon 11 Apr 2022 **MajorQ0** 5+9 (+3 for PwD) minutes Max marks 10

Instructions:

1. Download the paper and write your name and entry number in the designated space on top and *do not forget to sign the honour statement below.*
2. Answer the question(s). *Answers will be judged for correctness, efficiency and elegance.*
4. If there are minor mistakes in the question, correct them explicitly and answer the question accordingly. If the question is totally wrong, give adequate reasons why it is wrong with detailed counter-examples, if necessary.
4. Scan the paper with your completed answer.
5. Upload it on Gradescope 2102-COL226 page within the given time. *Make sure the first page with your name, entry no and signature is also the first page of your uploaded file*
6. Late submissions (within 2 minutes of submission deadline) on the portal will attract a penalty of 10% of the total marks allotted to the paper for each minute of delay and 20% for each minute of delay thereafter.
7. Email submissions after the closing of the portal will not be evaluated (You get a 0).
8. Uploads without the first page details (including signature) may be awarded 0 marks.

I abide by the Honour code that I have signed on my admission to IIT Delhi. I have neither given any help to anybody nor received any help from anybody nor from any site or other sources in solving the question(s) in this paper.

Signature: **Date:**

[**2 × 5 = 10 marks**]

Let $\rho, \sigma, \tau \subseteq A \times A$ be binary relations on any (finite or infinite) set A. We define

$$
\begin{array}{lcll}
\rho^{-1} & = & \{(b,a) \mid (a,b) \in \rho\} & \\
\rho^{0} & = & \{(a,a) \mid a \in A\} & \\
\sigma;\tau & = & \{(a,c) \mid \exists b \in A[(a,b) \in \sigma, (b,c) \in \tau]\} & \\
\rho^{n+1} & = & \rho^{n};\rho & \text{for any } n \geq 0 \\
\rho^{*} & = & \bigcup_{n \geq 0} \rho^{n} &
\end{array}
$$

1. Prove that $\rho^* \cup (\rho^*)^{-1} \subseteq (\rho \cup \rho^{-1})^*$.
2. Either prove that $(\rho \cup \rho^{-1})^* \subseteq \rho^* \cup (\rho^*)^{-1}$ or show by a counterexample that $(\rho \cup \rho^{-1})^* \not\subseteq \rho^* \cup (\rho^*)^{-1}$.