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16

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CSL302

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Programming Languages

10 minutes

Quiz 2

Maximum Marks: 10

Suppose Σ is a signature, and $\mathcal{A}, \mathcal{B}, \mathcal{C}$ are all Σ-algebras.

Q1. (3 marks) identity. Show that id_A, the identity function on the carrier set A of $\mathcal{A}$, is a Σ-homomorphism.

$id_A : A \to A$

(i) for $\forall c \in \Sigma^{(0)}$: $id_A(c_A) = c_A$, $c_A \in A$

(ii) for $f \in \Sigma^{(k)}$, $a_1 \ldots a_k \in A$

$id_A(f(a_1, a_2, \ldots, a_k)) = f(id_A(a_1), id_A(a_2), \ldots, id_A(a_k))$

$= a_j \in A$ $\quad$ $= f(a_1, a_2, \ldots, a_k)$, $a_j \in A$

$\therefore$ LHS = RHS, $\therefore$ Σ homomorphism

2 (∵ f not under graph defined) to be shown, not the alg...

Q2. (7 marks) Composition. Let $f : \mathcal{A} \to \mathcal{B}$ and $g : \mathcal{B} \to \mathcal{C}$ be Σ-homomorphisms. Show that their function composition $f;g : \mathcal{A} \to \mathcal{C}$ is a Σ-homomorphism.

$\mathcal{A} = \langle A, \ldots \rangle$

$\mathcal{B} = \langle B, \ldots \rangle$

$\mathcal{C} = \langle C, \ldots \rangle$

$f : \mathcal{A} \to \mathcal{B}$ $\quad$ Σ hom

$g : \mathcal{B} \to \mathcal{C}$ $\quad$ Σ hom.

Prove $\mathcal{A} \to \mathcal{C}$: Σ hom

(i) for $\forall c \in \Sigma^{(0)}$,

$g(f(c_A)) \to (c_B)$

c_C

3

(ii) for $\forall h \in \Sigma^{(k)}$, $a_1 \ldots a_k \in A$

$g(f(h_A(a_1, a_2, \ldots, a_k))) = h_C(g(f(a_1)), g(f(a_2)), \ldots, g(f(a_k)))$

$= h_C(gf(a_1), gf(a_2), \ldots, gf(a_k))$

Both condition satisfied

$\therefore f;g : \mathcal{A} \to \mathcal{C}$ is Σ hom