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COL226 Programming Languages Quiz 4
February 23, 2017 10 minutes Maximum Marks: 10

Q1 (6+4 = 10 marks) **Σ-homomorphisms.**
Consider the signature $\Sigma = \{0^{(0)}, 1^{(0)}, +^{(2)}\}$, where the arities of the symbols are as indicated. Trees over this signature can be represented in OCaml as

```
type numeral = Zero | One | Plus of numeral * numeral;;
```

1. Define in OCaml the (unique) Σ-homomorphism from $Trees_\Sigma$ to the Σ-algebra $\mathcal{A} = \langle \mathbf{2}, true, false, and \rangle$. Also describe in one English sentence what function this is.

```
let rec f num = match num with
    Zero -> true
  | One -> false
  | Plus(x, y) -> (f(x)) and (f(y))
;;
```

&& (4)

4.5/10

The function essentially ~~calculates~~ not of the Tree$_\Sigma$.

2. Consider the Σ-algebras $\mathcal{B} = \langle I\!N, zero, one, addition \rangle$ and $\mathcal{C} = \langle \mathbf{2}, false, true, xor \rangle$. Show that the function $odd : I\!N \to \mathbf{2}$ that returns *true* for odd and *false* for even natural numbers, is a Σ-homomorphism from $\mathcal{B} \to \mathcal{C}$.

for odd : ℕ → 2 to be a Σ-homomorphism, & n be any Natural Number

i) $odd(n) = \begin{cases} true & \text{if } n \text{ is odd} \\ false & \text{if } n \text{ is even.} \end{cases}$

(1/2)

Arity is 2 XOR

ii) $odd(n_1 + n_2 + n_3 + \dots) = \cancel{odd} \; xnot(odd(n_1), odd(n_2) \dots odd(n_m))$

Hence proved. Where is the proof?