Name: Aman Agrawal Entry: 2015CS10210 Gp: 1 1

Indian Institute of Technology Delhi
Department of Computer Science and Engineering

COL226 Programming Languages Quiz 1
January 20, 2017 10 minutes Maximum Marks: 10

Instructions. Write your name, entry number and group number in the space provided at the top of the page. Write in black or blue ink. Manage your time effectively.

Consider the set $A = \{0, 1\}$ and let us define A^* inductively as the (smallest) set such that

$$\text{The empty sequence } \epsilon \in A^*$$
$$\text{For all } b \in A, \sigma \in A^*, \quad b\sigma \in A^*$$

$6\frac{1}{2}$

A^* can be represented in OCaml, for example, as `bool list`.

Q1 (6+4 marks) **Denumerability of Sequences.**

Define in OCaml a 1-1 total function *inj* : `bool list` → `int`, which represents an enumeration of strings in A^*. [Hints: How many strings are of length k? Within strings of length k, what is a simple and intuitive way of enumerating the strings? How many strings are shorter than any string of length k?]

~~let len list = match type of l with~~
~~ε → 0~~
~~1 →~~

$4\frac{1}{2}$

inj(ε) = ?
inj(0) = $2^1 - 2 + 0 = 0$
inj(1) = $2^1 - 2 + 1 = 1$

Let the length be k there are 2^k strings of length k.

So. Let the number be given by $2^k - 2 + \text{int}(\text{binary string})$ — why −2 ?

Here int(binary) is the decimal representation of the string

Now prove that function *inj* is 1-1, justifying each step.

The function is injective as for 2 strings l_1 & l_2

if $inj(l_1) = inj(l_2)$,

then

$$2^{len(l_1)} - 2 + int(l_1) = 2^{len(l_2)} - 2 + int(l_2)$$

$$2^{len(l_1)} + int(l_1) = 2^{len(l_2)} + int(l_2)$$

$$\Rightarrow l_1 = l_2$$

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How?
What evidence supports this claim?

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