COMP1006 Foundations of Computer Science
Assignment 1, 2020
@ Computing, Curtin University

Weighting:
This assignment contains 12 questions, for a total of 100 points, which weights for 25% of the final mark.

Submission:
You can only submit **a single PDF** file containing neatly typed answers. **No photos or scans** are accepted. Name the file as `<studentID>_<name>_Assignment01.pdf`. Use the `Declaration_of_originality.pdf` as the cover page of your assignment. Submit your assignment via the **Turnitin** link on Blackboard. The due date is **18 September 2020 11:59 PM**.

Academic Integrity:
This is an **individual** assignment so that any form of collaboration is not permitted. This is an **open-book** assignment so that you are allowed to use external materials, but make sure you properly **cite the references**. It is your responsibility to understand Curtin's Academic Misconduct Rules, for example, post assessment questions online and ask for answers is considered as contract cheating and not permitted.

Truth Table and Logic Equivalence

1. (6 points) Use **truth table** to show that the following propositions are tautology, contradiction, or contingency.
   (a) $p \oplus (p \vee q)$
   (b) $(p \rightarrow \neg q) \wedge (p \wedge q)$
   (c) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

2. (6 points) Use **logical equivalence** to verify the following propositions are logically equivalent.
   (a) $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
   (b) $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

3. (5 points) How many of the propositions $p \vee q$, $\neg p \vee q$, $p \vee r$, $q \vee r$, $q \vee \neg r$, $\neg q \vee \neg r$ can be made simultaneously true by an assignment of truth values to p, q, and r? Explain your reasoning.

4. (5 points) A compound proposition is **satisfiable** if there is at least one truth assignment to its variables that makes it true. Assume there is a program that can test whether a given proposition is satisfiable or not. Explain how such a program can be used to determine whether a given proposition is a tautology.

Translation between Logic and English

5. (7 points) let p and q be the propositions:
   p: You drive over 70 km per hour.
   q: You get a speeding ticket.

   Write the following statements using p, q, and logical connectives.
   (a) You do not drive over 70 km per hour.
   (b) You drive over 70 km per hour, but you do not get a speeding ticket.
   (c) You will get a speeding ticket if you drive over 70 km per hour.
   (d) If you do not drive over 70 km per hour, then you will not get a speeding ticket.

(e) Driving over 70 km per hour is sufficient for getting a speeding ticket.

(f) You get a speeding ticket, but you do not drive over 70 km per hour.

(g) Whenever you get a speeding ticket, you are driving over 70 km per hour.

6. (12 points) Let $L(x, y)$ be the statement "student x likes cuisine y", where the domain for x includes all students at Curtin and the domain for y consists of all cuisines. Express each of these statements by a simple English sentence.

   (a) $\exists x L(x, Chinese) \wedge \forall x L(x, Mexican)$

   (b) $\exists y (L(Monica, y) \vee L(Jay, y))$

   (c) $\forall x \forall y \exists z ((x \neq y) \rightarrow \neg(L(x, z) \wedge L(y, z)))$

   (d) $\exists x \exists y (L(x, y) \wedge \forall z ((z \neq y) \rightarrow \neg L(x, z)))$

   (e) $\exists x \exists y ((x \neq y) \wedge \forall z (L(x, z) \leftrightarrow L(y, z)))$

   (f) $\forall x \forall y \exists z (L(x, z) \leftrightarrow L(y, z))$

7. (12 points) Let $A(x)$ denote "x has a Facebook account" and $F(x, y)$ be the statement "x is a friend of y on Facebook", where the domain of x and y is all people of Computing. Express the following statements by logics.

   (a) Ross is not a friend of Rachel on Facebook.

   (b) No one in Computing is a Facebook friend of Joe.

   (c) Everyone in Computing has a Facebook account except Monica.

   (d) There is a person who is a Facebook friend of anyone else in Computing.

   (e) Everyone with a Facebook account is a friend of at least one other person in Computing.

   (f) There are two persons in Computing who doesn't have common friends on Facebook at all.

Argument and Proof

8. (12 points) For each of the following arguments, determine whether the argument is valid or not. If valid, what rule(s) of inference used? If invalid, what fallacy is it? Note that you need to define propositions or propositional functions and show the argument form first.

   (a) "All men are mortal. Joseph is a man. Therefore, Joseph is mortal."

   (b) "No man is an island. Manhattan is an island. Therefore, Manhattan is not a man."

   (c) "Chandler likes all action movies. Chandler likes the movie *Twelve Angry Men*. Therefore, *Twelve Angry Men* is an action movie."

   (d) "Phoebe, a student in Computing, knows how to write programs in Python. Everyone who knows how to write programs in Python can get a high-paying job. Therefore, someone in Computing can get a high-paying job."

   (e) "Any convertible car is fun to drive. Isaac's car is not a convertible. Therefore, Isaac's car is not fun to drive."

   (f) "If superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be impotent. If Superman were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent. Therefore, Superman does not exist." (Hint: resolution rule, $(a \vee b) \wedge (c \vee \neg b) \rightarrow (a \vee c)$ is a tautology)

9. (15 points) Proof the following statements, where you may or may not be advised which proof method to use.

   (a) Show that the sum of two odd integers is even, using a direct proof.

   (b) Show that if n is an integer and $n^3 + 5$ is odd, then n is even, using a proof by contraposition.

   (c) Show that the sum of an irrational number and rational number is irrational, using a proof by contradiction.

   (d) Prove that $n^2 + 1 \geq 2^n$ when n is a positive integer with $1 \leq n \leq 4$.

   (e) Prove that there is no positive integer n such that $n^2 + n^3 = 100$.

Mathematical Induction

10. (8 points) Let $P(n)$ be the statement that $1^2 + 2^2 + 3^2 + \ldots + n^2 = n(n+1)(2n+1)/6$ for the positive integer n.
    (a) What is the statement $P(1)$?
    (b) Show that $P(1)$ is true, completing the basis step of the proof.
    (c) What is the inductive hypothesis?
    (d) What do you need to prove in the inductive step?
    (e) Complete the inductive step.
    (f) Explain in your own words why these steps show that this formula is true whenever n is a positive integer.

11. (6 points) Use mathematical induction to proof that $1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{n^2} < 2 - \frac{1}{n}$, where n is an integer greater than 1.

12. (6 points) Use strong mathematical induction to proof that a postage of n cents can be formed using just 3-cent stamps and 5-cent stamps, for $n \geq 8$.