Venue ______________________

Student Number |__|__|__|__|__|__|__|__|

Family Name ______________________

First Name ______________________

Curtin University

Faculty of Science and Engineering

EXAMINATION

End of Semester 1, 2019

MATH1019 Linear Algebra and Statistics for Engineers

This paper is for Bentley Campus and Miri Sarawak Campus students

This is a RESTRICTED BOOK examination

Examination paper IS to be released to student

Examination Duration 2 hours

Reading Time 10 minutes

Students may write notes in the margins of the exam paper during reading time

Total Marks 100

Supplied by the University

1 x 16 page answer book

Supplied by the Student

Materials

One A4 sheet of handwritten or typed notes (both sides)

Calculator

A calculator displaying 'Engineering Approved Calculator' sticker

Instructions to Students

Attempt as many questions or part questions as possible.

SHOW ALL WORKING.

For Examiner Use Only

Q	Mark
1	
2	
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Total ________

Question 1

(a) Given the vectors $\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$, $\boldsymbol{b} = 2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{c} = [2, 1, 0]$ determine the following:

(i) $\boldsymbol{a} + 3\boldsymbol{b}$. (2 marks)

(ii) A vector of twice the length of $\boldsymbol{a}$ but in the direction of vector $\boldsymbol{b}$. (3 marks)

(iii) $||(\boldsymbol{c}.\boldsymbol{a})\boldsymbol{b}||$. (3 marks)

(iv) The scalar projection of $\boldsymbol{a}$ on $\boldsymbol{b}$. (2 marks)

(v) The area of the parallelogram formed by the vectors $\boldsymbol{a}$ and $\boldsymbol{c}$. (4 marks)

(b) Determine whether the four points $A(3, 2, 1)$, $B(3, 0, -1)$, $C(2, 2, -3)$ and $D(0, 4, 1)$ are coplanar or not. (6 marks)

(A total of 20 marks for this question.)

QUESTION 2 IS ON THE FOLLOWING PAGE.

Question 2

Given the matrices,

$$A = \begin{bmatrix} 4 & 1 \\ -1 & 2 \\ 0 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 2 \\ 4 & -1 \\ 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & 3 \\ -2 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -3 \\ -1 & 0 & 2 \end{bmatrix}$$

find the following, or briefly justify why it cannot be found,

(a) $B - A$. (1 marks)

(b) B^2. (2 marks)

(c) AC. (3 marks)

(d) $3I_2C$. (3 marks)

(e) C^{-1}. (3 marks)

(f) D^{-1}. (8 marks)

(A total of 20 marks for this question.)

QUESTION 3 IS ON THE FOLLOWING PAGE.

Question 3

(a) Determine whether the following two lines are parallel, skew or intersecting,

$$L_1 \begin{cases} x = 3+4t \\ y = 10+3t \\ z = 1+t \end{cases} \qquad L_2 \begin{cases} x = \tau \\ y = -1+2\tau \\ z = 2+\tau \end{cases}$$

If they do intersect then find the point of intersection. (7 marks)

(b) Find the shortest distance from the point $P(0,3,2)$ to the plane $4x-2y+z=-8$. (5 marks)

(c) Find the point at which the line $x=2+t$, $y=1-t$, $z=-4t$ intersects the plane $x+2y-z=10$. (4 marks)

(d) Given the planes,

$$P_1: \; -2x+y-z=0$$
$$P_2: \; 6x-3y+3z=-1$$
$$P_3: \; 4x+5y-3z=2$$

(i) Show the planes P_1 and P_2 are parallel. (2 marks)

(ii) Show the planes P_1 and P_3 are perpendicular. (2 marks)

(A total of 20 marks for this question.)

QUESTION 4 IS ON THE FOLLOWING PAGE.

Question 4

(a) Find the determinant of the matrix $A = \begin{bmatrix} 1 & 2 & -3 \\ 8 & 4 & 3 \\ -1 & 0 & -2 \end{bmatrix}$. Based on the determinant state whether the matrix A is singular or non-singular. (6 marks)

(b) Use Cramer's rule to solve the following system of linear equations. (Make sure you use Cramer's rule in solving both x_1 and x_2).

$$\begin{aligned} 3x_1 + 2x_2 &= 4 \\ -x_1 + x_2 &= -3 \end{aligned}$$

(6 marks)

(c) Let $v_1 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ -2 \\ 7 \end{bmatrix}$ and $v_3 = \begin{bmatrix} -2 \\ 1 \\ -11 \end{bmatrix}$. By using Gaussian Elimination show that $w = \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$ is not a linear combination of v_1, v_2 and v_3. (8 marks)

(A total of 20 marks for this question.)

QUESTION 5 IS ON THE FOLLOWING PAGE.

Question 5

(a) Given the following homogenous system of linear equations:

$$\begin{aligned} x_1 - x_2 - x_4 &= 0 \\ x_2 + x_4 &= 0 \\ -x_1 + 3x_2 + x_3 &= 0 \\ x_2 + x_3 - x_4 &= 0 \end{aligned}$$

(i) Use the Gauss Jordan method to get the augmented matrix $[A|\mathbf{0}]$ into reduced row echelon form. (9 marks)

(ii) State the rank of A as well as the number of solutions, then determine the solution(s). (3 marks)

(b) By using the pseudoinverse, find the least squares solution for the following inconsistent system of linear equations.

$$\begin{aligned} x_1 + x_2 &= 1 \\ 2x_1 + x_2 &= 1 \\ -3x_1 + 2x_2 &= 0 \\ -x_1 - 4x_2 &= 2 \end{aligned}$$

(8 marks)

(A total of 20 marks for this question.)

END OF EXAMINATION