MATH1019 Linear Algebra and Statistics for Engineers
Laboratory Session 9

Learning outcomes for this session

At the end of this session, you will be able to
1. Calculate cofactor expansions and determinants with matlab.
2. Solve linear systems by using Cramer's Rule.

Overview

1. Using matlab to enhance our understanding of solving systems of equations and calculating determinants.
2. matlab commands introduced in this lab: `det`
3. matlab Symbolic Toolbox function `adjoint`

Determinants and Cramer's rule

1. Input to matlab the matrix A (which, incidentally, is the A2 used in the previous lab):

```
A= [1,2,3;5,6,7;9,10,12]
```

One can extract submatrices obtained by deleting the 1st row and j-th column, for $j = 1, 2, 3$ by using the following respective commands:

```
A11=A([2 3],[2 3])
A12=A([2 3],[1 3])
A13=A([2 3],[1 2])
```

Use this to calculate the determinant of A by doing a cofactor expansion along the first row. (i.e. you are allowed to use matlab's `det` function on the 2 by 2 matrices.)
Check your answer against matlab's `det(A)`.

2. By taking the determinant of an appropriate matrix, find the scalar triple product of $\boldsymbol{u} = [1, 4, -7]$, $\boldsymbol{v} = [2, -1, 4]$ and $\boldsymbol{w} = [0, -9, 18]$. What does the result tell you about the three vectors?

3. Define (using the Symbolic Toolbox) the following matrices, A and B:

```
A = sym('A', [2 2])  % general entries
B = sym('B', [2 2])
```

Use matlab to find the product AB and then to verify that,

$$\det(A\,B) = \det(A)\,\det(B).$$

4. Define the following matrices F and G,

$$F = \begin{bmatrix} 1 & x & x \\ -x & -2 & x \\ x & x & 3 \end{bmatrix} \qquad G = \begin{bmatrix} -x & 1 & 1 \\ 1 & -x & 1 \\ 1 & 1 & -x \end{bmatrix}$$

by using the commands:

```
syms x
F = sym([1,x,x;-x,-2,x;x,x,3])
G = sym([-x,1,1;1,-x,1;1,1,-x])
```

Use matlab (Symbolic Toolbox) to find (a) the value(s) of x that makes F singular and (b) the value(s) of x that make G singular.

5. Just so you will know the function is there, read the beginning of the help page on the matlab Symbolic Toolbox function `adjoint` (which might, more properly, have been called 'adjugate').

6. We begin with an example of solving a system of linear equations using Cramer's Rule. Suppose the system is:

$$\begin{aligned} 2x_1 + x_2 + x_3 &= 3 \\ x_1 - x_2 - x_3 &= 0 \\ x_1 + 2x_2 + x_3 &= 0 \end{aligned}$$

Then a naive implementation of Cramer's Rule is as follows:

```
A = [2 1 1; 1 -1 -1; 1 2 1]
b = [3; 0; 0]
A1 = [3 1 1; 0 -1 -1; 0 2 1]  % 1st col of A replaced by b
A2 = [2 3 1; 1 0 -1; 1 0 1]   % 2nd col of A replaced by b
A3 = [2 1 3; 1 -1 0; 1 2 0]   % 3rd col of A replaced by b
detA=det(A)
x1 = det(A1)/detA
x2 = det(A2)/detA
x3 = det(A3)/detA
[x1,x2,x3]
```

Check your answer against `sol=A\b`. Now apply this method to the system with the matrix A of Question 1 and column vector
`b = [4;8;12]`

7. Here is code implementing Cramer's rule. (Never use this for large systems as Gaussian elimination `lu` is much more efficient.)

```
n=3; % for 3 eqns in 3 unknowns
x = ones(n,1);
a_det = det(A);
for i = 1:n
    C = A;
    C(:,i) = b;
    x(i) = det(C)/a_det;
end
[x(1),x(2),x(3)]
```

Run this code with the A and b of the preceding question. Check that your answer agrees with `A\b`.