Question 1 (25 marks)

(a) Explain the difference between $p \leftrightarrow q$ *and* $p \equiv q$. Further address their relationship.

(5 marks)

(b) Represent the following statements in a mathematical logic.

(i) All dogs cannot fly.
(ii) There is one female student such that none of her male friends are also friends.
(iii) Some students never attend lectures.
(iv) There is only one system administrator in department of computing.
(v) Alice loves everyone who hates cats.

(10 marks)

(c) First state the procedure of strong mathematical induction and then use this procedure to prove the following assertion.

Every integer greater than 1 can be written as a product of primes

(7 marks)

(d) Prove the following assertion.

$$\neg(x \rightarrow \neg y) \rightarrow (x \wedge (w \vee y)) \equiv T$$

(3 marks)

Questions continue on next page.

Question 2 (30 marks)

(a) For set $A_i=\{1, 2, 3,\ldots.100-i+1\}$ with i=1, 2, 3,..100, and $B_j=\{10, 11,\ldots j\}$ with j=10, 11, …, 50. Find

(i) $\bigcap_{i=10}^{12} A_i$

(ii) $\bigcap_{j=10}^{20} B_j$

(iii) $P(\{\phi, 1\})$

(iv) $|P(A_{10} \cap B_{20})| = ?$

(7 marks)

(b) Let ***A={1, 2, 3, 4}***. Give examples of relations which satisfy each of the following requirements for (i)-(iii) (explain the relations first and justify your answers) and then find a solution for (iv).

(i) The relation is symmetric and anti-symmetric;
(ii) The relation is reflexive, anti-symmetric and transitive, but not symmetric;
(iii) The relation is neither symmetric nor anti-symmetric, but is reflexive.
(iv) Find an equivalence relationship $\Re$ from $A \times A$ and compute $[2]_R$

(15 marks)

(c) Let ***A = {2, 4, 6}***.
(i) Give the definition for a function, and then construct a function from ***A×A*** to ***A***.
(ii) Is it possible to construct an **onto function** from ***A to A×A***? Construct such a function if it exists. Give the reason if such a function does not exist.
(iii) Is it possible to construct an **one-to-one function** from ***A×A to A***? Construct such a function if it exists. Give the reason if such a function does not exist.

(8 marks)

Questions continue on next page.

Question 3 (20 marks)

(a) (i) Find a recurrence relation for the number of bit strings of length n that contain two consecutive zeroes.
(ii) What are the initial conditions for part (i)?
(iii) How many bit strings of length **five** that contain two consecutive zeroes there in part (i)?

(10 marks)

(b) A class consists of 8 men and 5 women. Find the number of ways that the people in the class can arrange themselves in each of the following cases.

(i) How many groups can be chosen from this class which consists of 4 men and 4 women?

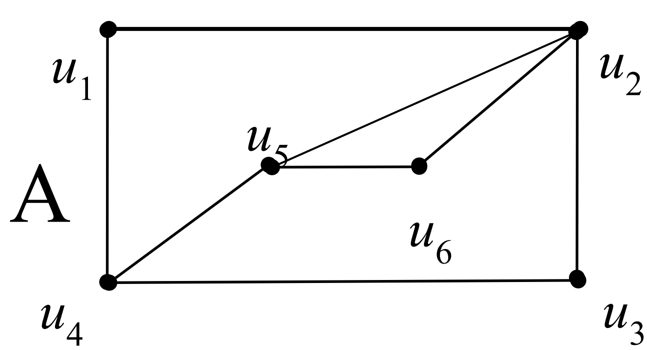
(ii) If two students have to be in the same group, how many groups of 8 students can be formed from this class?

(iii) If one male A and one female B cannot be in the same group, how many ways can a group, consisting of 4 men and 4 women, be chosen from the class?

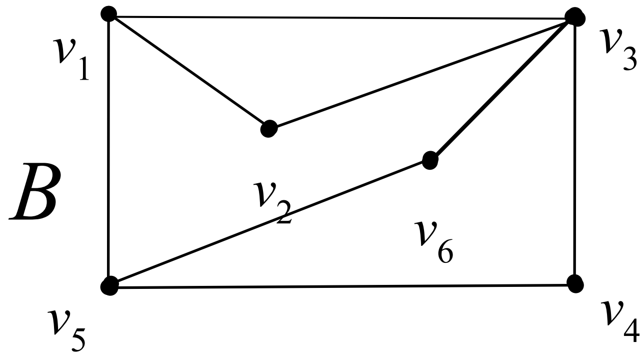
(10 marks)

Question 4 (25 marks)

(a) (i) Explicitly explain the concept of isomorphism for two graphs.
(ii) Give two graphs A and B as below



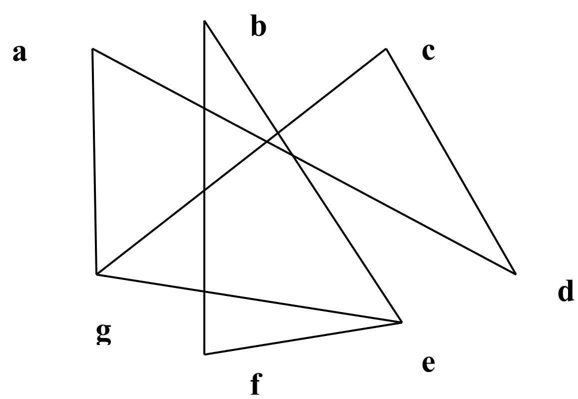
Questions continue on next page



Prove or disprove whether the two graphs ***A*** and ***B*** are isomorphic?

(10 marks)

(b) Consider the following graph:



(i) Write the adjacency matrix of the graph.

(ii) Is there an Euler circuit or Euler path in the graph? If yes, list one. Otherwise explain why not.

Is there a Hamilton circuit or Hamilton path in the graph? If yes, list one. Otherwise explain why not.

(15 marks)

Useful information continues on next page.