

# MATH1019 Linear Algebra and Statistics for Engineers

## Workshop 3

### Learning outcomes for this session

At the end of this session, you should be able to

1. Understand and work with the sampling distribution of the mean.
2. Determine probabilities using the Central Limit Theorem.
3. Find confidence intervals.

### Overview

We will work through and discuss exercises based on lecture 3.

### Exercises

1. A certain type of thread is manufactured with a mean tensile strength of 78.3 kilograms and a standard deviation of 5.6 kilograms. How is the variance of the sample mean changed when the sample size is
  - (a) increased from 64 to 196?
  - (b) decreased from 784 to 49?
2. The heights of 1000 students are approximately normally distributed with a mean of 174.5 centimetres and a standard deviation of 6.9 centimetres. Suppose 200 random samples of size 25 are drawn from this population and the means recorded to the nearest tenth of a centimetre. Determine
  - (a) the mean and standard deviation of the sampling distribution of  $\bar{X}$ ;
  - (b) the number of sample means that fall between 172.5 and 175.8 centimetres inclusive;
  - (c) the number of sample means falling below 172.0 centimetres.
3. The random variable  $X$ , representing the number of cherries in a cherry puff, has the following probability distribution:

$x$	4	5	6	7
$P(X = x)$	0.2	0.4	0.3	0.1

- (a) Find the mean  $\mu$  and the variance  $\sigma^2$  of  $X$ .
- (b) Find the mean  $\mu_{\bar{X}}$  and the variance  $\sigma_{\bar{X}}^2$  of the mean  $\bar{X}$  for random samples of 36 cherry puffs.

- (c) Find the probability that the average number of cherries in 36 cherry puffs will be less than 5.5.
4. If a certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms, what is the probability that a random sample of 36 of these resistors will have a combined resistance of more than 1458 ohms?
  5. Shear strength measurements for spot welds of a certain type have been found to have a standard deviation of approximately 10 psi. If 100 test welds are to be measured, find the approximate probability that the sample mean will be within 1 psi of the true population mean.
  6. If shear strength measurements have a standard deviation of 10 psi, how many test welds should be used in the sample if the sample mean is to be within 1 psi of the population mean with probability approximately 0.95?
  7. The fracture strengths of a certain type of glass have a mean of 14 (thousands of pounds per square inch) and a standard deviation of 2. What is the probability that the mean fracture strength for 100 pieces of this glass exceeds 14.5?
  8. An important manufacturing process produces cylindrical component parts for the automotive industry. It is important that the process produce parts having a mean diameter of 5.0 millimetres. The engineer involved conjectures that the population mean is 5.0 millimetres. An experiment is conducted in which 100 parts produced by the process are selected randomly and the diameter measured on each. It is known that the population standard deviation is  $\sigma = 0.1$  millimetre. The experiment indicates a sample mean diameter of  $\bar{x} = 5.027$  millimetres. Does this sample information appear to support or refute the engineer's conjecture? This exercise reflects the kind of problem often posed and solved with hypothesis testing introduced in lecture 12; and we will leave formality to that lecture. The question can be rephrased as follows: How likely is it that we can obtain  $\bar{x} \geq 5.027$  with  $n = 100$  if the population mean is  $\mu = 5.0$ ? If this probability suggests that  $\bar{x} = 5.027$  is not unreasonable, the conjecture is not refuted. If the probability is quite low, we could argue that the data do not support the conjecture that  $\mu = 5.0$ .
  9. Resistors of a certain type have resistances that average 200 ohms with a standard deviation of 10 ohms. Twenty-five of these resistors are to be used in a circuit.
    - (a) Find the probability that the average resistance of the 25 resistors is between 199 and 202 ohms.
    - (b) Find the probability that the *total* resistance of the 25 resistors does not exceed 5100 ohms.
    - (c) What assumptions are necessary for the answers in (a) and (b) to be good approximations?
  10. The average zinc concentration recovered from a sample of measurements taken in 36 different locations in a river is found to be 2.6 grams per millilitre. Suppose that it is known that  $\sigma = 0.3$ . Find
    - (a) the 95% confidence interval for the mean zinc concentration in the river

- (b) the 99% confidence interval for the mean zinc concentration in the river
  - (c) How large a sample is required if we want to be 95% confident that our estimate of  $\mu$  is off by less than 0.05?
11. An important property of plastic clays is the percent of shrinkage on drying. For a certain type of plastic clay 45 test specimens showed an average shrinkage percentage of 18.4. Suppose that it is known that the population standard deviation is 1.2. Estimate the true average percent of shrinkage for specimens of this type in a 98% confidence interval.