WORKSHOP 10

This workshop will build on material from Lecture 10: Lines and Planes in 3 Space.

During this workshop, students will work towards the following learning outcomes:

- find the vector, parametric and cartesian equations of a line.
- determine if lines are parallel, intersecting or skew.
- find the general equation of a plane.
- find the line of intersection between planes.
- calculate distances between points, lines and planes.

Lines in $\mathbb{R}^3$

1. Find the Cartesian equations of the line passing through the points $(3, 5, -2)$ and $(2, 1, -1)$.
2. Find the vector and parametric equations for the line passing through the point $(2, 5, -2)$ and is parallel to the line $\dfrac{x+4}{3} = \dfrac{y}{-2} = z - 3$.
3. Show that the line through the points $(7, 2, 2)$ and $(1, 4, -2)$ is parallel to the line $x = -2 + 3t$, $y = 1 - t$, $z = 4 + 2t$.
4. Find the shortest distance from the point $(0, 0, 12)$ to the line $x = 4t$, $y = -2t$, $z = 2t$.
5. Determine whether the following lines are parallel, intersecting, or skew. If they intersect find the point of intersection, or if they are skew then find the shortest distance between them.

   (i) $L_1 \begin{cases} x &= 3 + 2t \\ y &= -1 + 4t \\ z &= 2 - t \end{cases}$ $\qquad L_2 \begin{cases} x &= 1 + 4\tau \\ y &= 1 + 2\tau \\ z &= -3 + 4\tau \end{cases}$

   (ii) $L_1 \begin{cases} x &= 1 + 2t \\ y &= -1 - t \\ z &= 3t \end{cases}$ $\qquad L_2 \begin{cases} x &= 2 - \tau \\ y &= 3\tau \\ z &= 1 + \tau \end{cases}$

Planes in $\mathbb{R}^3$

6. Find the equation of the plane containing the points $P(0,1,1)$, $Q(1,0,1)$ and $R(1,1,0)$.

7. Find the equation of the plane passing through the point $P(6,0,-2)$ and which contains the line $x = 4-2t$, $y = 3+5t$, $z = 7+4t$.

8. Find the parametric equations for the line of intersection of the planes $3x-2y+z = 1$ and $2x+y-3z = 3$.

9. For each of the following pairs of planes, decide whether they are parallel, perpendicular or neither.

   (i) $x+z = 1$, $y+z = 1$.

   (ii) $-8x-6y+2z = 1$, $z = 4x+3y$.

   (iii) $x+4y-3z = 1$, $-3x+6y+7z = 0$.

10. Find the angle between the planes $x+y+z = 0$ and $x+2y+3z = 1$.

11. Find the distance from the point $P(3,-2,7)$ to the plane $4x-6y+z = 5$.