Design and Analysis of Algorithms (COMP3001)

Tutorial 4
Priority Queues and Heaps

Question 1.

a) Is the sequence <23, 17, 14, 6, 13, 10, 1, 5, 7, 12> a max heap?

b) Is an array that is in sorted order a min-heap?

Question 2.

Show that an n element binary heap has height $\lfloor \log_2 n \rfloor$.

Question 3.

Prove that the best case running time of BUILD_HEAP is $\Omega(n)$.

Question 4.

Using Figure 6.2 (textbook) as a model, illustrate the operation of MAX-HEAPIFY(A, 3) on the array A= <27, 17, 3, 16, 13, 10, 1, 5, 7, 12, 4, 9, 0>

Question 5.

Illustrate the operation of HEAP-EXTRACT-MAX on the heap A = <15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1>

Question 6.

Using Figure 6.4 (textbook) as a model, illustrate the operation of HEAPSORT on the array A = <5, 13, 2, 25, 7, 17, 20, 8, 4>

Question 7.

Using Figure 6.5 (textbook) illustrate the operation of MAX_HEAP-INSERT(A, 10) on the heap A = <15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1>

Question 8.

a) Design an algorithm for merging two priority queues that are represented by heaps.
b) What is the worst case running time of your algorithm?
c) If the priority queues were represented by leftist trees, would merging the two queues be faster than your heap-based algorithm?