MATH1019 Mid-Semester Test

Semester 1, 2020

Statutory declaration: At the top of the first page of your submission you need to write the following statement:

"*I declare that this is my own original work, no part has been copied from another student's work. I acknowledge that plagiarism also includes the act of allowing or assisting another person to copy my work. I recognise that should this declaration be found to be false, disciplinary action could be taken and penalties imposed in accordance with Curtin University policy.* "

Write your name and signature below this statement.

Question 1

The time taken by students to complete an online assignment is Normally distributed. According to past records the completion time, in minutes, had a mean of 98.5 and a standard deviation of 6.7. A new similar online assignment is released. It is found that the total time taken to complete the new assignment by 10 randomly selected students is 1008.8 minutes. You plan to conduct a hypothesis test to study whether the mean completion time for the new assignment is different from the mean completion time of the old assignment, at the 10% significance level.

(You may assume that the times taken for the new assignment are Normally distributed, with standard deviation given as above.)

(a) Explain what is meant by the 10% significance level.   (2 marks)
(b) State the null and alternative hypotheses.   (2 marks)
(c) Carry out the hypothesis test (be sure to state the critical region, test statistic and conclusion with reasoning)   (6 marks)
(d) State the p-value for the test. Does it support your conclusion in part (c)?   (3 marks)

Question 2

The random variable X takes the values $-1, 1, 2$ and 3 only. X has probability function given by $\frac{c}{x^2}$, where c is a constant.

(a) Find the exact value of c. Hence, construct the probability distribution table for X.

(4 marks)

(b) Compute the expected value and variance of X.   (5 marks)

Question 3

Suppose there are two boxes, *A* and *B*, with numbered cards. Box *A* consists of ten cards numbered $1, 2, 2, 3, 3, 3, 4, 4, 4, 4$. Box *B* consists of five cards $0, 0, 5, 5, 5$. One card is chosen randomly from each box. The random variable *X* is defined as the sum of the two numbers on the cards. Construct the probability distribution table for *X* and hence find $E(X)$.   (6 marks)

Question 4

The Environmental Protection Agency has been studying Miller Creek regarding ammonia nitrogen concentration. For many years, the concentration has been 2.3 mg/L. However, a new golf course and housing developments are raising concern that the concentration may have changed because of lawn fertiliser. Any change (either an increase or a decrease) in the ammonia nitrogen concentration can affect plant and animal life in and around the creek. A sample of eight water tests from the creek gave a sample mean of 2.51 mg/L and a sample standard deviation of 0.35 mg/L.

(a) Construct a 95% confidence interval for μ, the mean ammonia nitrogen concentration.
(4 marks)
(b) State any assumptions you made in constructing the confidence interval above. (2 marks)
(c) Based on your interval in part (a), do you think that there is enough evidence in the sample to conclude that the ammonia nitrogen concentration has changed? Explain.   (2 marks)

Question 5

The amount of coffee dispensed by a coffee machine is normally distributed with mean 205 mL and standard deviation 20 mL.

(a) What is the probability that a cup contains between 191 mL and 209 mL of coffee?
(3 marks)
(b) Above what value do we get the biggest 25% of the drinks?   (3 marks)

For another coffee machine, the amount coffee is also normally distributed but with mean 210 mL.

(c) If 20% of 220 mL cups will overflow using this particular coffee machine, what is the population standard deviation?   (4 marks)
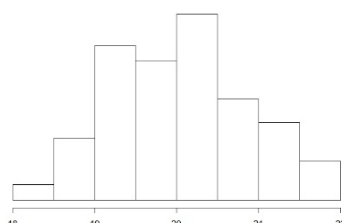
Question 6

Below are four plots. The top plot represents the distribution of a population with μ=20 and σ=4. The remaining three plots show a distribution of : (**1**) a single random sample of 100 values from this population; (**2**) a distribution of 100 sample means from random samples of size 5 from this population; and (**3**) a distribution of 100 sample means from random samples of size 25 from this population. Match each plot **A**, **B**, and **C** with the corresponding description **1**, **2**, and **3**. Justify your answer. (4 marks)
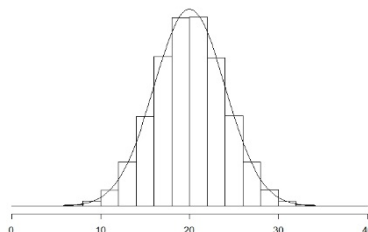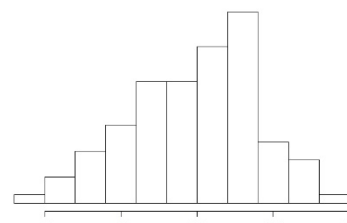

Plot A
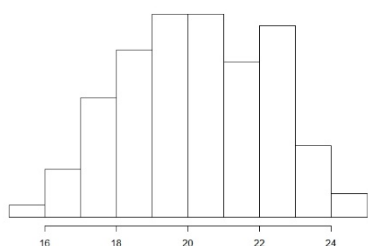
Plot B

Plot C

Question 7

Given the points,

$$A(1,-1,2), B(0,4,3), C(2,1,0), D(3,-3,-1), E(1,2,-2), F(3,0,2)$$

and also given the vectors,

$$\boldsymbol{a} = [-1,4,2], \boldsymbol{b} = [0,3,4], \boldsymbol{c} = \left[2,-2,\sqrt{8}\right], \boldsymbol{d} = [2,8,-4],$$

$$\boldsymbol{e} = [4,8,x], \boldsymbol{f} = [5,2,-1], \boldsymbol{g} = [1,y,-1]$$

Solve the following questions.

(a) Find the position vector of point A. (1 mark)

(b) Find a vector with the same length as $\boldsymbol{a}$ but in the opposite direction. (1 mark)

(c) Find the unit vector $\widehat{\overrightarrow{AB}}$. (2 marks)

(d) Find a vector with the same length as $\boldsymbol{b}$ but in the direction of $\boldsymbol{c}$. (3 marks)

(e) Determine whether the vectors $\boldsymbol{a}$ and $\boldsymbol{d}$ are parallel or not. (2 marks)

(f) Find the value of x that makes the vectors $\boldsymbol{b}$ and $\boldsymbol{e}$ perpendicular to each other. (2 marks)

(g) Find a non-zero vector perpendicular to both $\boldsymbol{d}$ and $\boldsymbol{f}$. (2 marks)

(h) If the scalar projection of the vector $\boldsymbol{g}$ on $\boldsymbol{f}$ is $\frac{1}{\sqrt{30}}$, find the value of y. (3 marks)

(i) Given a quadrilateral $ABCD$ is formed by the points, A, B, C and D, determine the angle at the vertex C. (4 marks)

(j) Determine whether the four points C, D, E and F are coplanar or not. (5 marks)