Question 1 (25 marks)

(a) List three basic techniques for the Proof of $P \to Q$ and briefly justify each technique.

(5 marks)

(b) Represent the following statements in a mathematical logic.

(i) Some people in this department are cheating.
(ii) There is one female student such that none of her male friends are also friends.
(iii) Everybody has a good hobby.
(iv) There are only one student in this class who can achieve the highest mark in COMP2001.
(v) Bob hates everyone who likes cats.

(10 marks)

(c) Prove the following is true by using mathematical induction.

$$4 \mid 3^{2n-1} + 1$$

for all positive integer n.

(7 marks)

(d) Calculate the negation for the following proposition.

$$\neg[\forall \varepsilon > 0, \exists \delta > 0,\ [(0 <| x - a |> \delta) \to (| f(x) - f(a) |< \varepsilon)\ ]\ ] = ?$$

(3 marks)

Questions continue in next page.

Question 2 (30 marks)

(a) For set $A_i=\{1,2,3,\ldots.i\}$ with $i=1,2,3,..100$, and $B_j=\{10,11,\ldots j\}$ with $j=10, 11, \ldots 50$. Find

(i) $\bigcup_{j=10}^{20} A_i$

(ii) $\bigcap_{j=10}^{20} B_j$

(iii) $P(\{\phi\})$

(iv) $|P(A_{90} \cap B_{20})| = ?$

(7 marks)

(b) Let ***A={1,2,3,4,5}***. Give examples of relations which satisfy each of the following requirements for (i)-(iii) and then find a solution for (iv).

(i) The relation is symmetric and anti-symmetric;

(ii) The relation is reflexive, anti-symmetric and transitive, but not symmetric;

(iii) The relation is neither symmetric nor anti-symmetric, but is reflexive.

(iv) Find an equivalence relationship $\Re$ from $A \times A$ and compute $[3]_R$

(15 marks)

(c) Let ***A = {a, b, c, d}***.

(i) Give the definition for a function, and then construct a function from ***A×A*** to ***A***.

(ii) Is it possible to construct an **onto function** from ***A to A×A***? Construct such a function if it exists. Give the reason if such a function does not exist.

(iii) Is it possible to construct an onto function from ***A×A to A***? Construct such a function if it exists. Give the reason if such a function does not exist.

(8 marks)

Questions continue in next page.

Question 3 (20 marks)

(a) (i) Find a recurrence relation for the number of bit strings of length n that **do not** contain three consecutive zeroes.
(ii) What are the initial conditions for part (i)?
(iii) How many bit strings of length **six** that **do not** contain three consecutive zeroes for part (i)?

(10 marks)

(b) A class consists of 10 men and 6 women. Find the number of ways that the people in the class can arrange themselves in the following cases.

(i) How many groups can be chosen from this class which consists of 7 men and 3 women?

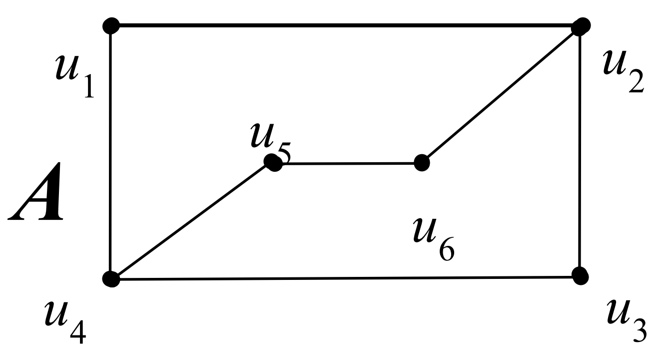
(ii) If two students have to be in the same group, how many groups of 12 students can be formed from this class?

(iii) If one male A and one female B cannot be in the same group, how many ways can a group, consisting of 4 men and 4 women, be chosen from the class?

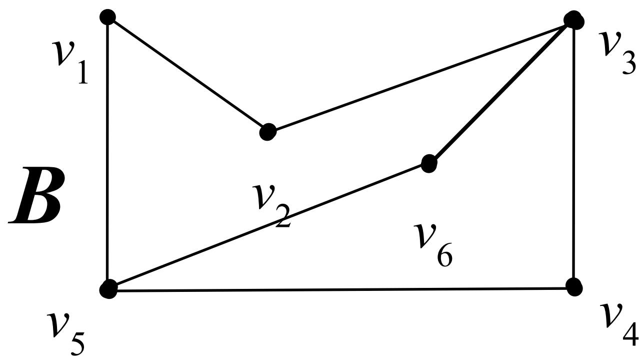
(10 marks)

Question 4 (25 marks)

(a) (i) Explicitly explain the concept of isomorphism for two graphs.
(ii) Give two graphs below.



Questions continue in next page.



Prove or disprove the two graphs ***A*** and ***B*** are isomorphic?

(10 marks)

(b) The complete 3-partite graph $K_{n,m,p}$, with $n, m, p \geq 1$, is a simple graph that has its vertex set partitioned into 3 disjoint non-empty subsets of n, m and p vertices, respectively. Two vertices are adjacent if and only if they are in different subsets in the partition.

(i) Draw $K_{3,2,2}$.

(ii) Can you find an Euler circuit in above graph? If so, find one; if not, justify your answer. .

(iii) For which values of n, m, p, does $K_{n,m,p}$ have an Euler Circuit ? Justify your answer.

(10 marks)

(c) Given a graph $G(V, E)$, give the definitions of Euler path and Hamilton path; Further, construct two illustrative examples for these two kind of paths using a graph with **six** vertices.

(5 marks)

END OF EXAMINATION PAPER