MATH1019 Mid-Semester Test

2018 Semester 1

Name: _____

Student number: _____

Please circle your workshop tutor and corresponding workshop time:

Samira Albrbar: | Monday 2–4pm | | Thursday 4–6pm |

Muhammad Kamran: | Monday 4–6pm | | Wednesday 8–10am |

Shuang Li: | Wednesday 10am–12pm | | Wednesday 2–4pm |

Grant Keady: | Thursday 8–10am | | Friday 2–4pm |

Lydia Jiang: | Thursday 10am–12pm |

Mehdi Moghadam: | Friday 10am–12pm |

Mikhail Dokuchaev: | Wednesday 4–6pm | | Friday 8–10am |

| Friday 12–2pm |

1. Consider the following system of equations:

$$x - 3y + 2z = 1$$
$$2x - 5y + 6z = 5$$
$$-x + 5y + 2z = 5$$

   (a) Write the system as an augmented matrix, and use the Gauss-Jordan method to manipulate the augmented matrix into reduced row echelon form.

   [6 marks]

   (b) State the number of solutions to the system. Find all solutions, or justify why there are none.

   [4 marks]

(A total of 10 marks for Question 1)

TEST CONTINUES OVER THE PAGE

2. If k is a real constant, then we can define a system by

$$
\begin{aligned}
2x &- 4y &+ 4z &= 12 \\
3x &+ y &- 8z &= 4 \\
-5x &+ 11y &+ kz &= -32
\end{aligned}
$$

(a) Write the system as an augmented matrix, and use Gaussian elimination to reduce the matrix to row-echelon form.

[6 marks]

(b) Briefly justify for which value(s) of k, if any, the system has:

   (i) a unique solution

  (ii) infinitely many solutions

 (iii) no solution

You do **not** need to solve the system in any of these cases.

[4 marks]

(A total of 10 marks for Question 2)

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3. Consider the system of equations

$$x - 2y - 4z = 1$$
$$2x - 3y - 6z = 2$$
$$-3x + 6y + 15z = 3$$

(a) Express the system in the form $A\underset{\sim}{x} = \underset{\sim}{b}$, where A is the matrix of coefficients, $\underset{\sim}{x}$ is the column vector of variables, and $\underset{\sim}{b}$ is the column vector of constants.

[1 mark]

(b) Use the inverse of A to solve the system of equations, or justify why there are no solutions.

[9 marks]

(A total of 10 marks for Question 3)

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4. Calculate the determinant of each of the following matrices, and justify if the matrix is invertible or singular. You do **not** need to find the inverse.

(a) $A = \begin{bmatrix} 1 & -3 & 7 \\ -3 & 9 & -21 \\ 2 & -6 & 14 \end{bmatrix}$

[5 marks]

(b) $B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

[3 marks]

(A total of 8 marks for Question 4)

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5. Use Cramer's rule to solve the following system:

$$\begin{aligned} 3x_1 - 2x_2 &= 6 \\ -5x_1 + 4x_2 &= 8 \end{aligned}$$

[7 marks]

(A total of 7 marks for Question 5)

END OF TEST