Venue _____

Student Number |__|__|__|__|__|__|__|__|

Family Name _____

First Name _____

Curtin University

Faculty of Science and Engineering

EXAMINATION

End of Semester 2, 2017

MATH1019 Linear Algebra and Statistics for Engineers

This paper is for Bentley Campus and Miri Sarawak Campus students

This is a RESTRICTED BOOK examination

Examination paper IS to be released to student

Examination Duration 2 hours

Reading Time 10 minutes

Notes in the margins of exam paper may be written by Students during reading time

Total Marks 100

Supplied by the University

1 x 16 page answer book

Formula sheet (included with exam paper)

Supplied by the Student

Materials

One A4 sheet of handwritten or typed notes (both sides)

Calculator

A calculator displaying 'Engineering Approved Calculator' sticker

Instructions to Students

SHOW ALL WORKING.

For Examiner Use Only

Q	Mark
1	
2	
3	
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Total _____

Question 1

(a) Let $\boldsymbol{a} = [-3, 4, 1]$, $\boldsymbol{b} = [2, -2, -1]$ and $\boldsymbol{c} = [x, 4, 6]$. Determine:

(i) A vector in the opposite direction to vector $\boldsymbol{a}$ but with the same length as vector $\boldsymbol{a}$. (1 mark)

(ii) A vector of length 4 in the direction of vector $\boldsymbol{b}$. (2 marks)

(iii) The angle between the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. (3 marks)

(iv) The vector projection of $\boldsymbol{a}$ on $\boldsymbol{b}$. (2 marks)

(v) Find the value of x that makes the vector $\boldsymbol{c}$ perpendicular to the vector $\boldsymbol{b}$. (2 marks)

(b) Determine whether the vectors $\boldsymbol{a} = [-3, 0, 1]$, $\boldsymbol{b} = [-3, 2, 1]$ and $\boldsymbol{c} = [2, 4, 0]$ are coplanar or not. (6 marks)

(c) Find the work done by a force $\boldsymbol{F} = 3\boldsymbol{i} - 4\boldsymbol{j} + 5\boldsymbol{k}$ in moving an object along the line from the the point $(1, 4, -1)$ to the point $(5, 2, 3)$ (distance in meters). (4 marks)

(A total of 20 marks for this question.)

Question 2

(a) Determine the parametric equations of the line passing through the point $P(-3, 4, 1)$ and which is parallel to the line $\dfrac{x-1}{4} = \dfrac{y+2}{3} = \dfrac{z}{-2}$. (3 marks)

(b) Determine whether the two lines are parallel, skew or intersecting,

$$L_1 \begin{cases} x &= -1 - t \\ y &= 4 + t \\ z &= -2 + 2t \end{cases} \qquad L_2 \begin{cases} x &= 2 + \tau \\ y &= 7 + 2\tau \\ z &= 2\tau \end{cases}$$

If they do intersect then find the point of intersection. (7 marks)

(c) Find the shortest distance from the point $(1, -1, 3)$ to the line $x = 2 - t$, $y = 4 + t$, $z = -3 + 2t$. (5 marks)

(d) Find the equation of the plane passing through the points $P(2, -1, -1)$, $Q(3, 0, 4)$ and $R(-2, -1, 2)$. (5 marks)

(A total of 20 marks for this question.)

QUESTION 3 IS ON THE FOLLOWING PAGE.

Question 3

(a) Use the Gauss-Jordan method to either calculate the inverse A^{-1} of the matrix A or to show that A has no inverse, where

$$A = \begin{bmatrix} 2 & 4 & 1 \\ -1 & 1 & -1 \\ 1 & 4 & 0 \end{bmatrix}$$

(8 marks)

(b) Find the determinant $|B|$ if $B = \begin{bmatrix} -4 & 0 & 6 \\ -1 & 0 & 5 \\ 2 & 3 & 1 \end{bmatrix}$. (4 marks)

(c) Consider the following systems of equations,

$$\begin{aligned} x_1 + x_2 + x_3 &= 5 \\ 3x_1 - x_2 - x_3 &= -1 \\ -2x_1 + 2x_2 + x_3 &= 3 \end{aligned}$$

Write down the augmented matrix of the above system then use the Gaussian Elimination method and the concept of rank of a matrix to solve the system of equations.
(8 marks)

(A total of 20 marks for this question.)

QUESTION 4 IS ON THE FOLLOWING PAGE.

Question 4

(a) Determine whether the following sets of vectors are linearly independent or linearly dependent. Give a reason for your decision.

(i) $\left\{ \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix} \right\}$ (2 marks)

(ii) $\left\{ \begin{bmatrix} -3 \\ 2 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 9 \\ -6 \\ -12 \\ 6 \end{bmatrix} \right\}$ (2 marks)

(b) Use Cramer's rule to solve the following system.

$$\begin{aligned} x_1 - 3x_2 &= -3 \\ 5x_1 - x_2 &= 13 \end{aligned}$$

(6 marks)

(c) A set of final examination grades in an introductory statistics course was found to be normally distributed with mean of 73 and a standard deviation of 8.

(i) What is the probability of getting a grade no higher than 91 on this exam? (3 marks)

(ii) If the lecturer curves the grades (gives A's to the top 10% of the class regardless of the score) are you better off with a score of 81 on this exam or a grade of 68 on a different exam where the mean is 62 and the standard deviation is 3? Explain your answer. (4 marks)

(d) During one stage in the manufacture of integrated circuit chips, a coating must be applied. If 70% of chips receive a thick enough coating, find the probability that among 15 chips at least 12 will have thick enough coatings. State the appropriate probability model for this scenario. (3 marks)

(A total of 20 marks for this question.)

QUESTION 5 IS ON THE FOLLOWING PAGE.

Question 5

(a) The following ten observations are a random sample from a normal population:

85	88	76	77	79	94	93	85	88	76

(i) Find the 98% confidence interval for the population mean μ. (4 marks)

(ii) Test $H_0 : \mu = 78$ versus $H_1 : \mu > 78$ at the 5% level of significance by using the following headings as a guide: Assumptions, Test Statistic, Critical Region(s), and Conclusion. (6 marks)

(b) One-hour carbon monoxide concentrations, measured in parts per million (ppm), in air samples from a large city are normally distributed with mean 12 ppm and standard deviation 9 ppm. Find the probability that the average concentration in 100 samples selected randomly will exceed 14 ppm. (4 marks)

(c) Consider the following sample of observations, with the assumption that they are a simple random sample.

36	27	48	51	25	75	99	34

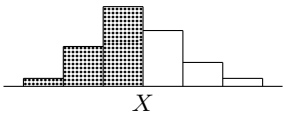
(i) Obtain the **five number summary** for the above data. (4 marks)

(ii) Use the **five number summary** to calculate the Interquartile Range (IQR) and determine if there are any outliers in the data. Justify your answer. (2 marks)

(A total of 20 marks for this question.)

END OF EXAMINATION PAPER

Cumulative probabilities for the Binomial distribution



X

$P(X \leq x)$ where $X \sim \text{Bin}(n, p)$ Read blank entries as 0.0000 or 1.0000 as appropriate

n	x \ p	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90
2	**0**	0.8100	0.6400	0.5625	0.4900	0.3600	0.2500	0.1600	0.0900	0.0625	0.0400	0.0100
	1	0.9900	0.9600	0.9375	0.9100	0.8400	0.7500	0.6400	0.5100	0.4375	0.3600	0.1900
	2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	**0**	0.7290	0.5120	0.4219	0.3430	0.2160	0.1250	0.0640	0.0270	0.0156	0.0080	0.0010
	1	0.9720	0.8960	0.8438	0.7840	0.6480	0.5000	0.3520	0.2160	0.1562	0.1040	0.0280
	2	0.9990	0.9920	0.9844	0.9730	0.9360	0.8750	0.7840	0.6570	0.5781	0.4880	0.2710
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	**0**	0.6561	0.4096	0.3164	0.2401	0.1296	0.0625	0.0256	0.0081	0.0039	0.0016	0.0001
	1	0.9477	0.8192	0.7383	0.6517	0.4752	0.3125	0.1792	0.0837	0.0508	0.0272	0.0037
	2	0.9963	0.9728	0.9492	0.9163	0.8208	0.6875	0.5248	0.3483	0.2617	0.1808	0.0523
	3	0.9999	0.9984	0.9961	0.9919	0.9744	0.9375	0.8704	0.7599	0.6836	0.5904	0.3439
	4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	**0**	0.5905	0.3277	0.2373	0.1681	0.0778	0.0312	0.0102	0.0024	0.0010	0.0003	0.0000
	1	0.9185	0.7373	0.6328	0.5282	0.3370	0.1875	0.0870	0.0308	0.0156	0.0067	0.0005
	2	0.9914	0.9421	0.8965	0.8369	0.6826	0.5000	0.3174	0.1631	0.1035	0.0579	0.0086
	3	0.9995	0.9933	0.9844	0.9692	0.9130	0.8125	0.6630	0.4718	0.3672	0.2627	0.0815
	4	1.0000	0.9997	0.9990	0.9976	0.9898	0.9688	0.9222	0.8319	0.7627	0.6723	0.4095
	5		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	**0**	0.5314	0.2621	0.1780	0.1176	0.0467	0.0156	0.0041	0.0007	0.0002	0.0001	0.0000
	1	0.8857	0.6554	0.5339	0.4202	0.2333	0.1094	0.0410	0.0109	0.0046	0.0016	0.0001
	2	0.9842	0.9011	0.8306	0.7443	0.5443	0.3438	0.1792	0.0705	0.0376	0.0170	0.0013
	3	0.9987	0.9830	0.9624	0.9295	0.8208	0.6562	0.4557	0.2557	0.1694	0.0989	0.0158
	4	0.9999	0.9984	0.9954	0.9891	0.9590	0.8906	0.7667	0.5798	0.4661	0.3446	0.1143
	5	1.0000	0.9999	0.9998	0.9993	0.9959	0.9844	0.9533	0.8824	0.8220	0.7379	0.4686
	6		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	**0**	0.4783	0.2097	0.1335	0.0824	0.0280	0.0078	0.0016	0.0002	0.0001	0.0000	
	1	0.8503	0.5767	0.4449	0.3294	0.1586	0.0625	0.0188	0.0038	0.0013	0.0004	0.0000
	2	0.9743	0.8520	0.7564	0.6471	0.4199	0.2266	0.0963	0.0288	0.0129	0.0047	0.0002
	3	0.9973	0.9667	0.9294	0.8740	0.7102	0.5000	0.2898	0.1260	0.0706	0.0333	0.0027
	4	0.9998	0.9953	0.9871	0.9712	0.9037	0.7734	0.5801	0.3529	0.2436	0.1480	0.0257
	5	1.0000	0.9996	0.9987	0.9962	0.9812	0.9375	0.8414	0.6706	0.5551	0.4233	0.1497
	6		1.0000	0.9999	0.9998	0.9984	0.9922	0.9720	0.9176	0.8665	0.7903	0.5217
	7			1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
8	**0**	0.4305	0.1678	0.1001	0.0576	0.0168	0.0039	0.0007	0.0001	0.0000	0.0000	
	1	0.8131	0.5033	0.3671	0.2553	0.1064	0.0352	0.0085	0.0013	0.0004	0.0001	
	2	0.9619	0.7969	0.6785	0.5518	0.3154	0.1445	0.0498	0.0113	0.0042	0.0012	0.0000
	3	0.9950	0.9437	0.8862	0.8059	0.5941	0.3633	0.1737	0.0580	0.0273	0.0104	0.0004
	4	0.9996	0.9896	0.9727	0.9420	0.8263	0.6367	0.4059	0.1941	0.1138	0.0563	0.0050
	5	1.0000	0.9988	0.9958	0.9887	0.9502	0.8555	0.6846	0.4482	0.3215	0.2031	0.0381
	6		0.9999	0.9996	0.9987	0.9915	0.9648	0.8936	0.7447	0.6329	0.4967	0.1869
	7		1.0000	1.0000	0.9999	0.9993	0.9961	0.9832	0.9424	0.8999	0.8322	0.5695
	8				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$P(X \leq x)$ where $X \sim \text{Bin}(n, p)$ Read blank entries as 0.0000 or 1.0000 as appropriate

	p	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90
n	x											
9	**0**	0.3874	0.1342	0.0751	0.0404	0.0101	0.0020	0.0003	0.0000	0.0000		
	1	0.7748	0.4362	0.3003	0.1960	0.0705	0.0195	0.0038	0.0004	0.0001	0.0000	
	2	0.9470	0.7382	0.6007	0.4628	0.2318	0.0898	0.0250	0.0043	0.0013	0.0003	0.0000
	3	0.9917	0.9144	0.8343	0.7297	0.4826	0.2539	0.0994	0.0253	0.0100	0.0031	0.0001
	4	0.9991	0.9804	0.9511	0.9012	0.7334	0.5000	0.2666	0.0988	0.0489	0.0196	0.0009
	5	0.9999	0.9969	0.9900	0.9747	0.9006	0.7461	0.5174	0.2703	0.1657	0.0856	0.0083
	6	1.0000	0.9997	0.9987	0.9957	0.9750	0.9102	0.7682	0.5372	0.3993	0.2618	0.0530
	7		1.0000	0.9999	0.9996	0.9962	0.9805	0.9295	0.8040	0.6997	0.5638	0.2252
	8			1.0000	1.0000	0.9997	0.9980	0.9899	0.9596	0.9249	0.8658	0.6126
	9					1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	**0**	0.3487	0.1074	0.0563	0.0282	0.0060	0.0010	0.0001	0.0000			
	1	0.7361	0.3758	0.2440	0.1493	0.0464	0.0107	0.0017	0.0001	0.0000	0.0000	
	2	0.9298	0.6778	0.5256	0.3828	0.1673	0.0547	0.0123	0.0016	0.0004	0.0001	
	3	0.9872	0.8791	0.7759	0.6496	0.3823	0.1719	0.0548	0.0106	0.0035	0.0009	0.0000
	4	0.9984	0.9672	0.9219	0.8497	0.6331	0.3770	0.1662	0.0473	0.0197	0.0064	0.0001
	5	0.9999	0.9936	0.9803	0.9527	0.8338	0.6230	0.3669	0.1503	0.0781	0.0328	0.0016
	6	1.0000	0.9991	0.9965	0.9894	0.9452	0.8281	0.6177	0.3504	0.2241	0.1209	0.0128
	7		0.9999	0.9996	0.9984	0.9877	0.9453	0.8327	0.6172	0.4744	0.3222	0.0702
	8		1.0000	1.0000	0.9999	0.9983	0.9893	0.9536	0.8507	0.7560	0.6242	0.2639
	9				1.0000	0.9999	0.9990	0.9940	0.9718	0.9437	0.8926	0.6513
	10					1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	**0**	0.3138	0.0859	0.0422	0.0198	0.0036	0.0005	0.0000				
	1	0.6974	0.3221	0.1971	0.1130	0.0302	0.0059	0.0007	0.0000	0.0000		
	2	0.9104	0.6174	0.4552	0.3127	0.1189	0.0327	0.0059	0.0006	0.0001	0.0000	
	3	0.9815	0.8389	0.7133	0.5696	0.2963	0.1133	0.0293	0.0043	0.0012	0.0002	
	4	0.9972	0.9496	0.8854	0.7897	0.5328	0.2744	0.0994	0.0216	0.0076	0.0020	0.0000
	5	0.9997	0.9883	0.9657	0.9218	0.7535	0.5000	0.2465	0.0782	0.0343	0.0117	0.0003
	6	1.0000	0.9980	0.9924	0.9784	0.9006	0.7256	0.4672	0.2103	0.1146	0.0504	0.0028
	7		0.9998	0.9988	0.9957	0.9707	0.8867	0.7037	0.4304	0.2867	0.1611	0.0185
	8		1.0000	0.9999	0.9994	0.9941	0.9673	0.8811	0.6873	0.5448	0.3826	0.0896
	9			1.0000	1.0000	0.9993	0.9941	0.9698	0.8870	0.8029	0.6779	0.3026
	10					1.0000	0.9995	0.9964	0.9802	0.9578	0.9141	0.6862
	11						1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
12	**0**	0.2824	0.0687	0.0317	0.0138	0.0022	0.0002	0.0000				
	1	0.6590	0.2749	0.1584	0.0850	0.0196	0.0032	0.0003	0.0000			
	2	0.8891	0.5583	0.3907	0.2528	0.0834	0.0193	0.0028	0.0002	0.0000	0.0000	
	3	0.9744	0.7946	0.6488	0.4925	0.2253	0.0730	0.0153	0.0017	0.0004	0.0001	
	4	0.9957	0.9274	0.8424	0.7237	0.4382	0.1938	0.0573	0.0095	0.0028	0.0006	0.0000
	5	0.9995	0.9806	0.9456	0.8822	0.6652	0.3872	0.1582	0.0386	0.0143	0.0039	0.0000
	6	0.9999	0.9961	0.9857	0.9614	0.8418	0.6128	0.3348	0.1178	0.0544	0.0194	0.0005
	7	1.0000	0.9994	0.9972	0.9905	0.9427	0.8062	0.5618	0.2763	0.1576	0.0726	0.0043
	8		0.9999	0.9996	0.9983	0.9847	0.9270	0.7747	0.5075	0.3512	0.2054	0.0256
	9		1.0000	1.0000	0.9998	0.9972	0.9807	0.9166	0.7472	0.6093	0.4417	0.1109
	10				1.0000	0.9997	0.9968	0.9804	0.9150	0.8416	0.7251	0.3410
	11					1.0000	0.9998	0.9978	0.9862	0.9683	0.9313	0.7176
	12						1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$P(X \leq x)$ where $X \sim \text{Bin}(n, p)$ Read blank entries as 0.0000 or 1.0000 as appropriate

	p	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90
n	x											
14	**0**	0.2288	0.0440	0.0178	0.0068	0.0008	0.0001	0.0000				
	1	0.5846	0.1979	0.1010	0.0475	0.0081	0.0009	0.0001				
	2	0.8416	0.4480	0.2811	0.1608	0.0398	0.0065	0.0006	0.0000			
	3	0.9559	0.6982	0.5213	0.3552	0.1243	0.0287	0.0039	0.0002	0.0000		
	4	0.9908	0.8702	0.7415	0.5842	0.2793	0.0898	0.0175	0.0017	0.0003	0.0000	
	5	0.9985	0.9561	0.8883	0.7805	0.4859	0.2120	0.0583	0.0083	0.0022	0.0004	
	6	0.9998	0.9884	0.9617	0.9067	0.6925	0.3953	0.1501	0.0315	0.0103	0.0024	0.0000
	7	1.0000	0.9976	0.9897	0.9685	0.8499	0.6047	0.3075	0.0933	0.0383	0.0116	0.0002
	8		0.9996	0.9978	0.9917	0.9417	0.7880	0.5141	0.2195	0.1117	0.0439	0.0015
	9		1.0000	0.9997	0.9983	0.9825	0.9102	0.7207	0.4158	0.2585	0.1298	0.0092
	10			1.0000	0.9998	0.9961	0.9713	0.8757	0.6448	0.4787	0.3018	0.0441
	11				1.0000	0.9994	0.9935	0.9602	0.8392	0.7189	0.5519	0.1584
	12					0.9999	0.9991	0.9919	0.9525	0.8990	0.8021	0.4154
	13					1.0000	0.9999	0.9992	0.9932	0.9822	0.9560	0.7712
	14						1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
15	**0**	0.2059	0.0352	0.0134	0.0047	0.0005	0.0000					
	1	0.5490	0.1671	0.0802	0.0353	0.0052	0.0005	0.0000				
	2	0.8159	0.3980	0.2361	0.1268	0.0271	0.0037	0.0003	0.0000			
	3	0.9444	0.6482	0.4613	0.2969	0.0905	0.0176	0.0019	0.0001	0.0000		
	4	0.9873	0.8358	0.6865	0.5155	0.2173	0.0592	0.0093	0.0007	0.0001	0.0000	
	5	0.9978	0.9389	0.8516	0.7216	0.4032	0.1509	0.0338	0.0037	0.0008	0.0001	
	6	0.9997	0.9819	0.9434	0.8689	0.6098	0.3036	0.0950	0.0152	0.0042	0.0008	
	7	1.0000	0.9958	0.9827	0.9500	0.7869	0.5000	0.2131	0.0500	0.0173	0.0042	0.0000
	8		0.9992	0.9958	0.9848	0.9050	0.6964	0.3902	0.1311	0.0566	0.0181	0.0003
	9		0.9999	0.9992	0.9963	0.9662	0.8491	0.5968	0.2784	0.1484	0.0611	0.0022
	10		1.0000	0.9999	0.9993	0.9907	0.9408	0.7827	0.4845	0.3135	0.1642	0.0127
	11			1.0000	0.9999	0.9981	0.9824	0.9095	0.7031	0.5387	0.3518	0.0556
	12				1.0000	0.9997	0.9963	0.9729	0.8732	0.7639	0.6020	0.1841
	13					1.0000	0.9995	0.9948	0.9647	0.9198	0.8329	0.4510
	14						1.0000	0.9995	0.9953	0.9866	0.9648	0.7941
	15							1.0000	1.0000	1.0000	1.0000	1.0000
20	**0**	0.1216	0.0115	0.0032	0.0008	0.0000						
	1	0.3917	0.0692	0.0243	0.0076	0.0005	0.0000					
	2	0.6769	0.2061	0.0913	0.0355	0.0036	0.0002					
	3	0.8670	0.4114	0.2252	0.1071	0.0160	0.0013	0.0000				
	4	0.9568	0.6296	0.4148	0.2375	0.0510	0.0059	0.0003				
	5	0.9887	0.8042	0.6172	0.4164	0.1256	0.0207	0.0016	0.0000			
	6	0.9976	0.9133	0.7858	0.6080	0.2500	0.0577	0.0065	0.0003	0.0000		
	7	0.9996	0.9679	0.8982	0.7723	0.4159	0.1316	0.0210	0.0013	0.0002	0.0000	
	8	0.9999	0.9900	0.9591	0.8867	0.5956	0.2517	0.0565	0.0051	0.0009	0.0001	
	9	1.0000	0.9974	0.9861	0.9520	0.7553	0.4119	0.1275	0.0171	0.0039	0.0006	
	10		0.9994	0.9961	0.9829	0.8725	0.5881	0.2447	0.0480	0.0139	0.0026	0.0000
	11		0.9999	0.9991	0.9949	0.9435	0.7483	0.4044	0.1133	0.0409	0.0100	0.0001
	12		1.0000	0.9998	0.9987	0.9790	0.8684	0.5841	0.2277	0.1018	0.0321	0.0004
	13			1.0000	0.9997	0.9935	0.9423	0.7500	0.3920	0.2142	0.0867	0.0024
	14				1.0000	0.9984	0.9793	0.8744	0.5836	0.3828	0.1958	0.0113
	15					0.9997	0.9941	0.9490	0.7625	0.5852	0.3704	0.0432
	16					1.0000	0.9987	0.9840	0.8929	0.7748	0.5886	0.1330
	17						0.9998	0.9964	0.9645	0.9087	0.7939	0.3231
	18						1.0000	0.9995	0.9924	0.9757	0.9308	0.6083
	19							1.0000	0.9992	0.9968	0.9885	0.8784
	20								1.0000	1.0000	1.0000	1.0000

Cumulative probabilities for the Standard Normal distribution

$P(Z \leq z)$ where $Z \sim \mathrm{N}(0,1)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002
−3.3	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9986	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Critical points of the t-distribution

Entry is t where $P(T \geq t) = p$ for t-distribution with u degrees of freedom

ν | p	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
31	1.309	1.696	2.040	2.453	2.744	3.375	3.633
32	1.309	1.694	2.037	2.449	2.738	3.365	3.622
33	1.308	1.692	2.035	2.445	2.733	3.356	3.611
34	1.307	1.691	2.032	2.441	2.728	3.348	3.601
35	1.306	1.690	2.030	2.438	2.724	3.340	3.591
36	1.306	1.688	2.028	2.434	2.719	3.333	3.582
37	1.305	1.687	2.026	2.431	2.715	3.326	3.574
38	1.304	1.686	2.024	2.429	2.712	3.319	3.566
39	1.304	1.685	2.023	2.426	2.708	3.313	3.558
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
41	1.303	1.683	2.020	2.421	2.701	3.301	3.544
42	1.302	1.682	2.018	2.418	2.698	3.296	3.538
43	1.302	1.681	2.017	2.416	2.695	3.291	3.532
44	1.301	1.680	2.015	2.414	2.692	3.286	3.526
45	1.301	1.679	2.014	2.412	2.690	3.281	3.520
46	1.300	1.679	2.013	2.410	2.687	3.277	3.515
47	1.300	1.678	2.012	2.408	2.685	3.273	3.510
48	1.299	1.677	2.011	2.407	2.682	3.269	3.505
49	1.299	1.677	2.010	2.405	2.680	3.265	3.500
50	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	1.290	1.660	1.984	2.364	2.626	3.174	3.390
200	1.286	1.652	1.972	2.345	2.601	3.131	3.340
500	1.283	1.648	1.965	2.334	2.586	3.107	3.310
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291