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Curtin University – Department of Computing

Assignment Cover Sheet / Declaration of Originality

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Unit name:	Foundations of Computer Science	Unit ID:	COMP1006
Lecturer / unit coordinator:	Antoni Liu	Tutor:	IDK
Date of submission:	23/10/2020	Which assignment?	2 (Leave blank if the unit has only one assignment.)

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Question 1

a. 1010100000 (Liu 2020a)
b. $01\ 0110\ 1100$ (Liu 2020a)
c. $\{2, 4, 5, 7, 9\}$ (Liu 2020a)
d. $\{1, 7, 8, 9, 10\}$ (Liu 2020a)

Question 2

a. $A \cap B = \{\emptyset\}$ (Liu 2020a)
b. $A \cup B = \{\emptyset, a, \{a, b\}, \{a\}, \{b\}\}$ (Liu 2020a)
c. $A \times B = \{(\emptyset, \emptyset), (\emptyset, \{a\}), (\emptyset, \{b\}), (a, \emptyset), (a, \{a\}), (a, \{b\}), (\{a, b\}, \emptyset), (\{a, b\}, \{a\}), (\{a, b\}, \{b\})\}$ (Liu 2020a)
d. $P(A) = \{\emptyset, \{a\}, \{\{a, b\}\}, \{\emptyset, a\}, \{\emptyset, \{a, b\}\}, \{a, \{a, b\}\}, \{\emptyset, a, \{a, b\}\}\}$ (Liu 2020a)

Question 3

a.

$$\begin{aligned}
\overline{A \cap B \cap C} &= \{x \mid x \notin A \cap B \cap C\} \\
&= \{x \mid \neg(x \in A \cap B \cap C)\} \\
&= \{x \mid \neg(x \in A \land x \in B \land x \in C)\} \\
&= \{x \mid \neg x \in A \lor \neg x \in B \lor \neg x \in C\} \\
&= \{x \mid x \notin A \lor x \notin B \lor x \notin C\} \\
&= \{x \mid x \in \bar{A} \lor x \in \bar{B} \lor x \in \bar{C}\} \\
&= \{x \mid x \in \bar{A} \cup \bar{B} \cup \bar{C}\} \\
&= \bar{A} \cup \bar{B} \cup \bar{C}
\end{aligned}$$

$\therefore \overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$

(Liu 2020a)

b.

A	B	C	$A \cap B \cap C$	$\overline{A \cap B \cap C}$
1	1	1	1	0
1	1	0	0	1
1	0	1	0	1
1	0	0	0	1
0	1	1	0	1
0	1	0	0	1
0	0	1	0	1
0	0	0	0	1

A	B	C	$\bar{A} \cup \bar{B} \cup \bar{C}$
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	1

$\therefore \overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$
(Liu 2020a)

Question 4

a. **Antisymmetric**, as if person a is older than person b, then person b is not older than person a.
Transitive, because if person a is older than person b and person b is older than person c, then person a is older than person c.
(Rosen 2007)

b. **Reflexive**, as if person a was born on the same day as person b, then the day person a was born on is equal to itself.
Symmetric, because if person a was born on the same day as person b, then person b was born on the same day as person a.
Transitive, as if person a was born on the same day as person b and person b was born on the same day as person c, then person a was born on the same day as person c.
(Rosen 2007)

c. **Reflexive**, as if person a has the same first name as person b, then person a's first name is equal to itself.
Symmetric, because if a person a has the same first name as person b, then person b has the same first name as person a.
Transitive, as if person a has the same first name as person b and person b has the same first name as person c, then person a has the same first name as person c.
(Rosen 2007)

d. **Reflexive**, as if person a has a grandparent in common with person b, then person a's grandparent is in common with itself.
Symmetric, because if a person a has a grandparent in common with person b, then person b has a grandparent in common with person a.
(Rosen 2007)

Question 5

a. $R_1 \cup R_2$ contains all ordered pairs (a, b), such that student a is either required to read book b in a unit or has read book b.
(Rosen 2007)

b. $R_1 \oplus R_2$ contains all ordered pairs (a, b), where a is a student who is required to read book b in a unit but has not read it or needs to read book b but has not read it.
(Rosen 2007)

c. $R_1 - R_2$ is the set of all ordered pairs (a, b), such that student a is required to read book b in a unit but has not read it.
(Rosen 2007)

d. $R_2 - R_1$ contains all ordered pairs (a, b), such that student a has read book b but is not required to read in a unit.
(Rosen 2007)

Question 6

a. Since A contains n elements (and, in turn, its length is n), the matrix representing R, $\mathbf{M}_R$ will be of order $n \times n$ and will be defined by $\mathbf{M}_{R_{aa}} = 1$ if (a_i, a_j) is in R and $\mathbf{M}_{R_{ab}} = 0$ otherwise. Moreover, in order to find $\mathbf{M}_{R^{-1}}$, we need to interchange the rows and columns of $\mathbf{M}_R$. Therefore, since the entries of $\mathbf{M}_R$ only have their positions changed, the number of nonzero entries in $\mathbf{M}_{R^{-1}}$ will be given by the number of nonzero entries in $\mathbf{M}_R$ (i.e. k).
(Rosen 2007)

b. Since A contains n elements (and, in turn, its length is n), the matrix representing R, $\mathbf{M}_R$ will be of order $n \times n$ and will be defined by $\mathbf{M}_{R_{aa}} = 1$ if (a_i, a_j) is in R and $\mathbf{M}_{R_{ab}} = 0$ otherwise. Moreover, in order to find $\mathbf{M}_{\bar{R}}$, we need to change each 0 and 1 in $\mathbf{M}_R$ to 1 and 0, respectively. Therefore, the number of nonzero entries in $\mathbf{M}_{\bar{R}}$ will be given by the number of entries in $\mathbf{M}_R$ less its number of nonzero entries
(i.e. $n^2 - k$).
(Rosen 2007)

Question 7

Definition: A relation R on a set S is called an equivalence relation if it is reflexive, symmetric and transitive

a. **Reflexivity**
i.e. Show that $(a, b) \sim (a, b)$
$a + b = b + a$
$\quad\quad = a + b$
$\therefore (a, b) \sim (a, b)$
$\therefore$ Since $(a, b) \sim (a, b)$, R is reflexive

Symmetry
i.e. Show that $((a,b)\sim(c,d)) \rightarrow ((c,d)\sim(a,b))$
$((a,b)\sim(c,d)) \Leftrightarrow\ a+d=b+c$
$((c,d)\sim(a,b)) \Leftrightarrow\ c+b=d+a$
$\therefore ((a,b)\sim(c,d)) \rightarrow ((c,d)\sim(a,b))$
$\therefore$ Since $((a,b)\sim(c,d)) \rightarrow ((c,d)\sim(a,b))$, R is symmetric

Transitivity
i.e. Show that $[((a,b)\sim(c,d)) \wedge ((c,d)\sim(e,f))] \rightarrow\ ((a,b)\sim(e,f))$
$((a,b)\sim(c,d)) \Leftrightarrow\ a+d=b+c$
$((c,d)\sim(e,f)) \Leftrightarrow\ c+f=d+e$

$$\begin{aligned}\big((a,b)\sim(e,f)\big) &\Leftrightarrow a+f=b+e\\ &\Leftrightarrow (b+c-d)+(d+e-c)=(a+d-c)+(c+f-d)\\ &\Leftrightarrow b+c-d+d+e-c=a+d-c+c+f-d\\ &\Leftrightarrow b-d+d+e=a+d-c+c+f-d\\ &\Leftrightarrow b+e=a+d-c+c+f-d\\ &\Leftrightarrow b+e=a-c+c+f\\ &\Leftrightarrow b+e=a+f\\ &\Leftrightarrow a+f=b+e\end{aligned}$$

$\therefore ((a,b)\sim(c,d)) \rightarrow ((c,d)\sim(a,b))$
$\therefore$ Since $((a,b)\sim(c,d)) \rightarrow ((c,d)\sim(a,b))$, R is transitive

$\therefore$ Since R is reflexive, symmetric and transitive, it is an equivalence relation

(The Math Sorcerer, 2018)

b. $[a]=\{(c,d) \mid \big((2,1),(c,d)\big) \in R\}$ (Rosen 2007)

Question 8

Definition: A relation R on a set S is called partial order if it is reflexive, antisymmetric and transitive (Rosen 2007).

a. **Reflexive**, as each vertex in the digraph features an arc in the form (x,x) (i.e. a loop)
Antisymmetric, as each vertex in the digraph features an arc in the form (x,y), but not an arc in the form (y,x)
Not **Transitive**, as there is an edge from a to b and an edge from b to c, but there is no edge from a to c.
(Liu 2020b)

b. **Reflexive**, as each vertex in the digraph features an arc in the form (x, x) (i.e. a loop)
Antisymmetric, as each vertex in the digraph features an arc in the form (x, y), but not an arc in the form (y, x)
Transitive, as whenever there is an edge (x, y) and an edge (y, z), there is an edge (x, z)
(Liu 2020b)

Question 9

a. For any n bits, there are 2^n possible bit strings. If a string is length n, then we can write its length as being either $2k$, if n is even or $2k + 1$, if n is odd, where $k \in \mathbb{Z}$

Regardless, the bits of the first half of the bit string determine the bits of the second half. Therefore, let

$$(s_i)_{i=1}^{n}$$

be a bit string that is a palindrome.

Therefore, there are 2 choices (0 or 1) for each s_i where $i \leq k$. Furthermore, if n is odd, then there are also 2 choices for s_{k+1}. Resultantly, we see that if n is even, we have a total of 2^k bit strings and if n is odd, we have a total of 2^{k+1}. Therefore, if n is even then we have a total of $2^{\frac{n}{2}}$ palindromic sub strings and if n is odd then we have a total of $2 \cdot 2^{\frac{n+1}{2}}$.
(Teja713 2015)

Let's assume that "contains two consecutive 0s" implies that a bit string contains two *and only two* consecutive 0s (i.e. one set of two consecutive 0s), as the question does not specify if we should take into account only bit strings with one and only one set of two consecutive 0s or all bit strings with at least one set of consecutive 0s. Since the length of the bit string is relatively small, we can determine the solution manually. Therefore, let's place 2 consecutive 0s at all indexes, marking bits at all other indexes with x (to represent either 0 or 1) and then determine the number of bit strings with a set of consecutive zeros. Let n be the number of bit strings that contain one and only one set of two consecutive 0s:

$$s_1 = \text{00xxx00} \Rightarrow n = 0$$
$$s_2 = \text{x00x00x} \Rightarrow n = 0$$
$$s_3 = \text{xx000xx} \Rightarrow n = 0$$
$$s_4 = s_3 = \text{xx000xx}$$
$$s_5 = s_2 = \text{x00x00x}$$
$$s_6 = s_1 = \text{00xxx00}$$

∴ The number of bit strings that are palindromes if the bit string is of length 7 and contains two consecutive 0s is 0.

b. Let's assume that "containing three consecutive 1s" implies that a bit string contains three *and only three* consecutive 1s (i.e. one set of three consecutive 1s), as the question does not specify if we should take into account only bit strings with one and only one set of three consecutive 1s or all bit strings with at least one set of consecutive 1s. Since the length of the bit string is relatively small, we can determine the solution manually. However, it would be easier to determine the solution by subtracting the number of bit strings which contain three consecutive 0s from the total number of bit strings. Let n be the number of bit strings that contain one and only one set of three consecutive 1s:

$$
\begin{aligned}
s_1 &= 111\mathrm{xx}111 \Rightarrow n = 0 \\
s_2 &= \mathrm{x}111111\mathrm{x} \Rightarrow n = 0 \\
s_3 &= \mathrm{xx}1111\mathrm{xx} \Rightarrow n = 0 \\
s_4 &= s_3 = \mathrm{xx}1111\mathrm{xx} \\
s_5 &= s_2 = \mathrm{x}111111\mathrm{x} \\
s_6 &= s_1 = 111\mathrm{xx}111
\end{aligned}
$$

∴ The number of bit strings that are palindromes if the bit string is of length 8 and does not contain three consecutive 1s is $2^8 - 0 = 256$

Question 10

a. Firstly, we need to seat people at the 8-seater table. However, we first need to select 8 people to seat at that table. The number of ways in which we can select 8 people from the original 15 is given by $C_8^{15} = 6{,}435$.

Next, we need to seat the 8 people we selected around the 8-seater table. The number of ways in which we can seat these 8 people is given by $(8-1)! = 7! = 5{,}040$. Given that "seatings are considered to be the same if they can be obtained from each other by rotating the table" we needed to discount arrangements which can be obtained by rotating a particular arrangement. Hence, the reason for $8-1$.

Thereafter, we need to seat the remaining 7 people around the 7-seater table. The number of ways we can seat these 7 people is given by $(7-1)! = 6! = 720$. Again, the reason for $7-1$ is the same as above.

Consequently, the number of ways is given by $C_8^{15} \times 7! \times 6! = 23{,}351{,}328{,}000$ (Liu 2020c)

b. Firstly, we need to seat people at the 8-seater table. However, we first need to select 8 people to seat at that table. Given that 2 particular people have to sit next to each other, we can treat these 2 people as 1. Therefore, we need to select 7 people and thus, the number of ways in which we can select 7 people from the original 15 is given by $C_7^{15} = 6{,}435$.

   Next, we need to seat the 7 people we selected around the 8-seater table, where the "7" selected includes of 2 people that have to sit next to each other being counted as 1. The number of ways in which we can seat these 7 people is given by $2! \times (7-1)! = 2! \times 6! = 1440$. Given that "seatings are considered to be the same if they can be obtained from each other by rotating the table" we needed to discount arrangements which can be obtained by rotating a particular arrangement. Hence, the reason for $7-1$. Additionally, we need to consider the number of permutations of persons a and b. Hence, 2!.

   Thereafter, we need to seat the remaining 7 people around the 7-seater table. The number of ways we can seat these 7 people is given by $(7-1)! = 6! = 720$.

   Consequently, the number of ways is given by $C_7^{15} \times 2! \times 6! \times 6! = 6{,}671{,}808{,}000$ (Liu 2020c)

c. ...

Question 11

a. Let P be the total number of possible passwords, and P_8, P_9, P_{10}, P_{11} and P_{12} denote the number of possible passwords of length 8, 9, 10, 11 and 12, respectively. Using the sum rule $P = P_8 + \ P_9 + P_{10} + P_{11} + P_{12}$. We will now find P_8, P_9, P_{10}, P_{11} and P_{12}. Using the sum and product rules, we obtain the number of passwords of n characters:

$$\begin{aligned} P_n &= (26 + 26 + 10 + 6)^8 \\ &= 68^n \end{aligned}$$

   Then using the general formula for calculating P_n, we calculate P:

$$\begin{aligned} P &= 68^8 + 68^9 + 68^{10} + 68^{11} + 68^{12} \\ &= 9{,}920{,}671{,}339{,}261{,}325{,}541{,}376 \end{aligned}$$

   Let T be the time taken, in years, to try every password.

$$T = \frac{\text{Time per password} \times P}{60 \times 60 \times 24 \times 365}$$
$$T = \frac{1 \times 9{,}920{,}671{,}339{,}261{,}325{,}541{,}376}{60 \times 60 \times 24 \times 365}$$
$$T \approx 314{,}582$$

(matin 2014; Rosen 2007)

b. Let Q be the total number of possible passwords which do not contain any of the six special characters and Q_8, Q_9, Q_{10}, Q_{11} and Q_{12} denote the number of possible passwords of length 8, 9, 10, 11 and 12, respectively. Using the sum rule $Q = Q_8 + Q_9 + Q_{10} + Q_{11} + Q_{12}$. We will now find Q_8, Q_9, Q_{10}, Q_{11} and Q_{12}. Using the sum and product rules, we obtain the number of passwords of n characters:

$$Q_n = (26 + 26 + 10)^8$$
$$= 62^n$$

Then using the general formula for calculating Q_n, we calculate Q:

$$Q = 62^8 + 62^9 + 62^{10} + 62^{11} + 62^{12}$$
$$= 3{,}279{,}156{,}377{,}874{,}257{,}103{,}616$$

Finally, Let R be the total number of possible passwords which contain at least one of the six special characters:

$$R = P - Q$$
$$= 6{,}641{,}514{,}961{,}387{,}068{,}437{,}760$$

(Rosen 2007)

c. Let P be the total number of possible passwords and P_8, P_9, P_{10}, P_{11} and P_{12} denote the number of possible passwords of length 8, 9, 10, 11 and 12, respectively. Furthermore, let Q be the total number of possible passwords which contain at least one uppercase letter, one lowercase letter, one digit and one special character and Q_8, Q_9, Q_{10}, Q_{11} and Q_{12} denote the number of possible passwords of length 8, 9, 10, 11 and 12, respectively. Using the sum rule $Q = Q_8 + Q_9 + Q_{10} + Q_{11} + Q_{12}$. We will now find Q_8, Q_9, Q_{10}, Q_{11} and Q_{12}. Using the sum and product rules, we obtain all of the number of passwords of n characters:

$$P_n = (26 + 26 + 10 + 6)^8$$
$$= 68^n$$

Then using the general formula for calculating P_n, we get $P_8 = 68^8$

Thereafter, we remove all passwords with no lowercase letter (42^8), all passwords with no uppercase letter (42^8), all passwords with no digit (58^8), and all passwords with no special character (62^8).

However, we have removed some passwords multiple times. Thus, we must add back all passwords with:

- No lowercase letter and no uppercase letter (16^8)
- No lowercase letter and no digit (32^8)
- No lowercase letter and no special character (36^8)
- No uppercase letter and no digit (32^8)
- No uppercase letter and no special character (36^8)
- No digit and no special character (52^8)

However, we have added back some passwords multiple times. For example, an all-digit password was removed three times in the first step, then put back three times in the second step. Thus, it must be removed again:

- Only lowercase letters (26^8)
- Only uppercase letters (26^8)
- Only digits (10^8)
- Only special characters (6^8)

Then, using the above we obtain the number of passwords of 8 characters which contain at least one uppercase letter, one lowercase letter, one digit and one special character:

$$\begin{aligned} Q_8 &= 68^8 - 42^8 - 42^8 - 58^8 - 62^8 + 16^8 + 32^8 + 36^8 + 32^8 + 36^8 + 52^8 - 26^8 - 26^8 - 10^8 - 6^8 \\ &= 152{,}282{,}259{,}118{,}080 \end{aligned}$$

Similarly,

$$\begin{aligned} Q_9 &= 68^9 - 42^9 - 42^9 - 58^9 - 62^9 + 16^9 + 32^9 + 36^9 + 32^9 + 36^9 + 52^9 - 26^9 - 26^9 - 10^9 - 6^9 \\ &= 12{,}351{,}615{,}486{,}013{,}440 \end{aligned}$$

and

$$\begin{aligned} Q_{10} &= 68^{10} - 42^{10} - 42^{10} - 58^{10} - 62^{10} + 16^{10} + 32^{10} + 36^{10} + 32^{10} + 36^{10} + 52^{10} - 26^{10} - 26^{10} - 10^{10} - 6^{10} \\ &= 963{,}496{,}829{,}048{,}832{,}000 \end{aligned}$$

and

$$\begin{aligned} Q_{11} &= 68^{11} - 42^{11} - 42^{11} - 58^{11} - 62^{11} + 16^{11} + 32^{11} + 36^{11} + 32^{11} + 36^{11} + 52^{11} - 26^{11} - 26^{11} - 10^{11} - 6^{11} \\ &= 73{,}133{,}653{,}758{,}389{,}452{,}800 \end{aligned}$$

and

$$
\begin{aligned}
Q_{12} &= 68^{12} - 42^{12} - 42^{12} - 58^{12} - 62^{12} + 16^{12} + 32^{12} + 36^{12} + 32^{12} + 36^{12} + 52^{12} - 26^{12} - 26^{12} - 10^{12} - 6^{12} \\
&= 5{,}441{,}497{,}101{,}757{,}745{,}233{,}920
\end{aligned}
$$

Consequently,

$$
\begin{aligned}
Q &= Q_8 + \ Q_9 + Q_{10} + Q_{11} + Q_{12} \\
&= 5{,}515{,}606{,}756{,}242{,}928{,}650{,}240
\end{aligned}
$$

(user2192774 2014)

Question 12

a. Let F be the event that a person has COVID-19 and E be the event that this person tests positive for the disease. Consequently, we want to determine $p(F|E)$ and to do so, we use Bayes' Theorem. In order to determine $p(F|E)$ we need to find $p(E|F)$, $p(E|\bar{F})$, $p(F)$ and $p(\bar{F})$.

   We know that the chance of having COVID-19 is 0.0001, therefore, $p(F) = 0.0001$ and $p(\bar{F}) = 1 - 0.0001 = 0.9999$. Because someone who has COVID-19 tests positive 99% of the time, we know that $p(E|F) = 0.99$; this is probability of a true positive (i.e. that someone with COVID-19 tests positive). We also know that $p(\bar{E}|F) = 0.01$; this is the probability of a false negative (i.e. that someone with COVID-19 tests negative). Furthermore, because someone who does not have COVID-19 tests positive 0.2% of the time, we know that $p(E|\bar{F}) = 0.002$; this is the probability of a false positive (i.e. that someone without COVID-19 tests positive).

   The probability that someone who tests positive for COVID-19 actually has the disease is given by $p(F|E)$. By Bayes' Theorem, we know that

$$
\begin{aligned}
p(F|E) &= \frac{p(E|F)p(F)}{p(E|F)p(F) + p(E|\bar{F}\ )p(\bar{F})} \\
&= \frac{(0.99)(0.0001)}{(0.99)(0.0001) + (0.002)(0.9999)} \\
&\approx 0.05
\end{aligned}
$$

   This means that only 5% of people who test positive for COVID-19 actually have the disease. Because COVID-19 is rare, the number of false positives from the diagnostic test is greater than the number of true positives, making the percentage of people who test positive who actually have the disease quite small. People who test positive for COVID-19 should be relatively concerned that they actually have the disease.

(Rosen 2007)

b. The probability that someone who tests negative for COVID-19 has the disease is given by $p(F|\bar{E})$. By Bayes' Theorem, we know that

$$\begin{aligned} p(F|\bar{E}) &= \frac{p(E|F)p(F)}{1-\left(p(\bar{E}|\bar{F})p(\bar{F})+p(\bar{E}|F)p(F)\right)} \\ &= \frac{(0.99)(0.0001)}{1-\left((0.998)(0.9999)+(0.01)(0.0001)\right)} \\ &\approx 0.05 \end{aligned}$$

Consequently, 5% of people who test negative have COVID-19.
(Liu 2020d; Rosen 2007)

Question 13

a. The information can be summarised in a tabular format, where the ij entry gives the probability that i is received, given that j was sent:

	0 Received	1 Received
0 Sent	0.8	0.2
1 Sent	0.1	0.9

Let $P(x)$ be the probability that x is received, $s(x)$ be the fact that x was sent and $r(x)$ be the fact that r was received.

$$\begin{aligned} \therefore P(0) &= P\left(r(0) \wedge s(0)\right) + P\left(r(0) \wedge s(1)\right) \\ &= P\left(r(0) \mid s(0)\right)P\left(s(0)\right) + P\left(r(0) \mid s(1)\right)P\left(s(1)\right) \\ &= 0.8\left({}^{2}/_{3}\right) + 0.1\left({}^{1}/_{3}\right) \\ &= {}^{17}/_{30} \end{aligned}$$

(Department of Computer Science 2009)

b. i.e. Determine $P\left(r(0) \mid s(0)\right)$. By using Bayes' theorem, we know that

$$\begin{aligned} P\left(r(0) \mid s(0)\right) &= \frac{P\left(r(0) \mid s(0)\right)P\left(s(0)\right)}{P\left(r(0) \mid s(0)\right)P\left(s(0)\right) + P\left(r(0) \mid s(1)\right)P\left(s(1)\right)} \\ &= \frac{0.8\left({}^{2}/_{3}\right)}{{}^{17}/_{30}} \\ &= {}^{16}/_{17} \end{aligned}$$

(Department of Computer Science 2009)

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