COMP1006 Foundations of Computer Science
Final assessment, 2020
@ Bentley, Curtin University

Weighting:
This assessment contains 13 questions, for a total of 100 points, which weights for 50% of the final mark.

Submission:
You can only submit **a single PDF** file containing neatly typed answers or images. Name the file as `<studentID>_<name>_exam.pdf`. Use the signed `Declaration_of_originality.pdf` as the cover page of your submission.

Academic Integrity:
This is an **individual** assessment so that any form of collaboration is not permitted. This is an **open-book** assignment so that you are allowed to use external materials, but make sure you properly **cite the references**. It is your responsibility to understand Curtin's Academic Misconduct Rules, for example, post assessment questions online and ask for answers is considered as contract cheating and not permitted.

Logics and Proofs (20 marks)

1. (3 points) Show that $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$ are logically equivalent.

2. Express each of these English sentences using propositions, predicates, quantifiers, and logic operators, if necessary.
   (a) (2 points) There is a person who runs faster than anyone else.
   (b) (2 points) It is impossible for anyone to be loved by everyone.
   (c) (2 points) There is exactly one king.
   (d) (2 points) Everyone is either a friend of Alice or a friend of someone who is a friend of Alice.

3. (4 points) Prove that $|x| + |y| \geq |x + y|$ where x and y are real numbers (Note that $|x|$ is the absolute value of x, which equals x if $x \geq 0$ and equals $-x$ if $x < 0$).

4. (5 points) Use mathematical induction to prove for all integers $n \geq 1$ that

$$\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \cdots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

Sets and Relations (20 marks)

5. Let the set $A_i = \{-i, ..., -2, -1, 0, 1, 2, ..., i\}$. Find
   (a) (3 points) $\bigcap_{i=1}^{100} A_i$ (the intersection of sets $A_1, A_2, ..., A_{100}$)
   (b) (3 points) $\bigcup_{i=1}^{100} A_i$ (the union of sets $A_1, A_2, ..., A_{100}$)

6. Let R be the relation on the set $\{1, 2, 3, 4\}$ containing the ordered pairs $R = \{(1, 1), (1, 3), (2, 3), , (3, 4), (4, 2)\}$. Find
   (a) (3 points) R^2
   (b) (3 points) R^3

7. Let R be the relation on the set of all bit strings such that $(a, b) \in R$ if and only if a and b contain the same number of 0s.
   (a) (6 points) Show that R is an equivalence relation.
   (b) (2 points) What is the equivalence class of 10001?

Counting and Probabilities (20 marks)

8. Suppose there are 9 students and 6 staffs. We are going to select a group of 5 people to form a committee.
   (a) (3 points) How many ways are there if it consists of 3 students and 2 staffs?
   (b) (3 points) How many ways are there if it consists of 3 students, 2 staffs, and one of the staffs is the chair?
   (c) (3 points) How many ways are there if two particular students cannot be selected at the same time?
   (d) (3 points) What is the probability of the committee consisting of at least one student and one staff?

9. (8 points) Suppose we are building a Bayesian spam filters based on occurrence of the word "prize". Given the statistics of past emails, we have found that "prize" occurs in 150 of 1000 emails known to be spam and in 10 of 2000 emails known not to be spam. We also know that 10 out of 100 incoming emails are spam. Estimate the probability that an incoming email containing the word "prize" is spam.

Recurrence Relation (20 marks)

10. (a) (5 points) Find a recurrence relation for the number of ways to climb n stairs if the person can take one, two or three stairs at a time.
    (b) (3 points) What are the initial conditions?
    (c) (2 points) How many ways can this person climb a flight of 6 stairs?

11. Consider the number of bit strings of length n that contain the substring 10,

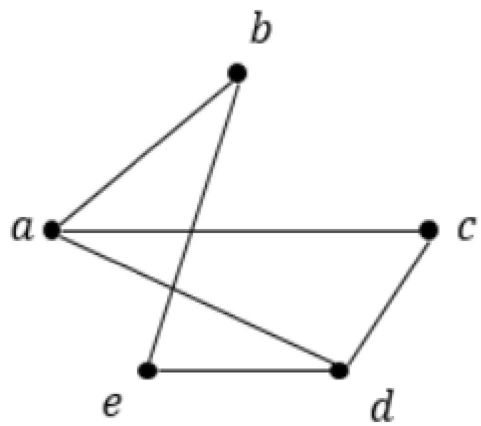
(a) (5 points) find a recurrence relation.

(b) (3 points) What are the initial conditions?

(c) (2 points) How many bit strings of length 6 contain the substring 10?

Graphs and Trees (20 marks)

12. Consider the following graph,



(a) (4 points) Determine if there is a Euler circuit or Euler path in the graph? If yes, list one example. Otherwise explain why not.

(b) (4 points) Determine if there is a Hamilton circuit or Hamilton path in the graph? If yes, list one example. Otherwise explain why not.

13. Consider the compound propositions $(p \vee (q \rightarrow r)) \vee (q \wedge (p \oplus r))$,

(a) (4 points) represent the expression using ordered rooted trees.

(b) (4 points) write the expression in prefix notation.

(c) (4 points) write the expression in postfix notation.