MATH1019 Assessed Online Assignment
Tanaka Chitete

Due date: 17:00:00 27 Nov 2020

Remark on notation.   Your answers will be accepted in *either* row vector syntax (which is the same as a list), i.e. in the form

$$[\, a, b, c\,]$$

or in **i**, **j**, **k** notation, e.g. $a\,\mathbf{i} + b\,\mathbf{j} + c\,\mathbf{k}$ is obtained by entering:

$$a * i + b * j + c * k$$

for real numbers a, b, c.
Please don't confuse the two notations. The *list* syntax has square brackets and commas, whereas the **i**, **j**, **k** notation has neither.
Note. You may prefer the **i**, **j**, **k** notation, since it permits the taking out of a common factor, e.g.

$$\frac{1}{\sqrt{102}}\,(\mathbf{i} + \mathbf{j} + 10\,\mathbf{k})$$

may be entered as:

$$(i + j + 10 * k)/\text{sqrt}(102)$$

(Note the *round* brackets here!)
The *list* syntax does *not* permit the taking out of a common factor!

Given a vector

$$\mathbf{u} = -2\,\mathbf{i} - 8\,\mathbf{j} + 8\,\mathbf{k}$$

1.1 (1 mark)
Find a unit vector having the same direction as **u**.
Answer:
You have not attempted this yet

1.2 (1 mark)
Find a unit vector oppositely directed to **u**.
Answer:
You have not attempted this yet

1.3 (1 mark)
Find a vector in the same direction as **u** but with 8 times the length of **u**.
Answer:
You have not attempted this yet

Remark on notation.   A vector may be represented as either a row or a column in square brackets, round brackets or angle brackets. Whether vectors are represented as rows or columns they have similar properties. The current textbook represents vectors as rows in angle brackets; Maple, can represent them as either rows or columns (but the syntax is longer for columns), but always uses *square* brackets (as for lists). For the purposes of the current quiz, vectors will be represented as

***rows** in **square** brackets with entries separated by commas*

(or, in later questions, in **i**, **j**, **k** notation.)

Given vectors

$$\mathbf{a} = [6, -3, 3], \quad \mathbf{b} = [8, 2, 6], \quad \mathbf{c} = [-2, 1, -5]$$

2.1 (1 mark)

3 **a** + **b** = [   ]
You have not attempted this yet

2.2 (1 mark)

3 **a** − 5 **b** = [   ]
You have not attempted this yet

2.3 (1 mark)

2 **a** − 4 (**b** + **c**) = [   ]
You have not attempted this yet

2.4 (1 mark)

|**b**| = [   ]
You have not attempted this yet

2.5 (1 mark)

|−5 **c**| = [   ]
You have not attempted this yet

Find **u** • **v** where

$$\mathbf{u} = 8\,\mathbf{i} + 7\,\mathbf{j} + \mathbf{k} \quad \text{and} \quad \mathbf{v} = -3\,\mathbf{i} - 8\,\mathbf{j} - 5\,\mathbf{k}$$

u • **v** = [   ]
You have not attempted this yet

Suppose

$$\mathbf{a} = 6\,\mathbf{i} - 2\,\mathbf{j} \quad \text{and} \quad \mathbf{b} = -\,\mathbf{i} + 5\,\mathbf{j}$$

4.1 (1 mark)

a • **b** = [   ]
You have not attempted this yet

4.2 (1 mark)
Also give the angle between the vectors in degrees to one decimal place.
Answer: [   ] °

You have not attempted this yet

Find the scalar and vector projections of **b** onto **a**, where

$$\mathbf{b} = -6\,\mathbf{i} - 5\,\mathbf{j} - 8\,\mathbf{k} \quad \text{and} \quad \mathbf{a} = 8\,\mathbf{i} + 4\,\mathbf{j} - 3\,\mathbf{k}$$

5.1 (1 mark)

(scalar projection) comp$_\mathbf{a}\mathbf{b}$ = [   ]
You have not attempted this yet

5.2 (1 mark)

(vector projection) proj$_\mathbf{a}\mathbf{b}$ = [   ]
You have not attempted this yet

Suppose

$$\mathbf{a} = -26\,\mathbf{i} - 3\,\mathbf{j} \quad \text{and} \quad \mathbf{b} = k\,\mathbf{i} + \mathbf{j}$$

6.1 (1 mark)
Find the *exact* value of k such that **a** and **b** are parallel.
Answer: [   ]
You have not attempted this yet

6.2 (1 mark)
Find the *exact* value of k such that **a** and **b** are perpendicular.
Answer: [   ]
You have not attempted this yet

Find a unit vector **u** that is orthogonal to **a** and **b** where

$$\mathbf{a} = 8\,\mathbf{i} + 5\,\mathbf{j} - 7\,\mathbf{k} \quad \text{and} \quad \mathbf{b} = -2\,\mathbf{i} + 4\,\mathbf{j} + 2\,\mathbf{k}$$

u = [   ]
You have not attempted this yet

Find the area of the triangle whose vertices are A(2,3,1), B(2,4,–4) and C(5,1,–1).
Area = [   ]
You have not attempted this yet

Determine the direction cosines and direction angles of the vector

$$\mathbf{u} = -8\,\mathbf{i} - 7\,\mathbf{j} - 6\,\mathbf{k}$$

9.1 (1 mark)

Enter the direction cosines [as a list] in square brackets in x, y, z order, leaving them in the *exact* form.

direction cosines = [_____]

You have not attempted this yet

9.2 (1 mark)

Estimate the direction angles to the nearest degree and enter them [as a list] in square brackets in x, y, z order.

direction angles (°) ≈ [_____]

You have not attempted this yet

Find the vector equation of the line through the point (4,–6,–8) parallel to the vector [2,6,9].

You may use `r` to abbreviate the general position vector [x, y, z]. Both the row vector notation or **i**, **j**, **k** notation of vectors (used in previous questions) may be used. Any single lowercase letter except `i`, `j`, `k`, `r`, `x`, `y` or `z` may be used as a parametric variable, e.g. `t` is a nice choice.

Answer: [_____]

You have not attempted this yet

Determine whether the lines L_1, L_2 are parallel, skew or intersecting, where

$$L_1: x = -1-8\,t, \quad y = -4\,t, \quad z = 1+3\,t$$
$$L_2: x = -1-6\,s, \quad y = 4\,s, \quad z = 1+5\,s$$

Enter `p` (for *parallel*) or `s` (for *skew*) or the point of intersection as a list of coordinates [in square brackets], i.e. use row vector notation, if the lines intersect.

Answer: [_____]

You have not attempted this yet

Find an equation of the plane which passes through the point (–9,9,7) that is parallel to the plane $-2\,x+9\,y-7\,z = -9$.

Leave your answer in the form $A\,x + B\,y + C\,z = D$, where A, B, C are integers.

Answer: [_____]

You have not attempted this yet

Find an equation of the plane that passes through the point (–9,7,5) and contains the line

$$x = -4-5\,t, \quad y = -2\,t, \quad z = 2+7\,t$$

Answer: [_____]

You have not attempted this yet

Given the point P(2, 0, –4) and the plane with equation

$$3 \; x+4 \; y+z=4$$

14.1 (1 mark)

enter the coordinates of a point Q that lies on the plane, below.

Q = ([____] [____] [____])

You have not attempted this yet

14.2 (1 mark)

Write down a vector **u** that is parallel to a normal vector to the plane. Please enter a row vector of form [A, B, C], i.e. use the *list* syntax.

u = [_____]

You have not attempted this yet

14.3 (1 mark)

Find the exact value of the shortest distance d from the point P to the plane.

d = [_____]

You have not attempted this yet

Consider the following matrices:

$$A = \begin{bmatrix} 1 & 2 & 4 \\ -9 & 15 & -6 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -7 & -10 \\ -5 & 14 & -3 \end{bmatrix}, \quad C = \begin{bmatrix} -10 & 9 \\ 11 & -10 \end{bmatrix}$$

$$D = \begin{bmatrix} -9 & -4 \\ 3 & -1 \end{bmatrix}, \quad E = \begin{bmatrix} 15 & -3 \\ 1 & 4 \\ -14 & 7 \end{bmatrix}$$

From the following statements select those that are true.

A ☐ The matrix product B C is well defined.

B ☐ The matrix product E D is not well defined.

C ☐ The matrix product A E is well defined and is of order 2 × 2.

D ☐ 3 C − 8 D is well defined and is of order 2 × 2.

E ☐ A + B is well defined and is of order 2 × 3.

F ☐ B + C is well defined.

☐ None of the above

☑ I do not want to answer this yet

You have not attempted this yet

Warning: Penalties are higher for multiple choice/response. Also, note that you get just one hint at a time. Don't guess!

If

$$A = \begin{bmatrix} -3 & 1 & -1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 5 \\ 7 \\ -9 \end{bmatrix}$$

then BA is

A ○ undefined

B ○
$$\begin{bmatrix} -15 \ 8 \ -10 \end{bmatrix}$$

C ○
$$\begin{bmatrix} 2 \\ 8 \\ -10 \end{bmatrix}$$

D ○ 1

E ○
$$\begin{bmatrix} -15 & 5 & -5 \\ -21 & 7 & -7 \\ 27 & -9 & 9 \end{bmatrix}$$

F ○ None of the above

◉ I do not want to answer this yet

You have not attempted this yet

Find the Reduced Row Echelon form of the following matrix:

$$\begin{bmatrix} 1 & -2 & -2 & -2 & -10 & -23 \\ 3 & -5 & -3 & -3 & -19 & -43 \\ -3 & 4 & 1 & 3 & 11 & 37 \\ -4 & 6 & 5 & 12 & 27 & 106 \end{bmatrix}$$

and fill its entries in the spaces provided below.

Answer:

You have not attempted this yet

What is the rank of the following matrix?

$$\begin{bmatrix} -9 & -18 & 7 \\ 5 & 10 & 9 \end{bmatrix}$$

$$\begin{bmatrix} -8 & -16 & -3 \end{bmatrix}$$

Answer: [   ]
You have not attempted this yet

How many solutions do the following equations have?

$$6x - 4y + 4z = 24$$
$$-9x + 6y - 6z = -36$$
$$6x - 4y + 4z = 24$$

A ○ none

B ○ precisely one

C ○ precisely two

D ○ infinitely many

⦿ I do not want to answer this yet

You have not attempted this yet

The following equations have infinitely many solutions.

$$9z = -18$$
$$3x + 9y - 6z = 12$$
$$2x + 6y + 8z = -16$$

Give the right hand side of the vector form of the general solution, using a parameter such as s or t. (Any lowercase letter will do as a parameter, so long as it is not x, y or z.)

For example, for the equations

$$x = y + 1, y = z + 1, z = x - 2$$

one correct answer is

$$[x, y, z] = [0, -1, -2] + t\,[1, 1, 1]$$

and you would enter the right hand side of this equation in the space provided:

$$[0, -1, -2] + t * [1, 1, 1]$$

or, if you prefer:

$$[t, t - 1, t - 2]$$

$[x, y, z] =$ [   ]
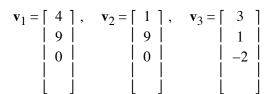You have not attempted this yet

Find coefficients c_1, c_2, c_3 for which

$$\mathbf{u} = \begin{bmatrix} 8 \\ 19 \\ -2 \end{bmatrix}$$

is a linear combination $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3$ of the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 4 \\ 9 \\ 0 \\ \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 9 \\ 0 \\ \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 3 \\ 1 \\ -2 \\ \end{bmatrix}$$

Enter your answer as a list in the form: [c_1, c_2, c_3]

Answer: [_____]

You have not attempted this yet

Let

$$A = \begin{bmatrix} -4 & 0 \\ -9 & 9 \end{bmatrix}$$

Find the value of the determinant of A. Does this matrix have an inverse?

Please enter your answer as a list of the form: [d, i]
where d is the exact value of the determinant of A, and i is either y (for `yes', A has an inverse) or n (for `no', A doesn't have an inverse).

Answer: [_____]

You have not attempted this yet

Find the determinant of the following matrix:

$$\begin{bmatrix} 0 & -4 & -3 \\ -4 & -4 & -1 \\ 3 & -3 & 3 \end{bmatrix}$$

Answer: [_____]

You have not attempted this yet

Let

$$P = \begin{bmatrix} 5 & -4 \\ 0 & 2 \end{bmatrix}$$

Determine the inverse of P, and enter your matrix entries in the cells below.

Answer: [_____] [_____]
[_____] [_____]

You have not attempted this yet

Find the inverse of the following matrix:

$$\begin{bmatrix} -27 & -6 & 9 \\ -6 & -3 & 3 \\ 12 & 0 & -3 \\ & & \end{bmatrix}$$

Answer:

You have not attempted this yet

Determine which of the following sets of vectors in $\mathbf{R}^3$ are linearly dependent.

A ☐ { [0,5,–3],   [–7,6,7],   [–21,3,30] }

B ☐ { [–3,2,5],   [5,–6,–3],   [–6,0,–1],   [3,–1,4] }

C ☐ { [a,b,c],   [u,v,w],   [–5 u–2 a,–5 v–2 b,–5 w–2 c] }

D ☐ { [0,–2,–6],   [–7,6,4],   [–7,0,4] }

E ☐ { [3,4,5],   [–6,12,15] }

☐ None of the above

☑ I do not want to answer this yet

You have not attempted this yet

Given the system of equations $A\mathbf{v} = \mathbf{b}$, where

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ -1 & 1 \\ 1 & 2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}$$

find the optimal solution, in the least squares sense.

Your answer here should be a column vector. A column vector

$$\begin{bmatrix} a \\ b \\ \dots \end{bmatrix}$$

is obtained by entering

$$\texttt{Vector([}a, b, \dots \texttt{])}$$

which is [$a, b, \dots$] (the usual *list* syntax), enclosed by `Vector( )`.

Your answer should be *exact;* i.e. do *not* use decimals.

Answer:

You have not attempted this yet

Mark summary:

Question	Value	Your mark

1.1	1.00	-
1.2	1.00	-
1.3	1.00	-
2.1	1.00	-
2.2	1.00	-
2.3	1.00	-
2.4	1.00	-
2.5	1.00	-
3	1.00	-
4.1	1.00	-
4.2	1.00	-
5.1	1.00	-
5.2	1.00	-
6.1	1.00	-
6.2	1.00	-
7	1.00	-
8	1.00	-
9.1	1.00	-
9.2	1.00	-
10	1.00	-
11	1.00	-
12	1.00	-
13	1.00	-
14.1	1.00	-
14.2	1.00	-
14.3	1.00	-
15	1.00	-
16	1.00	-
17	1.00	-
18	1.00	-
19	1.00	-
20	1.00	-
21	1.00	-
22	1.00	-
23	1.00	-
24	1.00	-
25	1.00	-
26	1.00	-
27	2.00	-
Total	40.00	0.00

Validate — Click here to check that your answers have no syntax errors and that Maple interprets them in the way that you intended.

Mark — Click here to mark your answers.

If you have technical problems, you can send email to the [administrator](#).
Mathematical questions can be sent to the [teacher](#).