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Housekeeping

- Emergencies
- Consultation Hours
- Unit Outline
- Attendance (Lectures, Tutorials)
- Class Rules (Punctuality, Mobile Phones, *etc*).

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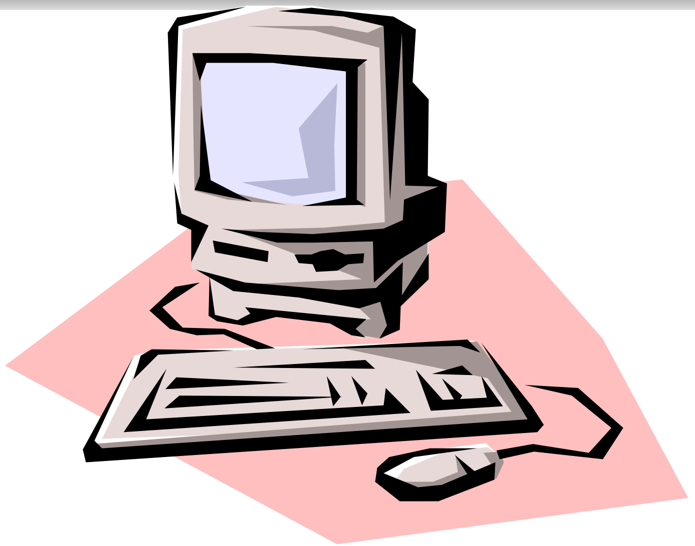
THEORY OF COMPUTATION



Module 1: Automata and Languages

Lecture 1: Deterministic Finite Automata

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Readings

- Sisper – Sections 0 & 1.1

  OR

- Savage – 1.1 to 1.3, 4.1

  (the rest of Ch 1 is also useful)

- Simulation – jFAST

  (http://sourceforge.net/projects/jfast-fsm-sim/)

- Simulation - JFLAP

  (http://www.jflap.org/)

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Topics

- Deterministic Finite Automata (DFAs)
- Computation with DFAs
- Designing DFAs
- Regular Language

Unit Learning Outcomes

Outcome 1

Synthesize Finite Automata, Push-Down Automata, Context Free Grammars, and Turing Machines with specific properties, and relate one form to another.

Assessment Criteria

- You need to be able to:
  - ➢ **Model** a specification as a Deterministic Finite Automata (DFA).
  - ➢ **Explain** the operation of a machine on an input string.
  - ➢ **Classify** a problem as belonging to a class able to be modelled by a DFA.

BACKGROUND

Simple model of computing

Example

Representations

What is a computer ?

- The Theory of Computing begins with the question above.
  - ➢ Real computers are too complex for direct mathematical representation

- Use of a computational model
  - ➢ An idealized computer
    - Accurate in some ways but not in others
    - Use different models depending on the features we want to focus on
  - ➢ One simple model: a Finite Automaton (FA)
    - Note that Savage also discusses an even simpler model – basic electrical circuits.

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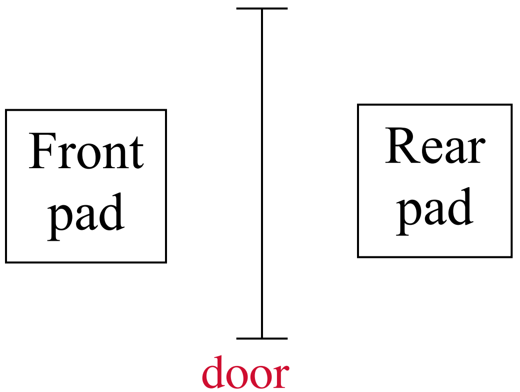
Finite Automata

- Finite Automata concept based on
  - ➢ Models of computers with an extremely limited amount of memory
    - What can such a computer do?
      - Many useful things!
    - We interact with such computers all the time
      - The heart of many electromechanical devices
    - An example:
      - Controller of an automatic door

Example: Automatic door control

- Front pad to detect the presence of a person about to walk through
- Rear pad to hold the door long enough for the person to pass all the way through
- **States** of controller and door:
  - ➢ OPEN or CLOSED
- Four possible **input conditions**:
  - ➢ FRONT (person standing in front), REAR, BOTH, or NEITHER

- Top view of an automatic door:



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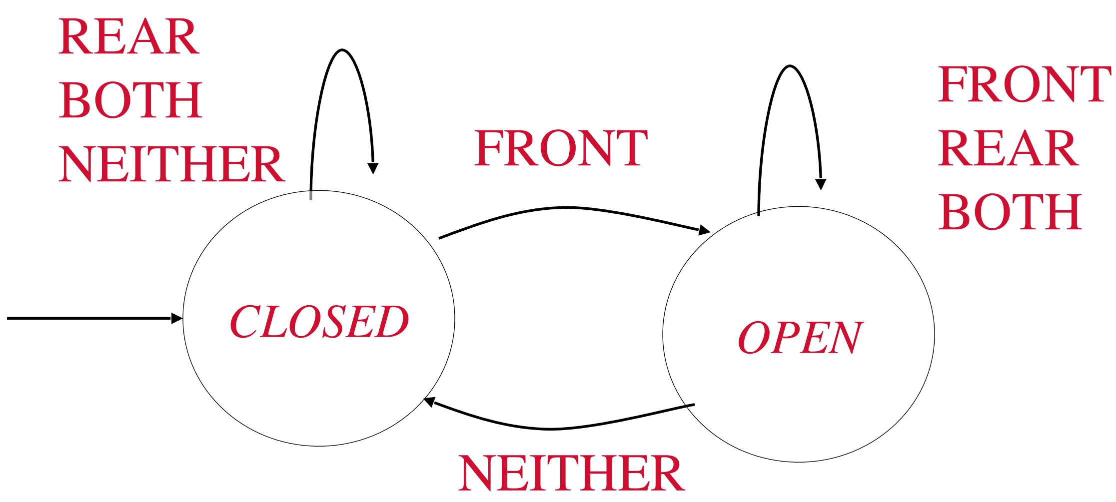
State Diagram



- State diagram for automatic door controller
- Each **state** is represented by a circle. The **initial state** has an arrow entering on the left.
- Each state has an arrow leading from it for each possible **input condition**.

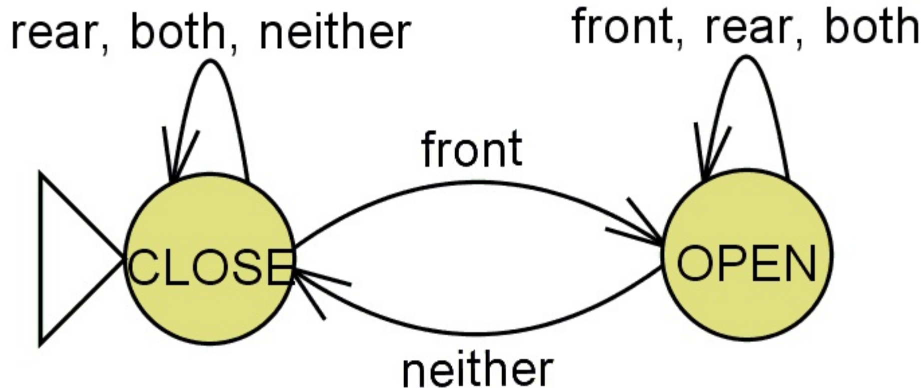
State transition table

- Input conditions and state transitions for automatic door controller can also be shown in a table.
- Note that there are a **finite** number of **states**, and movement between them is fully **deterministic**.

	Input signal			
State	**NEITHER**	**FRONT**	**REAR**	**BOTH**
CLOSED	CLOSED	OPEN	CLOSED	CLOSED
OPEN	CLOSED	OPEN	OPEN	OPEN

Simulation

- This door controller can be built using the JFLAP simulator.



- The .xml file containing the controlled is available on Blackboard. Feel free to experiment with it.

- JFLAP Note – This simulator doesn't like input conditions of more than one character. To run a simulation use jFAST instead.

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Finite Automaton As Controller

- Provides a standard way of representation
  - ➢ A computer with just a single bit of memory
  - ➢ Records which of the two states the controller is in

- Controllers for other common devices
  - ➢ Have slightly larger memories
  - ➢ *e.g.*, elevator controller:
    - states represent floors
    - inputs are signals from buttons
    - several bits to keep track of the information

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Other Applications

- Markov chains
  - ➢ Probabilistic counterpart of finite automata

- Finite Automata & Markov Chains useful for recognizing patterns in data
  - ➢ speech processing and optical character recognition
  - ➢ Markov Chains used for predicting price changes in financial markets

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FORMAL DEFINITION OF A DFA

Formal Definition

Transition Function

Example

Formal Definition: Concept

- Need for formal definition
  - ➢ It is precise: resolves any uncertainties about what is allowed in a finite automaton
  - ➢ Provides notation: good notation helps to think and express thoughts clearly
- A finite automaton consists of
  - ➢ A set of states
  - ➢ An input alphabet (the allowed input symbols)
  - ➢ Rules for going from one state to another, based on the input symbol
  - ➢ A start state and a set of accept/final states

Transition Function

- Denoted by δ (delta)
  - ➢ Defines the rules for moving from one state to another
  - ➢ An arrow from state x to y with input symbol 1, indicated by $\delta(x, 1)=y$

- A mathematical shorthand
- Deterministic computation: δ is a function
  - ➢ When a machine in a given state reads the next input symbol, the next state is unique.

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Definition of Finite Automaton

- A Finite Automaton is a 5-tuple (Q, Σ, δ, q_0, F), where:
  - ➢ Q is a finite set called the *states*.
  - ➢ Σ Is a finite set called the *alphabet*.
  - ➢ $\delta : Q\times\Sigma\rightarrow Q$ is the *transition function*.
  - ➢ $q_0 \in Q$ is the *start state*, and
  - ➢ $F \subseteq Q$ is the *set of accept states*.

- This precisely describes what is meant by a finite automaton.
- Precision at this level is important for mathematical proof.

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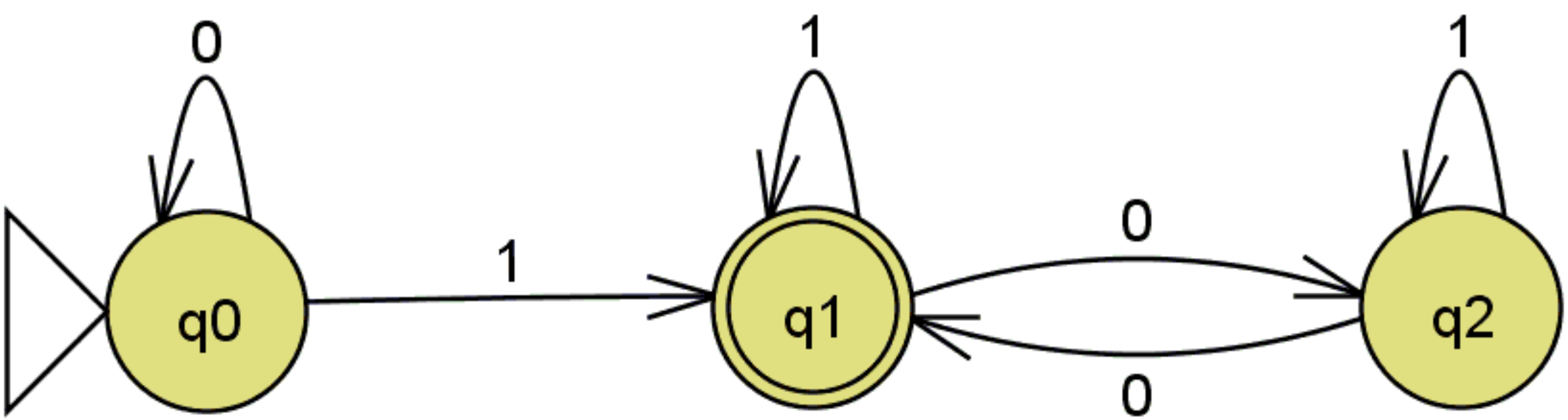
Example Machine M

- To fully describe a machine, specify each of the five elements in the definition.



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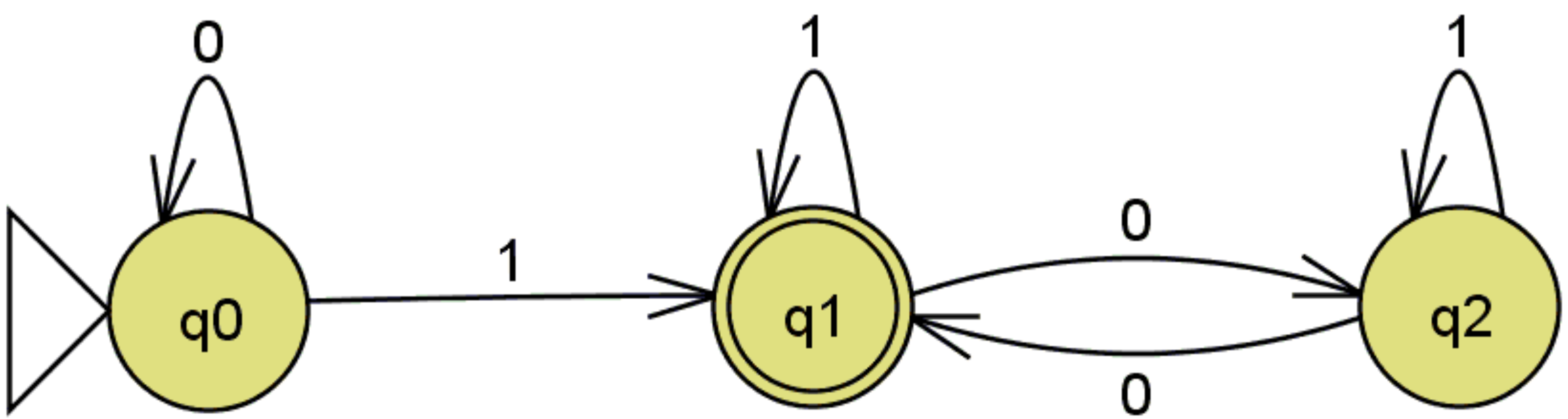
State Diagram of M_1

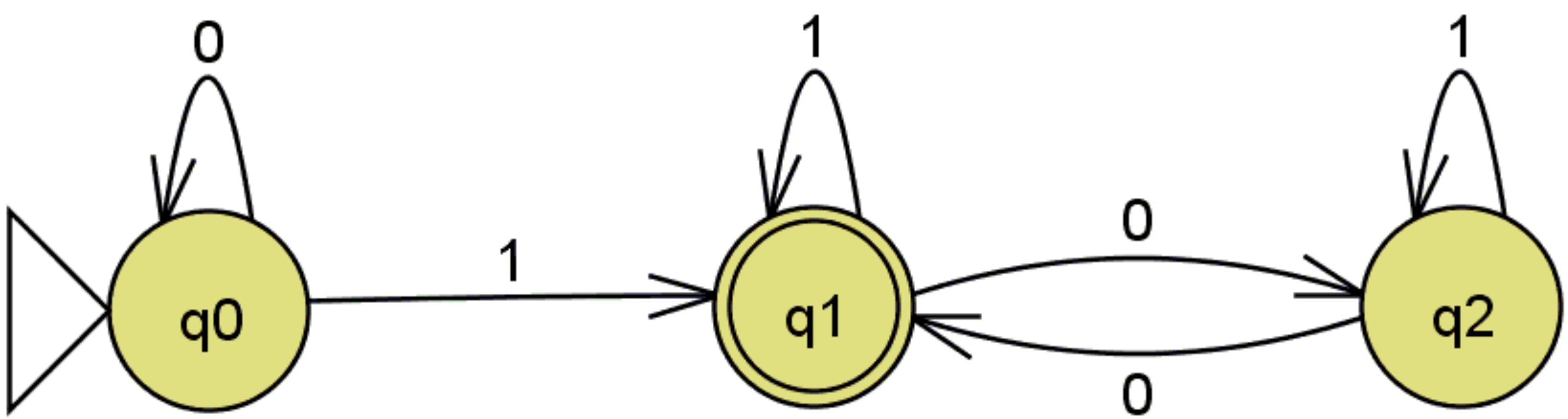
- A labeled directed graph
- 3 states: q_0, q_1, q_2
  - ➢ Start state q_0, accept state q_1
- Transitions
  - ➢ Arrows from one state to another



Specifying M

- M= (Q, Σ, δ, q_0, F), where
  - $Q=\{q_0,q_1,q_2\}$
  - $\Sigma = \{0,1\}$
  - $q_0=q_1$
  - $F=\{q_2\}$

δ	0	1
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_1	q_1



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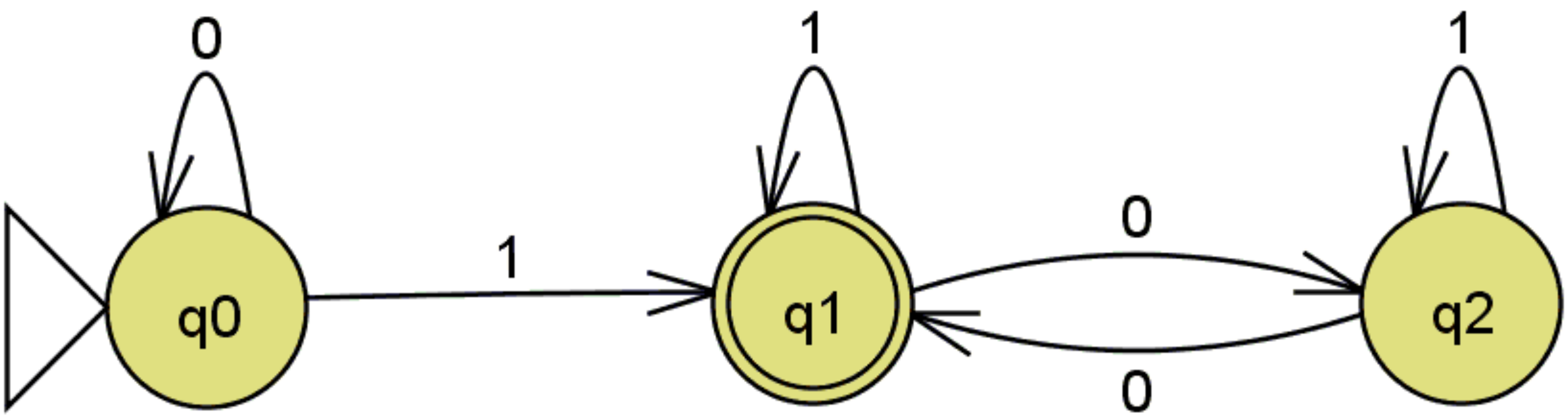
Language of M

- *Let A* = {*w* | *w* contains at least one 1 and an even number of 0s after the first 1}
- We say *L*(*M*) = *A*, or *M* recognizes *A*.
- Note that the machine we've been looking at does exactly this.
- What level of task is this?



COMPUTATION WITH FINITE AUTOMATA

Formal Definition

Inputs and Outputs

Example

Language of a Machine

- If A is the set of all strings that machine M accepts, then we say that A is the *language of machine M.*
  - Written as $L(M) = A$
  - *M recognizes A* or *M accepts A*
  - To avoid confusion of machines accepting strings and accepting languages, we use the term *recognize* for languages
  - A machine may accept several strings, but recognizes only one language.

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Definition of Computation

- Let $M = (Q, \Sigma, \delta, q_0, F)$ be a finite automaton and $w = w_1 w_2 \ldots w_n$ be a string over the alphabet Σ
- M <u>accepts</u> w if a sequence of states $r_0, r_1, \ldots, r_n$ exists in Q such that
  - ➢ $r_0 = q_0$,
  - ➢ $\delta(r_i, w_{i+1}) = r_{i+1}$ for $i = 0, \ldots, n\text{-}1$, and
  - ➢ $r_n \in F$.
- M <u>recognizes</u> language A if
  - ➢ $A = \{w \mid M \text{ accepts } w\}$

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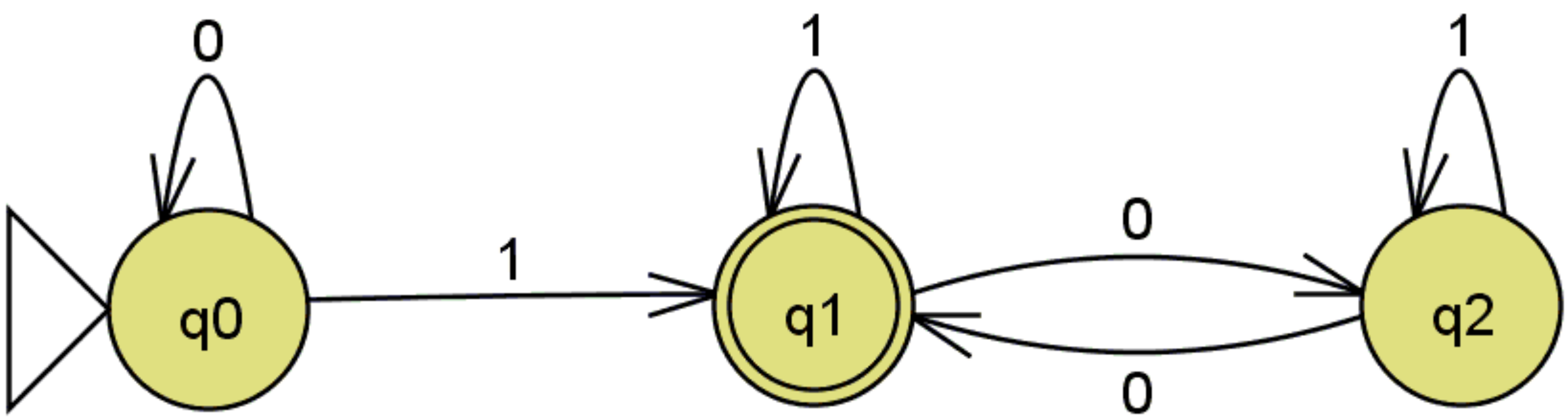
Inputs and Outputs



- Receives an input string (*e.g.*, 1101)
- Processes the string one symbol at a time
- Produces output after reading last symbol
  - ➢ *Accept* if M in accept state, otherwise *reject*

Strings Accepted by M

Check M on 11, 100, and 10100

1 1:	$q_0\ q_1\ q_1$	q_1 is an accept state
	1 1	11 accepted by M
100:	$q_0\ q_1\ q_2\ q_1$	q_1 is an accept state
	1 0 0	100 accepted by M
10100:	$q_0\ q_1\ q_2\ q_2\ q_1\ q_2$	q_2 is not an accept state
	1 0 1 0 0	10100 rejected by M

Strings Accepted by M

- We can see that these computation examples agree with our earlier interpretation of M.
- M_1 accepts any string in $\{w \mid w$ contains at least one 1 and an even number of 0s follow the first1$\}$
- Want to formalize this.

Language of a Machine

- If A is the set of all strings that machine M accepts, then we say that A is the *language of machine M.*
  - Written as $L(M) = A$
  - *M recognizes A* or *M accepts A*
  - To avoid confusion of machines accepting strings and accepting languages, we use the term *recognize* for languages
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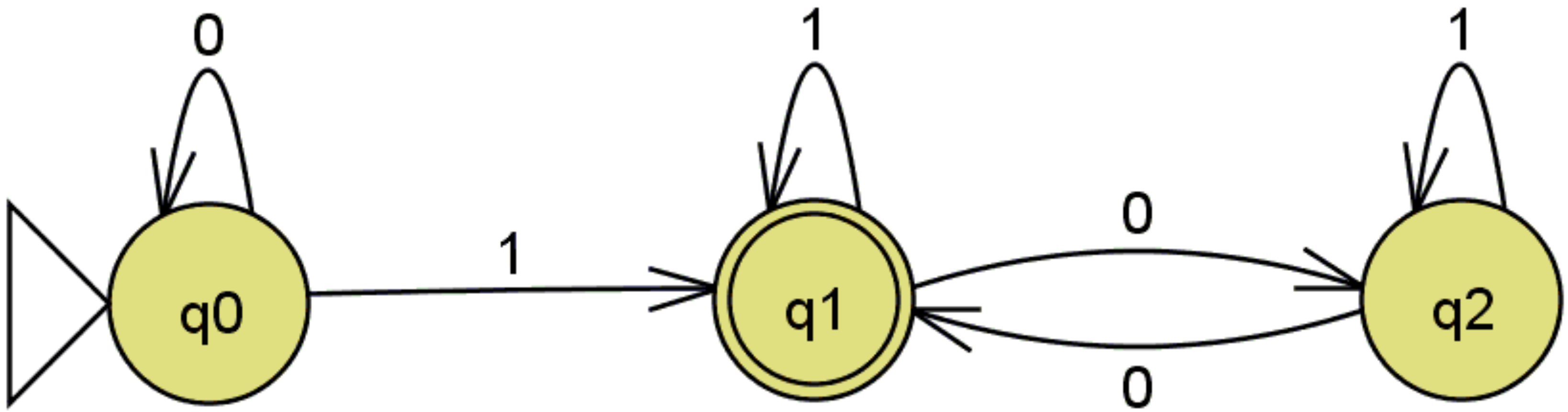
Language of M



- Let $A = \{w \mid w$ contains at least one 1 and an even number of 0s follow the first1$\}$
- L(M) = A, or M recognises A.
- This is a *regular* language. Discussed in Week 3.

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DESIGNING A FINITE AUTOMATA

Concept

Example

How to Design Finite Automata?

- Pretend to be the automaton
  - ➢ You receive an input string
  - ➢ Need to determine whether it is a member of the language the automaton is to recognize
  - ➢ See the symbols one at a time
  - ➢ After each symbol, decide whether the string seen so far is in the language
    - the string may end at any point, so must be ready with an answer
    - if the string seen so far is in the language, the current state should be an accept state, otherwise it should not be an accept state

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Designing Finite Automata

- What is to be remembered about the string you are reading ?
  - ➢ The Automaton has a finite set of states or a finite memory
  - ➢ For many languages, only need to remember the crucial information
  - ➢ What is crucial depends on the language being considered

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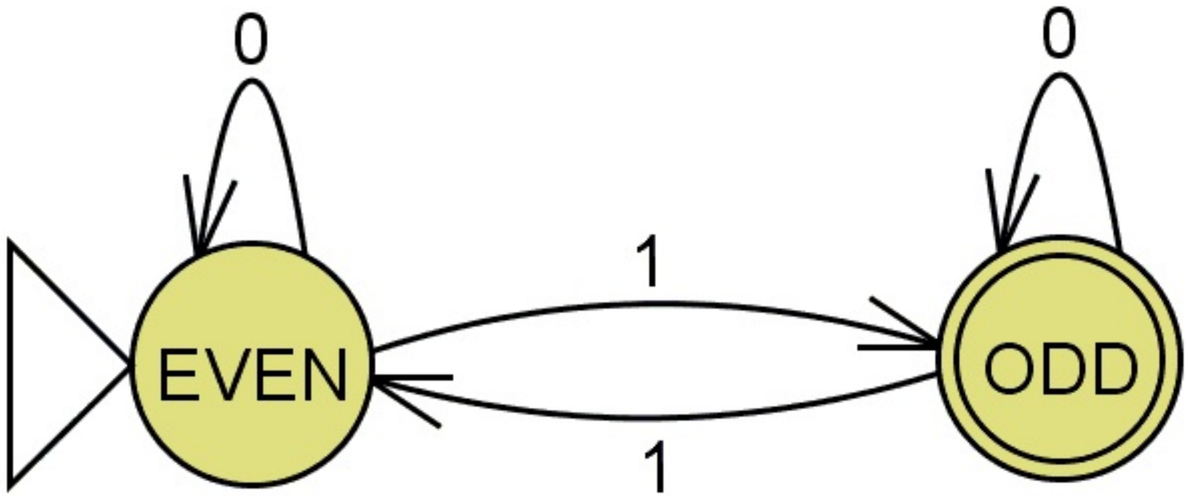
Design Example

- Alphabet $\Sigma = \{0,1\}$
- Language consists of all strings with an odd number of 1s
- Construct an automaton to recognize this language
- What is to be remembered ?
  - Whether the number of 1s seen so far is odd or even
  - If you read a 1 flip the answer, if you read a 0 leave the answer as is

Design Example

- A finite list of possibilities
  - ➢ Two states *even* and *odd*
- How to go from one possibility to another
  - ➢ Transitions for each input symbol
- Set state for 0 symbols seen so far (empty string ε)
  - ➢ start state is *even* (0 is even)
- Set accept states to possibilities where you want to accept the input string
  - ➢ *odd* (odd number of 1s are seen)



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Deterministic Finite Automata Task Level

- If a problem can be solved using a DFA, how hard is it to write a program?

- Answer – very easy.

- It can probably be handled by a simple script or module.
  - ➢ There should be no need to undergo the full software design process.
  - ➢ You may consider PERL, PYTHON or a shell script for the task.

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Deterministic Finite Automata Task Level

- You will probably never have to design a DFA in order to help you finish a task like this.

- However, designing such a DFA proves that you have correctly classified the task.

- It can be easy to be wrong.
  - Consider the problem "Accept any string that has the same number of 1s and 0s".
  - This problem looks similar to the problems we have seen so far.
  - This problem **can not** be solved with a DFA.

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Summary

- Finite automata
  - ➢ Definition Examples
  - ➢ Be able to state the formal definition
- Designing FA
  - ➢ <ULO> Generate State Diagram based on informal (English) specification
  - ➢ <ULO> Translate between Transition Function and definition.
- Computation with FA
  - ➢ <ULO> Explain the operation of a FA
- Regular language
  - ➢ Introduction to a theme
  - ➢ <ULO> Generate State Diagram based on formal specification

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