Curtin University

MATH1019 Linear Algebra and Statistics for Engineers

Mid-Semester Test, S2 2019; Time Allowed: **1** Hour + **5** minutes reading time

This paper contains 8 pages (including this cover sheet), 5 questions, worth a total of 45 marks

Write your answers in the spaces provided. Write your name and student number on this cover sheet. If pages become separated write your name on all separated sheets. A blank page is attached should you require additional space, however if you need more paper than this, please ask.

NAME: SOLUTIONS
STUDENT NUMBER:

Please circle your workshop tutor and corresponding workshop time:

Karo Fathollahzadeh: Monday 4–6pm | Thursday 8–10am | Thursday 12–2pm

Mikhail Dokuchaev: Tuesday 8–10am | Tuesday 2–4pm

Muhammad Kamran: Tuesday 10–12pm | Tuesday 2–4pm | Tuesday 4–6pm | Friday 8–10am | Friday 10–12pm

Shuang Li: Wednesday 2–4pm | Thursday 2–4pm | Thursday 4–6pm | Friday 4–6pm

Question 1.

(a) A set of five positive whole numbers: a, b, c, d, e has the following statistical measures: Mean = 31, Median = 33, Mode = 34, Range = 8. Use the given data to determine the values of a, b, c, d and e. (Hint: you may assume the data are in ascending order) (5 marks)

(b) The following data represent bone densities of ten individuals:

611	621	614	593	593	653	600	554	603	569

(i) Obtain the five-number summary for the above data. (4 marks)

(ii) Are there any outliers? Justify your answer. (3 marks)

(iii) Sketch a boxplot of the above data, indicating any outliers. (3 marks)

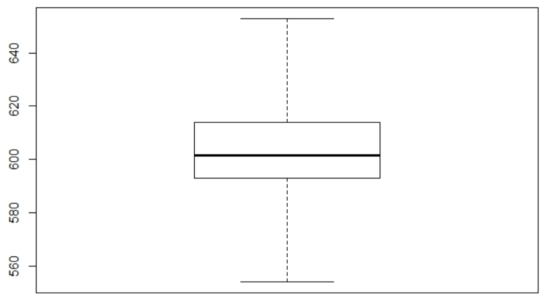
Solution

(a) Median $= 33 \Rightarrow c = 33$ **[0.5 mark]**
Range $= 8 \Rightarrow e - a = 8 \ldots (*)$ **[1 mark]**
Mean $= 31 \Rightarrow a + b + d + e = 31(5) - 33 = 122 \ldots (**)$ **[1 mark]**
Mode = 34, need at least two values of 34 $\Rightarrow d = e = 34$ **[1 mark]**
From $(*)$, $a = e - 8 = 34 - 8 = 26$ **[1 mark]**
Substituting this into $(**)$, we get $b = 122 - 26 - 2(34) = 28$ **[0.5 mark]**
So the numbers are: $26, 28, 33, 34, 34$

(b) (i) Ordered data set: $554, 569, 593, 593, 600, 603, 611, 614, 621, 653$
Min = 554 **[0.5 mark]**
$\frac{1}{4}(10+1) = 2\frac{3}{4} \Rightarrow Q_1 = 569 + \frac{3}{4}(593 - 569) = 587$ **[1 mark]**
Median $= Q_2 = \dfrac{600 + 603}{2} = 601\frac{1}{2}$ **[1 mark]**
$\frac{3}{4}(10+1) = 8\frac{1}{4} \Rightarrow Q_3 = 614 + \frac{1}{4}(621 - 614) = 615\frac{3}{4}$ **[1 mark]**
Max = 653 **[0.5 mark]**

(ii) $IQR = Q_3 - Q_1 = 28.75$, $Q_1 - 1.5 \cdot IQR = 543.875$, $Q_3 + 1.5 \cdot IQR = 658.875$ **[2 marks]**
Since there are no values below 543.875 and no values above 658.875, there are no outliers. **[1 mark]**

(iii) whiskers - **[1 mark]**; box - **[1 mark]**; scale - **[1 mark]**



Question 2. A process has been set up to manufacture polypropylene capacitors with a 25 micro-Farad capacitance. The process mean is 25.08 and the standard deviation is 0.98. The capacitors are to be marketed with a tolerance of $\pm 10\%$. Assume that capacitances are normally distributed.

(a) Calculate the proportion of production, in parts per million (ppm), that will lie outside the tolerance range. (4 marks)

(b) Suppose now that the process mean is 25.00. What does the standard deviation need to be reduced to for only 5000 ppm to be outside the tolerance interval? (4 marks)

Solution

(a) Tolerance range: (L, U), where $L = 25 - 0.1(25) = 22.5$ and $U = 25 + 0.1(25) = 27.5$. **[0.5 mark]**
Next we standardise L and U and workout the proportions
$\frac{22.5 - 25.08}{0.98} = -2.63265 \Rightarrow P(Z < -2.63) \approx 0.0043$ **[1.5 marks]**
$\frac{27.5 - 25.08}{0.98} = 2.46938 \Rightarrow P(Z > 2.47) = 1 - P(Z < 2.47) = 1 - 0.9932 \approx 0.0068$ **[1.5 marks]**
This gives us $(0.0043 + 0.0068) \times 10^6 = 11100$ppm. **[0.5 mark]**

(b) Now we have symmetry, so need only look at one tail end of the distribution. The required proportion is $(5000 \times 10^{-6})/2 = 0.0025$ **[1 mark]**
Now, $P(Z < \frac{22.5 - 25}{\sigma}) = 0.0025$, **[1 mark]**
i.e. $\frac{22.5 - 25}{\sigma} \approx -2.81$, from tables **[1 mark]**
Hence, $\sigma = \frac{22.5 - 25}{-2.81} = 0.8897$ **[1 mark]**

Question 3. In a particular game involving eight-sided dice, three fair eight-sided dice are rolled after the player has placed a bet on the occurrence of a particular face of the dice. For every \$1 bet that you place: you can lose the \$1 if none of the three dice shows the face; you can win \$1 if one die shows the face; you can win \$2 if two of the dice show the face; or you can win \$3 if three dice show the face.

(a) Form and identify the probability distribution function representing the different monetary values (winnings or losses) that are possible from one roll of the three dice. (4 marks)

(b) What is the player's expected long-run profit (or loss) from a \$1 bet? (2 marks)

Solution

(a) Let X be the number of dice showing the number the player has placed a bet on. Then

$P(X = x) = \binom{3}{x}\left(\frac{1}{8}\right)^{x}\left(\frac{7}{8}\right)^{3-x}$. **[2 marks]** This is the Binomial distribution. **[1 mark]**

win/loss	-\$1	\$1	\$2	\$3
x	0	1	2	3
$P(X = x)$	0.6699	0.2871	0.0410	0.0020

[1 mark]

(b) Expected profit $= (-1)(0.6699) + (1)(0.2871) + (2)(0.0410) + (3)(0.0020) = -\$0.29$ **[2 marks]**

Question 4. The friction between a vehicle's tyres and a bitumen road is due to the aggregate that is bound with the tar. A good crushed stone for use as aggregate will maintain frictional forces despite the polishing action of tyres. Samples of aggregate from a large road building project were sent to four independent laboratories for friction test readings (FTR). The FTR were:

$$62.15, 53.50, 55.00, 61.50$$

(a) Calculate a 95% confidence interval for the mean FTR μ of the notional population of all such aggregate samples. (6 marks)

(b) What assumptions were required in order for you to be able to calculate the confidence interval above? (2 marks)

(c) In general, what is the interpretation of a 95% confidence interval? (2 marks)

Solution

(a) $n = 4, df = 4 - 1 = 3, \alpha = 0.05$ **[1 mark]**
$\bar{x} = 58.0375$ to 4 dps., $s = 4.4241$ to 4 dps. **[2 marks]**
$t_{\alpha/2,n-1} = t_{0.025,3} = 3.182$, (from tables) **[1 mark]**
95% CI: $\bar{x} \pm t_{\alpha/2,n-1}\frac{s}{\sqrt{n}} = 58.0375 \pm 3.182\frac{4.4241}{\sqrt{4}} = (50.999, 65.076)$ (to 3dps.) **[2 marks]**

(b) (1) Data can be viewed as a simple random sample; (2) The population is approximately normally distributed (since the sample size is small). **[1 mark each]**

(c) If we had 100 samples taken from this population then approximately 95 of the corresponding confidence intervals would contain the population mean μ and 5 would not. Other answers are possible, such as we are 95% confident that the population mean μ in contained in our CI. *Note that CI makes no probabilistic statements about μ, so deduct a mark if the word probability is used to describe the CI.* **[1 mark for main idea; 1 mark for mentioning population mean]**

Question 5. An inductor is manufactured to a specified inductance of 470 microhenrys. A customer tests a sample of 20 inductors and finds the sample mean and standard deviation are 465.8 and 8.7, respectively. If we assume the sample is a simple random sample from production is there evidence that the required specification is not met? To answer this question set up and conduct a hypothesis test at the 5% level of significance, stating the hypotheses, test statistic, p-value or critical region, and your conclusion.

(6 marks)

Solution

$n = 20, d.f. = 20 - 1 = 19, \alpha = 0.05, \bar{x} = 465.8, s = 8.7$

Hypotheses:

$H_0 : \mu = 470$ **[1 mark]**
$H_A : \mu \neq 470$ **[1 mark]**

Test statistic

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{465.8 - 470}{8.7/\sqrt{20}} = -2.15896$$ **[2 marks]**

p-value:

p-value $= 2P(T < -2.15896) < 0.05 \Rightarrow$ Reject H_0 **[1 mark]**
Note that this value cannot be obtained exactly from the tables.
Alternatively, we could have used the critical region approach: $t < -t_{0.025,19} = -2.093$ or $t > t_{0.025,19} = 2.093$

Conclusion:

There is sufficient evidence in the data at the 5% significance level to reject the null hypothesis. Hence, the population mean is lower than the specified 470 microhenrys. **[1 mark]**

END OF TEST PAPER