Question 1 (25 marks)

(a) State two types of Fallacies in proof process, and illustrative them with concrete examples.

(5 marks)

(b) Represent the following statements in a mathematical logic.

(i) Some people are cheating.
(ii) There is only one student in this class such that none of his/her friends are also friends.
(iii) Everybody has a bad hobby.
(iv) There are only one student in this class who can achieve the highest mark in COMP2001
(v) Bob hates everyone who likes cats.

(10 marks)

(c) Prove by using Mathematical Induction, that if h>-1, then

$$1+nh \leq (1+h)^n$$

for all nonnegative n. This is the famous **Bernoulli** inequality.

(7 marks)

(d) Calculate the negation for following proposition.

$$\neg(\forall \varepsilon > 0, \exists \delta > 0\ (\neg(0 <| x-a |< \delta) \vee (| f(x)-f(a) |< \varepsilon)\ )\ ) = ?$$

(3 marks)

Question 2 (30 marks)

(a) For set $A_i=\{1,2,3,\ldots.i\}$ with i=1,2,3,..100. Compute

(i) $\bigcup_{i=1}^{100} A_i$
(ii) $\bigcap_{i=1}^{100} A_i$
(iii) $P(\{\{\phi\}\})$
(iv) $|P(A_{100})|$

(7 marks)

Questions continue in next page.

(b) Let A={a,b,c}. Give examples of relations which satisfy each of the following requirements for (i)-(iii) and then find a solution for (iv).

(i) The relation is symmetric and anti-symmetric;
(ii) The relation is reflexive, symmetric and transitive, but not anti-symmetric;
(iii) The relation is neither symmetric nor anti-symmetric, but is reflexive.
(iv) Find an equivalence relationship $\Re$ from $A \times A$ and compute $[b]_R$

(15 marks)

(c) Let A = {1, 2, 3, 4,}.

(i) Construct a function from A×A to A.
(ii) Is it possible to construct an onto function from A to A×A? Construct such a function if it exists. Give the reason if such a function does not exist.
(iii) Is it possible to construct an onto function from A×A to A? Construct such a function if it exists. Give the reason if such a function does not exist.

(8 marks)

Question 3 (20 marks)

(a) (i) Find a recurrence relation for the number of bit strings of length n that contain three consecutive zeroes.
(ii) What are the initial conditions for part (i)?
(iii) How many bit strings of length seven contain three consecutive zeroes for part (i)?

(10 marks)

(b) A class consists of 14 men and 12 women. Find the number of ways that the people in the class can arrange themselves in the following cases.

(i) How many groups can be chosen from this class which consists of 7 men and 10 women?

Questions continue in next page.

(ii) If two students have to be in the same group, how many groups of 10 students can be formed from this class?

(iii) If one male A and one female B cannot be in the same committee, how many ways can a committee consisting of 6 men and 4 women be chosen from the class?

(10 marks)

Question 4 (25 marks)

(a) (i) Give the definition of a tree.

(ii) Does there exist a tree with five vertices of the following degrees? Either draw a tree with the specific properties or justify why such tree does not exist.

- 3, 1, 2, 1, 1
- 2, 3, 2, 1, 2
- 1,4,1,3,1

(8 marks)

(b) The complete 3-partite graph $K_{n,m,p}$, with $n, m, p \geq 1$, is a simple graph that has its vertex set partitioned into 3 disjoint non-empty subsets of n, m and p vertices, respectively. Two vertices are adjacent if and only if they are in different subsets in the partition.

(i) Draw $K_{4,2,1}$.

(ii) Give the definition of Euler circuit.

(iii) For which values of n, m, p, does $K_{n,m,p}$ have an Euler Circuit ? Justify your answer.

(12 marks)

(c) Given a graph $G(V, E)$, give the definitions of Euler path and Hamilton path; Further, construct two illustrative examples for these two kind of paths using a graph with five vertices.

(5 marks)

END OF EXAMINATION PAPER