MATH1019 LINEAR ALGEBRA & STATISTICS for ENGINEERS

Workshop Assessment (Week 12)

STUDENT NAME: **STUDENT NUMBER:**

WORKSHOP DAY & TIME:

OVERALL MARK AWARDED (out of 30):

1. Find a vector in the opposite direction of $\boldsymbol{a} = 2\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k}$ but with 3 times the length of $\boldsymbol{b} = [-1, 2, 2]$. (4 marks)

$$-3\|\underset{\sim}{b}\|\hat{\underset{\sim}{a}} = -3\|\underset{\sim}{b}\|\frac{\underset{\sim}{a}}{\|\underset{\sim}{a}\|} \overset{1/2}{=} -3 \overset{1/2}{} \sqrt{(-1)^2+2^2+2^2} \overset{1/2}{} \frac{[2,-3,1]}{\sqrt{2^2+(-3)^2+1^2}} \ 1/2$$

$$= -3\sqrt{1+4+4} \ 1/2 \ \frac{[2,-3,1]}{\sqrt{4+9+1}} \ 1/2 = -3\sqrt{9}\frac{[2,-3,1]}{\sqrt{14}} = \left[\frac{-18}{\sqrt{14}}, \frac{27}{\sqrt{14}}, \frac{-9}{\sqrt{14}}\right] \ 1$$

2. Find the vector projection of $\boldsymbol{c} = [-2, 4, 1]$ onto $\boldsymbol{d} = [1, -1, 2]$. (3 marks)

Scalar proj,

$$p = \underset{\sim}{c}\cdot\hat{\underset{\sim}{d}} = \frac{\underset{\sim}{c}\cdot\underset{\sim}{d}}{\|\underset{\sim}{d}\|} = \frac{[-2,4,1]\cdot[1,-1,2]}{\sqrt{1^2+(-1)^2+2^2}} \ 1/2 = \frac{-2-4+2}{\sqrt{1+1+4}} \ 1/2 = \frac{-4}{\sqrt{6}} \ 1/2$$

Vector proj,

$$\underset{\sim}{p} = p\hat{\underset{\sim}{d}} = p\frac{\underset{\sim}{d}}{\|\underset{\sim}{d}\|} = \frac{-4}{\sqrt{6}} \ 1/2 \ \frac{[1,-1,2]}{\sqrt{6}} \ 1/2 = \left[-4/6, 4/6, -8/6\right] \ 1/2$$

3. Determine a non-zero vector perpendicular to $\boldsymbol{e} = \boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$ and $\boldsymbol{f} = 2\boldsymbol{j} + 4\boldsymbol{k}$. (3 marks)

$$\underset{\sim}{e}\times\underset{\sim}{f} \ 1/2 = \begin{matrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} & \underset{\sim}{i} & \underset{\sim}{j} \\ 1 & 1 & -3 & 1 & 1 \\ 0 & 2 & 4 & 0 & 2 \end{matrix}$$

$$= \underset{\sim}{i}(4 \overset{1/2}{-} (-6)) + \underset{\sim}{j}(0 \overset{1/2}{-} 4) + \underset{\sim}{k}(2 \overset{1/2}{-} 0)$$

$$= 10\underset{\sim}{i} - 4\underset{\sim}{j} + 2\underset{\sim}{k} \ 1$$

4. Given the following matrices

$$A = \begin{bmatrix} 3 & 1 \\ 5 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 4 & 0 \\ -1 & -2 \end{bmatrix} \quad C = \begin{bmatrix} -4 & 2 \\ -11 & 6 \end{bmatrix}$$

determine each of the following. If an operation is undefined, explain why.

(i) $2B$. (1 mark)

(ii) $A - 4I_2$. (3 marks)

(iii) C^T. (1 mark)

(iv) BA. (3 marks)

(v) C^{-1}. (3 marks)

(i). $2B = 2\begin{bmatrix} 1 & 2 \\ 4 & 0 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & 0 \\ -2 & -4 \end{bmatrix}$ 1 mark

(ii). $A - 4I_2 = \begin{bmatrix} 3 & 1 \\ 5 & -2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 1 mark $= \begin{bmatrix} 3 & 1 \\ 5 & -2 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ 1 $= \begin{bmatrix} -1 & 1 \\ 5 & -6 \end{bmatrix}$ 1

(iii). $C^T = \begin{bmatrix} -4 & -11 \\ 2 & 6 \end{bmatrix}$ 1

(iv). $BA = \begin{bmatrix} 1 & 2 \\ 4 & 0 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 5 & -2 \end{bmatrix} = \begin{bmatrix} 3+10 & 1-4 \\ 12+0 & 4+0 \\ -3-10 & -1+4 \end{bmatrix} = \begin{bmatrix} 13 & -3 \\ 12 & 4 \\ -13 & 3 \end{bmatrix}$ ½ mark for each collect entry

(v). $C^{-1} \Rightarrow [C \mid I] = \left[\begin{array}{cc|cc} -4 & 2 & 1 & 0 \\ -11 & 6 & 0 & 1 \end{array}\right]$ 1 $R_2 = 4R_2 - 11R_1$

$\sim \left[\begin{array}{cc|cc} -4 & 2 & 1 & 0 \\ 0 & 2 & -11 & 4 \end{array}\right]$ $R_1 = R_1 - R_2$ ½ $\sim \left[\begin{array}{cc|cc} -4 & 0 & 12 & -4 \\ 0 & 2 & -11 & 4 \end{array}\right]$ $R_1 \div -4$ ½ $R_2 \div 2$

$\sim \left[\begin{array}{cc|cc} 1 & 0 & -3 & 1 \\ 0 & 1 & -11/2 & 2 \end{array}\right]$ 1 $\therefore C^{-1} = \begin{bmatrix} -3 & 1 \\ -11/2 & 2 \end{bmatrix}$

5. Use the Gauss Jordan method to solve the following system of linear equations:

$$\begin{aligned} x_1 - 3x_2 + x_3 &= 2 \\ -3x_1 + x_2 + x_3 &= 6 \\ 4x_1 + x_2 - x_3 &= -10 \end{aligned}$$

(9 marks)

$$[A|\underset{\wedge}{b}] = \left[\begin{array}{ccc|c} 1 & -3 & 1 & 2 \\ -3 & 1 & 1 & 6 \\ 4 & 1 & -1 & -10 \end{array}\right] \begin{array}{l} R_2 = R_2 + 3R_1 \\ R_3 = R_3 - 4R_1 \end{array} \quad 1$$

$$\sim \left[\begin{array}{ccc|c} 1 & -3 & 1 & 2 \\ 0 & -8 & 4 & 12 \\ 0 & 13 & -5 & -18 \end{array}\right] R_3 = 8R_3 + 13R_2 \quad 1 \quad \sim \left[\begin{array}{ccc|c} 1 & -3 & 1 & 2 \\ 0 & -8 & 4 & 12 \\ 0 & 0 & 12 & 12 \end{array}\right] R_3 \div 12 \quad 1$$

$$\sim \left[\begin{array}{ccc|c} 1 & -3 & 1 & 2 \\ 0 & -8 & 4 & 12 \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_1 = R_1 - R_3 \\ R_2 = R_2 - 4R_3 \end{array} \quad 1 \quad \sim \left[\begin{array}{ccc|c} 1 & -3 & 0 & -1 \\ 0 & -8 & 0 & 8 \\ 0 & 0 & 1 & 1 \end{array}\right] R_2 \div -8 \quad 1$$

$$\sim \left[\begin{array}{ccc|c} 1 & -3 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right] R_1 = R_1 + 3R_2 \quad 1 \quad \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad 1$$

$r(A) = 3 = r([A|\underset{\wedge}{b}]) = n = 3$ $\therefore$ Unique solution $\frac{1}{2}$

Row 3: $x_3 = 1$ $\frac{1}{2}$

Row 2: $x_2 = -1$ $\frac{1}{2}$

Row 1: $x_1 = -2$ $\frac{1}{2}$