CURTIN UNIVERSITY
Faculty of Science and Engineering

Final Assessment

End of Semester 1, 2020

MATH1019 Linear Algebra and Statistics for Engineers

This is an OPEN BOOK assessment

To obtain full marks for a question you must **clearly** show appropriate working.

TIME ALLOWED: 4 hours (+ an additional 30 mins for submission)

TOTAL MARKS: 100

INSTRUCTIONS TO STUDENTS:

1. Attempt as many questions or part questions as possible.
2. SHOW ALL WORKING OUT.
3. Your submission should be a single pdf file which is a scan of your handwritten work.
4. Name your submission/solution pdf file as MATH1019_Examination_[yourStudentID].pdf (i.e. MATH1019_Examination_20145327.pdf).

Student declaration: At the top of the first page of your submission you must write the following statement:

"*I declare that this assessment item is my own unassisted work, and it has not been submitted in any form for assessment or academic credit elsewhere.*"

I certify that I have read and understood Curtin University policies on Academic Misconduct and declare that this assessment item complies with these policies.

I certify that I will/have adhered to the time duration limit prescribed for the completion of this assessment item.

I recognise that should this declaration be found to be false, disciplinary action could be taken and penalties imposed in accordance with Curtin University policy."

Write your Name, Student ID Number, Signature and Date below this statement.

Question 1

(a) Find the distance between the points $A(-1,1,2,4)$ and $B(2,0,-3,3)$. (2 marks)

(b) Find the scalar product of $\boldsymbol{a} = [2,-1,4]$ and $\boldsymbol{b} = [3,5,-1]$. (2 marks)

(c) Find the cosine of the angle between $\boldsymbol{c} = [3,1,1,-1]$ and $\boldsymbol{d} = [2,0,-4,1]$. (5 marks)

(d) Find the direction cosines of $\boldsymbol{e} = [3,2,-1]$. (2 marks)

(e) Find a unit vector perpendicular to $\boldsymbol{f} = [3,2,0]$ and $\boldsymbol{g} = [-1,1,1]$. (4 marks)

(f) Given $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are vectors in 3 space, also A and B are points in 3 space, determine whether the following expressions result in either: a scalar, a vector, or the expression is meaningless (i.e. it is not possible). If the expression is meaningless explain why the expression cannot be determined.

(i) $\overrightarrow{AB} - \overrightarrow{AB}$ (1 mark)

(ii) $\|\overrightarrow{AB}\|\boldsymbol{b}$ (1 mark)

(iii) $\dfrac{\boldsymbol{a}}{\|\boldsymbol{b}\times\boldsymbol{c}\|}$ (1 mark)

(iv) $\boldsymbol{a}.\boldsymbol{b} - \boldsymbol{b}.\boldsymbol{c}$ (1 mark)

(v) $\boldsymbol{a} \times \boldsymbol{b} + \|\boldsymbol{c}\|$ (1 mark)

Question 2 is on the next page…

Question 2

(a) Given the line in parametric form,

$$x = -2 + t$$
$$y = -1 - 2t$$
$$z = 3 + 3t$$

Determine whether the following points are on the line or not on the line.

(i) $(0, -4, 9)$ (1 mark)

(ii) $(-3, 1, 0)$ (1 mark)

(b) Given the plane,

$$(x + 2) + 4(y - 2) - 2z = 0$$

Determine whether the following points are on the plane or not on the plane.

(i) $(2, 1, 0)$ (1 mark)

(ii) $(-3, 0, -4)$ (1 mark)

(c) Find the vector equation of the line passing through the points $(-4, 0, 2)$ and $(-1, 3, 5)$. (3 marks)

(d) Find a pair of planes (i.e. two planes) whose intersection is the line,

$$\frac{x + 3}{-2} = \frac{y + 1}{3} = \frac{z - 5}{2}$$

(4 marks)

(e) Find the equation of the plane that is parallel to the plane $x + 4y - z = 6$ and which also contains a point from the plane $-2x - y + 3z = -6$. (4 marks)

(f) Show that the following parametric equations define the same line,

$$\begin{aligned} x &= -4 + 2t \\ y &= 5 - t \\ z &= 6 - 3t \end{aligned} \qquad \text{and} \qquad \begin{aligned} x &= -10 - 4t \\ y &= 8 + 2t \\ z &= 15 + 6t \end{aligned}$$

(5 marks)

Question 3 is on the next page…

Question 3

(a) Determine whether the vector $\boldsymbol{w} = \begin{bmatrix} 0 \\ -1 \\ -3 \end{bmatrix}$ can or cannot be written as a linear combination of $\boldsymbol{v}_1 = \begin{bmatrix} 2 \\ -4 \\ -2 \end{bmatrix}$, $\boldsymbol{v}_2 = \begin{bmatrix} -1 \\ 3 \\ 3 \end{bmatrix}$ and $\boldsymbol{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}$. (7 marks)

(b) For each of the following sets of vectors, decide whether they are linearly independent or linearly dependent, giving a reason for your decision.

(i) $\{[-1], [2], [4]\}$ (2 marks)

(ii) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \\ -3 \end{bmatrix} \right\}$ (6 marks)

(c) Show why the following sets of vectors are not subspaces.

(i) $U = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} \in \mathbb{R}^2 \middle| a \geq 0, b \leq 0 \right\}$ (2 marks)

(ii) $V = \left\{ \begin{bmatrix} a \\ a^2 \\ a^3 \end{bmatrix} \in \mathbb{R}^3 \middle| a \in \mathbb{R} \right\}$ (3 marks)

Question 4 is on the next page…

Question 4

(a) Consider the following homogeneous system of linear equations,

$$\begin{aligned} x_1 + 3x_2 + x_3 - 2x_4 &= 0 \\ -2x_1 + x_2 - 2x_3 &= 0 \\ -x_1 + 4x_2 - x_3 - 2x_4 &= 0 \end{aligned}$$

Solve the system by first writing it in the form of an augmented matrix $[A|\mathbf{0}]$ and then using the Gaussian Elimination method (make sure you state the rank of A, as well as the number of parameters required to describe the infinite solutions).

(7 marks)

(b) Find the determinant of the following matrix,

$$A = \begin{bmatrix} 0 & 2 & 0 & 0 & 1 \\ 4 & 0 & -2 & 0 & 0 \\ 0 & -3 & 0 & 2 & 0 \\ 6 & 0 & 0 & 0 & -2 \\ 0 & 0 & 3 & 2 & 0 \end{bmatrix}$$

(7 marks)

(c) Use Cramer's rule to solve the following system for x_1 without solving for the remaining variables.

$$\begin{aligned} x_1 - x_2 + 3x_3 &= -6 \\ 2x_1 + x_2 + 2x_3 &= 2 \\ -x_1 + 3x_2 + x_3 &= 2 \end{aligned}$$

(6 marks)

Question 5 is on the next page…

Question 5

(a) Use the inverse of the coefficient matrix to solve the following system of linear equations,

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ 2x_1 + 3x_2 + 2x_3 &= 5 \\ 3x_1 + 8x_2 + 2x_3 &= 13 \end{aligned}$$

(7 marks)

(b) By using Gaussian Elimination solve the normal equations $A^T A\boldsymbol{x} = A^T\boldsymbol{b}$ to determine the cubic least squares polynomial $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ for the data points $(-2,1)$, $(-1,-2)$, $(0,0)$, $(1,2)$ and $(2,0)$. (13 marks)

~END OF ASSESSMENT~