Curtin University

Department of Computing

Quiz 1 – Semester 2, 2018

Subject: **Foundations of Computer Science**

Index No.: COMP1006

Name: ______________________________

Student ID: ______________________________

Prac. Time: ______________________________

Time Allowed: 45 MINUTES

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1. Represent the following statements in a propositional logic. You are required to define all necessary propositions and predicates used in your answers.

   (i) The difference of two positive even numbers are always even.
   (ii) No two students in this class have the same name.
   (iii) You will fail this exam unless you have done all the past paper questions and the tutorial questions.
   (iv) All Curtin students have a unique student number and a health insurance number.
   (v) Not everyone can sing.

(5 marks)

2. **(5 marks)**

a) Write out the truth tables for the following propositions

i. $\mathrm{P} \oplus \mathrm{Q}$

ii. $\mathrm{P} \leftrightarrow \mathrm{Q}$

b) Using the logical equivalences ($\mathrm{P} \leftrightarrow \mathrm{Q} \equiv (\mathrm{P} \rightarrow \mathrm{Q}) \wedge (\mathrm{Q} \rightarrow \mathrm{P})$), express $\mathrm{P} \leftrightarrow \mathrm{Q}$ by using only disjunction, conjunction and negation (standard logical operators). ($\wedge$, $\vee$, $\neg$)

c) With the results observed for part (a (i)), (a (ii)), and (b) above, express $\mathrm{P} \oplus \mathrm{Q}$ by using only disjunction, conjunction and negation (standard logical operators). ($\wedge$, $\vee$, $\neg$)

d) Determine whether the following argument is **true or false**. State all the rules of inference in each step used in the proof.

There will be contestants for the sailing race only if it is sunny or not foggy. The sailing race will be held or the life sailing demonstration will go on only if there are contestants. The trophy will not be awarded unless the sailing race is held. It is a not a sunny day and the trophy is awarded. Therefore, it is foggy.

3.

a) Using the absorption rule

$$A \lor (A \land B) \equiv A$$
$$A \land (A \lor B) \equiv A$$

Prove the following assertion:

(**Hint: A and B are not restricted to only a single element**)

$$(P \land Q) \lor (P \land Q \land R \land S \land T) \lor \Big(\big((P \lor Q) \land (P \lor R)\big) \land P\Big) \equiv P$$

b) Write the **negation** of the following statement using propositional logic and proper quantifier:

There is no difficult question on this quiz.

(5 marks)

4. Prove or disprove the following statements.

(i) $\exists x \in \mathbb{Z},\ x^2 = -1 \rightarrow (\forall y\ \varepsilon\ \mathrm{Prime}, \mathrm{Even}(y)) \equiv \mathrm{T}$

(ii) Given $\lfloor x \rfloor,\ x \in \mathbb{R}$ is the largest integer $\leq x$, prove

$$\forall y \in \mathbb{Z}, y^2 \neq -1 \rightarrow \forall x \in \mathbb{R}, (\lfloor x - 1 \rfloor = \lfloor x \rfloor - 1)$$

(iii) If Dr. Wanquan Liu does not continue to teach this unit in this semester, I will not get a high distinction.

(iv) If $3\mathrm{n} + 3$ is odd, then $\mathrm{n} + 5$ is odd.

(5 marks)

5. Prove using mathematical induction:

Sum of first *n* **even** positive integers is $\mathbf{n(n+1)}$, $n \geq 1$

(5 marks)

End of Test

Rule of Inference	**Name**
$\begin{array}{c} p \\ \hline \therefore (p \vee q) \end{array}$	Addition
$\begin{array}{c} (p \wedge q) \\ \hline \therefore p \end{array}$	Simplification
$\begin{array}{c} p \\ p \rightarrow q \\ \hline \therefore q \end{array}$	Modus Ponens
$\begin{array}{c} \neg q \\ p \rightarrow q \\ \hline \therefore \neg p \end{array}$	Modus Tollens
$\begin{array}{c} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$	Hypothetical syllogism
$\begin{array}{c} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$	Disjunctive syllogism

$$\boldsymbol{p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)}$$

$$\boldsymbol{p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)}$$