MATH1019 Linear Algebra and Statistics for Engineers

Workshop 4

Learning outcomes for this session
At the end of this session, you should be able to

1. Find confidence intervals when population standard deviation is unknown.

2. Set up and conduct hypothesis testing for a single population.

Overview
The following exercises cover the material from Lecture 4.

Exercises

1. Prestressing wire for wrapping concrete pipe is manufactured in large rolls. A quality-control inspection required 5 specimens from a roll to be tested for ultimate tensile strength (UTS). The UTS measurements (in 1000 psi) turned out to be 253, 261, 258, 255, 256. Use these data to construct a 95% confidence interval estimate of the true mean UTS for the sampled roll.

2. A random sample of 40 engineers was selected from among the large number employed by a corporation engaged in seeking new sources of petroleum. The hours worked in a particular week was determined for each engineer selected. These data had a mean of 46 hours and a standard deviation of 3 hours. For that particular week, estimate the mean hours worked for all engineers in the corporation using a 95% confidence interval.

3. In the setting of the previous exercise, how many engineers should be sampled if it is desired to estimate the mean number of hours worked to within 0.5 hour with confidence coefficient 0.95?

4. The warpwise breaking strength measured on five specimens of a certain cloth gave a sample mean of 180 psi and a standard deviation of 5 psi. Estimate the true mean warpwise breaking strength for cloth of this type in a 95% confidence interval. What assumption is necessary for your answer to be valid?

5. Answer the previous exercise if the same data had resulted from a sample of

    (a) 10 specimens   (b) 100 specimens

6. Fatigue behaviour of reinforced concrete beams in sea water was studied by T. Hodgkiess, et al. (*Materials Performance*, July 1984, pp. 27–29). The number of cycles to failure in sea water for beams subjected to certain bending and loading stress are as follows (in thousands):

774, 633, 477, 268, 407, 576, 659, 963, 193

Construct a 90% confidence interval estimate of the average number of cycles to failure for beams of this type.

7. In a power-generating plant, pressure in a certain line is supposed to maintain an average of 100 psi over any four-hour period. If the average pressure exceeds 100 psi for a four-hour period, serious complications can evolve. During a given four-hour period 30 random measurements are to be taken, giving a sample mean of 103 psi. Suppose that it is known that $\sigma = 4$. Set up and conduct an appropriate hypothesis test at the 1% level of significance.

8. The output voltage for a certain electric circuit is specified to be 130. A sample of 40 independent readings on the voltage for this circuit gave a sample mean of 128.6 and a standard deviation of 2.1. Test the hypothesis that the average output voltage is 130 against the alternative that it is less than 130. Use a 5% significance level.

9. The Rockwell hardness index for steel is determined by pressing a diamond point into the steel and measuring the depth of penetration. For 50 specimens of a certain type of steel the Rockwell hardness index averaged 62 with a standard deviation of 8. The manufacturer claims that this steel has an average hardness index of 64. Test this claim at the 1% significance level. Find the p-value for this test

10. The manufacture of large liquid crystal displays (LCD's) is difficult. Some defects are minor and can be removed; others are unremovable. The number of unremovable defects for each of 45 displays has $\bar{x} = 2.667$ and $s = 3.057$ unremovable defects.

    (a) Conduct a test of hypothesis with the intent of showing that the mean number of unremovable defects is less than 3.6. Use $\alpha = 0.025$.

    (b) Based on your conclusion in part (a), what error could you have made. Explain in the context of the problem.

11. To monitor complex chemical processes, chemical engineers will consider key process indicators, which may be just yield but most often depend on several quantities. Before trying to improve a process, 9 random measurements were made on a key performance indicator:

    123, 106, 114, 128, 113, 109, 120, 102, 111

    (a) Conduct a test of hypothesis with the intent of showing that the mean key performance indicator is different from 107. Take $\alpha = 0.05$ and assume a normal population.

    (b) Based on your conclusion in part (a), what error could you have made. Explain in the context of the problem.

12. A manufacturer claims that the average tar content of a certain kind of cigarette is $\mu = 14.0$ In an attempt to show that it differs from this value, five random measurements were made of the tar content (mg per cigarette):

14.5   14.2   14.4   14.3   14.6

(a) Show the the difference between the mean of this sample and the average tar claimed by the manufacturer is statistically significant at $\alpha = 0.05$. Assume normality.

(b) suppose that the first measurement is recorded incorrectly as 16.0 instead of 14.5. Show that even though the mean of the sample increases, the null hypothesis is not rejected at $\alpha = 0.05$. Explain the apparent paradox that even though the difference between observed $\bar{x}$ and μ has increased, the null hypothesis is no longer rejected.