Venue _____

Student Number |__|__|__|__|__|__|__|__|

Family Name _____

First Name _____

Curtin University

Faculty of Science and Engineering

EXAMINATION

End of Semester 2, 2019

MATH1019 Linear Algebra and Statistics for Engineers

This paper is for Bentley Campus and Curtin Malaysia students

This is a RESTRICTED BOOK examination

Examination paper IS to be released to student

Examination Duration 2 hours

Reading Time 10 minutes

Students may write notes in the margins of the exam paper during reading time

Total Marks 100

Supplied by the University

1 x 16 page answer book

Supplied by the Student

Materials

One A4 sheet of handwritten or typed notes (both sides)

Calculator

A calculator displaying 'Engineering Approved Calculator' sticker

Instructions to Students

Attempt as many questions or part questions as possible.

SHOW ALL WORKING.

For Examiner Use Only

Q	Mark
1	
2	
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Total _____

Examination Cover Sheet

Question 1

Given the four points $A(2,1,1)$, $B(0,2,-1)$, $C(3,-1,0)$ and $D(6,-1,5)$, as well as the vectors $\boldsymbol{a} = [3,-2,1]$ and $\boldsymbol{b} = [-2,1,0]$, determine:

(a) The position vector of A. (1 mark)

(b) The distance from A to B. (2 marks)

(c) The length of $\boldsymbol{a}$. (1 mark)

(d) The dot product $\boldsymbol{b}.\boldsymbol{a}$. (1 mark)

(e) The direction cosines and direction angles of $\boldsymbol{b}$. (4 marks)

(f) A non-zero vector that is orthogonal to $\boldsymbol{b}$ and $\overrightarrow{DB}$. (3 marks)

(g) A vector in the direction of $\boldsymbol{b}$ but with a length of 2. (2 marks)

(h) If the four points A, B, C and D are coplanar or not. (6 marks)

(A total of 20 marks for this question.)

QUESTION 2 IS ON THE FOLLOWING PAGE.

Question 2

(a) Given that $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are vectors in 3 space, determine whether the following expressions results in either: a scalar, a vector, or is the expression meaningless (i.e. it's not possible). If the expression is meaningless explain why the expression cannot be determined.

(i) $(\boldsymbol{a} \times \boldsymbol{b}).\boldsymbol{c}$ (1 mark)

(ii) $||\boldsymbol{a}||.\boldsymbol{b}$ (1 mark)

(iii) $\boldsymbol{a} \times \boldsymbol{b} - \boldsymbol{a} \times \boldsymbol{b}$ (1 mark)

(iv) $||\boldsymbol{a}||\boldsymbol{b} + \boldsymbol{c} \times \boldsymbol{a}$ (1 mark)

(v) $(\boldsymbol{c}.\boldsymbol{c})||\boldsymbol{c}||$ (1 mark)

(vi) $\dfrac{\boldsymbol{a} + \boldsymbol{b}}{(\boldsymbol{a} \times \boldsymbol{b})}$ (1 mark)

(b) Determine the angle between the direction vector of the line $\dfrac{x-1}{2} = \dfrac{y+2}{-4} = z$ and the normal vector to the plane $-x + z = 3$. (5 marks)

(c) Determine if the line $x = 1 + 3t$, $y = -t$, $z = -4 - 2t$ is parallel or not parallel to the plane $-6x + 2y - 4z = 1$. (4 marks)

(d) Find the shortest distance from the point $P(-1, 1, 0)$ to the line $\boldsymbol{r} = [2, 2, -1] + t[3, 0, -2]$. (5 marks)

(A total of 20 marks for this question.)

QUESTION 3 IS ON THE FOLLOWING PAGE.

Question 3

Given the matrices,

$$A = \begin{bmatrix} -1 & 2 \\ -4 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 3 \end{bmatrix}, \quad D = \begin{bmatrix} -1 & 2 & 3 & 0 \\ 4 & 1 & 1 & 0 \end{bmatrix}, E = \begin{bmatrix} -9 & -15 \\ 3 & 5 \end{bmatrix}$$

find the following, or briefly justify why it cannot be found,

(a) $B - A$. (2 marks)

(b) BA. (2 marks)

(c) CD. (2 marks)

(d) AC^T. (3 marks)

(e) $\det(C)$. (2 marks)

(f) A^{-1}. (3 marks)

(g) E^{-1}. (3 marks)

(h) $|B|I_3$. (3 marks)

(A total of 20 marks for this question.)

QUESTION 4 IS ON THE FOLLOWING PAGE.

Question 4

(a) Solve the following linear system with two variables by using Cramer's rule. (Make sure you use Cramer's rule in solving both variables p and q).

$$\begin{aligned} 3p - 4q &= 8 \\ p - 2q &= 5 \end{aligned}$$

(5 marks)

(b) Let $\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, $\boldsymbol{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ and $\boldsymbol{v}_3 = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$. Decide whether the set of vectors $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ is linearly dependent or linearly independent. (5 marks)

(c) Consider the following system of linear equations,

$$\begin{aligned} x_1 + x_2 - x_3 + x_4 &= 2 \\ 2x_2 + 4x_3 + 2x_4 &= 3 \\ x_1 + 2x_2 + x_3 + 2x_4 &= k \end{aligned}$$

By using Gaussian Elimination, identify the value of k that makes the system consistent. Hence, find the solution of the system. (10 marks)

(A total of 20 marks for this question.)

QUESTION 5 IS ON THE FOLLOWING PAGE.

Question 5

(a) Given the matrices:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}, \ \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(i) Use the Gauss-Jordan method to either calculate the inverse A^{-1} of the matrix A or to show that A has no inverse. Is A a singular matrix? (8 marks)

(ii) If $A\boldsymbol{x} = \boldsymbol{b}$ is the matrix form of a homogeneous system, where $\boldsymbol{b}$ is the column vector of constants, determine whether the system has a trivial or non-trivial solution. (1 mark)

(b) Is the set $U = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \in \mathbb{R}^3 \mid b = a^2, c = a + b \text{ where } a, b, c \in \mathbb{R} \right\}$ a subspace of $\mathbb{R}^3$? Justify why it is or isn't a subspace. (4 marks)

(c) Using the pseudoinverse, determine the least squares line $y = a_0 + a_1 x$ that best fits the four-point data set: $(-1, 6), (0, 3), (2, 2)$ & $(3, -1)$. (7 marks)

(A total of 20 marks for this question.)

END OF EXAMINATION