Curtin University – Department of Computing

Assignment Cover Sheet / Declaration of Originality

Complete this form if/as directed by your unit coordinator, lecturer or the assignment specification.

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Unit name:	Foundations of Computer Science	Unit ID:	COMP1006
Lecturer / unit coordinator:	Antoni Liang	Tutor:	IDK
Date of submission:	20/11/2020	Which assignment?	FA (Leave blank if the unit has only one assignment.)

I declare that:

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- I have taken (and will continue to take) all reasonable steps to ensure my work is *not accessible* to any other students who may gain unfair advantage from it.
- I have *not previously submitted* this work for any other unit, whether at Curtin University or elsewhere, or for prior attempts at this unit, except where clearly indicated otherwise.

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- It is my responsibility to ensure that my submission is complete, correct and not corrupted.

Signature: TANAKA CHITETE   Date of signature: 20/11/2020

(By submitting this form, you indicate that you agree with all the above text.)

Question 1

p	q	r	(p -> r)	(q -> r)	(p -> r) v (q -> r)
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

p	q	r	(p ^ q)	(p ^ q) -> r
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Therefore, these statements are logically equivalent

Question 2

a. $\exists x(\forall y\big(x \neq y \wedge \text{Faster}(x, y)\big)$, where the domain of x and y is all people and Faster(x, y) denotes that person x runs faster than person y.
b. $\neg\exists x(\forall y\big(x \neq y \wedge \text{Likes}(y, x)\big)$, where the domain of x and y is all people and Likes(y, x) denotes that person y loves person x
c. $\exists!\, x\big(\text{King}(x)\big)$, where the domain of x is all people and King(x) denotes that person x is a king
d. $\forall x(\text{Friend}(x, Alice) \vee \exists y\big(x \neq y \wedge Friend(y, Alice) \wedge Friend(x, y)\big)$

Question 3

i.e. Show that $(|x| + |y|)^2 \geq |x + y|^2$
Note: $\forall a \in \mathbb{R}(|b|^2 = b^2)$

$$
\begin{gathered}
|x|^2 + 2|x||y| + |y|^2 \\
\geq x^2 + 2xy + y^2, \text{ where } |a| \geq a \text{ and } |b| \geq b \\
= (a + b)^2 \\
= |a + b|^2
\end{gathered}
$$

$$
\begin{gathered}
\therefore (|x| + |y|)^2 \geq |x + y|^2 \\
\therefore |x| + |y| \geq |x + y|
\end{gathered}
$$

(The Math Sorcerer, 2015)

Question 4

For n > 1, let P(n) denote the statement

$$P(n) \equiv \frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\cdots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$$

Base step

$$\begin{aligned} P(2) &\equiv \frac{1}{1\times 3}+\frac{1}{3\times 5}=\frac{2}{2(2)+1} \\ &\equiv \frac{1}{3}+\frac{1}{15}=\frac{2}{5} \\ &\equiv \frac{5}{15}+\frac{1}{15}=\frac{6}{15} \\ &\equiv \text{True} \end{aligned}$$

Inductive step

For an arbitrary k > 2, assuming that P(k) is true, it remains to prove that P(k + 1), given below, holds:

$$P(k+1) \equiv \frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\cdots+\frac{1}{(2(k+1)-1)(2(k+1)+1)}=\frac{k+1}{2(k+1)+1}$$

Starting with the LHS of P(k + 1),

$$\begin{aligned} P(k+1) \equiv \frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\cdots+\frac{1}{(2k-1)(2k+1)}+\frac{1}{(2(k+1)-1)(2(k+1)+1)} &= \frac{k}{2k+1}+\frac{1}{(2(k+1)-1)(2(k+1)+1)} \\ &= \frac{k+1}{2(k+1)+1} \end{aligned}$$

we see that the RHS of $P(k+1)$ follows.

∴ By completing the inductive step, we have proven that $P(k+1)$ is true

∴ By mathematical induction, we have also proven that for any $n > 1$, the statement $P(n)$is true

i.e. We have proven that the implication $P(k) \rightarrow P(k+1)$ is true

(Rosen 2007)

Question 5

a. …
b. …

Question 6

Definition: A relation R on a set S is called an equivalence relation if it is reflexive, symmetric and transitive

a. **Reflexivity**

   i.e. Show that $a \sim b$

$$\text{zeros}(a) = \text{zeros}(b)$$
$$= \text{zeros}(a)$$
$$\therefore a \sim b$$

$\therefore$ Since $a \sim b, R$ is reflexive

Symmetry

i.e. Show that $a \sim b \rightarrow b \sim a$

$$(a \sim b) \Leftrightarrow \text{zeros(a)} = \text{zeros(b)}$$
$$(b \sim a) \Leftrightarrow \text{zeros(b)} = \text{zeros(a)}$$
$$\therefore (a \sim b) \rightarrow (b \sim a)$$

$\therefore$ Since $(a \sim b) \rightarrow (b \sim a), R$ is symmetric

Transitivity

i.e. Show that $[(a \sim b) \wedge (b \sim c)] \rightarrow (a \sim c)$

$$(a \sim b) \Leftrightarrow \text{zeros(a)} = \text{zeros(b)}$$
$$(b \sim c) \Leftrightarrow \text{zeros(b)} = \text{zeros(c)}$$

$$(a \sim c) \Leftrightarrow \text{zeros(a)} = \text{zeros(c)}$$

$$\therefore [(a \sim b) \wedge (b \sim c)] \rightarrow (a \sim c)$$

$\therefore$ Since $[(a \sim b) \wedge (b \sim c)] \rightarrow (a \sim c), R$ is transitive

$\therefore$ Since R is reflexive, symmetric and transitive, it is an equivalence relation.

(The Math Sorcerer, 2018)

b. $[a] = \{s | (10001, s) \in R\}$ (Rosen, 2007)

Question 7

a. ...
b. ...

Question 8

a. Firstly, we need to select 3 students. The number of ways in which we can select 3 students from the original 9 is given by $C_3^9 = 84$
   Next, we need to select 2 staff members. The number of ways in which we can 2 staff members from the original 6 is given by $C_2^6 = 15$
   Consequently, the number of ways is given by $84 \times 15 = 1260$
b. ...
c. ...
d. ...

Question 9

...

Question 10

a. $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for $n \geq 3$
(Rosen 2007)

b. $a_0 = 1$, $a_1 = 1$ and $a_2 = 2$
(Rosen 2007)

c.

n	a_n
0	1
1	1
2	2
3	4
4	7
5	13
6	24

(Rosen 2007)

Question 11

a. Suppose that a given bit string is *valid* if it contains the substring 10 and *invalid* otherwise. Moreover, suppose that s is a string of length n – 1.

If s is valid, we have two possible options:
1. We can append a 0
2. We can append a 1

Either way, seeing as the string was already valid, no change to its validity will occur with either option; this accounts for $2a_{n-1}$ good strings of length n.

If s is invalid, we can only form a valid string by appending one more bit, is for the final bit of s to be 1 and then for us to append a 0. Conclusively, this scenario will grant us one valid string for every invalid string with a length of $n - 1$ that concludes with a 1. Seeing as there are 2^{n-1} strings of length $n - 1$ and a_{n-1} of them are valid, there will be a total of $2^{n-1} - a_{n-1}$ strings that are invalid. The only invalid string of length $n - 1$ ends in 0, shown below:

000…000,

so there are $2^{n-1} - a_{n-1} - 1$ invalid strings with a length of $n - 1$ which can be modified into a valid string with a length of n

Thus, the total number of valid strings is given by:

$$a_n = 2a_{n-1} + (2^{n-1} - a_{n-1} - 1) = a_{n-1} + 2^{n-1} - 1$$

(foobar512 2016)

b. $a_0 = 0$, $a_1 = 0$ and $a_2 = 1$

c.

n	a_n
0	0
1	0
2	1
3	4
4	11
5	26
6	57

(foobar512 2016)

Question 12

a. Euler Circuit
   No, as not every vertex has even degree. e.g. deg(a) = 3
   (Liau 2020)
   Euler Path
   Yes, a, b, e, d, a, c, d
   (Liau 2020)
b. Hamilton Circuit
   Yes, a, b, e, d, c, a
   (Liau 2020)
   Hamilton Path
   Yes, a, b, e, d, c
   (Liau 2020)

Question 13

a. …
b. …
c. …

References

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