Venue _____

Student Number |__|__|__|__|__|__|__|__|__|

Family Name _____

First Name _____

End of Semester 1, 2018
MATH1019 Linear Algebra and Statistics for Engineers

Curtin University

Faculty of Science and Engineering

EXAMINATION

End of Semester 1, 2018

MATH1019 Linear Algebra and Statistics for Engineers

This paper is for Bentley Campus and Miri Sarawak Campus students

This is a RESTRICTED BOOK examination

Examination paper is NOT to be released to student

Examination Duration 2 hours

Reading Time 10 minutes

Notes in the margins of exam paper may be written by Students during reading time

Total Marks 100

Supplied by the University

1 x 16 page answer book

Formula sheet (included with exam paper)

Supplied by the Student

Materials

One A4 sheet of handwritten or typed notes (both sides)

Calculator

A calculator displaying 'Engineering Approved Calculator' sticker

Instructions to Students

Please attempt all questions, and show all working.

For Examiner Use Only

Q	Mark
1	
2	
3	
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Total _____

Examination Cover Sheet

Question 1

(a) Consider the vectors $\underset{\sim}{\boldsymbol{a}} = 2\underset{\sim}{\boldsymbol{i}} - 3\underset{\sim}{\boldsymbol{j}} + 6\underset{\sim}{\boldsymbol{k}}$, $\underset{\sim}{\boldsymbol{b}} = \underset{\sim}{\boldsymbol{i}} + 2\underset{\sim}{\boldsymbol{j}} + 3\underset{\sim}{\boldsymbol{k}}$ and $\underset{\sim}{\boldsymbol{c}} = \underset{\sim}{\boldsymbol{i}} + x\underset{\sim}{\boldsymbol{j}} + 3\underset{\sim}{\boldsymbol{k}}$.

(i) Find a unit vector in the direction of $\underset{\sim}{\boldsymbol{a}}$. (1 mark)

(ii) Calculate the scalar projection of $\underset{\sim}{\boldsymbol{b}}$ onto $\underset{\sim}{\boldsymbol{a}}$. (2 marks)

(iii) Find a non-zero vector that is orthogonal to both vectors $\underset{\sim}{\boldsymbol{a}}$ and $\underset{\sim}{\boldsymbol{b}}$. (4 marks)

(iv) Find the value of x that makes the vector $\underset{\sim}{\boldsymbol{c}}$ parallel to the vector $\underset{\sim}{\boldsymbol{a}}$. (2 marks)

(v) Determine if the set $\left\{\underset{\sim}{\boldsymbol{a}}, \underset{\sim}{\boldsymbol{b}}\right.$ is linearly dependent or independent. (2 marks)

(b) Calculate the volume of the parallelepiped with edges defined by the vectors $\underset{\sim}{\boldsymbol{a}} = [2, -1, 3]$, $\underset{\sim}{\boldsymbol{b}} = [4, 1, 0]$ and $\underset{\sim}{\boldsymbol{c}} = [\ \ 3, 2, 2]$.

(6 marks)

(c) A force of magnitude 10 Newtons moves an object through a distance of 7 metres. If there is a 45° angle between the direction of the force and the displacement direction of the object, find the work done by the force in moving the object.

(3 marks)

(A total of 20 marks for this question.)

Question 2

(a) Find parametric and cartesian equations of the line that passes through the points $(7, -2, 4)$ and $(5, -5, 3)$.

(4 marks)

(b) Consider the plane that passes through the point $(1, 3, -2)$, and contains the line

$$\underset{\sim}{\boldsymbol{r}} = [1, 2, 0] + t\,[-1, -1, 1]\,,\ t \in \mathbb{R}$$

Find vector and cartesian equations of this plane. (8 marks)

(c) Determine if the following planes intersect:

$$x + 2y + 3z = 10 \ \text{ and } \ 4x + 5y + 6z = 52$$

If the planes do intersect, give the vector form of the intersection and justify what this means physically. If the planes do not intersect, find the distance between them.

(8 marks)

(A total of 20 marks for this question.)

QUESTION 3 IS ON THE FOLLOWING PAGE.

Question 3

(a) Consider the following system of equations:

$$\begin{aligned} x_1 - 3x_2 + 2x_3 &= 1 \\ 2x_1 - 5x_2 + 6x_3 &= 5 \\ -x_1 + 5x_2 + 2x_3 &= 6 \end{aligned}$$

Use Gaussian elimination to find the solution(s), or justify why there are no solutions.

(6 marks)

(b) Determine the inverse of the following matrix, or justify why it does not exist.

$$\begin{bmatrix} 1 & -2 & -4 \\ 2 & -3 & -6 \\ -3 & 6 & 15 \end{bmatrix}$$

(7 marks)

(c) Use Cramer's rule to solve the following system for x_2 without solving for the remaining variables.

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 0 \\ -x_1 + x_2 + x_3 &= 0 \\ 2x_1 - x_2 - x_3 &= 1 \end{aligned}$$

(7 marks)

(A total of 20 marks for this question.)

QUESTION 4 IS ON THE FOLLOWING PAGE.

Question 4

(a) Muzzle velocities of eight shells tested with a new gunpowder yield a sample mean of $\bar{x} = 2959$ feet per second and a standard deviation of $s = 39.4$. The manufacturer claims that the new gunpowder produces an average velocity of more than 3000 feet per second.

Does the sample provide enough evidence to support the manufacturer's claim? Set up and test an appropriate hypothesis at the 5% level of significance.

Use the following headings as a guide:

- Assumptions
- Test Statistic
- Critical Region(s)
- Conclusion.

(10 marks)

(b) The downtime per day of a certain computing facility averages 4.0 hours with a standard deviation of 0.8 hour.

(i) Find the probability that the average daily downtime for a period of 30 days is between 1 and 5 hours.

(4 marks)

(ii) Find the probability that the total downtime for the 30 days is less than 115 hours.

(6 marks)

(A total of 20 marks for this question.)

QUESTION 5 IS ON THE FOLLOWING PAGE.

Question 5

(a) During a manufacturing process, 15 units are randomly selected each day from the production line to check the percent defective. From historical information it is known that the probability of a defective unit is 0.05. Any time that two or more defectives are found in the sample of 15, the process is stopped. This procedure is used to provide a signal in case the probability of a defective has increased. What is the probability that on any given day the production process will be stopped?

(5 marks)

(b) The yield stress in steel bars reported by P. Booster (*Journal of Quality Technology*, 15, No. 4, 1983, p.191) gave the following data for one test:

$$n = 150, \qquad \bar{x} = 477 \text{ N/mm}^2, \qquad s = 13$$

Estimate the true yield stress for bars of this grade in a 90% confidence interval. Are any assumptions necessary for your answer to be valid?

(6 marks)

(c) Consider the following sample of observations, with the assumption that they are a simple random sample.

49	34	19	56	43	97	81	85	77	69	31

Obtain the **five number summary** for the above data. (6 marks)

(c) For a distribution that is skewed with a long right tail, indicate which of the following statements is correct:

(i) The mean is less than the median;

(ii) The mean is equal to the median;

(iii) The mean is greater than the median;

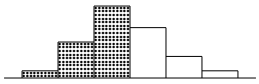
(iv) Skewed distributions do not have a median.

Justify your answer, making use of a sketch. (3 marks)

(A total of 20 marks for this question.)

END OF EXAMINATION

Cumulative probabilities for the Binomial distribution



X

$P(X \leq x)$ where $X \sim \text{Bin}(n, p)$ Read blank entries as 0.0000 or 1.0000 as appropriate

	p	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90
n	x											
2	**0**	0.8100	0.6400	0.5625	0.4900	0.3600	0.2500	0.1600	0.0900	0.0625	0.0400	0.0100
	1	0.9900	0.9600	0.9375	0.9100	0.8400	0.7500	0.6400	0.5100	0.4375	0.3600	0.1900
	2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	**0**	0.7290	0.5120	0.4219	0.3430	0.2160	0.1250	0.0640	0.0270	0.0156	0.0080	0.0010
	1	0.9720	0.8960	0.8438	0.7840	0.6480	0.5000	0.3520	0.2160	0.1562	0.1040	0.0280
	2	0.9990	0.9920	0.9844	0.9730	0.9360	0.8750	0.7840	0.6570	0.5781	0.4880	0.2710
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	**0**	0.6561	0.4096	0.3164	0.2401	0.1296	0.0625	0.0256	0.0081	0.0039	0.0016	0.0001
	1	0.9477	0.8192	0.7383	0.6517	0.4752	0.3125	0.1792	0.0837	0.0508	0.0272	0.0037
	2	0.9963	0.9728	0.9492	0.9163	0.8208	0.6875	0.5248	0.3483	0.2617	0.1808	0.0523
	3	0.9999	0.9984	0.9961	0.9919	0.9744	0.9375	0.8704	0.7599	0.6836	0.5904	0.3439
	4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	**0**	0.5905	0.3277	0.2373	0.1681	0.0778	0.0312	0.0102	0.0024	0.0010	0.0003	0.0000
	1	0.9185	0.7373	0.6328	0.5282	0.3370	0.1875	0.0870	0.0308	0.0156	0.0067	0.0005
	2	0.9914	0.9421	0.8965	0.8369	0.6826	0.5000	0.3174	0.1631	0.1035	0.0579	0.0086
	3	0.9995	0.9933	0.9844	0.9692	0.9130	0.8125	0.6630	0.4718	0.3672	0.2627	0.0815
	4	1.0000	0.9997	0.9990	0.9976	0.9898	0.9688	0.9222	0.8319	0.7627	0.6723	0.4095
	5		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	**0**	0.5314	0.2621	0.1780	0.1176	0.0467	0.0156	0.0041	0.0007	0.0002	0.0001	0.0000
	1	0.8857	0.6554	0.5339	0.4202	0.2333	0.1094	0.0410	0.0109	0.0046	0.0016	0.0001
	2	0.9842	0.9011	0.8306	0.7443	0.5443	0.3438	0.1792	0.0705	0.0376	0.0170	0.0013
	3	0.9987	0.9830	0.9624	0.9295	0.8208	0.6562	0.4557	0.2557	0.1694	0.0989	0.0158
	4	0.9999	0.9984	0.9954	0.9891	0.9590	0.8906	0.7667	0.5798	0.4661	0.3446	0.1143
	5	1.0000	0.9999	0.9998	0.9993	0.9959	0.9844	0.9533	0.8824	0.8220	0.7379	0.4686
	6		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	**0**	0.4783	0.2097	0.1335	0.0824	0.0280	0.0078	0.0016	0.0002	0.0001	0.0000	
	1	0.8503	0.5767	0.4449	0.3294	0.1586	0.0625	0.0188	0.0038	0.0013	0.0004	0.0000
	2	0.9743	0.8520	0.7564	0.6471	0.4199	0.2266	0.0963	0.0288	0.0129	0.0047	0.0002
	3	0.9973	0.9667	0.9294	0.8740	0.7102	0.5000	0.2898	0.1260	0.0706	0.0333	0.0027
	4	0.9998	0.9953	0.9871	0.9712	0.9037	0.7734	0.5801	0.3529	0.2436	0.1480	0.0257
	5	1.0000	0.9996	0.9987	0.9962	0.9812	0.9375	0.8414	0.6706	0.5551	0.4233	0.1497
	6		1.0000	0.9999	0.9998	0.9984	0.9922	0.9720	0.9176	0.8665	0.7903	0.5217
	7			1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
8	**0**	0.4305	0.1678	0.1001	0.0576	0.0168	0.0039	0.0007	0.0001	0.0000	0.0000	
	1	0.8131	0.5033	0.3671	0.2553	0.1064	0.0352	0.0085	0.0013	0.0004	0.0001	
	2	0.9619	0.7969	0.6785	0.5518	0.3154	0.1445	0.0498	0.0113	0.0042	0.0012	0.0000
	3	0.9950	0.9437	0.8862	0.8059	0.5941	0.3633	0.1737	0.0580	0.0273	0.0104	0.0004
	4	0.9996	0.9896	0.9727	0.9420	0.8263	0.6367	0.4059	0.1941	0.1138	0.0563	0.0050
	5	1.0000	0.9988	0.9958	0.9887	0.9502	0.8555	0.6846	0.4482	0.3215	0.2031	0.0381
	6		0.9999	0.9996	0.9987	0.9915	0.9648	0.8936	0.7447	0.6329	0.4967	0.1869
	7		1.0000	1.0000	0.9999	0.9993	0.9961	0.9832	0.9424	0.8999	0.8322	0.5695
	8				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$P(X \leq x)$ where $X \sim \text{Bin}(n, p)$ Read blank entries as 0.0000 or 1.0000 as appropriate

n	x \ p	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90
9	**0**	0.3874	0.1342	0.0751	0.0404	0.0101	0.0020	0.0003	0.0000	0.0000		
	1	0.7748	0.4362	0.3003	0.1960	0.0705	0.0195	0.0038	0.0004	0.0001	0.0000	
	2	0.9470	0.7382	0.6007	0.4628	0.2318	0.0898	0.0250	0.0043	0.0013	0.0003	0.0000
	3	0.9917	0.9144	0.8343	0.7297	0.4826	0.2539	0.0994	0.0253	0.0100	0.0031	0.0001
	4	0.9991	0.9804	0.9511	0.9012	0.7334	0.5000	0.2666	0.0988	0.0489	0.0196	0.0009
	5	0.9999	0.9969	0.9900	0.9747	0.9006	0.7461	0.5174	0.2703	0.1657	0.0856	0.0083
	6	1.0000	0.9997	0.9987	0.9957	0.9750	0.9102	0.7682	0.5372	0.3993	0.2618	0.0530
	7		1.0000	0.9999	0.9996	0.9962	0.9805	0.9295	0.8040	0.6997	0.5638	0.2252
	8			1.0000	1.0000	0.9997	0.9980	0.9899	0.9596	0.9249	0.8658	0.6126
	9					1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	**0**	0.3487	0.1074	0.0563	0.0282	0.0060	0.0010	0.0001	0.0000			
	1	0.7361	0.3758	0.2440	0.1493	0.0464	0.0107	0.0017	0.0001	0.0000	0.0000	
	2	0.9298	0.6778	0.5256	0.3828	0.1673	0.0547	0.0123	0.0016	0.0004	0.0001	
	3	0.9872	0.8791	0.7759	0.6496	0.3823	0.1719	0.0548	0.0106	0.0035	0.0009	0.0000
	4	0.9984	0.9672	0.9219	0.8497	0.6331	0.3770	0.1662	0.0473	0.0197	0.0064	0.0001
	5	0.9999	0.9936	0.9803	0.9527	0.8338	0.6230	0.3669	0.1503	0.0781	0.0328	0.0016
	6	1.0000	0.9991	0.9965	0.9894	0.9452	0.8281	0.6177	0.3504	0.2241	0.1209	0.0128
	7		0.9999	0.9996	0.9984	0.9877	0.9453	0.8327	0.6172	0.4744	0.3222	0.0702
	8		1.0000	1.0000	0.9999	0.9983	0.9893	0.9536	0.8507	0.7560	0.6242	0.2639
	9				1.0000	0.9999	0.9990	0.9940	0.9718	0.9437	0.8926	0.6513
	10					1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	**0**	0.3138	0.0859	0.0422	0.0198	0.0036	0.0005	0.0000				
	1	0.6974	0.3221	0.1971	0.1130	0.0302	0.0059	0.0007	0.0000	0.0000		
	2	0.9104	0.6174	0.4552	0.3127	0.1189	0.0327	0.0059	0.0006	0.0001	0.0000	
	3	0.9815	0.8389	0.7133	0.5696	0.2963	0.1133	0.0293	0.0043	0.0012	0.0002	
	4	0.9972	0.9496	0.8854	0.7897	0.5328	0.2744	0.0994	0.0216	0.0076	0.0020	0.0000
	5	0.9997	0.9883	0.9657	0.9218	0.7535	0.5000	0.2465	0.0782	0.0343	0.0117	0.0003
	6	1.0000	0.9980	0.9924	0.9784	0.9006	0.7256	0.4672	0.2103	0.1146	0.0504	0.0028
	7		0.9998	0.9988	0.9957	0.9707	0.8867	0.7037	0.4304	0.2867	0.1611	0.0185
	8		1.0000	0.9999	0.9994	0.9941	0.9673	0.8811	0.6873	0.5448	0.3826	0.0896
	9			1.0000	1.0000	0.9993	0.9941	0.9698	0.8870	0.8029	0.6779	0.3026
	10					1.0000	0.9995	0.9964	0.9802	0.9578	0.9141	0.6862
	11						1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
12	**0**	0.2824	0.0687	0.0317	0.0138	0.0022	0.0002	0.0000				
	1	0.6590	0.2749	0.1584	0.0850	0.0196	0.0032	0.0003	0.0000			
	2	0.8891	0.5583	0.3907	0.2528	0.0834	0.0193	0.0028	0.0002	0.0000	0.0000	
	3	0.9744	0.7946	0.6488	0.4925	0.2253	0.0730	0.0153	0.0017	0.0004	0.0001	
	4	0.9957	0.9274	0.8424	0.7237	0.4382	0.1938	0.0573	0.0095	0.0028	0.0006	0.0000
	5	0.9995	0.9806	0.9456	0.8822	0.6652	0.3872	0.1582	0.0386	0.0143	0.0039	0.0000
	6	0.9999	0.9961	0.9857	0.9614	0.8418	0.6128	0.3348	0.1178	0.0544	0.0194	0.0005
	7	1.0000	0.9994	0.9972	0.9905	0.9427	0.8062	0.5618	0.2763	0.1576	0.0726	0.0043
	8		0.9999	0.9996	0.9983	0.9847	0.9270	0.7747	0.5075	0.3512	0.2054	0.0256
	9		1.0000	1.0000	0.9998	0.9972	0.9807	0.9166	0.7472	0.6093	0.4417	0.1109
	10				1.0000	0.9997	0.9968	0.9804	0.9150	0.8416	0.7251	0.3410
	11					1.0000	0.9998	0.9978	0.9862	0.9683	0.9313	0.7176
	12						1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$P(X \leq x)$ where $X \sim \text{Bin}(n, p)$ Read blank entries as 0.0000 or 1.0000 as appropriate

	p	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90
n	x											
14	**0**	0.2288	0.0440	0.0178	0.0068	0.0008	0.0001	0.0000				
	1	0.5846	0.1979	0.1010	0.0475	0.0081	0.0009	0.0001				
	2	0.8416	0.4480	0.2811	0.1608	0.0398	0.0065	0.0006	0.0000			
	3	0.9559	0.6982	0.5213	0.3552	0.1243	0.0287	0.0039	0.0002	0.0000		
	4	0.9908	0.8702	0.7415	0.5842	0.2793	0.0898	0.0175	0.0017	0.0003	0.0000	
	5	0.9985	0.9561	0.8883	0.7805	0.4859	0.2120	0.0583	0.0083	0.0022	0.0004	
	6	0.9998	0.9884	0.9617	0.9067	0.6925	0.3953	0.1501	0.0315	0.0103	0.0024	0.0000
	7	1.0000	0.9976	0.9897	0.9685	0.8499	0.6047	0.3075	0.0933	0.0383	0.0116	0.0002
	8		0.9996	0.9978	0.9917	0.9417	0.7880	0.5141	0.2195	0.1117	0.0439	0.0015
	9		1.0000	0.9997	0.9983	0.9825	0.9102	0.7207	0.4158	0.2585	0.1298	0.0092
	10			1.0000	0.9998	0.9961	0.9713	0.8757	0.6448	0.4787	0.3018	0.0441
	11				1.0000	0.9994	0.9935	0.9602	0.8392	0.7189	0.5519	0.1584
	12					0.9999	0.9991	0.9919	0.9525	0.8990	0.8021	0.4154
	13					1.0000	0.9999	0.9992	0.9932	0.9822	0.9560	0.7712
	14						1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
15	**0**	0.2059	0.0352	0.0134	0.0047	0.0005	0.0000					
	1	0.5490	0.1671	0.0802	0.0353	0.0052	0.0005	0.0000				
	2	0.8159	0.3980	0.2361	0.1268	0.0271	0.0037	0.0003	0.0000			
	3	0.9444	0.6482	0.4613	0.2969	0.0905	0.0176	0.0019	0.0001	0.0000		
	4	0.9873	0.8358	0.6865	0.5155	0.2173	0.0592	0.0093	0.0007	0.0001	0.0000	
	5	0.9978	0.9389	0.8516	0.7216	0.4032	0.1509	0.0338	0.0037	0.0008	0.0001	
	6	0.9997	0.9819	0.9434	0.8689	0.6098	0.3036	0.0950	0.0152	0.0042	0.0008	
	7	1.0000	0.9958	0.9827	0.9500	0.7869	0.5000	0.2131	0.0500	0.0173	0.0042	0.0000
	8		0.9992	0.9958	0.9848	0.9050	0.6964	0.3902	0.1311	0.0566	0.0181	0.0003
	9		0.9999	0.9992	0.9963	0.9662	0.8491	0.5968	0.2784	0.1484	0.0611	0.0022
	10		1.0000	0.9999	0.9993	0.9907	0.9408	0.7827	0.4845	0.3135	0.1642	0.0127
	11			1.0000	0.9999	0.9981	0.9824	0.9095	0.7031	0.5387	0.3518	0.0556
	12				1.0000	0.9997	0.9963	0.9729	0.8732	0.7639	0.6020	0.1841
	13					1.0000	0.9995	0.9948	0.9647	0.9198	0.8329	0.4510
	14						1.0000	0.9995	0.9953	0.9866	0.9648	0.7941
	15							1.0000	1.0000	1.0000	1.0000	1.0000
20	**0**	0.1216	0.0115	0.0032	0.0008	0.0000						
	1	0.3917	0.0692	0.0243	0.0076	0.0005	0.0000					
	2	0.6769	0.2061	0.0913	0.0355	0.0036	0.0002					
	3	0.8670	0.4114	0.2252	0.1071	0.0160	0.0013	0.0000				
	4	0.9568	0.6296	0.4148	0.2375	0.0510	0.0059	0.0003				
	5	0.9887	0.8042	0.6172	0.4164	0.1256	0.0207	0.0016	0.0000			
	6	0.9976	0.9133	0.7858	0.6080	0.2500	0.0577	0.0065	0.0003	0.0000		
	7	0.9996	0.9679	0.8982	0.7723	0.4159	0.1316	0.0210	0.0013	0.0002	0.0000	
	8	0.9999	0.9900	0.9591	0.8867	0.5956	0.2517	0.0565	0.0051	0.0009	0.0001	
	9	1.0000	0.9974	0.9861	0.9520	0.7553	0.4119	0.1275	0.0171	0.0039	0.0006	
	10		0.9994	0.9961	0.9829	0.8725	0.5881	0.2447	0.0480	0.0139	0.0026	0.0000
	11		0.9999	0.9991	0.9949	0.9435	0.7483	0.4044	0.1133	0.0409	0.0100	0.0001
	12		1.0000	0.9998	0.9987	0.9790	0.8684	0.5841	0.2277	0.1018	0.0321	0.0004
	13			1.0000	0.9997	0.9935	0.9423	0.7500	0.3920	0.2142	0.0867	0.0024
	14				1.0000	0.9984	0.9793	0.8744	0.5836	0.3828	0.1958	0.0113
	15					0.9997	0.9941	0.9490	0.7625	0.5852	0.3704	0.0432
	16					1.0000	0.9987	0.9840	0.8929	0.7748	0.5886	0.1330
	17						0.9998	0.9964	0.9645	0.9087	0.7939	0.3231
	18						1.0000	0.9995	0.9924	0.9757	0.9308	0.6083
	19							1.0000	0.9992	0.9968	0.9885	0.8784
	20								1.0000	1.0000	1.0000	1.0000

Cumulative probabilities for the Standard Normal distribution

$P(Z \leq z)$ where $Z \sim \mathrm{N}(0,1)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002
−3.3	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9986	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Critical points of the t-distribution

Entry is t where $P(T \geq t) = p$ for t-distribution with u degrees of freedom

ν | p	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
31	1.309	1.696	2.040	2.453	2.744	3.375	3.633
32	1.309	1.694	2.037	2.449	2.738	3.365	3.622
33	1.308	1.692	2.035	2.445	2.733	3.356	3.611
34	1.307	1.691	2.032	2.441	2.728	3.348	3.601
35	1.306	1.690	2.030	2.438	2.724	3.340	3.591
36	1.306	1.688	2.028	2.434	2.719	3.333	3.582
37	1.305	1.687	2.026	2.431	2.715	3.326	3.574
38	1.304	1.686	2.024	2.429	2.712	3.319	3.566
39	1.304	1.685	2.023	2.426	2.708	3.313	3.558
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
41	1.303	1.683	2.020	2.421	2.701	3.301	3.544
42	1.302	1.682	2.018	2.418	2.698	3.296	3.538
43	1.302	1.681	2.017	2.416	2.695	3.291	3.532
44	1.301	1.680	2.015	2.414	2.692	3.286	3.526
45	1.301	1.679	2.014	2.412	2.690	3.281	3.520
46	1.300	1.679	2.013	2.410	2.687	3.277	3.515
47	1.300	1.678	2.012	2.408	2.685	3.273	3.510
48	1.299	1.677	2.011	2.407	2.682	3.269	3.505
49	1.299	1.677	2.010	2.405	2.680	3.265	3.500
50	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	1.290	1.660	1.984	2.364	2.626	3.174	3.390
200	1.286	1.652	1.972	2.345	2.601	3.131	3.340
500	1.283	1.648	1.965	2.334	2.586	3.107	3.310
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291