## Curtin University MATH1019 Linear Algebra and Statistics for Engineers

Mid-Semester Test, S1 2019; Time Allowed:  $\mathbf{1}$  Hour +  $\mathbf{5}$  minutes reading time

This paper contains 10 pages (including this cover sheet), 5 questions, worth a total of 45 marks

Write your answers in the spaces provided. Write your name and student number on this cover sheet. If pages become separated write your name on all separated sheets. A blank page is attached should you require additional space, however if you need more paper than this, please ask.

NAME:		
STUDENT NUMBER:		
Please circle your workshop tutor and corresponding workshop time:		
Grant Keady:	Monday 10–12noon Friday 10–12noon	
Cathy (Shican) Liu:	Monday 2–4pm Thursday 2–4pm Thursday 4–6pm	
Shuang Li:	Tuesday 10–12noon Tuesday 4–6pm	
Muhammad Kamran:	Tuesday 12–2pm	
Mikhail Dokuchaev:	Wednesday 10–12noon Wednesday 2–4pm	
Karo Fathollahzadeh:	Thursday 8–10am Friday 12–2pm Friday 4–6pm	

**Question 1.** A shop would like to estimate its average number of customers per hour. After a simple random sample process, the shop has found the following sample of customer numbers per hour for 10 hours.

109   66   49   23   89   99   70	0   88   92   85
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- (a) Find the mean and standard deviation of the sample. (3 marks)
- (b) Find the five number presentation of the above data set. (7 marks)
- (c) Use the results in part (b) to draw the boxplot of the above sample, indicating clearly the whiskers and outlier(s), if any. (4 marks)

Question 2. A company offers you the job to design a payoff function for a casino game machine. It is known that the game has three outcomes A, B and C and their respective probabilities are 0.2, 0.5 and 0.3. The payoff function for the game player equals  $P_1$  for A, 0 for B and  $P_2$  for C. The company requires that the expected value of the player's payoff is -\$0.1 (unfair game). If it is also required that  $P_1 + P_2 = -0.04$ . Help the company to determine  $P_1$  and  $P_2$ . Also, find the variance of the payoff function.

(4 marks)

Question 3. Kellogg's produces boxed breakfast cereals. The weight of cereals w in each box is a random variable satisfying the normal distribution with the mean  $\mu = 500g$  and standard deviation  $\sigma$ . From time to time The Australian Competition & Consumer Commission (ACCC) randomly chooses 20 boxes of the product and calculates the average weight  $\bar{w}$  of the box contents. If ACCC finds  $P(\bar{w} < 485g) > 0.1$ , the company will be fined. What is the maximum standard deviation  $\sigma$  of w so as for Kellogg's to avoid a fine by ACCC? (5 marks)

- **Question 4.** This is a continuation of Question 3 above. ACCC wants to assess independently whether there is any significant evidence to support Kellogg's claim that each cereal box contains 500g of cereal. Using the randomly chosen 20 boxes of the product, ACCC finds that the average weight per box is  $\bar{x} = 495$ g with the sample standard deviation s = 6g.
- (a) Perform a test of hypothesis at the 5% significance level with the intent to show that Kellogg over-estimates the average weight of cereal in a box. (6 marks)
- (b) If we accept Kellogg's claim of the average cereal weight per box is  $\mu = 500$ g with the standard deviation  $\sigma = 10$ g, how large a sample is required if we want a 98% confidence interval for the mean  $\mu$  to have a margin of error of  $\pm 5$ g? (4 marks)

Question 5. City Toyota sales department receives on average 5 customers per hour.

- (a) Let X be the number of customers visiting the department. What probability distribution does X satisfy? (1 mark)
- (b) Find the probability that there are 6 to 8 customers inclusive in one hour, i.e.  $P(6 \le X \le 8)$ . (3 marks)
- (c) What is the probability that the department has 6 to 8 customers inclusive in 2 hours? (4 marks)
- (d) Using the probability found in part (b), find the probability that any 2 of the 4 chosen hours the department has 6 to 8 customers. (4 marks)

## Additional working space if required

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