MATH1019 Linear Algebra and Statistics for Engineers
Laboratory Session 8

Learning outcomes for this session

At the end of this session, you will be able to

1. Use matlab to solve homogeneous systems by Gaussian Elimination.
2. Solve systems by using the Gauss Jordan method.
3. Calculate inverses with matlab.

Overview

1. Using matlab to enhance our understanding of solving systems of equations and calculating inverses.
2. matlab commands introduced in this worksheet: `rref`, `inv`.

Gauss-Jordan `rref` and matrix inverses

1. In the last lab we defined the following matrices M and submatrices A and column vectors $\boldsymbol{b}$:

```
M0= [1,2,3,4;5,6,7,8;9,10,11,12]; A0=M0(:,[1:3]);  b0=M0(:,4)
M1= [1,2,3,4;5,6,7,8;9,10,11,11]; A1=M1(:,[1:3]);  b1=M1(:,4)
M2= [1,2,3,4;5,6,7,8;9,10,12,12]; A2=M2(:,[1:3]);  b2=M2(:,4)
```

and performed row operations as required for Gaussian Elimination on matrix $M0$ and obtained a matrix row equivalent to $M0$ which is in row echelon form. We also saw that the solution of linear systems $A_j\boldsymbol{x} = \boldsymbol{b}_j$ is easy after one has the augmented matrix $A|\boldsymbol{b}$ in row echelon form.
In this lab we explore Gauss Jordan reduction to what is called 'Reduced Row Echelon Form', abbreviated rref.. The distinction should be clear, but if not, look at
https://en.wikipedia.org/wiki/Row_echelon_form

So Gauss Jordan is applying yet more row operations to get to the rref of a matrix, which is unique.
The matlab command to produce a rref is `rref`.
(a) Apply the `rref` command to each of the matrices $M0$, $M1$, and $M2$.
(b) Considering the M_j matrices as augmented matrices of systems $A_j\boldsymbol{x} = \boldsymbol{b}_j$, without using any further matlab commands, write down the solutions (when they exist) of each of the systems.

2. Use the `A2\b2` command to verify the solution you found for this system in Question 1.

3. Define the matrix,
`A2I=[A2,eye(3)]`
Apply `rref` to $A2I$. Look at the result, and next run `inv(A2)` to find the inverse of $A2$. You should see this matrix at the right of the rref result.

4. By using the inverse of $A2$ solve the system of linear equations $A\boldsymbol{x} = \boldsymbol{b}$ by premultiplying $b2$ by the inverse of $A2$.

5. There are lots of properties of inverses.
(a) The inverse of an invertible lower triangular matrix is lower triangular. Verify this with the matrix,

$$L = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

(b) The inverse of an invertible upper triangular matrix is upper triangular. Verify this with the matrix,

$$U = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

(c) The inverse of the transpose of a matrix A is the transpose of A^{-1}. Look at the results of (a) and (b) to see an example of this.
(d) For invertible matrices A and B of the same size, $(A\,B)^{-1} = B^{-1}\,A^{-1}$. Verify this with the matrices L and U of this qustion, i.e. , $(L\,U)^{-1} = U^{-1}\,L^{-1}$.