Curtin University – Department of Computing

Assignment Cover Sheet / Declaration of Originality

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Question 1

a.

p	q	$p \vee q$	$p \oplus (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

∴ Since $p \oplus (p \vee q)$ is neither always true nor always false, it is a contingency.

(Liu 2020a)

b.

p	q	$p \to \neg q$	$p \wedge q$	$(p \to \neg q) \wedge (p \wedge q)$
T	T	F	T	F
T	F	T	F	F
F	T	T	F	F
F	F	T	F	F

∴ Since $(p \to \neg q) \wedge (p \wedge q)$ is always false, it is a contradiction.

(Liu 2020a)

c.

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

∴ Since $[(p \to 1) \wedge (q \to r)] \to (p \to r)$ is always true, it is a tautology.

(Liu 2020a)

Question 2

a. $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
$(\neg p \vee q) \wedge (\neg p \vee r) \equiv \neg p \vee (q \wedge r)$
$(\neg p \vee q) \wedge (\neg p \vee r) \equiv (\neg p \vee q) \wedge (\neg p \vee r)$

$\therefore (p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

(Liu 2020a)

b. $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$(p \rightarrow q) \wedge (q \rightarrow p) \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$(\neg p \vee q) \wedge (\neg q \vee p) \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$((\neg p \vee q) \wedge \neg q) \vee ((\neg p \vee q) \wedge p) \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$((\neg p \wedge \neg q) \vee (q \wedge \neg q)) \vee ((\neg p \wedge p) \vee (q \wedge p)) \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$(\neg p \wedge \neg q) \vee (q \wedge p) \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$(q \wedge p) \vee (\neg p \wedge \neg q) \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$(p \wedge q) \vee (\neg p \wedge \neg q) \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\therefore p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

(Gray 2014)

Question 3

p	q	r	$p \vee q$	$\neg p \vee q$	$p \vee r$	$q \vee r$	$q \vee \neg r$	$\neg q \vee \neg r$
T	T	T	T	T	T	T	T	F
T	T	F	T	T	T	T	T	T
T	F	T	T	F	T	T	F	T
T	F	F	T	F	T	F	T	T
F	T	T	T	T	T	T	T	F
F	T	F	T	T	F	T	T	T
F	F	T	F	T	T	T	F	T
F	F	F	F	T	F	F	T	T

$\therefore$ From the table, we notice that all 5 of the propositions to be can be made simultaneously true (when p is true, q is true and r is false)

Question 4

"A compound proposition is satisfiable if there is an assignment of truth values to the variables in the compound proposition that makes the statement form true (Rosen 2007, 30). Moreover, "A compound proposition that is always true, no matter what the truth values of the propositions that occur in it, is called a *tautology*." (Rosen 2007, 21). Therefore, in accordance with the above definitions, a compound proposition which is deemed to be a tautology is also satisfiable.

With the aforementioned in mind, in order to determine whether a given compound proposition is a tautology, such a program could, firstly, check whether the proposition is satisfiable. Thereafter, if the proposition is deemed unsatisfiable, it is by definition, also not a tautology and, therefore, program execution should stop. Otherwise, if the proposition is satisfiable, program execution should continue, checking if there is an assignment of truth values that makes the compound proposition false. If there is such an assignment, the proposition is merely satisfiable and program execution should stop. Otherwise, the proposition is tautological and by definition, satisfiable and since its tautology has been determined, program execution should also stop.

Question 5

a. $\neg p$
b. $p \wedge \neg q$
c. $p \rightarrow q$
d. $\neg p \rightarrow \neg q$
e. $p \rightarrow q$
f. $q \wedge \neg p$
g. $q \rightarrow p$

Question 6

a. Some student likes Chinese and all students like Mexican
b. Some cuisine is liked by Monica or Jay
c. Amongst all students, there is a cuisine that two students don't like
d. For some student, there is a cuisine that this student likes and this student doesn't like any other cuisine
e. For some student, there is another student where amongst all cuisines, both students like a particular cuisine or neither students like that cuisine
f. Amongst all students, there is a cuisine that two students like or neither students like

(Kenneth 2007)

Question 7

a. $\neg F(Ross, Rachel)$
b. $\neg\exists x(F(x, Joe))$
c. $\forall x(x \neq Monica \rightarrow A(x))$
d. $\exists x(\exists y(F(y, x))$
e. $\forall x(A(x) \rightarrow \exists y(F(x, y) \wedge x \neq y))$
f. $\exists x \exists y(\neg\exists z(F(x, z) \wedge F(y, z)))$

(Kenneth 2007)

Question 8

a. Let $P(x) =$ "x is mortal", where the domain of x is all men

$\forall x(P(x));$
$Joseph \in M;$
$\therefore P(Joseph)$

Valid, Universal Instantiation

(Liu 2020b)

b. Let $P(x) =$ "x is a man", where the domain of x is all men
Let $Q(x) =$ "x is an island", where the domain of x is all islands

$\forall x(P(x) \rightarrow \neg Q(x))$
$Q(Manhattan)$
$\therefore \neg P(Manhatten)$

Valid, Universal Instantiation and Modus Tollens

(Liu 2020b)

c. Let $P(x)$ = "x is an action movie"
Let $Q(x)$ = "Chandler likes movie x"
The domain of x is all movies

$\forall x(P(x) \rightarrow Q(x))$
$Q(Twelve\ Angry\ Men)$
$\therefore P(Twelve\ Angry\ Men)$

Invalid, Fallacy of Converse

(Liu 2020b)

d. Let $P(x)$ = "student x knows how to write programs in Python"
Let $Q(x)$ = "student x can get a high-paying job",
The domain of x is all students in Computing

$\forall x(P(x) \rightarrow Q(x))$
$P(Phoebe)$
$\therefore Q(Phoebe)$

$Q(Phoebe)$
$\therefore \exists x(Q(x))$

Valid, Modus Ponens and Existential Generalisation

(Liu 2020b)

e. Let $P(x)$ = "car x is a convertible"
Let $Q(x)$ = "car x is fun to drive"
The domain of x is all cars

$\forall x((P(x) \rightarrow Q(x))$
$\neg P(Isaac's\ car)$
$\therefore \neg Q(Isaac's\ car)$

Invalid, Fallacy of Inverse

(Liu 2020b)

f. Let a = “Superman is able to prevent evil”
Let w = “Superman is willing to prevent evil”
Let p = “Superman prevents evil”
Let i = “Superman is impotent”
Let m = “Superman is malevolent”
Let e = “Superman exists”

First, let’s denote the givens:

1. $(a \wedge w) \rightarrow p$
   $\neg(a \wedge w) \vee p$
   $\neg a \vee \neg w \vee p$
2. $\neg a \rightarrow i$
   $\neg(\neg a) \vee i$
   $a \vee i$
3. $\neg w \rightarrow m$
   $\neg(\neg w) \vee m$
   $w \vee m$
4. $\neg p$
5. $e \rightarrow (\neg i \wedge \neg m)$
   $\neg e \vee (\neg i \wedge \neg m)$
   $(\neg e \vee \neg i) \wedge (\neg e \vee \neg m)$

Second, let’s put it all together:

6. $\neg a \vee \neg w$, from 1, 4
7. $i \vee m$, from 2, 3, 6 (Resolution)
8. $\neg e$ from 5, 6 (Resolution)

∴ Since $\neg e \equiv$ "Superman does not exist”, this argument is valid.

(Rosen 2007)

Question 9

a. Show that $Odd(x) \wedge Odd(y) \rightarrow Even(x\ +\ y)$
   - Assume $Odd(x) \Leftrightarrow x \in \mathbb{Z} \wedge \exists m \in \mathbb{Z}(x = 2m + 1)$
   - Assume $Odd(y) \Leftrightarrow y \in \mathbb{Z} \wedge \exists n \in \mathbb{Z}(y = 2n + 1)$
   - $Even(z) \Leftrightarrow z \in \mathbb{Z} \wedge \exists o \in \mathbb{Z}(z = 2o)$

   Proof

   $z = x + y$
   $= (2m + 1) + (2n + 1)$
   $= 2m + 1 + 2n + 1$
   $= 2m + 2n + 2$
   $= 2(m + n + 1)$
   $= 2o$ where $o = m + n + 1$ and $o \in \mathbb{Z}$

   $\therefore$ the sum of two odd integers is even

   (Maths and Stats 2017)

b. i.e. Show that $Odd(n^3 + 5) \rightarrow Even(n)$
   - $p \rightarrow q \equiv {\sim}q \rightarrow {\sim}p$
   - $\therefore \neg Odd(n^3 + 5) \rightarrow \neg Even(n) \equiv Even(n^3 + 5) \rightarrow Odd(n)$
   - $Even(n^3 + 5) \Leftrightarrow (n^3 + 5) \in \mathbb{Z} \wedge \exists y \in \mathbb{Z}(n^3 + 5 = 2y)$
   - $Odd(n) \Leftrightarrow n \in \mathbb{Z} \wedge \exists x \in \mathbb{Z}(n = 2x + 1)$
   - Assume $Odd(n)$

   Proof

   $n^3 + 5 = (2x + 1)^3 + 5$
   $= (2x + 1)(2x + 1)(2x + 1) + 5$
   $= (4x^2 + 4x + 1)(2x + 1) + 5$
   $= (8x^3 + 8x^2 + 2x) + (4x^2 + 4x + 1) + 5$
   $= 8x^3 + 8x^2 + 2x + 4x^2 + 4x + 1 + 5$
   $= 8x^3 + 12x^2 + 6x + 6$
   $= 2(4x^3 + 6x^2 + 3x + 3)$
   $= 2y$
   where $y = 4x^3 + 6x^2 + 3x + 3$ and $y \in \mathbb{Z}$

   $Even(n^3 + 5)$ by definition
   Since ${\sim}q \rightarrow {\sim}p$, it follows that $p \rightarrow q$
   $\therefore Odd(n^3 + 5) \rightarrow Even(n)$

   (maths gotserved 2014)

c. i.e. Show that $Irrational(x) \wedge Rational(y) \rightarrow Irrational(x+y)$

- $Irrational(x) \Leftrightarrow x \in \mathbb{Z} \wedge \neg\exists m, n \in \mathbb{Z}\left(x = \frac{a}{b}\right)$
- $Rational(y) \Leftrightarrow y \in \mathbb{Z} \wedge \exists m, n \in \mathbb{Z}\left(y = \frac{m}{n}\right)$
- Assume $Irrational(x) \wedge Rational(y) \rightarrow \neg Irrational(x+y) \equiv Irrational(x) \wedge Rational(y) \rightarrow Rational(x+y)$
- $Rational(x+y) \Leftrightarrow (x+y) \in \mathbb{Z} \wedge \exists o, p \in \mathbb{Z}\left(y = \frac{o}{p}\right)$

Proof

$$x + \frac{m}{n} = \frac{o}{p}$$

$$x = \frac{o}{p} - \frac{m}{n}$$

$$= \frac{on - pm}{pn}$$

$$= \frac{a}{b}$$

where $a = on - pm$ and $b = pn$ and $a \wedge b \in \mathbb{Z}$

Since $Irrational(x)$ by definition

$\therefore Irrational(x) \wedge Rational(y) \rightarrow Irrational(x+y)$

(Kahn Academy 2013)

d. i.e. Show that $\forall n \in \mathbb{Z}(n \in D \wedge P(n))$, where $P(n) = n^2 + 1 \geq 2^n$ and $D = \{1, 2, 3, 4\}$

Proof

$P(1) \equiv (1)^2 + 1 \geq 2^{(1)}$
$\equiv 1 + 1 \geq 2$
$\equiv 2 \geq 2$
$\therefore P(1)$ is true

$P(2) \equiv (2)^2 + 1 \geq 2^{(2)}$
$\equiv 4 + 1 \geq 4$
$\equiv 5 \geq 4$
$\therefore P(2)$ is true

$P(3) \equiv (3)^2 + 1 \geq 2^{(3)}$
$\equiv 9 + 1 \geq 8$
$\equiv 10 \geq 8$
$\therefore P(3)$ is true

$P(4) \equiv (4)^2 + 1 \geq 2^{(4)}$
$\equiv 16 + 1 \geq 16$
$\equiv 17 \geq 16$
$\therefore P(4)$ is true

$\therefore \forall\, n \in \mathbb{Z}(n \in D \wedge P(n))$ is true

e. i.e. Show that $\neg\exists n \in \mathbb{Z}\big(n \in \mathbb{Z}^+ \wedge P(n)\big)$ where $P(n)$ denotes $n^2 + n^3 = 100$

Proof

$P(1): (1)^2 + (1)^3 = 100$
$= 1 + 1 = 100$
$= 2 = 100$
$= 2 \neq 100$
$\therefore \neg P(1)$ is true

$P(2): (2)^2 + (2)^3 = 100$
$= 4 + 8 = 100$
$= 12 = 100$
$= 12 \neq 100$
$\therefore \neg P(2)$ is true

$P(3): (3)^2 + (3)^3 = 100$
$= 9 + 27 = 100$
$= 36 = 100$
$= 36 \neq 100$
$\therefore \neg P(3)$ is true

$P(4): (4)^2 + (4)^3 = 100$
$= 16 + 64 = 100$
$= 80 = 100$
$= 80 \neq 100$
$\therefore \neg P(4)$ is true

$P(5): (5)^2 + (5)^3 = 100$
$= 25 + 125 = 100$
$= 150 = 100$
$= 150 \neq 100$
$\therefore \neg P(5)$ is true

$\therefore$ Since $P(n) \leq 80$ for $1 \leq n \leq 4$ and $P(n) \geq 150$ for $5 \leq n \leq \infty$, $\neg\exists n \in \mathbb{Z}(n \in \mathbb{Z}^+ \wedge P(n))$ is true

Question 10

a. $P(1)$ is the base step of $P(n)$.

b. $P(1): 1^2 = \frac{(1)\big((1)+1\big)(2(1)+1)}{6}$

$$P(1): 1 = \frac{(1)\big((1)+1\big)(2(1)+1)}{6}$$
$$= \frac{(2)(2+1)}{6}$$
$$= \frac{(2)(3)}{6}$$
$$= \frac{6}{6}$$
$$= 1$$

$\therefore P(1)$ is true

c. The inductive hypothesis is the implication $P(k) \rightarrow P(k+1)$

d. You need to prove that $P(k) \rightarrow P(k+1)$ is true

e. For $n > 0$, let $P(n)$ denote the statement

$$P(n) \equiv 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Base Step
View Question 10b

Inductive Step
For an arbitrary $k > 0$, assuming that $P(k)$ is true, it remains to prove that, $P(k+1)$, given below, holds.

$$P(k+1) \equiv 1^2 + 2^2 + 3^2 + \cdots + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$

Starting with the LHS of $P(k+1)$,

$$\begin{aligned} P(k+1) \equiv 1^2 + 2^2 + 3^2 + \cdots + k^2 + (k+1)^2 &= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \\ &= \frac{(k+1)(k+2)(2k+3)}{6} \end{aligned}$$

we see that the RHS of $P(k+1)$ follows.

$$\begin{aligned} P(k+1) &\equiv \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6} \\ &\equiv \frac{(k+1)(k+2)(2k+3)}{6} \end{aligned}$$

∴ By completing the inductive step, we have proven that $P(k+1)$ is true
∴ By mathematical induction, we have also proven that for any $n > 0$, the statement $P(n)$is true
i.e. We have proven that implication $P(k) \rightarrow P(k+1)$ is true

(Rosen 2007)

Question 11

For $n > 1$, let $P(n)$ denote the statement

$$P(n) \equiv 1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2} < 2 - \frac{1}{n}$$

Base Step

$$\begin{aligned}
P(2) &\equiv 1 + \frac{1}{4} < 2 - \frac{1}{2} \\
&\equiv 1 + \frac{1}{4} < 2 - \frac{1}{2} \\
&\equiv \frac{4}{4} + \frac{1}{4} < 2 - \frac{1}{2} \\
&\equiv \frac{4}{4} + \frac{1}{4} < \frac{4}{2} - \frac{1}{2} \\
&\equiv \frac{5}{4} < \frac{3}{2} \\
&\equiv \frac{5}{4} < \frac{6}{4} \\
&\equiv \frac{5}{4} < \frac{3}{2}
\end{aligned}$$

Inductive Step

For an arbitrary $k > 2$, assuming that $P(k)$ is true, it remains to prove that $P(k+1)$, given below, holds.

$$P(k+1) \equiv 1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{(k+1)^2} < 2 - \frac{1}{k+1}$$

Starting with the LHS of $P(k+1)$,

$$\begin{aligned}
P(k+1) \equiv 1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{k^2} + \frac{1}{(k+1)^2} &< 2 - \frac{1}{k} + \frac{1}{(k+1)^2} \\
&< 2 - \frac{1}{k+1}
\end{aligned}$$

we see that the RHS of $P(k+1)$ follows.

∴ By completing the inductive step, we have proven that $P(k+1)$ is true

∴ By mathematical induction, we have also proven that for any $n > 1$, the statement $P(n)$is true

i.e. We have proven that the implication $P(k) \to P(k+1)$ is true

(Rosen 2007)

Question 12

For $n \geq 8$, let $P(n)$ denote the statement "Postage of n cents can be formed using just 3-cent stamps and 5-cent stamps".

Base Steps

f. $P(8)$: $8 = 5(1) + 3(1)$
$8 = 5 + 3$
$8 = 8$
$\therefore P(8)$ is true

g. $P(9)$: $9 = 5(0) + 3(3)$
$9 = 0 + 9$
$9 = 9$
$\therefore P(9)$ is true

h. $P(10)$: $10 = 5(2) + 3(0)$
$10 = 10 + 0$
$10 = 9$
$\therefore P(10)$ is true

From the above, we notice above that $n = 5x + 3y$ where $x \geq 0$ and $y \geq 0$.

Inductive Step

For an arbitrary $k \geq 10$, assuming that $P(8)$, $P(9)$, $P(10)$, ..., $P(k)$ are true, show that $[P(8) \wedge P(9) \wedge P(10) \wedge \ldots \wedge P(k)] \rightarrow P(k+1)$.

Since we seek to show that $P(k+1)$ is true, we can use $P(k-2)$ in our proof, which was proven to be true by inductive hypothesis as $8 \leq k-2 \leq k$

$$P(k-2): k - 2 = 5x + 3y$$
$$= 5x + 3y$$

$$P(k+1): k - 2 + 3 = 5x + 3y + 3$$
$$k - 1 = 5x + 3y + 3$$
$$k - 1 = 5x + 3(y + 1)$$

Our base cases of $P(8)$, $P(9)$ and $P(10)$ can be used to form any $n \geq 11$ when a multiple of 3 is added.

e.g.

$8 + 3 = 11$

$9 + 3 = 12$

$10 + 3 = 13$

$8 + 6 = 14$

$9 + 6 = 15$

$10 + 6 = 16$

∴ For $n \geq 8$, postage of n-cents can be formed using just 3-cent stamps and 5-cent stamps.

(Rosen 2007)

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