WORKSHOP 11a

This workshop will build on material from Lecture 11: Euclidean Vector Spaces, Linear Dependence & Independence.

During this workshop, students will work towards the following learning outcomes:

- extend ideas from $\mathbb{R}^2$ and $\mathbb{R}^3$ to $\mathbb{R}^n$.
- identify subspaces of $\mathbb{R}^n$.
- determine whether a given vector is a linear combination of other vectors or not.
- establish the linear dependence or independence of a given set of vectors.

Euclidean vector spaces

1. Given the vectors $\boldsymbol{a} = [1, 2, 0, 2]$ and $\boldsymbol{b} = [-2, 0, 1, 1]$, find:

   (i) $\boldsymbol{a} + 2\boldsymbol{b}$

   (ii) The unit vector $\hat{\boldsymbol{b}}$

   (iii) A vector in the same direction as $\boldsymbol{b}$ but has the same length of $\boldsymbol{a}$

2. Given the points $A(2, 4, 3, -1, 1)$ and $B(3, 1, 1, 0, -2)$ in $\mathbb{R}^5$, find the distance between the points A and B.

3. For the vectors $\boldsymbol{a} = [4, 1, -2, 2]$ and $\boldsymbol{b} = [1, 0, 3, 2]$ determine the vector projection of $\boldsymbol{a}$ on $\boldsymbol{b}$.

4. Find the angle between the hyperplanes $2x_1 - x_2 - 2x_3 + x_4 = -1$ and $x_1 + 3x_2 - x_4 = 2$.

Vector subspaces

5. For each of the following sets of vectors, determine whether or not it is a subspace of $\mathbb{R}^3$, giving reasons for your answer.

   (i) $V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \mid x - 2y = 0 \right\}$ (ii) $U = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \mid x^2 = 2y \right\}$

   (iii) $W = \left\{ \begin{bmatrix} 2 \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \mid y, z \in \mathbb{R} \right\}$

Linear combinations

6. Let $\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\boldsymbol{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\boldsymbol{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \in \mathbb{R}^3$. Show that $\boldsymbol{w} = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$ is a linear combination of $\boldsymbol{v}_1$, $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$.

7. Let $\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\boldsymbol{v}_2 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$ and $\boldsymbol{v}_3 = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix} \in \mathbb{R}^3$. Show that $\boldsymbol{w} = \begin{bmatrix} 3 \\ -3 \\ 1 \end{bmatrix}$ can not be written as a linear combination of $\boldsymbol{v}_1$, $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$.

Linear dependence / independence

8. For each of the following sets of vectors, decide whether they are l.i. or l.d.

(i) $\left\{ \begin{bmatrix} -10 \\ 15 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \end{bmatrix} \right\}$ (ii) $\left\{ \begin{bmatrix} 7 \\ 3 \end{bmatrix}, \begin{bmatrix} 21 \\ 12 \end{bmatrix} \right\}$

(iii) $\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix} \right\}$ (iv) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$

(v) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \right\}$ (vi) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 4 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 6 \\ -2 \end{bmatrix} \right\}$