Lecture 3. Methods of Proof

Ref.: K H Rosen Section 1.5, 1.6, 1.7

Terminologies

THEOREM

A statement that can be shown to be true

PROOF

A sequence of statements that form an argument

AXIOMS/POSTULATES

Statements used in a proof

RULES OF INFERENCE

The means used to draw conclusions from other assertions…. Tie together the steps of a proof

Terminologies

LEMMA

A 'pre-theorem' needed to prove a theorem

COROLLARY

A 'post-theorem' which follows directly from a theorem

CONJECTURE

Statements whose truth value is unknown

Rule of Inference

ADDITION

Implication :

It is below freezing now. Therefore, it is either below freezing (p) or raining now(q).

Rule of Inference	Tautology	Name
$\dfrac{p}{\therefore (p \vee q)}$	$p \rightarrow (p \vee q)$	Addition

Rule of Inference

SIMPLIFICATION

Argument :

It is below freezing(p) and raining (q) now. Therefore, it is below freezing now.

Rule of Inference	Tautology	Name
$\dfrac{(p \wedge q)}{\therefore\ p}$	$(p \wedge q) \rightarrow p$	Simplification

Rule of Inference

MODUS PONENS

Implication :

If it snows today, then we will go skiing = T

Hypothesis :

It is snowing today = T

By modus ponens the conclusion of the implication

"We will go skiing" = T

Rule of Inference

Modus Ponens states that if both an implication and its hypothesis are known to be true, then the conclusion of this implication is true

Rule of Inference	Tautology	Name
p $p \rightarrow q$ $\therefore q$	$[p \wedge (p \rightarrow q)] \rightarrow q$	Modus Ponens

Rule of Inference

MODUS TOLLENS

Rule of Inference	Tautology	Name
$\neg q$ $p \to q$ $\therefore \neg p$	$[\neg q \wedge (p \to q)] \to \neg p$	**Modus Tollens**

p	q	$\neg q$	$p \to q$	$\ldots \wedge \ldots$	$\neg p$	$\ldots \to \ldots$
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T

Rule of Inference

HYPOTHETICAL SYLLOGISM

Argument :

If it rains today(p), then we will not have a BBQ today(q). If we do not have a BBQ today, then we will have a BBQ tomorrow(r). Therefore, if it rains today, then we will have a BBQ tomorrow.

Rule of Inference	**Tautology**	**Name**
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	**Hypothetical syllogism**

Rule of Inference

DISJUNCTIVE SYLLOGISM

Rule of Inference	Tautology	Name
$p \lor q$ $\neg p$ $\therefore q$	$[(p \lor q) \land \neg p] \rightarrow q$	Disjunctive syllogism

p	q	$p \lor q$	$\neg p$	… $\land$ …	… $\rightarrow$ …
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

Rule of Inference

Rule of Inference	Tautology	Name
$\dfrac{p}{\therefore (p \vee q)}$	$p \rightarrow (p \vee q)$	Addition
$\dfrac{(p \wedge q)}{\therefore\ p}$	$(p \wedge q) \rightarrow p$	Simplification
$\dfrac{\begin{array}{l} p \\ p \rightarrow q \end{array}}{\therefore q}$	$[\, p \wedge (p \rightarrow q)] \rightarrow q$	Modus Ponens
$\dfrac{\begin{array}{l} \neg q \\ p \rightarrow q \end{array}}{\therefore\ \neg p}$	$[\neg\, q \wedge (p \rightarrow q)] \rightarrow \neg\, p$	Modus Tollens
$\dfrac{\begin{array}{l} p \rightarrow q \\ q \rightarrow r \end{array}}{\therefore p \rightarrow r}$	$[(\, p \rightarrow q) \wedge (\, q \rightarrow r)] \rightarrow (\, p \rightarrow r)$	Hypothetical syllogism
$\dfrac{\begin{array}{l} p \vee q \\ \neg\, p \end{array}}{\therefore\ q}$	$[(\, p \vee q\,) \wedge \neg\, p] \rightarrow q$	Disjunctive syllogism

Rule of Inference: Example

Hypothesis:

It is not sunny this afternoon($\neg p$) and it is colder than yesterday(q). We will go swimming (r) only if it is sunny. If we do not go swimming then we take a canoe trip (s). If we take a canoe trip then we will be home by sunset(t).

Conclusion:

We will be home by sunset.

Hypothesis: $\neg p \wedge q \quad r \rightarrow p \quad \neg r \rightarrow s \quad s \rightarrow t$

Conclusion: t

$\neg p \wedge q \xrightarrow{\text{simplification}} \neg p$

$\neg p \xrightarrow{\text{modus tollens}} \neg r \xrightarrow{\text{modus ponens}} s \xrightarrow{\text{modus ponens}} t$

Rule of Inference for Quantified Statement

- **Universal instantiation**

  $\forall x P(x)$

  $\therefore P(c)$

- **Universal generalization**

  $P(c)$ for an arbitrary $c \in U$

  $\therefore \forall x P(x)$

Rule of Inference for Quantified Statement

- **Existential instantiation**

  $\exists x P(x)$

  $\therefore P(c)$ for some $c \in U$

- **Existential generalization**

  $P(c)$ for some $c \in U$

  $\therefore \exists x P(x)$

Argument: Definition

An argument is a sequence of statements; all but the last one are called premises (or assumptions or hypotheses); the final statement is called the conclusion.

Example 1

E.g. "if you have paid up in full then I'll deliver it"
"you have paid up in full"

$\therefore$ I'll deliver it ———————— valid

in form: $p \to d$;
p; — premises
$\therefore d$ — conclusion

p	d	$p \to d$
T	T	T
T	F	F
F	T	T
F	F	T

Example 2

E.g. "If it is sunny and hot then I'll be at the pool"
"It is sunny" and "It is not hot"
$\therefore$ "I will not be at the pool" ——— invalid

$s \wedge h \to p;$ $\longleftarrow (x)$
$s;$
$\neg h;$
$\therefore \neg p.$

invalid

Example 2 (cont.)

s h p	$s \wedge h$	x	$\neg h$	$\neg p$
T T T	T	T	F	F
T T F	T	F	F	T
T F T	F	T	T	$\boxed{F}$
T F F	F	T	T	$\boxed{T}$
F T T	F	T	F	F
F T F	F	T	F	T
F F T	F	T	T	F
F F F	F	T	T	T

Argument validity

- An argument form is valid means that no matter what particular statements are substituted for the statement symbols, if the resulting premises are all true, then the conclusion is also true
- In a valid arguments: the conclusion follows necessarily or inescapably or without doubt from the truth of its premises. With a valid argument, whenever all the premises are true, the truth of the conclusion is inferred or deduced from the truth of the premises.
- A valid argument need not be meaningful

Testing validity of an argument

Validity of an argument form can be tested using truth analysis

1. construct a truth table showing the truth values of all premises and the conclusion;
2. find all critical rows (those with all premises are true);
3. if in each critical row, the conclusion is also true, then the argument form is valid;
   if $\geq$**1** critical row has a corresponding false conclusion, then the form is invalid.

Example

$p \vee q \vee r$;
$\neg q$;
$\therefore p \vee r$

valid!

p	q	r	$p \vee q \vee r$	$\neg q$	$p \vee r$	
T	T	T	T	F	T	
T	T	F	T	F	T	
T	F	T	**T**	**T**	**T**	√
T	F	F	**T**	**T**	**T**	√
F	T	T	T	F	T	
F	T	F	T	F	F	
F	F	T	**T**	**T**	**T**	√
F	F	F	F	T	F	

Example

$$p \to (r \to q)$$
$$q \to p \wedge r;$$
$$\therefore p \to r$$

Note: use the equivalence

$$(p\wedge r)\to q \equiv p\to(r\to q)$$

$$\begin{aligned}(p\wedge r)\to q &\equiv \neg(p\wedge r) \vee q \\ &\equiv (\neg p \vee \neg r) \vee q \\ &\equiv \neg p \vee (\neg r \vee q) \\ &\equiv \neg p \vee (r\to q) \\ &\equiv p\to(r\to q)\end{aligned}$$

Example

$p\ \ q\ \ r$	$p \wedge q$	$p \wedge r$	$p \wedge q \to r$	$q \to p \wedge r$	$p \to r$
T T T	T	T	T	T	T
T T F	T	F	F	F	F
T F T	F	T	T	T	T
T F F	F	F	T	T	$\boxed{F}$
F T T	F	F	T	F	T
F T F	F	F	T	F	T
F F T	F	F	T	T	T
F F F	F	F	T	T	T

Counter-example

- A critical row with corresponding false conclusion is a counter example.

  A counter example symbolizes a situation whereby the conclusion does not necessarily follow from the premises, hence show the invalidity of the argument (i.e. conclusion cannot be inferred from the premises)

Examples

If the last digit of this number is 0, then this number is divisible by 10;

The last digit of this number is 0;

$\therefore$ This number is divisible by 10.

Valid!

Examples

If Zeus is human then Zeus is mortal;

Zeus is not mortal;

$\therefore$ Zeus is not human

Valid!

Examples

If this number is divisible by 6, then it is divisible by 2;

This number is not divisible by 6;

$\therefore$ this number is not divisible by 2.

Invalid!

Examples

For a particular number x, $x-3=0$ or $x+2=0$;

x is not negative;

$\therefore\ x=3$

Valid!

Universal Instantiation

If some property is true of everything in a domain, then it is true of any particular thing in the domain;

let *x*: a variable; *a*: a constant, $a \in D$ in the following statements accordingly:

Universal Instantiation: Valid argument forms

1. $\forall x \in D\ [P(x)]$;

   $a \in D$;

   $\therefore\ P(a)$

e.g. 'All human beings are mortal';

'John is a human being';

$\therefore$ "John is mortal".

Universal Instantiation: Valid argument forms

2. **Syllogism**: universal Modus Ponens

$$\forall x \in D\,[P(x) \rightarrow Q(x)]$$

major premises

$$P(a);$$

minor premises

$$\therefore Q(a).$$

e.g. 'The square of an even number is even';

'*k* is a particular number that is even';

$\therefore$ 'k^2 is even'

Universal Instantiation: Valid argument forms

3. **Syllogism**: universal Modus Tollens

$$\forall \mathbf{x} \in \mathbf{D}[\mathbf{P(x)} \to \mathbf{Q(x)}];$$

$$\neg Q(a);$$

$$\therefore \neg P(a).$$

e.g. $\forall x \in \text{people } [BorninPerth(x) \to BorninOz(x)];$

$$\neg BorninOZ(\text{ David });$$

$$\therefore \neg BorninPerth(\text{David}).$$

Validity of arguments with quantifiers

- ***Valid argument form*: no matter what particular predicates are substituted for the predicate symbols in its premises, if the resulting premises are all true, then the conclusion is also true.**

an argument is <u>valid</u> iff its <u>form</u> is valid

Some Invalid Argument forms

- Converse (quantified form)

$$\forall x \in D[P(x) \rightarrow Q(x)]$$
$$Q(a);$$
$$\therefore P(a).$$

Some Invalid Argument forms

- Inverse (quantified form)

$$\forall x \in D\left[P(x) \rightarrow Q(x)\right]$$

$$\neg P(a);$$

$$\therefore \neg Q(a).$$

Some Invalid Argument forms

- Inverse (quantified form)

  $\forall x \in D[P(x) \rightarrow Q(x)]$

  $\neg P(a)$;

  $\therefore \neg Q(a)$.

e.g. $\forall x \in$ students [$InCS(x) \rightarrow InCurtin(x)$];

$\neg InCS(x)$;

$\therefore \neg InCurtin(x)$.

Invalid

Some usage of errors

Suppose a doctor knows:

$\forall x \in \text{patients}\ [\ \mathit{Pneumonia}(x) \rightarrow (\ \mathit{Fever}(x)$

$\wedge\ \mathit{Chills}(x) \wedge \mathit{DeepCough}(x) \wedge \mathit{Miserable}(x)\ )$

$\wedge\ \mathit{ExceptionallyTired}(x)\ ]$ And

$(\ \mathit{Fever}(\text{Tim}) \wedge \mathit{Chills}(\text{Tim}) \wedge \mathit{DeepCough}(\text{Tim}) \wedge$

$\mathit{ExceptionallyTired}(\text{Tim}) \wedge \mathit{Miserable}(\text{Tim})\ ) \wedge$

$(\ \neg \mathit{Fever}(\text{Min}) \vee \neg \mathit{Chills}(\text{Min}) \vee$

$\neg \mathit{DeepCough}(\text{Min}) \vee \neg \mathit{Miserable}(\text{Min})$

$\vee\ \neg \mathit{ExceptionallyTired}(\text{Min}))$

Some usage of errors

He can conclude that Min does not have pneumonia; but he can only diagnose that there is strong possibility, **not with certainty**, that Tim has pneumonia.

Converse error are often used, with care, as reasoning tools for diagnosis, debugging, scientific experiments, guide for police investigation etc.

Proof strategy of propositional statements

- Assertion form: $\neg P$;
  to prove: establish P is false
- Assertion form: $P \wedge Q$;
  to prove: establish both P and Q are true
- Assertion form: $P \vee Q$;
  to prove: establish
  - P is true or Q is true OR
  - $(\neg P \rightarrow Q)$ is true OR
  - $(\neg Q \rightarrow P)$ is true

Proof of $P \rightarrow Q$

Four basic techniques; 3 of which are based on the truth table of $\rightarrow$ and the remaining one is based upon its contrapositive.

Proof of $P \to Q$

1. **Vacuous proof**: establish $\neg P$ is true;

E.g.

Prove $(\exists x \in \boldsymbol{Z}$ s.t. $x^2 = -1) \to (\forall y \in Prime, Even(y)) \equiv \boldsymbol{T}$

it's known $\boldsymbol{T} \equiv (\forall x \in \boldsymbol{Z}, x^2 \neq -1) \equiv \neg(\exists x \in \boldsymbol{Z}$ s.t. $x^2 = -1)$

$\therefore (\exists x \in \boldsymbol{Z}$ s.t. $x^2 = -1) \to (\forall y \in Prime, Even(y)) \equiv \boldsymbol{T}$

Proof of $P \to Q$

2. **Trivial proof**: used when Q can be established to be true;

E.g.

Given $\lfloor x \rfloor$, $x \in R$ is the largest integer $\leq x$

Prove $(\forall y \in Z, y^2 \neq -1) \to \forall x \in R, (\lfloor x - 1 \rfloor = \lfloor x \rfloor - 1)$

$$\forall x \in R, (\lfloor x - 1 \rfloor = \lfloor x \rfloor - 1)$$

$$\equiv \forall x \in Z, (\lfloor x - 1 \rfloor = \lfloor x \rfloor - 1) \wedge$$

$$\forall x \in R - Z, (\lfloor x - 1 \rfloor = \lfloor x \rfloor - 1)$$

Proof of $P \rightarrow Q$

(i) when ($x \in \boldsymbol{Z} \wedge x \in \boldsymbol{R}$), $(x\text{-}1) \in \boldsymbol{Z}$

by definition $\forall x \in \boldsymbol{Z}\ (\lfloor x \rfloor = x) \wedge (\lfloor x - 1 \rfloor = x - 1)$

$$\therefore \lfloor x - 1 \rfloor = x - 1 = \lfloor x \rfloor - 1$$

(ii) when ($x \notin \boldsymbol{Z} \wedge x \in \boldsymbol{R}$), $(x\text{-}1) \notin \boldsymbol{Z}$

let $x = w + z$; where $w \in \boldsymbol{Z} \wedge (0 < \mathrm{z} < 1)$

by definition, $\lfloor x \rfloor = \lfloor w + z \rfloor = w$ and

$$\lfloor x - 1 \rfloor = \lfloor w + z - 1 \rfloor = \lfloor w - 1 + z \rfloor = w - 1$$

$$\therefore \lfloor x - 1 \rfloor = w - 1 = \lfloor x \rfloor - 1$$

$$\therefore (\forall y \in \boldsymbol{Z}, y^2 \neq -1) \rightarrow \forall x \in \boldsymbol{R}, (\lfloor x - 1 \rfloor = \lfloor x \rfloor - 1)$$

Proof of $P \rightarrow Q$

3. **Direct proof:**

Begin by assuming P is true , then show that Q necessarily follows to be true.

E.g. Prove that the square of an even number is even; i.e. given $a \in \boldsymbol{Z}$, show $Even(a) \rightarrow Even(a^2) \equiv \boldsymbol{T}$

Assume $Even(a)$ is true, by definition:

$Even(a) \Leftrightarrow (a \in \boldsymbol{Z} \wedge (\exists k \in \boldsymbol{Z} \text{ s.t. } a = 2 \times k))$

thus $a^2 = a \times a = 2k \times 2k = 2 \times (2k^2)$

and $k \in \boldsymbol{Z}$ means $k^2 \in \boldsymbol{Z} \wedge (2 \times k^2 \in \boldsymbol{Z})$

$\therefore$ by definition: $Even(a^2)$ is true

Proof of $P \to Q$

E.g. Prove $x \wedge y \to x \wedge (w \vee y) \equiv \boldsymbol{T}$

suppose $x \wedge y \equiv$ $\boldsymbol{T}$; (assume P is true)

x; conjunctive simplification

y; conjunctive simplification

$w \vee y$; disjunctive addition

$x \wedge (w \vee y)$ conjunctive addition

(and Q follows necessarily)

$$\therefore x \wedge y \to x \wedge (w \vee y)$$

Proof of $P \rightarrow Q$

E.g. Prove $|x| < |y| \rightarrow x^2 < y^2;\ x \in R, y \in R$

Consider $x \in R \wedge y \in R \wedge (|x| < |y|)$

$$\Rightarrow |x|^2 = |x| \cdot |x| < |x| \cdot |y|$$

and $|x| \cdot |y| < |y| \cdot |y| = |y|^2$

hence $|x|^2 < |y|^2$

and $\forall z \in R,\ |z|^2 = z^2$

$$\therefore x^2 < y^2$$

Proof of $P \to Q$

4. **Contrapositive proof (Indirect proof):**

Establish $\neg Q \to \neg P$ using one of the 3 above techniques; e.g. assume $\neg Q$ is true, then show $\neg P$ necessarily follows.

Proof of $P \to Q$

E.g., Prove that "if 3n+2 is odd, then n is odd."

Assume "n is not odd", i.e., n is even.

Then n=2k for some integer k

$3n+2=3*(2k)+2=2(3k+1)$ is even.

Hence it has been proven that "if n is even, then 3n+2 is even", we can conclude that "if 3n+2 is odd, n is odd."

Proof by Contradiction

Suppose that the statement to be proven is false. Show that this supposition leads logically to a contradiction. Conclude that the statement to be prove is true.

Proof by Contradiction

E.g., Prove that "There is no greatest integer."

Suppose that there is a greatest integer N, i.e., N≥n for all integer n.
Let M=N+1. Obviously M is a integer and M>N. The existence of integer N+1 contradicts the supposition "N is the greatest integer". Hence the supposition is false.
Hence "There is no greatest integer." is proven.

Proof by Contradiction

E.g., Prove that "if 3n+2 is odd, then n is odd."

Suppose that "if 3n+2 is odd, then n is odd" is false. Hence 3n+2 is odd and n is not odd. [Remember $\neg(p \rightarrow q) \equiv \neg(\neg p \vee q) \equiv p \wedge \neg q$] Hence n is even. Then n=2k for some k. Then 3n+2=3(2k)+2=2(3k+1) which is even. Contradiction is resulted. Therefore, "if 3n+2 is odd, then n is odd" is true.

Proof of existential statements

Base on definition for $\exists x \in D$ s.t. $P(x)$, such a statement can be proven using a constructive proof or a non-constructive proof.

- **Constructive proof**

Find an x in D that makes $P(x)$ true or to give a set of directions for finding such an x.

E.g. Prove that $\exists x \in \boldsymbol{R}$ such that $Even(x) \wedge Prime(x)$ is true.

Proof: $x = 2$

Proof of existential statements

E.g. Prove that $\exists x \in \mathbf{Z}$ s.t. ($Even(x)$ and x can be written in two ways as a sum of two prime numbers)

Proof: $x = 10 = 5 + 5 = 3 + 7$

E.g. Given $r \in \mathbf{Z}$, $s \in \mathbf{Z}$, prove that $\exists k \in \mathbf{Z}$ s.t. $22r + 18s = 2k$ (is true)

Proof: Consider $r, s \in \mathbf{Z}$, let $k = 11r + 9s$ being sum & product of integers, k, $2k \in \mathbf{Z}$; and by distributive law of algebra:

$$2k = 2\times(11r + 9s) = 22r + 18s$$

Proof of existential statements

- **Non-constructive proof**

either

- show the existence of a value of *x* that makes *P*(*x*) true is guaranteed by axioms or proven theorems;

Or

- show that the assumption that there is no such *x* leads to a contradiction

Proof of universal statements

To prove $\forall x \in D[P(x)]$, we can use either the method of exhaustion or the method of generalizing from the generic particular

In both methods, it is likely to be easier by first converting the statement into the form $\forall x[D(x) \rightarrow P(x)]$ where $D(x)$ is true whenever $x \in D$

Proof of universal statements

- **Method of exhaustion**

  show $P(x)$ is true for every values in D;

E.g. Prove that: $\forall n \in \mathbf{Z}$ [$4 \le n \le 20 \wedge Even(n)$ $\rightarrow \exists a,b \in \mathbf{Z}$ s.t. $((n = a + b) \wedge a \in Prime \wedge b \in Prime)$]

$4=2+2$; $6=3+3$; $8=5+3$; $10=5+5$; $12=5+7$;
$14=11+3$; $16=5+11$; $18=11+7$; $20=13+7$;
all odd n between 4 & 20: $P(x)$ vacuously true

Method used for D with (small) finite elements

Proof of universal statements

- **Method of generalizing from the generic particular**

Universal generalization rule: if $P(x)$ is true for an x arbitrarily chosen from D, then one can assert that $\forall x \in D, [P(x)]$.

arbitrarily chosen $x \Leftrightarrow$ no special properties which are not also true of all other elements of D. (i.e. no special assumption about x)

Proof of universal statements

E.g. Show: $\forall x \in \boldsymbol{C}$ [$\exists y \in \boldsymbol{R}$ s.t. $x + \bar{x} = y$)];
where $\boldsymbol{C}$ is the set of all complex numbers

Consider an arbitrarily chosen $x \in \boldsymbol{C}$,

by definition of complex number:

$\exists a, b \in \boldsymbol{R}$ s.t. $x = a + bi$ and $\bar{x} = a - bi$

hence $x + \bar{x} = (a + bi) + (a - bi) = 2 \times a$

since $a \in \boldsymbol{R} \rightarrow 2a \in \boldsymbol{R}$ is a tautology;

$\therefore \forall x \in \boldsymbol{C}$ [$\exists y \in \boldsymbol{R}$ s.t. $x + \bar{x} = y$)]

Proof of universal statements

E.g. Show:

$$\forall i,j,k \in \mathbf{Z},\ k \neq 0\ [i \bmod k = 0 \wedge j \bmod k = 0 \rightarrow (i+j) \bmod k = 0]$$

Consider arbitrarily chosen $i,j,k \in \mathbf{Z}$,

$i \bmod k = 0 \wedge j \bmod k = 0$

$\exists m,n \in Z$ s.t. $i = mk \wedge j = nk$

$i+j = mk+nk = (m+n)k$ where $m+n \in \mathbf{Z}$

$(i+j) \bmod k = 0$

$\therefore$ Proven by direct proof

Disproof of universal statements by counter-examples

To prove that $\forall x \in D \cdot [P(x)]$ is <u>false</u>

$\equiv$ proving $\neg(\forall x \in D[P(x)])$ is true

i.e. prove $\exists x \in D$ s.t. $\neg P(x)$ is true

Just find a x such that $P(x)$ is false, this x is a counter-example

Disproof of universal statements by counter-examples

E.g. disprove $\forall x \in \boldsymbol{R}\ [x^2+10x+25 > 0]$;
i.e., prove $\exists x \in \boldsymbol{R}\ [x^2+10x+25 \leq 0]$

consider $x = -5$,

$$(x^2+10x+25) = (x+5)^2 \leq 0$$

$$\exists x \in \boldsymbol{R}\ [x^2+10x+25 \leq 0]$$

$$\therefore \neg(\forall x \in \boldsymbol{R}[x^2+10x+25 > 0])$$

Disproof of universal statements by counter-examples

E.g. disprove: $\forall a,b \in \boldsymbol{R}\ [\ a^2 = b^2 \rightarrow a = b\ ]$

i.e. prove $\exists a,b \in \boldsymbol{R}\ [\ a^2 = b^2 \wedge a \neq b\ ]$

consider a = -6, b = 6

$a^2 = b^2 = 36$

a ≠ b

$\exists a,b \in \boldsymbol{R}\ [\ a^2 = b^2 \wedge a \neq b\ ]$

$\therefore \neg \forall a,b \in \boldsymbol{R}\ [\ a^2 = b^2 \rightarrow a = b\ ]$

Fallacies

Fallacies: Incorrect reasoning

Fallacy of Affirming the Conclusion

Argument :

If you do every problem in this book (p), then you will learn discrete mathematics (q). You learned discrete mathematics. Therefore you did every problem in the book.

∴ Argument is of the form:

If $p \to q$ and q, then p

Fallacy of Affirming the Conclusion

Fallacy of affirming the conclusion	$[(p \rightarrow q) \wedge q] \rightarrow p$

NOT TRUE; false when p =F and q = T

Fallacy of Denying the Hypothesis

Argument :

If you are to graduate with a CS degree (p), you need to pass subject “Engineering Computing” (q). You are not to graduate with a CS degree. Therefore you do not need to pass “Engineering Computing”.

$\therefore$ Argument is of the form:

If $p \rightarrow q$ and $\neg p$, then $\neg q$

Fallacy of Denying the Hypothesis

Fallacy of denying the hypothesis	$[(p \to q) \wedge \neg\ p] \to \neg\ q$

NOT TRUE; false when p = F and q = T

Fallacy of Begging the Question

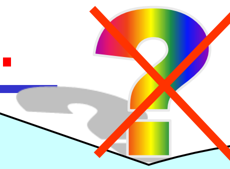
Argument :

If n^2 is even then n is also even.

n^2 is even, hence $n^2 = 2i$.

Let $n=2j$ for some integer j.

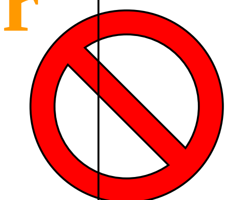
This shows that n is even.



Same as the statement being proven, i.e., "n is even"

Fallacy of Begging the Question

Begging the question	**Arises when a statement is proved using itself, or a statement equivalent to it.**



Also called Circular Reasoning

Summary

- Terminologies
- Rule of Inference
- Argument Validity

Summary

- Proof Strategies of propositional statements
- Proof of $P \to Q$
- Proof of existential statements
- Proof and disproof of universal statements
- Fallacies