Venue ____________________

Student Number |__|__|__|__|__|__|__|__|

Family Name ____________________

First Name ____________________

Curtin University

Faculty of Science and Engineering

EXAMINATION

End of Semester 2, 2018

MATH1019 Linear Algebra and Statistics for Engineers

This paper is for Bentley Campus and Miri Sarawak Campus students

This is a RESTRICTED BOOK examination

Examination paper IS to be released to student

Examination Duration 2 hours

Reading Time 10 minutes

Students may write notes in the margins of the exam paper during reading time

Total Marks 100

Supplied by the University

1 x 16 page answer book

Formula sheet (included with exam paper)

Supplied by the Student

Materials

One A4 sheet of handwritten or typed notes (both sides)

Calculator

A calculator displaying 'Engineering Approved Calculator' sticker

Instructions to Students

Attempt as many questions or part questions as possible.

SHOW ALL WORKING.

For Examiner Use Only

Q	Mark
1	
2	
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Total ________

Question 1

(a) Let $\boldsymbol{a} = [2, -1, 3]$, $\boldsymbol{b} = [4, 1, -2]$ and $\boldsymbol{c} = -3\boldsymbol{i} + \boldsymbol{k}$. Determine:

(i) A vector in the opposite direction to vector $\boldsymbol{a}$. (1 mark)

(ii) The length of $\boldsymbol{a}$. (1 mark)

(iii) $2\boldsymbol{a} - 3\boldsymbol{b}$. (2 marks)

(iv) The scalar projection of $\boldsymbol{b}$ on $\boldsymbol{a}$. (2 marks)

(v) Two unit vectors perpendicular to $\boldsymbol{b}$ and $\boldsymbol{c}$. (5 marks)

(b) Given the matrices,

$$A = \begin{bmatrix} 6 & -3 \\ 12 & -6 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 2 \\ 0 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ 0 & -1 \end{bmatrix}, \quad D = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

find the following, or briefly justify why it cannot be found,

(i) $A + 2B$. (2 marks)

(ii) $(BC^T)D$. (3 marks)

(iii) A^{-1}. (2 marks)

(iv) B^{-1}. (2 marks)

(A total of 20 marks for this question.)

QUESTION 2 IS ON THE FOLLOWING PAGE.

Question 2

(a) Three points, O, P and Q, are given such that $\overrightarrow{OP} = \boldsymbol{i} + 3\boldsymbol{j} + \alpha\boldsymbol{k}$ and $\overrightarrow{OQ} = -7\boldsymbol{i} + (1-\alpha)\boldsymbol{j} + \alpha\boldsymbol{k}$, where α is a constant.

(i) Find the unit vector in the direction of $\overrightarrow{OP}$ when $\alpha = -4$. (2 marks)

(ii) Determine the values of α for which $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{OQ}$. (4 marks)

(iii) Given the magnitudes of $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are x and y respectively, find the value of α if $y^2 = 2x^2$. (4 marks)

(b) The location of the vertices A, B, and C of a triangle are given by the position vectors $\overrightarrow{OA} = [1, 2, 3]$, $\overrightarrow{OB} = [2, 4, 1]$ and $\overrightarrow{OC} = [3, 5, -3]$.

(i) Find the exact value of the cosine of angle BAC, i.e. the angle at vertex A. (5 marks)

(ii) Find the area of triangle ABC. (5 marks)

(A total of 20 marks for this question.)

QUESTION 3 IS ON THE FOLLOWING PAGE.

Question 3
Given the point,

$$A(3, -1, 2)$$

The lines,

$$L_1 : \frac{x+1}{4} = \frac{y-4}{-8} = \frac{z}{3}$$

$$L_2 : x = 1 + 3t, \ y = 3t, \ z = 2 - t$$

And the planes,

$$P_1 : x + 3y - z = 7$$

$$P_2 : 2x - y + z = -2$$

$$P_3 : 3x + y + 2z = -2$$

Find the following:

(a) The parametric equations of a line which is perpendicular to the line L_1 and passes through the point A. (6 marks)

(b) The angle between the direction vector of the line L_2 and the normal vector to the plane P_1. (4 marks)

(c) The point at which the line L_2 intersects the plane P_2. (4 marks)

(d) The point of intersection of the planes P_1, P_2 and P_3. (6 marks)

(A total of 20 marks for this question.)

QUESTION 4 IS ON THE FOLLOWING PAGE.

Question 4

(a) Using the concept of determinant, determine whether the column vectors of the following matrix A are linearly dependent or linearly independent.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 2 & 1 \\ 4 & 3 & 2 \end{bmatrix}$$

(5 marks)

(b) Given the following system of homogeneous equations,

$$\begin{aligned} 2x_1 + x_2 + 2x_3 &= 0 \\ x_1 + x_2 + 3x_3 &= 0 \\ 4x_1 + 3x_2 + bx_3 &= 0 \end{aligned}$$

Write the augmented matrix of the above system and then use Gaussian Elimination to reduce the matrix into row echelon form. (4 marks)
Hence, using the concept of the rank of a matrix, determine the value(s) of b such that the system has:

(i) a trivial solution. (2 marks)

(ii) a non-trivial solution. (2 marks)

(c) The following system of linear equations has a non-trivial solution,

$$\begin{aligned} 5x_1 + 7x_2 + 3x_3 &= 4 \\ 3x_1 + 2x_2 + 26x_3 &= 9 \\ 7x_1 + 10x_2 + 2x_3 &= 5 \end{aligned}$$

Solve the system for x_1, x_2 and x_3, using an appropriate amount of parameters to describe the solutions. (7 marks)

(A total of 20 marks for this question.)

QUESTION 5 IS ON THE FOLLOWING PAGE.

Question 5

(a) Use the Gauss-Jordan method to either calculate the inverse A^{-1} of the matrix A or to show that A has no inverse, where

$$A = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & 3 \\ 5 & 6 & 0 \end{bmatrix}$$

(7 marks)

(b) For each of the following sets of vectors, determine whether or not it is a subspace of $\mathbb{R}^3$, giving reasons for your answer.

(i) $U = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \mid x + y = z;\ x, y, z \in \mathbb{R} \right\}$ (3 marks)

(ii) $V = \left\{ \begin{bmatrix} x \\ -2 \\ z \end{bmatrix} \in \mathbb{R}^3 \mid x, z \in \mathbb{R} \right\}$ (2 marks)

(c) For the set of data points $(-4, -1), (-2, 0), (2, 2)$ & $(4, 5)$ find the least squares line $y = a_0 + a_1 x$ by using the pseudoinverse. (8 marks)

(A total of 20 marks for this question.)

END OF EXAMINATION PAPER