CURTIN UNIVERSITY OF TECHNOLOGY (CRICOS number: 00301J)
Division of Engineering, Science and Computing
Department of Computing

Theoretical Foundations of Computer Science

(COMP3002 and COMP5001)

Work Sheet 5

AIM:

- To explore Turing machines.
- To determine the difference between Turing-decidable and Turing-recognizable problems.

You may undertake the work in this worksheet as a group activity; however each student is individually responsible for their own learning. The worksheet will not be submitted or marked, and no answers will be given directly. The questions in this sheet will be discussed in the tutorial in week 6 of semester, along with other questions of a similar nature.

ACTIVITY 1: Discussion Questions

It has been seen that a regular languages can only represent strings that don't involve counting an arbitrarily high number of characters.

a) If we can construct a Turing Machine that recognizes (but doesn't decide) the problem of finding a sub-string X in a binary string, does this mean that we can construct a Turing Machine that recognizes all binary strings that do not contain X?
b) Hilbert's axioms show (amongst other things) that the axioms of arithmetic are consistent. This means that anything proven using these axioms is correct. What does this mean for proofs by contradiction, which is used to prove something is not correct?
c) Hilbert's 10^{th} problem is to determine whether a polynomial has an integral root (*i.e.*, has a root that is an integer). The lecture notes give an algorithm but show that this cannot be a decider. Does this mean that the problem is not decidable?

ACTIVITY 2: Turing Machines

For each of the following problems, construct a Turing Machine that solves it. When a problem is decidable, show that the machine you have given is a decider.

a) Find all cliques of size 3 in a graph.
b) Accepting strings of the language of all binary strings of the form $0^i1^i0^i$ for some $i \geq 0$.
c) The problem 3-COLOR: {<G>: each vertex of G can be assigned one of three colours so that no two adjacent vertices have identical colours}.
d) Accepting strings of the language $0^i\#1^j$ where $i<j$.

End of Work Sheet 5