2018 - FCS Revision Exam

November 9, 2018

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Propositional Logics

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Represent the following statements in a mathematical logic with proper notations and meaningful predicate names:

- Some students love FCS.
- Not everyone loves FCS.
- No one loves FCS.
- Only 1 person in this room has the surname Smith.
- There is only one head of department in Department of Computing.
- Everyone loves everyone.
- Everyone only loves themselves.
- Everyone loves everyone except themselves.

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Mathematical Induction

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Prove using math induction:

- $3^n > n + 1, n \geq 2$
- $5^n + 2 < 6^n, n \geq 2$
- $2|(3^{4n} - 1), n \geq 0$

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Set Theory

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Consider set:

- $A_i = \{1, 2, 3, ..., i\},\ 1 \leq i \leq 100$
- $B_i = \{80, 79, 78, ..., i\},\ 10 \leq i \leq 80$
- $C_i = \{2, 4, 6, ..., 2i\},\ 1 \leq i \leq 100$

Evaluate:

- $\bigcup_{i=1}^{50} A_i$
- $\bigcap_{i=7}^{39} A_i$
- $\bigcup_{i=20}^{59} B_i$
- $\bigcap_{i=47}^{56} B_i$
- $\bigcup_{i=30}^{83} C_i$
- $\bigcap_{i=19}^{91} C_i$
- $P(A_2)$
- $P(C_3)$
- $P(\varnothing)$
- $P(\{\varnothing\})$

- $P(\{\varnothing, \{\varnothing\}\})$
- $|P(B_{35})|$
- $|P(C_{57})|$

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Relations
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Let A = {1, 2, 3, 4, 5}, B = {w, x, y, z}
For each set, derive:

- The relation which is reflexive and transitive.
- The relation which is NOT symmetric and NOT anti-symmetric, but transitive.
- The relation which is NOT reflexive, NOT symmetric, NOT anti-symmetric, NOT transitive.
- The relation which is a poset(partial order).
- Find an equivalence relationship R from
  - $A \times A$ and compute the equivalence class of $[3]_R$.
  - $B \times B$ and compute the equivalence class of $[\text{x}]_R$

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Discrete Probability
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You have an unbiased coin (50% change to land on either side). Calculate the probability of having:

- 1 head and 1 tail (any order) after flipping it 2 times.
- 4 tails after flipping it 4 times.
- at least 2 heads after flipping it 3 times.
- exactly 2 tails after flipping it 4 times, knowing that the first flip is head.

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Counting
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A class consists of 8 men and 6 women. Find the number of ways to form a group in the following cases:

- 7 people.
- 4 men and 3 women.
- 12 people, but 3 students have to be together (either all three in the group or not at all).
- 5 men and 3 women, but one of the men and one of the women are fighting, so they cannot be in the same group.

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Recursion
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- Find a recurrence relation and give initial conditions for the number of bit strings of length n that contain the pattern 000. How many such bit strings are there of length 5?
- Find a recurrence relation and give initial conditions for the number of bit strings of length n that contain the pattern 01. How many such bit strings are there of length 5?
- Find a recurrence relation and give initial conditions for the number of bit strings of length n that contain even number of zeros. How many such bit strings are there of length 5?

- Find a recurrence relation and give initial conditions for the number of bit strings of length n that contain exactly two of '1' bit. How many such bit strings are there of length 5?

- Find a recurrence relation and give initial conditions for the number of ways to climb stair with n steps if the person climbing the stair can take only two or three steps at a time. How many ways can this person climb a stair with 6 steps?

Tree

What is a definition of a tree? Draw a tree with the following degrees: (if impossible, justify the reason)
(The amount of the number determines the number of vertices)

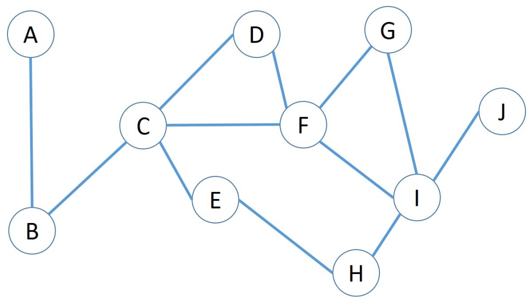
- 1, 2, 3
- 1, 2, 1
- 1, 1, 3
- 2, 2, 2, 1
- 1, 2, 1, 2
- 3, 1, 1, 1
- 1, 1, 1, 1, 1
- 3, 2, 2, 1, 1
- 1, 1, 4, 1, 1
- 1, 2, 3, 2, 1, 1
- 3, 2, 1, 2, 3, 1

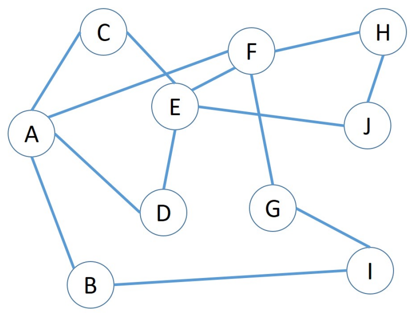
Path and Circuit

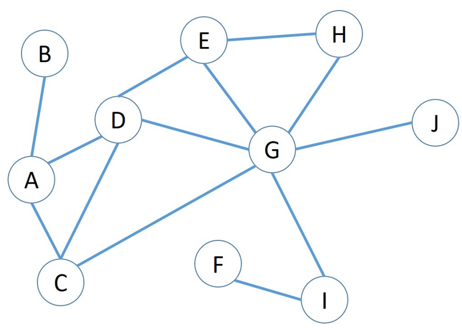
Look at the following 3 graphs:

- Is there any Euler circuit (draw it)? If not, why? How about Euler path (draw it)? If not, why?

- Is there any Hamilton circuit (draw it)? If not, why? How about Hamilton path (draw it)? If not, why?







The complete 3-partite graph $K_{n,m,p}$, with $n, m, p \geq 1$, is a simple graph that has its vertex set partitioned into 3 disjoint non-empty subsets of n, m and p vertices, respectively. Two vertices are adjacent if and only if they are in different subsets in the partition.

- Draw $K_{1,1,1}$, $K_{1,2,3}$, and $K_{3,3,3}$.
- What is the definition of Euler Circuit? What is the condition for it to exist?
- Give 3 examples of $K_{n,m,p}$ that has Euler circuit. What is the general formula to find value of n, m, p that still contains Euler circuit?