COMP1006 Foundations of Computer Science
Assignment 2, 2020
@ Computing, Curtin University

Weighting:
This assignment contains 13 questions, for a total of 100 points, which weights for 25% of the final mark.

Submission:
You can only submit **a single PDF** file containing neatly typed answers. **No photos or scans** are accepted. Name the file as `<studentID>_<name>_Assignment01.pdf`. Use the `Declaration_of_originality.pdf` as the cover page of your assignment. Submit your assignment via the **Turnitin** link on Blackboard. The due date is **23 October 2020 11:59 PM**.

Academic Integrity:
This is an **individual** assignment so that any form of collaboration is not permitted. This is an **open-book** assignment so that you are allowed to use external materials, but make sure you properly **cite the references**. It is your responsibility to understand Curtin's Academic Misconduct Rules, for example, post assessment questions online and ask for answers is considered as contract cheating and not permitted.

Set Theory

1. (8 points) Suppose that the universal set is $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Convert these sets to bit string representations (i-th bit in the string is 1 if i is in the set and 0 otherwise), or vice versa.
   (a) (2 points) $\{1, 3, 5\}$
   (b) (2 points) $\{2, 4, 5, 7, 8\}$
   (c) (2 points) 01 0110 1010
   (d) (2 points) 10 0000 1111
2. (8 points) Let $A = \{\emptyset, a, \{a, b\}\}, B = \{\emptyset, \{a\}, \{b\}\}$. Find
   (a) (2 points) $A \cap B$
   (b) (2 points) $A \cup B$
   (c) (2 points) $A \times B$
   (d) (2 points) $P(A)$
3. (8 points) Show that if A, B, C are sets, then $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$
   (a) (4 points) by showing that each side is a subset of the other side.
   (b) (4 points) using a membership table.

Relations

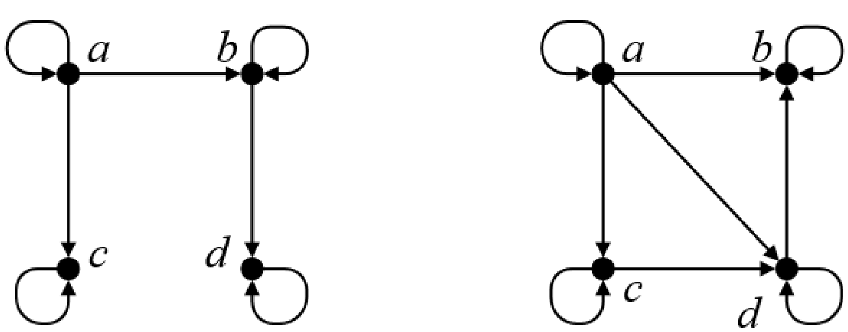
4. (8 points) Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ if and only if
   (a) (2 points) a is older than b.
   (b) (2 points) a and b were born on the same day.
   (c) (2 points) a has the same first name as b.
   (d) (2 points) a and b have a common grandparent.

5. (4 points) Let A be the set of students at Curtin and B be the set of books in the library. Let R_1 and R_2 be the relations consisting of all ordered pairs (a, b), where student a is required to read book b in a unit, and where student a has read book b, respectively. Describe the ordered pairs in each of these relations.
   (a) (1 point) $R_1 \cap R_2$
   (b) (1 point) $R_1 \oplus R_2$
   (c) (1 point) $R_1 - R_2$
   (d) (1 point) $R_2 - R_1$

6. (6 points) Let R be a relation on a set A with n elements. If there are k nonzero entries in $\mathbf{M}_R$, the matrix representing R,
   (a) (3 points) how many nonzero entries in $\mathbf{M}_{R^{-1}}$, the matrix representing R^{-1}, the inverse of R? Explain your reasoning.
   (b) (3 points) how many nonzero entries in $\mathbf{M}_{\overline{R}}$, the matrix representing $\overline{R}$, the complement of R? Explain your reasoning. (Note that: $\overline{R} = \{(a, b) \mid (a, b) \notin R\}$)

7. (5 points) Let R be a relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $a + d = b + c$.
   (a) (3 points) Show that R is an equivalence relation.
   (b) (2 points) What is the equivalence class of $(2, 1)$ with respect to R?

8. (6 points) Determine whether the relation represented by the digraph is a partial order. Explain your reasoning.



(a) **(b)**

Counting

9. (9 points) A palindrome is a string whose reversal is identical to the string. How many bit strings are palindromes if
   (a) (3 points) the bit string is of length n? Show your derivation.
   (b) (3 points) the bit string is of length 7 and contains two consecutive 0s? Show your derivation.
   (c) (3 points) the bit string is of length 8 and not containing three consecutive 1s? Show your derivation.

10. (9 points) 15 people are to be seated around two circular tables with 8 and 7 chairs, where seatings are considered to be the same if they can be obtained from each other by rotating the table.
    (a) (3 points) How many ways are there? Show your derivation.
    (b) (3 points) How many ways are there if two people have to sit next to each other? Show your derivation.
    (c) (3 points) How many ways are there if two people cannot sit in the same table? Show your derivation.

11. (9 points) Suppose that a password for a computer system must have at least 8 but no more than 12 characters, where each character in the password is a lowercase English letter, an uppercase English letter, a digit, or one of the six special characters $*, >, <, !, +, =$.
    (a) (3 points) How many different passwords are available for this computer system? If it takes one nanosecond for a hacker to check whether each possible password is your password, how long does it take to try every possible password? Show your derivation.
    (b) (3 points) How many of these passwords contain at least one of the six special characters? Show your derivation.
    (c) (3 points) How many of these passwords contain at least one uppercase letter, one lowercase letter, one digit, and one special character? Show your derivation.

Discrete Probability

12. (10 points) Suppose that the chance of having COVID-19 is 0.0001. There is an accurate test for COVID-19 that 99% of people with the disease test positive and only 0.2% who do not have the disease test positive.
    (a) (5 points) What is the probability that someone who tests positive has COVID-19? Show your derivation.
    (b) (5 points) What is the probability that someone who tests negative has COVID-19? Show your derivation.

13. (10 points) A space probe near Neptune communicates with Earth using bit strings. Suppose that in its transmissions it sends a 1 one-third of the time and 0 two-thirds of the time. But there are errors in the transmission. When a 0 is sent, the probability that it is received correctly is 0.8, and the probability that it is received incorrectly (as a 1) is 0.2. When a 1 is sent, the probability that it is received correctly is 0.9, and the probability that it is received incorrectly (as a 0) is 0.1.
    (a) (5 points) Calculate the probability that a 0 is received.
    (b) (5 points) Use Bayes' Theorem to find the probability that a 0 was transmitted, given that a 0 was received.