# MATH1019 Linear Algebra and Statistics for Engineers

## Workshop 2

**Learning outcomes for this session**
At the end of this session, you should be able to

1. Distinguish between discrete and continuous random variables.
2. Determine the mean and variance of random variables.
3. Define probability functions and probability density functions.
4. Recognise and apply some basic probability distributions.

**Overview**
We will work through and discuss exercises based on lecture 2.

**Exercises**

1. A car pooling study shows that the number of passengers, $X$, in a car (excluding the driver) is likely to assume the values 0,1,2,3 and 4 with probabilities given by the table

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X = x)$ | 0.7 | 0.1 | 0.1 | 0.05 | 0.05 |

   (a) Determine the probability of at least two passengers in a car.

   (b) Find the cumulative distribution function of $X$ and sketch it.

2. Consider a small post office with a single staff member operating a single postal counter. Suppose that the probability $p_k$, that there are $k$ customers in the post office, is given by $p_k = p_0 p^k, \quad k = 0, 1, 2, \ldots \qquad \text{where} \quad 0 < p < 1.$

   (a) Show that $p_0 = 1 - p$.

   (b) Determine the probability that a newly arriving customer has to wait to be served.

3. The pdf of random variable $X$ is given by

$$f(x) = \begin{cases} \frac{2}{9}(x+1), & -1 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

   (a) Sketch $f(x)$.

(b) Find the cdf of $X$ and sketch it.

(c) Calculate $P(X < 0.25)$ and show this on your two sketches.

4. The cdf of a random variable $Y$ is given by

$$F(y) = \begin{cases} 0 & \text{if } \; y < -1 \\ \frac{1}{2}(1+y) & \text{if } \; -1 \leq y < 0 \\ \frac{1}{2}(1+y^2) & \text{if } \; 0 \leq y < 1 \\ 1 & \text{if } \; y \geq 1 \end{cases}$$

(a) Find the pdf of $Y$.

(b) Sketch $f(y)$ and $F(y)$.

(c) Calculate $P(Y \leq 0.8)$ and show this on your two sketches.

5. Refer to Question 1. Calculate (i) $E(X)$ (ii) $E(X^2)$ (iii) $Var(X)$

6. Using the pdf in Question 3, find: $E(X)$ and $E(X^2)$, and hence $Var(X)$.

7. Suppose that an antique jewellery dealer is interested in purchasing a gold necklace for which the probabilities are 0.22, 0.36, 0.28, and 0.14, respectively, that she will be able to sell it for a profit of \$250, sell it for a profit of \$150, break even, or sell it for a loss of \$150. What is the expected profit?

8. The density function of the continuous random variable $X$, the total number of hours, in units of 100 hours, that a family runs a vacuum cleaner over a period of one year, is given by

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 \leq x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

Find the average number of hours per year that families run their vacuum cleaners.

9. Let $X$ be a random variable with the following probability distribution:

| $x$ | -3 | 6 | 9 |
|---|---|---|---|
| $f(x)$ | 1/6 | 1/2 | 1/3 |

Find $\mu_{g(X)}$, where $g(X) = (2X + 1)^2$.

10. Suppose that the probabilities are 0.4, 0.3, 0.2, and 0.1, respectively, that 0, 1,2, or 3 power failures will strike a certain subdivision in any given year. Find the mean and variance of the random variable $X$ representing the number of power failures striking the subdivision.

11. According to a survey by the Administrative Management Society, one-half of U.S. companies give employees four weeks of vacation after they have been with the company for 15 years. Find the probability that among 6 companies surveyed at random, the number that give employees 4 weeks of vacation after 15 years of employment is

(a) anywhere from 2 to 5.
(b) fewer than 3.

12. A manufacturer knows that on the average 20% of the electric toasters which he makes will require repairs within 1 year after they are sold. When 20 toasters are randomly selected, find the appropriate numbers $x$ and $y$ such that

    (a) The probability that at least $x$ of them will require repairs is less than 0.5.
    (b) The probability that at least $y$ of them will *not* require repairs is greater than 0.8.

13. The average number of field mice per acre in a 5-acre wheat field is estimated to be 12. Find the probability that fewer than 7 field mice are found

    (a) on a given acre;
    (b) on two of the next 3 acres inspected.

14. Changes in airport procedures require considerable planning. Arrival rates of aircraft is an important factor that must be taken into account. Suppose small aircraft arrive at a certain airport, according to a Poisson process, at the rate of 6 per hour. Thus the Poisson parameter for arrivals for a period of $t$ hours is $\lambda = 6t$.

    (a) What is the probability that exactly 4 small aircraft arrive during a 1-hour period?
    (b) What is the probability that at least 4 arrive during a 1-hour period?
    (c) If we define a working day as 12 hours, what is the probability that at least 75 small aircraft arrive during a day?

15. Given a standard normal distribution, find the value of $k$ such that

    (a) $P(Z < k) = 0.0427$
    (b) $P(Z > k) = 0.2946$
    (c) $P(-0.93 < Z < k) = 0.7235$

16. Given a standard normal distribution, find the area under the curve which lies

    (a) to the left of $z = 1.43$
    (b) to the right of $z = -0.89$
    (c) between $z = -2.16$ and $z = -0.65$
    (d) to the left of $z = -1.39$
    (e) to the right of $z = 1.96$
    (f) between $z = -0.48$ and $z = 1.74$

17. A soft drink machine is regulated so that it discharges an average of 200 millilitres per cup. If the amount of drink is normally distributed with a standard deviation equal to 15 millilitres,

    (a) what fraction of the cups will contain more than 224 millilitres?

(b) what is the probability that a cup contains between 191 and 209 millilitres?

(c) how many cups will probably overflow if 230 millilitre cups are used for the next 1000 drinks?

(d) below what value do we get the smallest 25% of the drinks?

18. The fracture strengths of a certain type of glass average 14 (thousands of pounds per square inch) and have standard deviation of 2.

    (a) What is the probability that the average fracture strength for 100 pieces of this glass exceeds 14.5?

    (b) Find the interval that includes the average fracture strength for 100 pieces of this glass with probability 0.95.