2018 - FCS Revision Quiz 2

October 18, 2018

Math Induction

1. $2^n \leq n!$, for $n \geq 4$
2. $5|6^n + 4$, for $n \geq 0$
3. $3|5^n + 2 \times 11^n$, for $n \geq 0$

(Note: $a|b$ means **a** divides **b**, OR **b** is divisible by **a**)

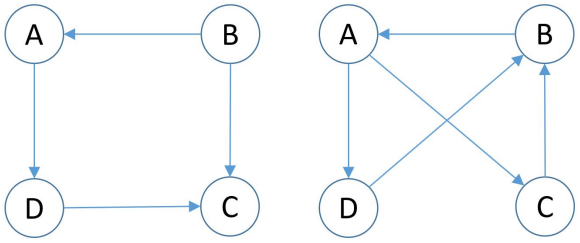
Set Theory

$A = \{\}$, $B = \{\varnothing\}$, $C = \{\varnothing, \{\varnothing\}\}$, $D = \{\varnothing, \{\varnothing, \varnothing\}, \{\{\varnothing\}\}\}$

1. $\varnothing \in A$, *true or false, why* ?
2. $|A|$?
3. $|D|$?
4. $|A \times B|$?
5. $|B \times C|$?
6. $B \cap C$
7. $C \cap D$
8. $C \cup D$
9. $P(A \times B)$?
10. $P(B)$?
11. $P(C)$?
12. $P(D)$?

Relations

1. Describe the definition of reflexive, symmetric, anti-symmetric, and transitive with its corresponding logic formula.
2. Prove that relation "|" in domain $\mathbb{Z}^+$ is a poset (partial order).
3. Prove that relation "has at least as many pages as ($\geq$)" in domain "All books" is NOT a poset.
4. Draw R^2 and R^3 of the following figures



Probability

1. You saw someone throw a six-faced dice into a box.
   - What is the probability of the dice showing 5?
   - What is the probability of the dice showing a number less than 4?

- That person told you "the number is even". What is the probability of the dice showing 6?
- That person told you "the number is not a prime number". What is the probability of the dice showing 1?

2. Suppose there is a family with 3 children, but you don't know their gender.
   - What is the probability of the family having 2 girls?
   - What is the probability of the family having at least 1 boy?
   - You knock at the door of their house. The one who opens the door is a girl (assuming it is one of their children). What is the probability of the family having 3 daughters?

Counting

1. There are 20 students in a class. Suppose 12 students are male and 8 are female. Provide more than 1 possible solution for following questions.
   (a) How many groups of seven can be chosen that contains at least 2 females?
   (b) How many groups of seven can be chosen that contains at least 2 males?
2. How many six-digit integers (integers from 100000 to 999999) are divisible by 2?

Recurrence Relations

1. Find a recurrence relation and give initial conditions for the number of bit strings of length n that contain the pattern 111. How many such bit strings are there of length 6?
2. Find a recurrence relation and give initial conditions for the number of bit strings of length n that contain the pattern 10. How many such bit strings are there of length 6?
3. Find a recurrence relation and give initial conditions for the number of bit strings of length n that contain odd number of ones. How many such bit strings are there of length 6?
4. Find a recurrence relation and give initial conditions for the number of ways to climb stair with n steps if the person climbing the stair can take only one or three steps at a time. How many ways can this person climb a stair with 6 steps?