WORKSHOP 6

This workshop will build on material from Lecture 6: Vectors & Introduction to Matrices.

During this workshop, students will work towards the following learning outcomes:

- calculate the cross product of two vectors, and use it to find areas and triple products.
- compute the sum, product, and transpose of matrices.
- identify properties of inverse matrices.

Cross product and applications

1. If $\boldsymbol{a} = 2\boldsymbol{i} - \boldsymbol{j}$ and $\boldsymbol{b} = \boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}$ find $\boldsymbol{a} \times \boldsymbol{b}$, then verify that $\boldsymbol{a} \times \boldsymbol{b}$ is orthogonal to $\boldsymbol{b}$.

2. Find the area of the triangle PQR determined by the points $P(1, -1, 2)$, $Q(2, 0, -1)$ and $R(0, 2, 1)$.

3. Find the area of the parallelogram formed by the two vectors $\boldsymbol{u}$ and $\boldsymbol{v}$, if $||\boldsymbol{u}|| = 16$, $||\boldsymbol{v}|| = 4$ and the cosine of the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is $\frac{1}{2}$.

4. Show that the vectors $\boldsymbol{a} = [1, 2, -1]$, $\boldsymbol{b} = [-2, 0, 3]$ and $\boldsymbol{c} = [2, -4, -4]$ are coplanar.

Matrix algebra

5. Given the following matrices

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$$

compute each of the following operations if it is defined. If an expression is undefined, explain why.

(i) $A + B$ (ii) $-4B$ (iii) AC (iv) CB (v) AB^T (vi) $C - 3I_2$ (vii) C^2

6. If a matrix A is 6×4 and the product AB is 6×8, what is the order (dimensions) of B?

7. How many rows does B have if BC is a 4×3 matrix?

8. Let $A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$. What value(s) of k, if any, will make $AB = BA$.

Inverse matrices

9. Verify that A and B are the inverse of one another, if $A = \begin{bmatrix} 2 & 5 \\ -3 & -7 \end{bmatrix}$ and $B = \begin{bmatrix} -7 & -5 \\ 3 & 2 \end{bmatrix}$.

10. Suppose that A and B are two square matrices such that $AB = 0$. Show that we must have $B = 0$ if A is invertible.