# Curtin University

**MATH1019 Linear Algebra and Statistics for Engineers**

Mid-Semester Test, Sem 2 2018; Time Allowed: **1** Hour + **5** minutes reading time

This paper contains 10 pages (including this cover sheet), 5 questions, worth a total of 45 marks

*Write your answers in the spaces provided. Write your name and student number on this cover sheet. If pages become separated write your name on all separated sheets. A blank page is attached should you require additional space, however if you need more paper than this, please ask.*

**NAME:**________________________________________

**STUDENT NUMBER:**__________________________________

**Please circle your workshop tutor and corresponding workshop time:**

| | | |
|---|---|---|
| Song Wang: | Monday 12–2pm | |
| Shuang Li: | Monday 12–2pm | Tuesday 8–10am |
| Muhammad Kamran: | Monday 4–6pm | Tuesday 1–3pm |
| | Tuesday 4–6pm | Thursday 1–3pm |
| Shican Liu: | Tuesday 8–10am | Thursday 8–10am |
| Mikhail Dokuchaev: | Tuesday 3–5pm | Thursday 4–6am |
| | Friday 4–6pm | |
| Grant Keady: | Wednesday 8–10am | |

**Question 1.** Ten students have just had a statistics test and their results are

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 75 | 66 | 49 | 23 | 89 | 55 | 60 | 72 | 80 | 62 |

(a) Find the mean and standard deviation of their grades. (2 marks)

(b) Find the lower quartile $Q_1$, median $Q_2$ and upper quartile $Q_3$ of the above data set.

(6 marks)

(c) Draw the boxplot of the above data set and indicating clearly the whiskers and outlier(s), if any. (4 marks)

**Question 2.** A survey result shows that hotline service receives on average 4 incoming phone calls in one period.

(a) Let $X$ be a random number representing the number of incoming phone calls. What probability distribution does $X$ satisfy? (1 mark)

(b) Find the probability that the hotline service receives 2 or fewer incoming calls in one period, i.e. $P(X \leq 2)$. (3 marks)

(c) What is the probability that the service receives 2 or fewer incoming calls in 2 consecutive periods? (4 marks)

(d) Using the probability found in part (b), find the probability that any three of the five chosen periods have 2 or fewer incoming calls. (4 marks)

**Question 3.** You are asked by your boss to design a game having three possible outcomes $-1$ (loss), $0$ (break-even) and $1$ (win) for its player with probabilities $p_1$, $p_2$ and $p_3$ respectively. Your boss would like to see that the expected value and variance of the random outcome are respectively $-0.1$ and $0.8$ (non-fair game). How do you choose $p_1$ and $p_3$ if $p_2 = 0.19$? (6 marks)

**Question 4.** Curtin University claims that the mean salary of its graduates is \$67K (\$67,000). You think Curtin over-estimates the average salary with the aim of attracting more students. You then conduct a survey by randomly sampling 5 Curtin's graduates' salaries and find that the mean salary is $\bar{x} = \$55\text{K}$ with the standard deviation $s = \$3\text{K}$.

(a) Perform a test of hypothesis at the 5% significance level with the intent to prove that Curtin University over-estimates the average salary. (6 marks)

(b) If we accept Curtin University's claim of the graduate's mean salary $\mu = \$67\text{K}$ with the standard deviation $\sigma = \$1\text{K}$, how large a sample is required if we want a 95% confidence interval for the mean $\mu$ to have a margin of error of $\pm 0.05$? (4 marks)

**Question 5.** You are an engineer of a bottled water supplier and you've been asked to set up a machine which fills 500ml water into a bottle. It is known that standard deviation of the volume filled into a bottle by the machine is $\sigma = 20$ml. From time to time The Australian Competition & Consumer Commission (ACCC) randomly chooses 30 bottles of your products and calculates the average volume $\bar{x}$ of the bottle contents. What is the minimum mean water volume $\mu$ in a bottle filled by the machine you need to set in order that $P(\bar{x} > 500\text{ml}) > 0.8$? (5 marks)

**END OF TEST PAPER**

**Additional working space if required**

## Additional working space if required

## Cumulative probabilities for the Standard Normal distribution

$P(Z \leq z)$ where $Z \sim \mathrm{N}(0, 1)$

| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| **−3.4** | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0002 | 0.0002 |
| **−3.3** | 0.0005 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0003 |
| **−3.2** | 0.0007 | 0.0007 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0005 | 0.0005 | 0.0005 |
| **−3.1** | 0.0010 | 0.0009 | 0.0009 | 0.0009 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0007 | 0.0007 |
| **−3.0** | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |
| **−2.9** | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| **−2.8** | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| **−2.7** | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| **−2.6** | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| **−2.5** | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| **−2.4** | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0066 | 0.0064 |
| **−2.3** | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| **−2.2** | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| **−2.1** | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| **−2.0** | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| **−1.9** | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| **−1.8** | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| **−1.7** | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| **−1.6** | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| **−1.5** | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |
| **−1.4** | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| **−1.3** | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| **−1.2** | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| **−1.1** | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| **−1.0** | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| **−0.9** | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
| **−0.8** | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| **−0.7** | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| **−0.6** | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| **−0.5** | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| **−0.4** | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
| **−0.3** | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 |
| **−0.2** | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
| **−0.1** | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 |
| **−0.0** | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 |
| **0.0** | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| **0.1** | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| **0.2** | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| **0.3** | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| **0.4** | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| **0.5** | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| **0.6** | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| **0.7** | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| **0.8** | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| **0.9** | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| **1.0** | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| **1.1** | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| **1.2** | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| **1.3** | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| **1.4** | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| **1.5** | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| **1.6** | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| **1.7** | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| **1.8** | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| **1.9** | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| **2.0** | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| **2.1** | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| **2.2** | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| **2.3** | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| **2.4** | 0.9918 | 0.9920 | 0.9922 | 0.9924 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| **2.5** | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| **2.6** | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| **2.7** | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| **2.8** | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| **2.9** | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| **3.0** | 0.9986 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| **3.1** | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| **3.2** | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| **3.3** | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 |
| **3.4** | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |

## Critical points of the $t$-distribution

Entry is $t$ where $P(T \geq t) = p$ for $t$-distribution with $\nu$ degrees of freedom

| $\nu$ \| $p$ | 0.1 | 0.05 | 0.025 | 0.01 | 0.005 | 0.001 | 0.0005 |
|---|---|---|---|---|---|---|---|
| **1** | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 318.309 | 636.619 |
| **2** | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 | 31.599 |
| **3** | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.215 | 12.924 |
| **4** | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.610 |
| **5** | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | 6.869 |
| **6** | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | 5.959 |
| **7** | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.785 | 5.408 |
| **8** | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 | 5.041 |
| **9** | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 | 4.781 |
| **10** | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 | 4.587 |
| **11** | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | 4.437 |
| **12** | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 | 4.318 |
| **13** | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 | 4.221 |
| **14** | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | 4.140 |
| **15** | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | 4.073 |
| **16** | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 | 4.015 |
| **17** | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 | 3.965 |
| **18** | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 | 3.922 |
| **19** | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 | 3.883 |
| **20** | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 | 3.850 |
| **21** | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 | 3.819 |
| **22** | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 | 3.792 |
| **23** | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 | 3.768 |
| **24** | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 | 3.745 |
| **25** | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 | 3.725 |
| **26** | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 | 3.707 |
| **27** | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 | 3.690 |
| **28** | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 | 3.674 |
| **29** | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 | 3.659 |
| **30** | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 | 3.646 |
| **31** | 1.309 | 1.696 | 2.040 | 2.453 | 2.744 | 3.375 | 3.633 |
| **32** | 1.309 | 1.694 | 2.037 | 2.449 | 2.738 | 3.365 | 3.622 |
| **33** | 1.308 | 1.692 | 2.035 | 2.445 | 2.733 | 3.356 | 3.611 |
| **34** | 1.307 | 1.691 | 2.032 | 2.441 | 2.728 | 3.348 | 3.601 |
| **35** | 1.306 | 1.690 | 2.030 | 2.438 | 2.724 | 3.340 | 3.591 |
| **36** | 1.306 | 1.688 | 2.028 | 2.434 | 2.719 | 3.333 | 3.582 |
| **37** | 1.305 | 1.687 | 2.026 | 2.431 | 2.715 | 3.326 | 3.574 |
| **38** | 1.304 | 1.686 | 2.024 | 2.429 | 2.712 | 3.319 | 3.566 |
| **39** | 1.304 | 1.685 | 2.023 | 2.426 | 2.708 | 3.313 | 3.558 |
| **40** | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 | 3.551 |
| **41** | 1.303 | 1.683 | 2.020 | 2.421 | 2.701 | 3.301 | 3.544 |
| **42** | 1.302 | 1.682 | 2.018 | 2.418 | 2.698 | 3.296 | 3.538 |
| **43** | 1.302 | 1.681 | 2.017 | 2.416 | 2.695 | 3.291 | 3.532 |
| **44** | 1.301 | 1.680 | 2.015 | 2.414 | 2.692 | 3.286 | 3.526 |
| **45** | 1.301 | 1.679 | 2.014 | 2.412 | 2.690 | 3.281 | 3.520 |
| **46** | 1.300 | 1.679 | 2.013 | 2.410 | 2.687 | 3.277 | 3.515 |
| **47** | 1.300 | 1.678 | 2.012 | 2.408 | 2.685 | 3.273 | 3.510 |
| **48** | 1.299 | 1.677 | 2.011 | 2.407 | 2.682 | 3.269 | 3.505 |
| **49** | 1.299 | 1.677 | 2.010 | 2.405 | 2.680 | 3.265 | 3.500 |
| **50** | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 | 3.261 | 3.496 |
| **60** | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 | 3.460 |
| **80** | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 | 3.195 | 3.416 |
| **100** | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 | 3.174 | 3.390 |
| **200** | 1.286 | 1.652 | 1.972 | 2.345 | 2.601 | 3.131 | 3.340 |
| **500** | 1.283 | 1.648 | 1.965 | 2.334 | 2.586 | 3.107 | 3.310 |
| $\infty$ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 | 3.291 |