WORKSHOP 8

This workshop will build on material from Lecture 8: More on Linear Systems & Inverses.

During this workshop, students will work towards the following learning outcomes:

- solve a homogeneous system of equations by performing Gaussian elimination.
- solve a system of equations by using the Gauss Jordan method.
- calculate the inverse (if it exists) of a square matrix.
- solve a system of equations by using the inverse of the coefficient matrix.

Systems of equations using matrices

1. For the following homogeneous systems of linear equations use the Gaussian Elimination method and the concept of rank of a matrix to solve the system of equations.

(i) $$\begin{aligned} 4x_1 + 3x_2 &= 0 \\ -2x_1 + x_2 &= 0 \end{aligned}$$

(ii) $$\begin{aligned} 2x_1 + 3x_2 &= 0 \\ 6x_1 + 9x_2 &= 0 \end{aligned}$$

(iii) $$\begin{aligned} 3x_1 + 5x_2 - 4x_3 &= 0 \\ -3x_1 - 2x_2 + 4x_3 &= 0 \\ 6x_1 + x_2 - 8x_3 &= 0 \end{aligned}$$

2. Solve the following systems of linear equations by using the Gauss Jordan method to manipulate the augmented matrix into reduced row echelon form.

(i) $$\begin{aligned} -2x_1 + 3x_2 &= 13 \\ 4x_1 + 2x_2 &= -2 \end{aligned}$$

(ii) $$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 6 \\ 2x_1 - 3x_2 + 2x_3 &= 14 \\ 3x_1 + x_2 - x_3 &= -2 \end{aligned}$$

Inverses of matrices

3. For the following matrices find the inverse of the matrix, if it exists.

(i) $A = \begin{bmatrix} 2 & -3 \\ 6 & -9 \end{bmatrix}$ (ii) $B = \begin{bmatrix} 2 & 5 \\ -3 & -7 \end{bmatrix}$ (iii) $C = \begin{bmatrix} -4 & -8 \\ -2 & -3 \end{bmatrix}$

(iv) $D = \begin{bmatrix} 5 & 0 & -1 \\ 1 & -3 & -2 \\ 0 & 5 & 3 \end{bmatrix}$ (v) $E = \begin{bmatrix} 1 & 5 & 0 \\ 2 & 4 & -1 \\ 0 & -2 & 0 \end{bmatrix}$

4. Solve the following systems of linear equations by using the inverse of the coefficient matrix (i.e. $\boldsymbol{x} = A^{-1}\boldsymbol{b}$).

(i) $$\begin{aligned} 2x_1 + x_2 &= 5 \\ 4x_1 + 3x_2 &= 9 \end{aligned}$$

(ii) $$\begin{aligned} 3x_1 + 5x_2 &= 0 \\ x_1 + 2x_2 &= 0 \end{aligned}$$

(iii) $$\begin{aligned} x_1 - x_2 + 3x_3 &= 8 \\ 2x_1 - x_2 + 4x_3 &= 11 \\ -x_1 + 2x_2 - 4x_3 &= -11 \end{aligned}$$