WORKSHOP 5

This workshop will build on material from Lecture 5: Vectors.

During this workshop, students will work towards the following learning outcomes:

- perform basic vector arithmetic and calculate vector properties.
- calculate the dot product of two vectors, and use it to find angles, projections, and work done.

Vector arithmetic and properties

1. Given the vectors $\boldsymbol{a} = [2, -1, 3]$, $\boldsymbol{b} = [4, 0, -3]$ and $\boldsymbol{c} = [1, -2, 2]$, find:
   (i) $\boldsymbol{a} + \boldsymbol{b}$
   (ii) $3\boldsymbol{a} - 4\boldsymbol{c}$
   (iii) The magnitude of $\boldsymbol{b}$
   (iv) $\hat{\boldsymbol{b}}$
   (v) A vector in the same direction as $\boldsymbol{b}$ but has the same length of $\boldsymbol{c}$
2. Given the points $A(2, -3)$ and $B(4, 1)$, find the vectors $\boldsymbol{a} = \overrightarrow{OA}$ and $\boldsymbol{b} = \overrightarrow{AB}$.

Dot product and applications

3. In each of the following cases, find (i) $\boldsymbol{a}.\boldsymbol{b}$, (ii) the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$, (iii) the scalar projection of $\boldsymbol{a}$ on $\boldsymbol{b}$, and (iv) the vector projection of $\boldsymbol{a}$ on $\boldsymbol{b}$.
   (a) $\boldsymbol{a} = [2, -4, \sqrt{5}]$, $\boldsymbol{b} = [-2, 4, -\sqrt{5}]$
   (b) $\boldsymbol{a} = 2\boldsymbol{i} + 10\boldsymbol{j} - 11\boldsymbol{k}$, $\boldsymbol{b} = 2\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$
   (c) $\boldsymbol{a} = \boldsymbol{i} + \boldsymbol{k}$, $\boldsymbol{b} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$
4. A truss takes the shape of a cube with a member on the main diagonal. Find the angle between this diagonal member and one of the edge members. (Hint: Consider the coordinates of the corners of a simple cube of side length one placed in three space.)
5. Show that the vectors $\boldsymbol{u} = [2, -2, -1]$ and $\boldsymbol{v} = [3, 5, -4]$ are orthogonal to one another.
6. Find the work done by a force $\boldsymbol{F} = 5\boldsymbol{j}$ (magnitude 5 Newtons) in moving an object along the line from the origin to the point $(1, 1, 1)$ (distance in meters).
7. How much work does it take to slide a crate 15m along a loading dock by pulling on it with a 150 Newton force at an angle of 45° from the horizontal?
8. Verify that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, where $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are the direction cosines of a vector $\boldsymbol{a}$ in 3 space.