Question 1 (25 marks)

(a) Prove that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

(5 marks)

(b) Represent the following statements in mathematical logic.

(i) Some people in this class are not working hard.
(ii) There is only one staff in this department such that none of his/her friends are also friends with each other. (You **can** use the notation $\exists!$ in this question)
(iii) Everybody has good personal characteristics.
(iv) There is only one student in this class who can achieve the highest mark in COMP1006. (You **cannot** use the notation $\exists!$ in this question)
(v) Mary hates everyone who does not like dogs.

(10 marks)

(c) Prove the following assertion using Mathematical Induction.

$$n < 2^n$$

for all positive n. **(7 marks)**

(d) Give the negation for the following proposition.

If you do not work hard, then you will not pass COMP1006.

(3 marks)

Question 2 (30 marks)

(a) Let set $A_i = \{1, 2, 3, \ldots, i\}$ with i=1, 2, 3,..100.

(i) $\bigcup_{i=1}^{100} A_i$ =? $\bigcap_{i=1}^{100} A_i$ =?

(ii) $P(\{\phi, 1, \{\phi\}\})$=?

(iii) $|P(A_{10})|$= ? **(7 marks)**

Question 2 continues in the next page.

(b) Let A= {1, 2, 3, 4}. Give examples of relations on A which satisfy each of the following requirements for (i)-(iii) and also find a solution for (iv).

(i) The relation is reflexive and transitive.

(ii) The relation is reflexive, symmetric and transitive, but not antisymmetric.

(iii) The relation is neither symmetric nor anti-symmetric but is reflexive.

(iv) Find an equivalence relationship $\Re$ from $A \times A$ and compute $[2]_{\Re}$

(15 marks)

(c) If one randomly generates a sequence of 11 bits, what is the probability that at least one of these bits is 1?

(8 marks)

Question 3 (20 marks)

(a) Suppose that **A** is the set of sophomores in the Department of Computer Science and **B** is the set of students who choose COMP1006 in the Department of Computer Science. Express each of the following sets in terms of **A,** and **B.**

(i) The set of sophomores taking COMP1006.

(ii) The set of sophomores who are not taking COMP1006.

(iii) The set of students in the Department of Computer Science who either are sophomores or are taking COMP1006.

(iv) The set of students in the Department of Computer Science who either are not sophomores and are taking COMP1006

(10 marks)

(b) A class consists of **12** men and **14** women.

(i) How many groups can be chosen from this class which consists of **6** men and **8** women?

(ii) If a specific male **A** and a specific female **B** have to be in the same group, how many groups of 12 students including **6** males and **6** females can be formed from this class?

Question 3 continues in the next page.

(iii) If a specific male **A** and a specific female **B** cannot be in the same committee, how many ways can a committee consisting of **6** men and **6** women be chosen from the class?

(10 marks)

Question 4 (25 marks)

(a) First, define what a tree is. And then state the hand-shaking theorem. Finally, decide whether there exists a tree with six vertices of the following degrees shown below. Either draw such a tree with the specific properties or explain why such tree does not exist.

(i) 2, 1, 2, 1, 1, 2

(ii) 1, 3, 1, 2, 1, 2

(8 marks)

(b) The complete 3-partite graph $K_{n, m, p}$, with $n, m, p \geq 1$, is a simple graph that has its vertex set partitioned into 3 disjoint non-empty subsets of n, m and p vertices, respectively. Two vertices are adjacent if and only if they are in different subsets in the partition.

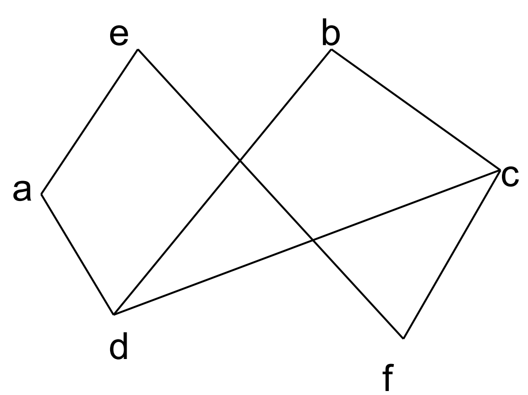
(i) Draw $K_{1,2,2}$.

(ii) Give the definition of Euler circuit.

(iii) Find a Euler circuit on $K_{1,2,2}$ if it exists. If not, justify your conclusion.

(5 marks)

(c) Given a graph $G(V, E)$ as below.



(12 marks)

Question 4 continues in the next page.

(i) Write the adjacency matrix.

(ii) Is there a Euler circuit or Euler path in the graph? If yes, list one. Otherwise explain why not.

(iii) Is there a Hamilton circuit or Hamilton path in the graph? If yes, list one. Otherwise, explain why not.

END OF EXAMINATION PAPER