Non-clairvoyant problems with Deadlines or Delay

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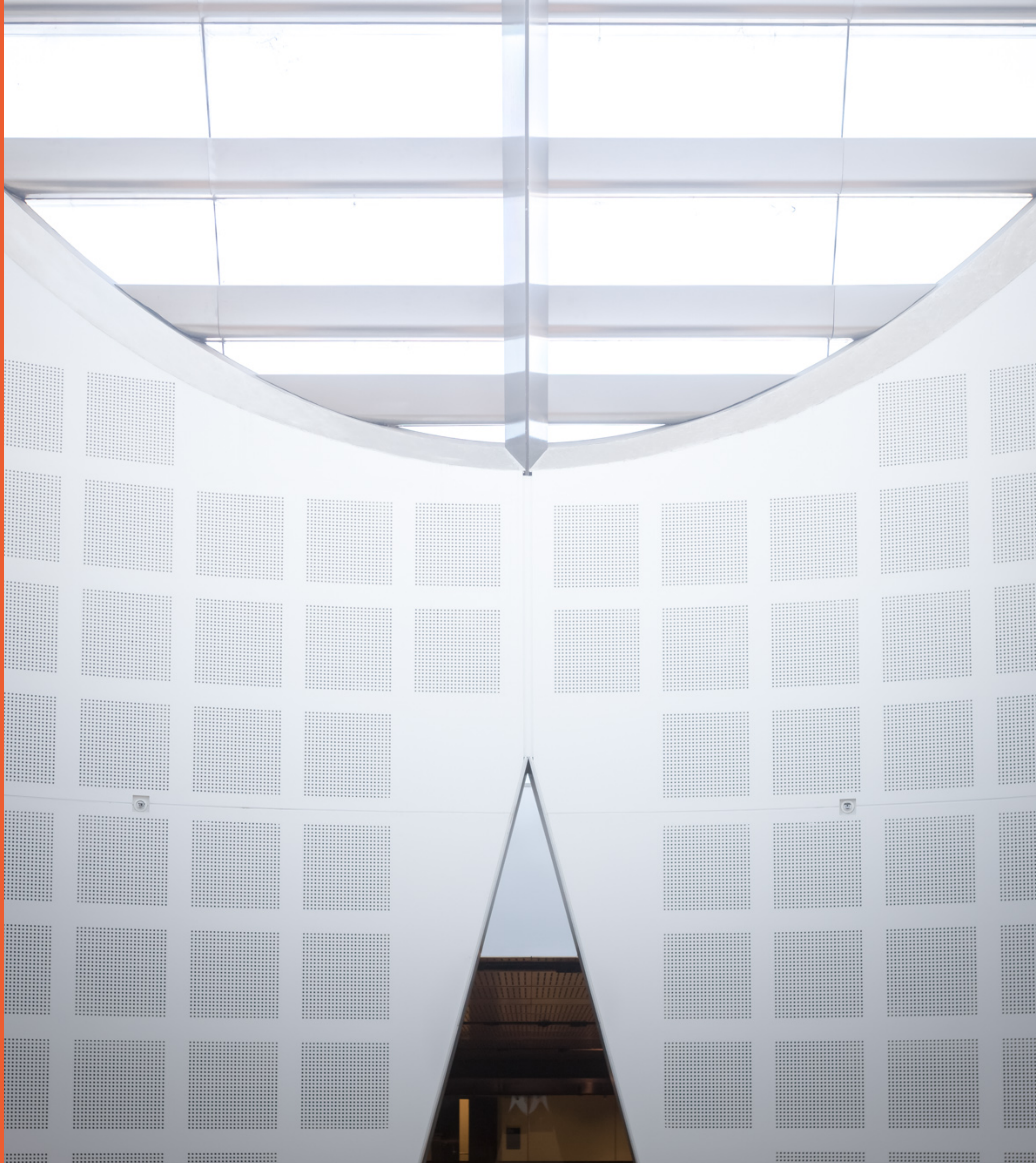
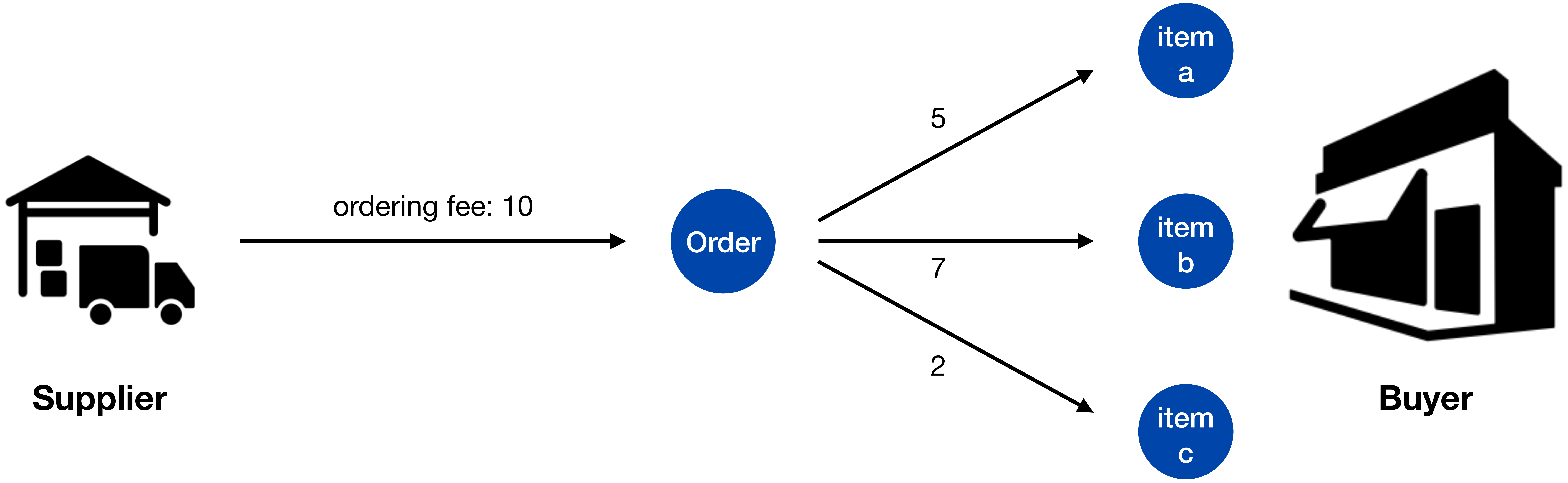
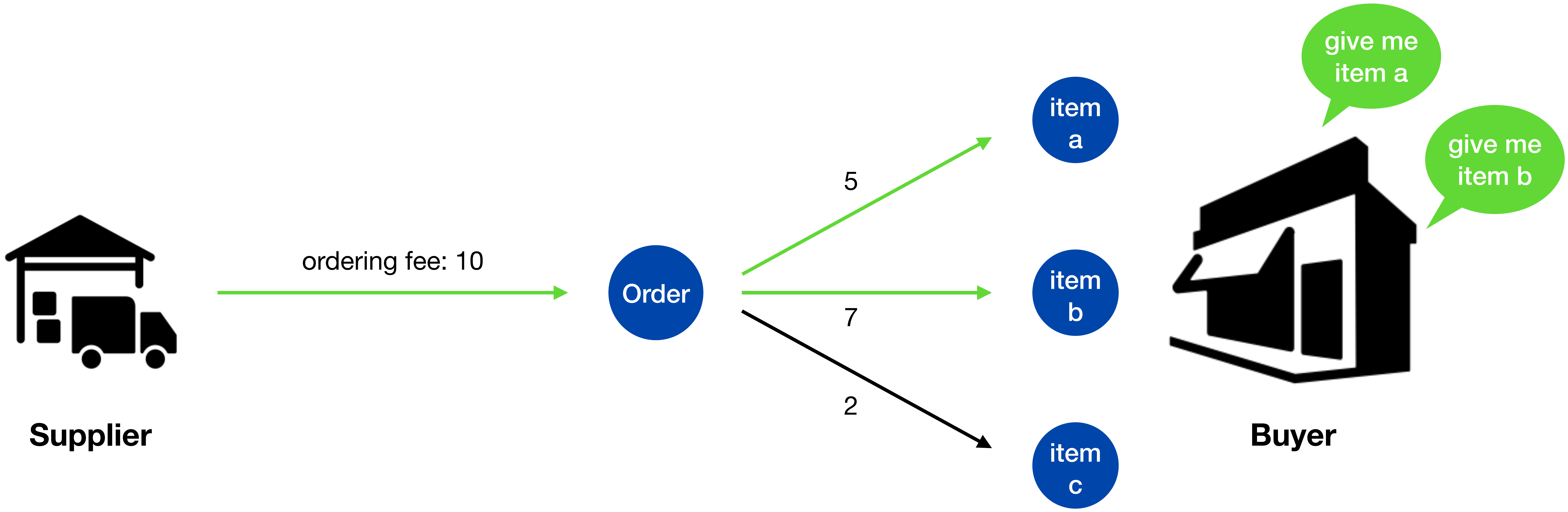


Joint replenishment problem (JRP)

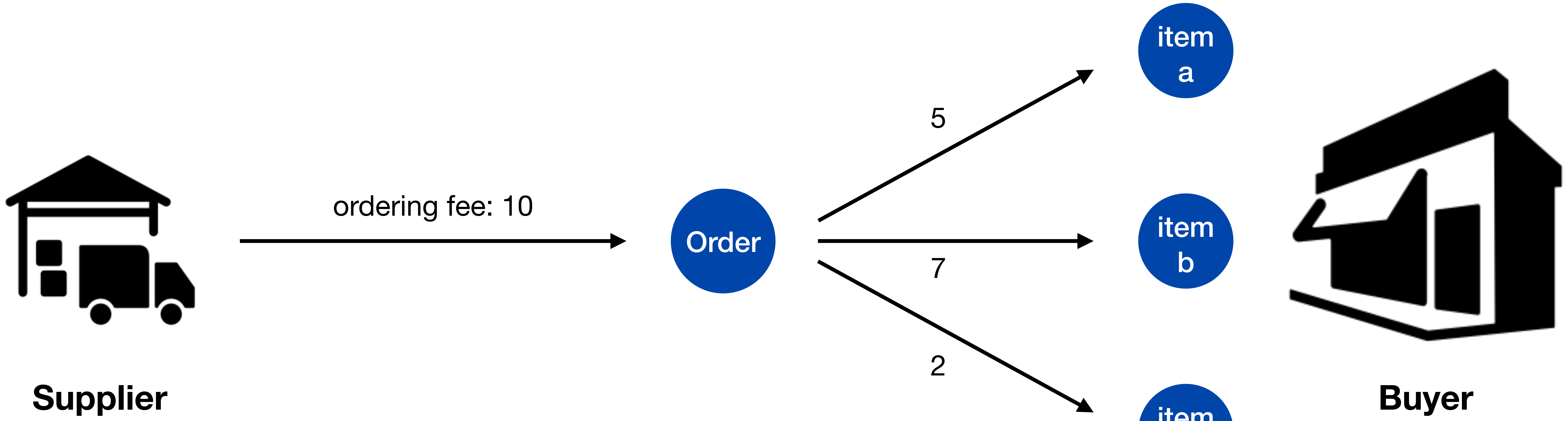


Joint replenishment problem



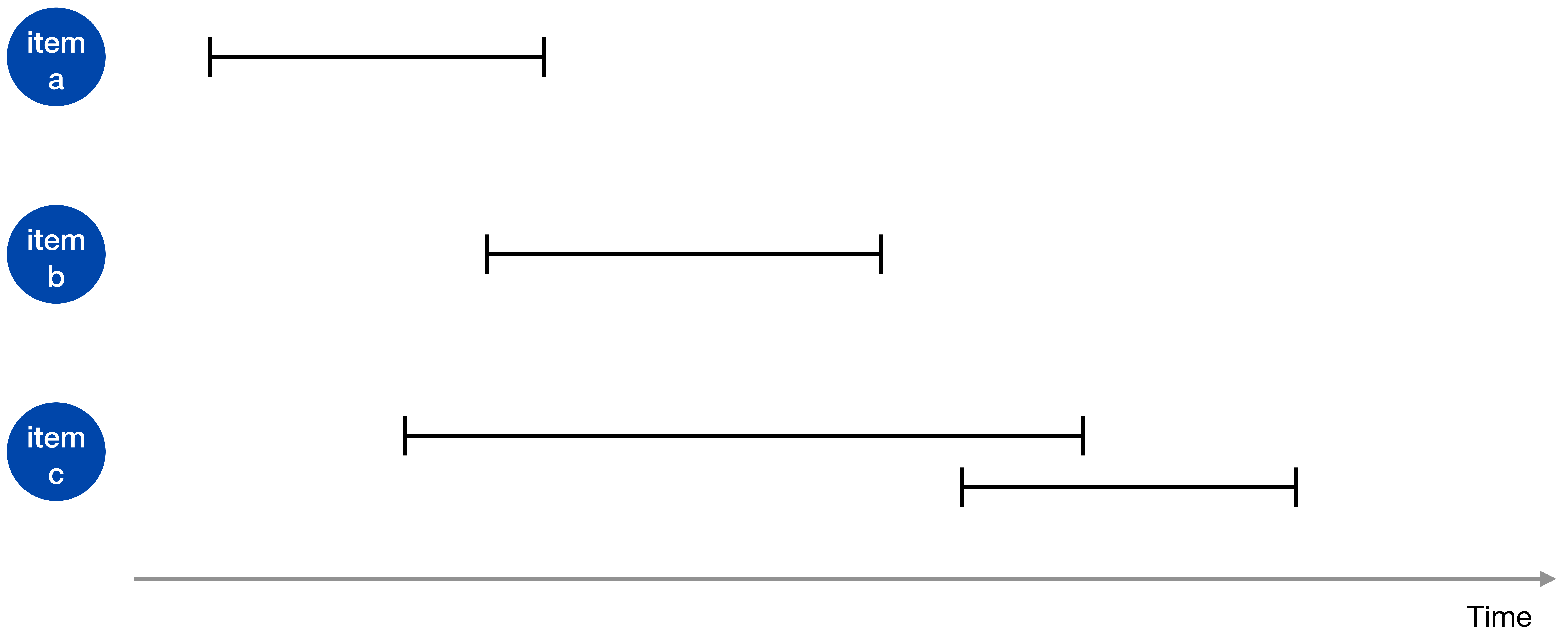
- Each request is on an item
- A transmission is a set of items delivered to the buyer
- A request is served if the item it occurs on is transmitted
- Transmitting a and b costs ordering fee + cost of a + cost of b = 10 + 5 + 7 = 22

Joint replenishment problem



- Each request is on an item
- A transmission is a set of items delivered to the buyer
- A request is served if the item it occurs on is transmitted
- Transmitting a and b costs 10 + 5 + 7 = 22

Problem #1: JRP with deadlines



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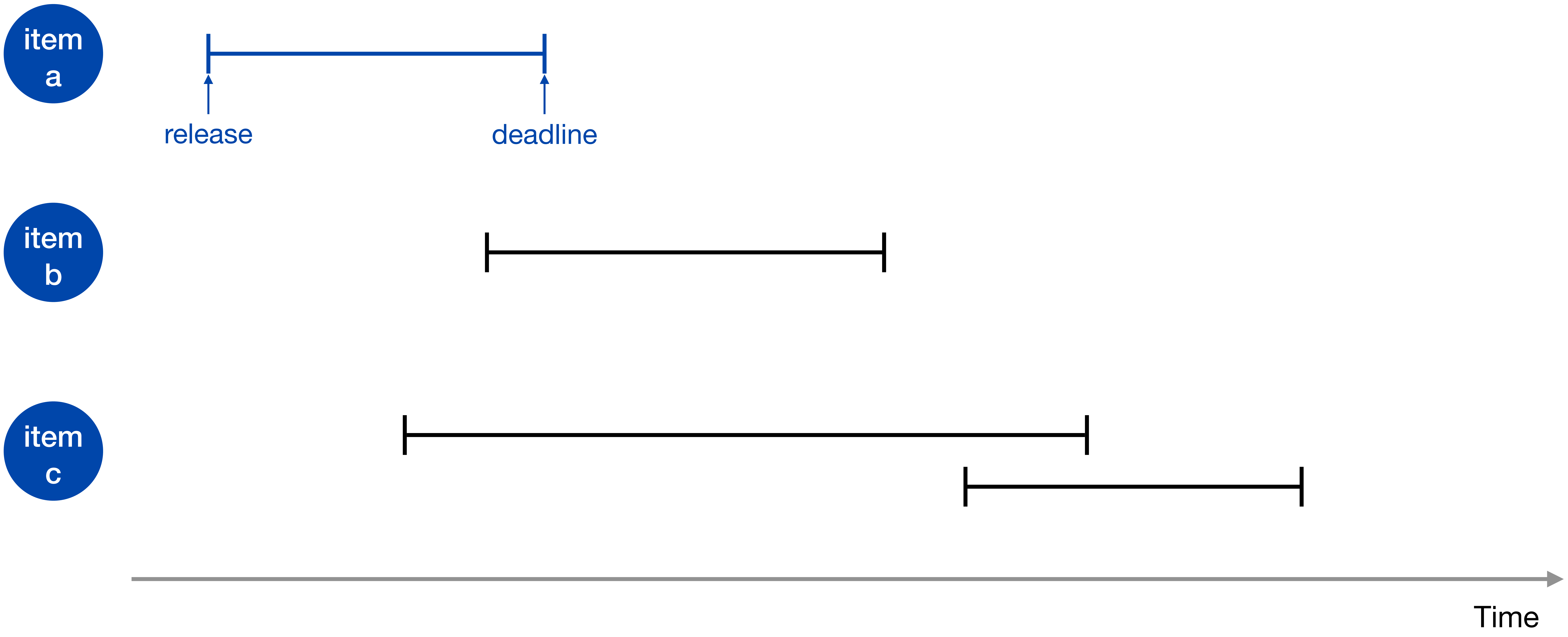
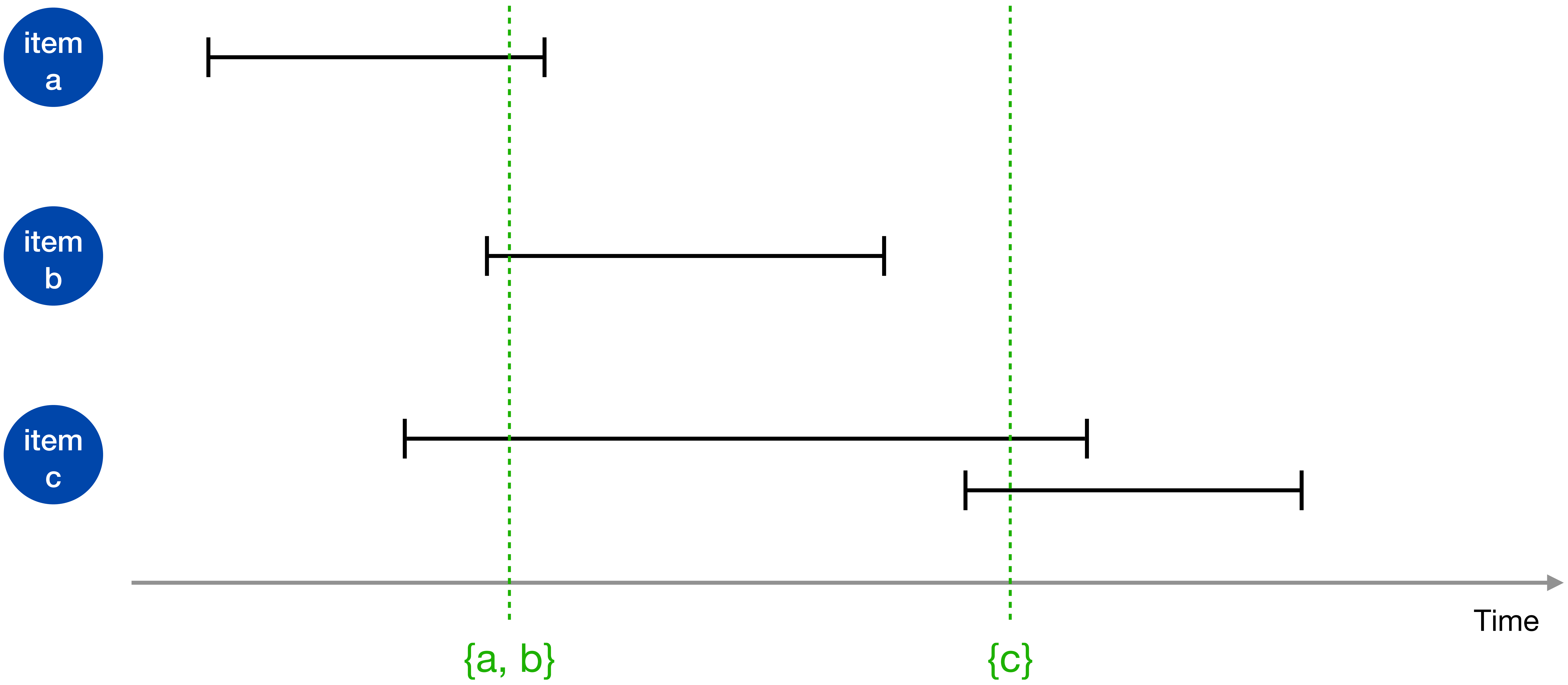


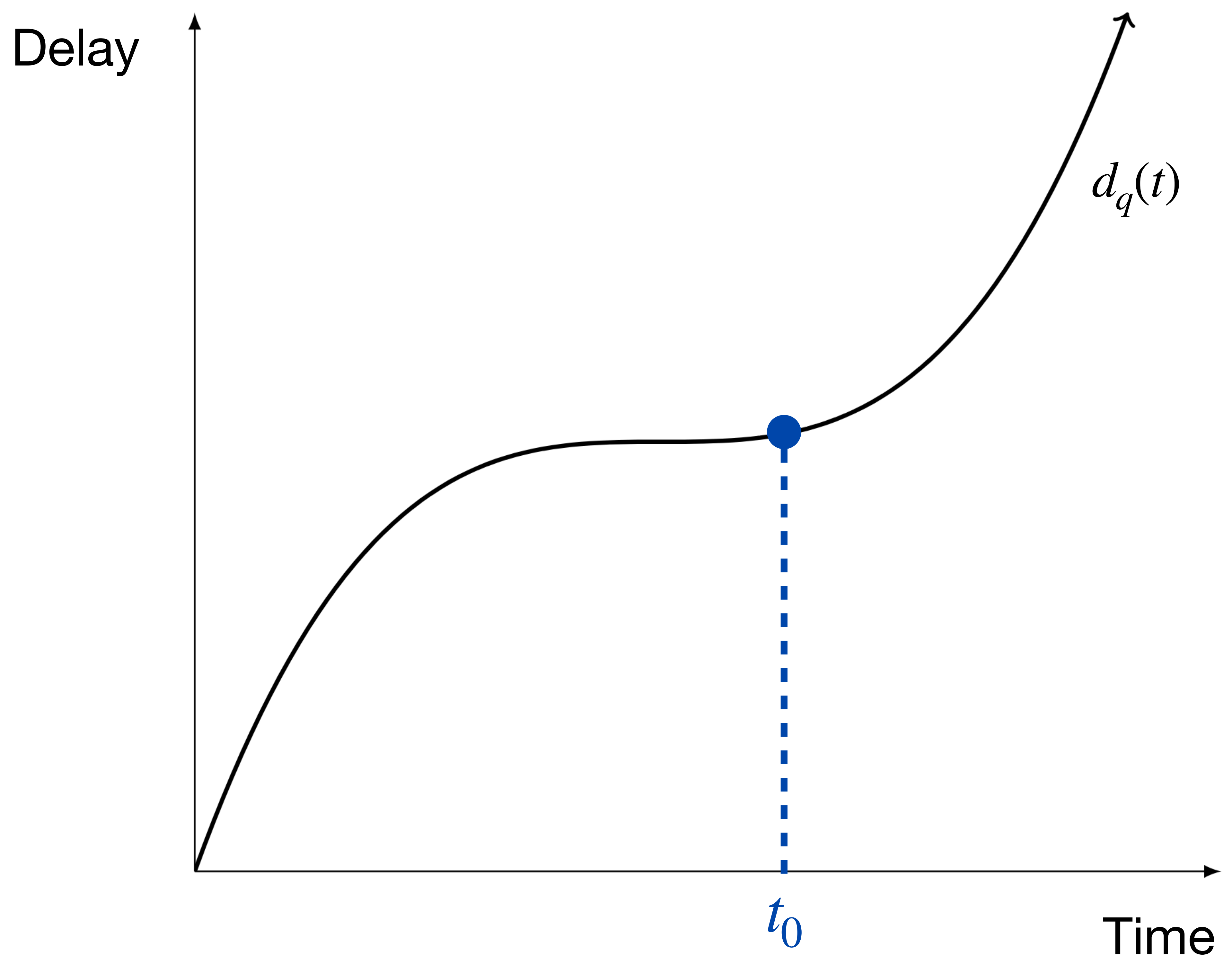
- Each request must be served after it's released and before its deadline

Problem #1: JRP with deadlines



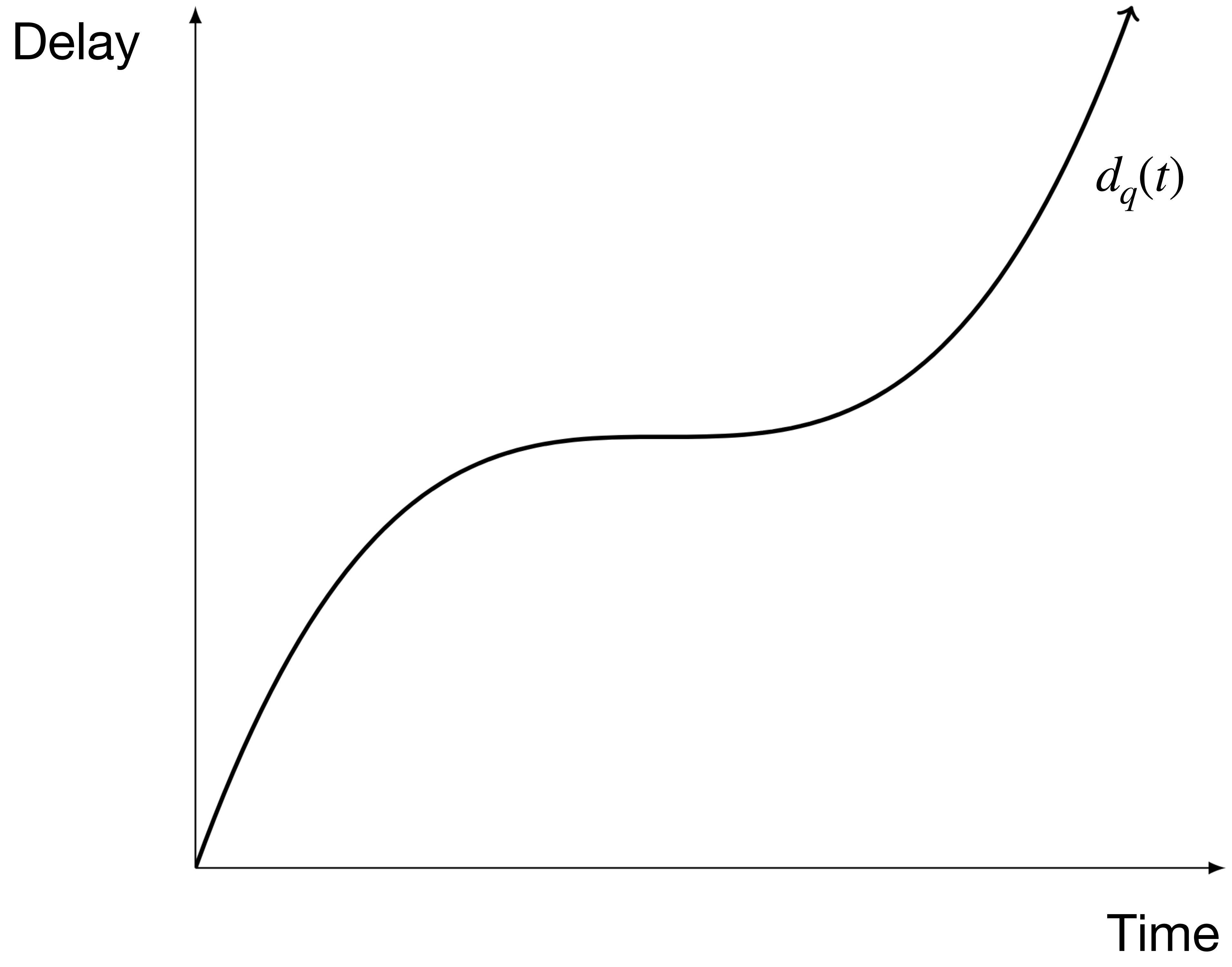
Goal: minimise total cost of all transmissions

Problem #2: JRP with delay



- Each request has a non-decreasing, continuous delay function $d_q(t)$
- We pay $d_q(t_0)$ if q is served at time t_0

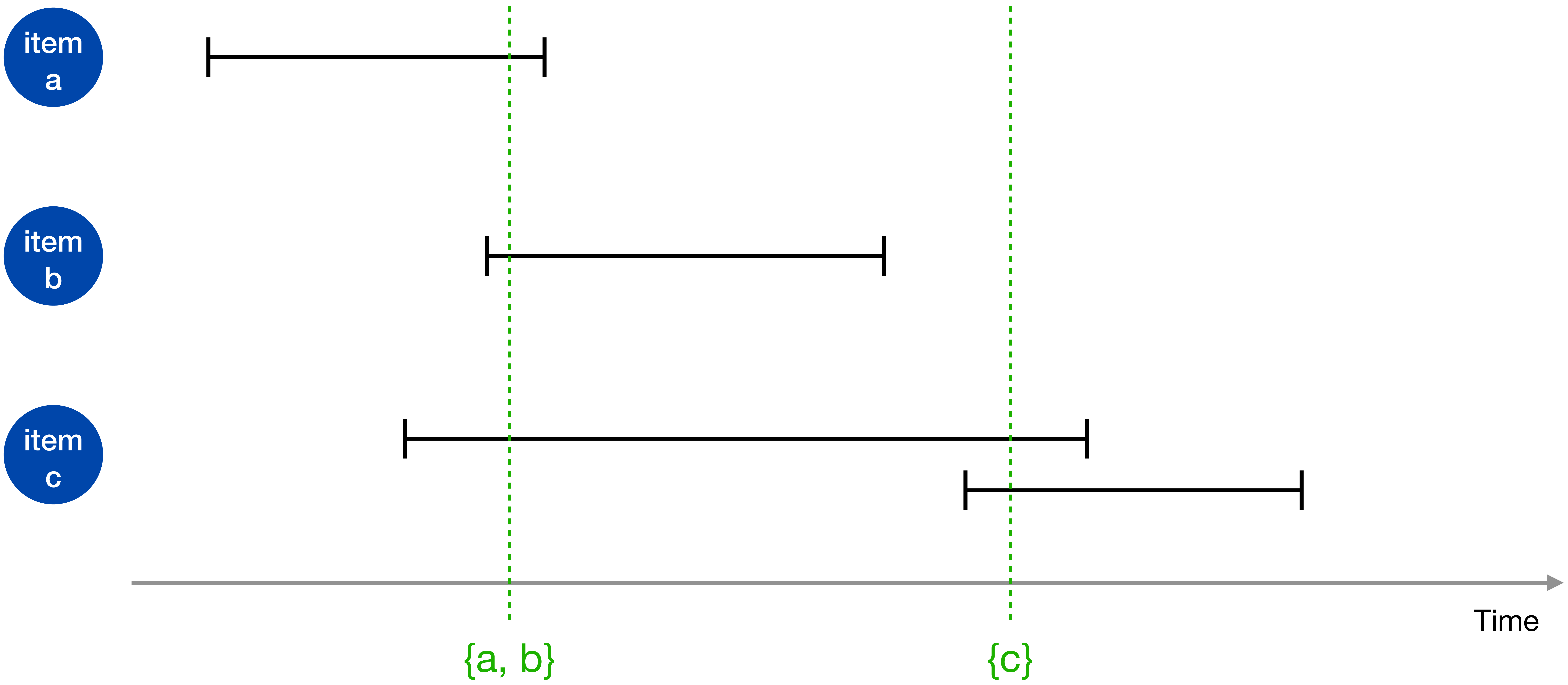
Problem #2: JRP with delay



- Each request has a non-decreasing, continuous delay function $d_q(t)$

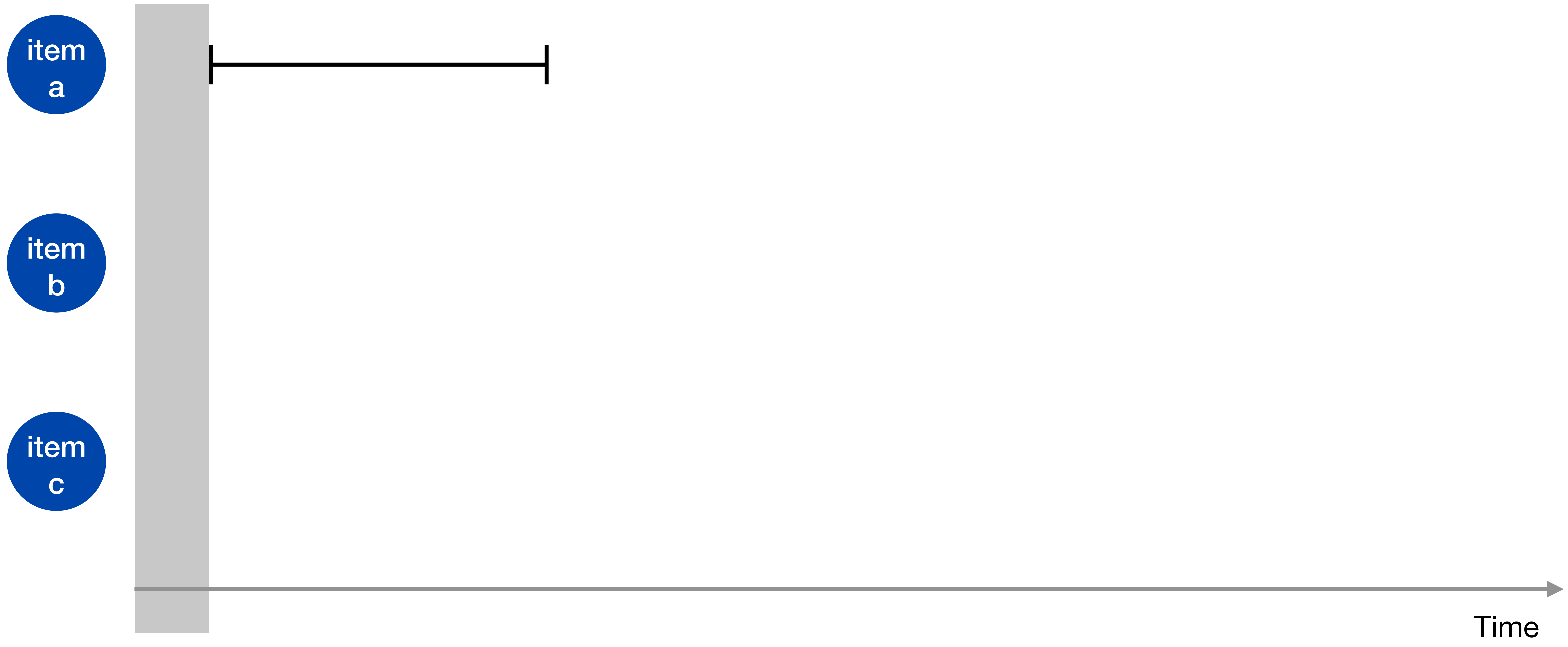
Goal: minimise total cost of all transmissions + delay costs

Offline Setting



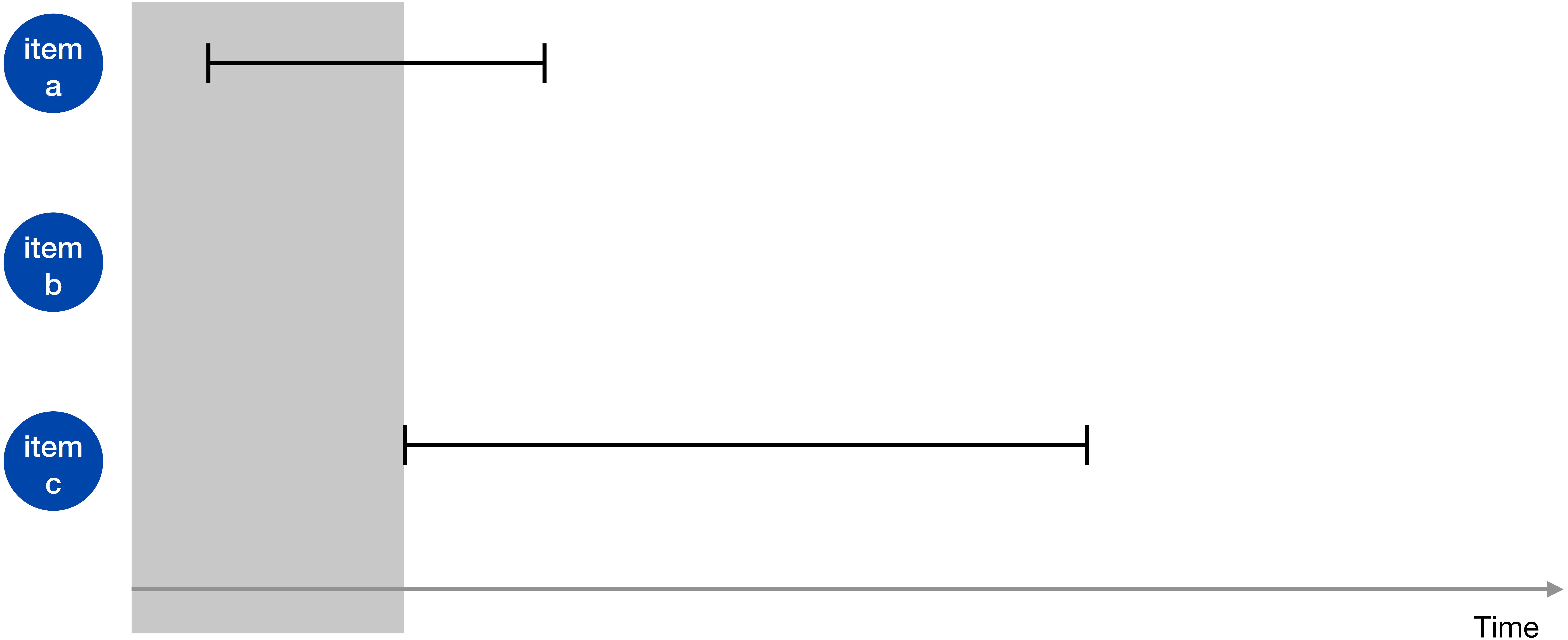
In the offline setting, all requests are known at the beginning

Online Setting

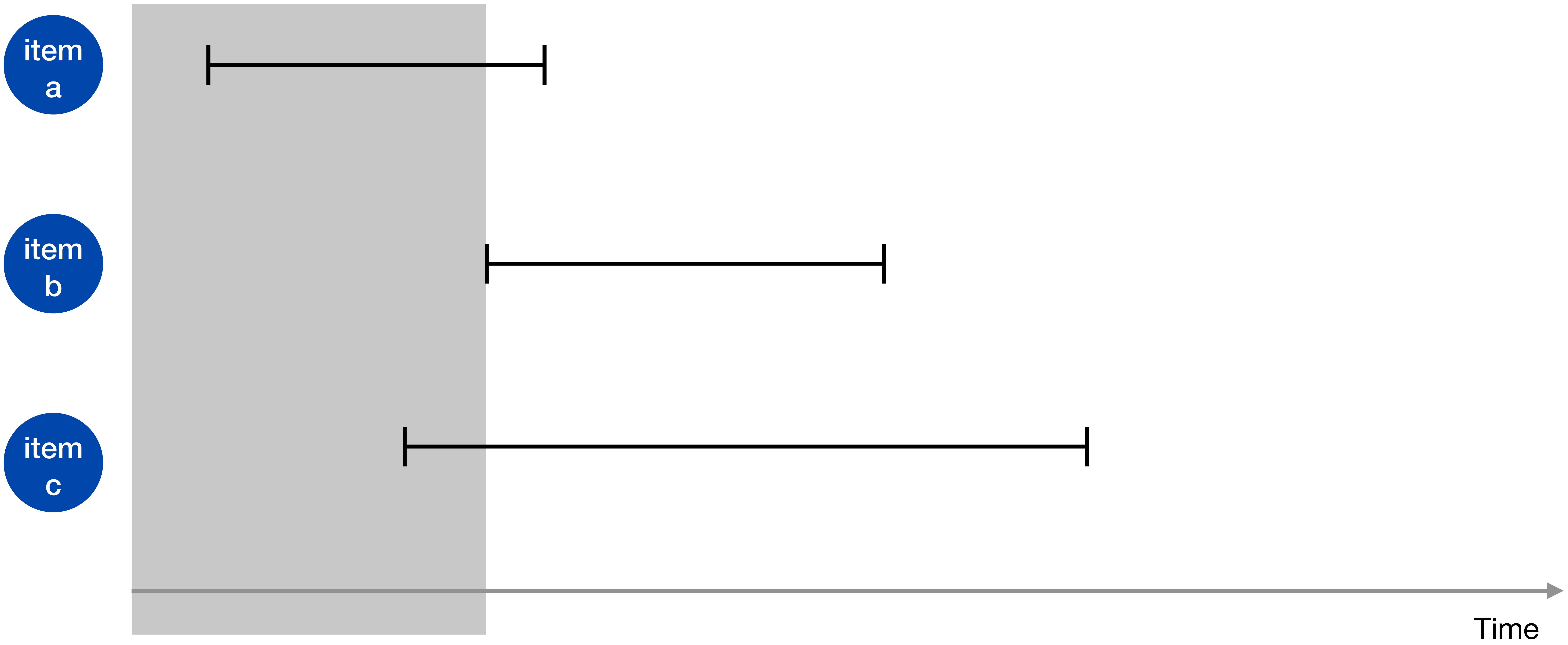


In the online setting, requests are released online

Online Setting



Online Setting



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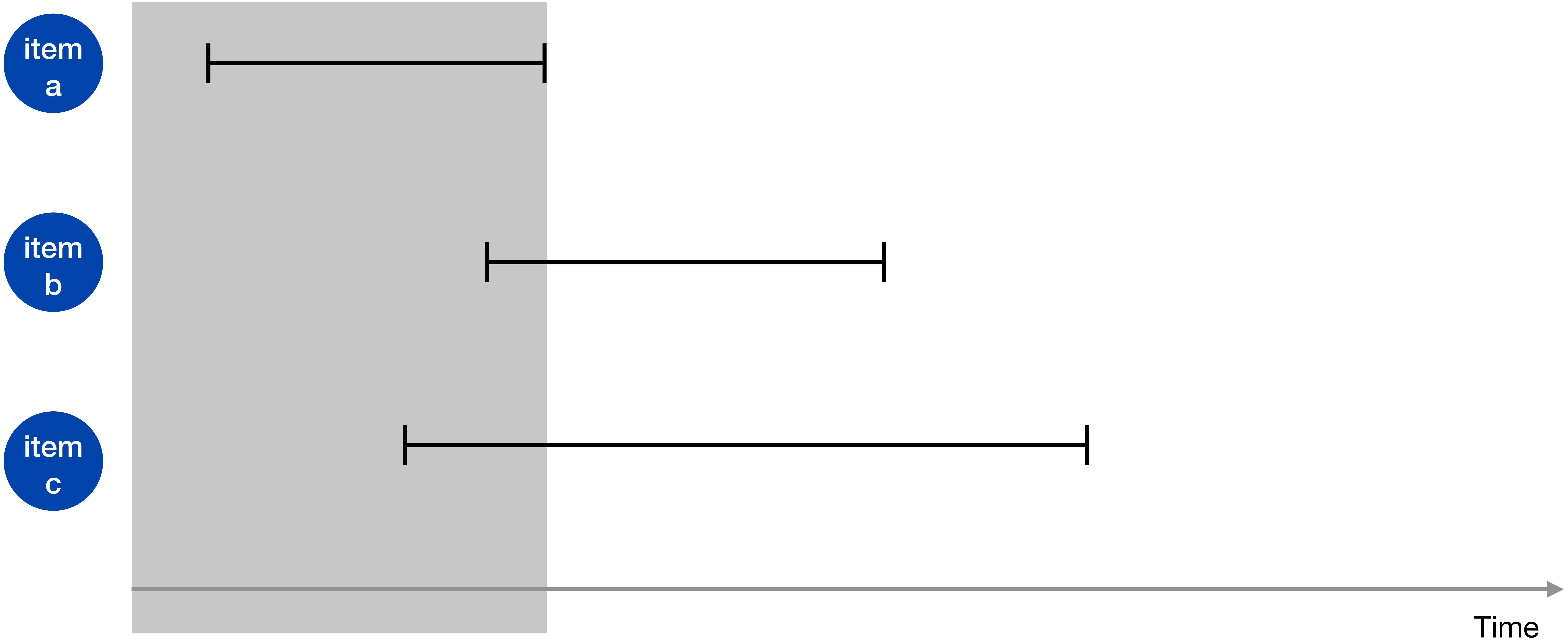

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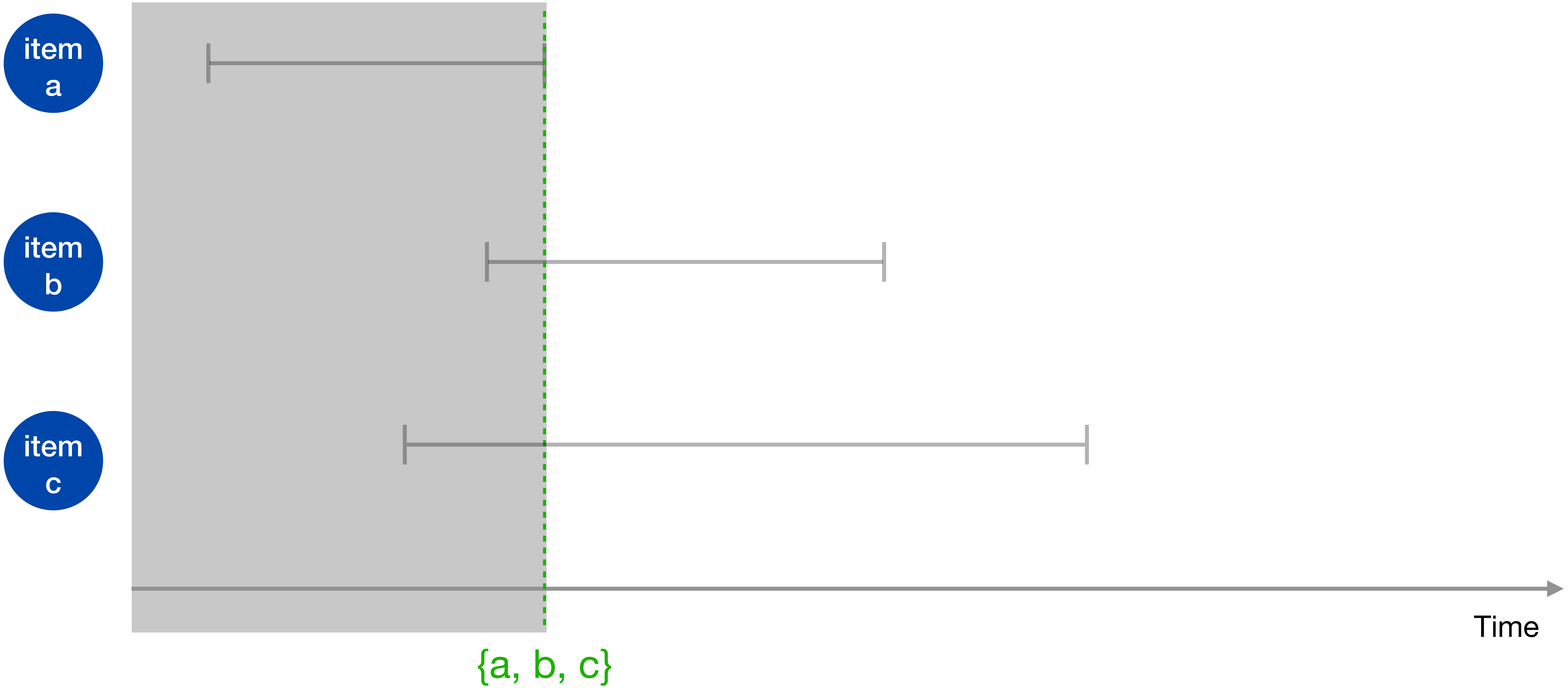

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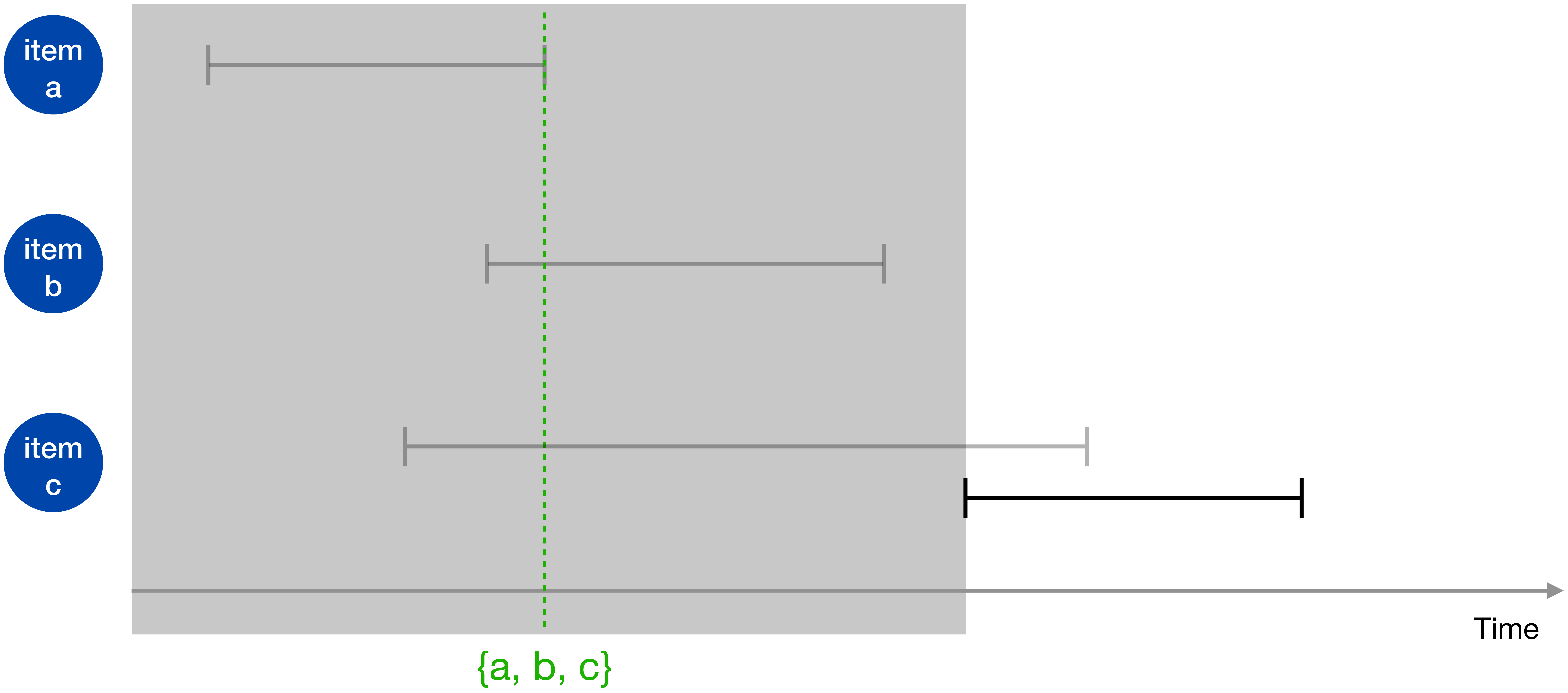
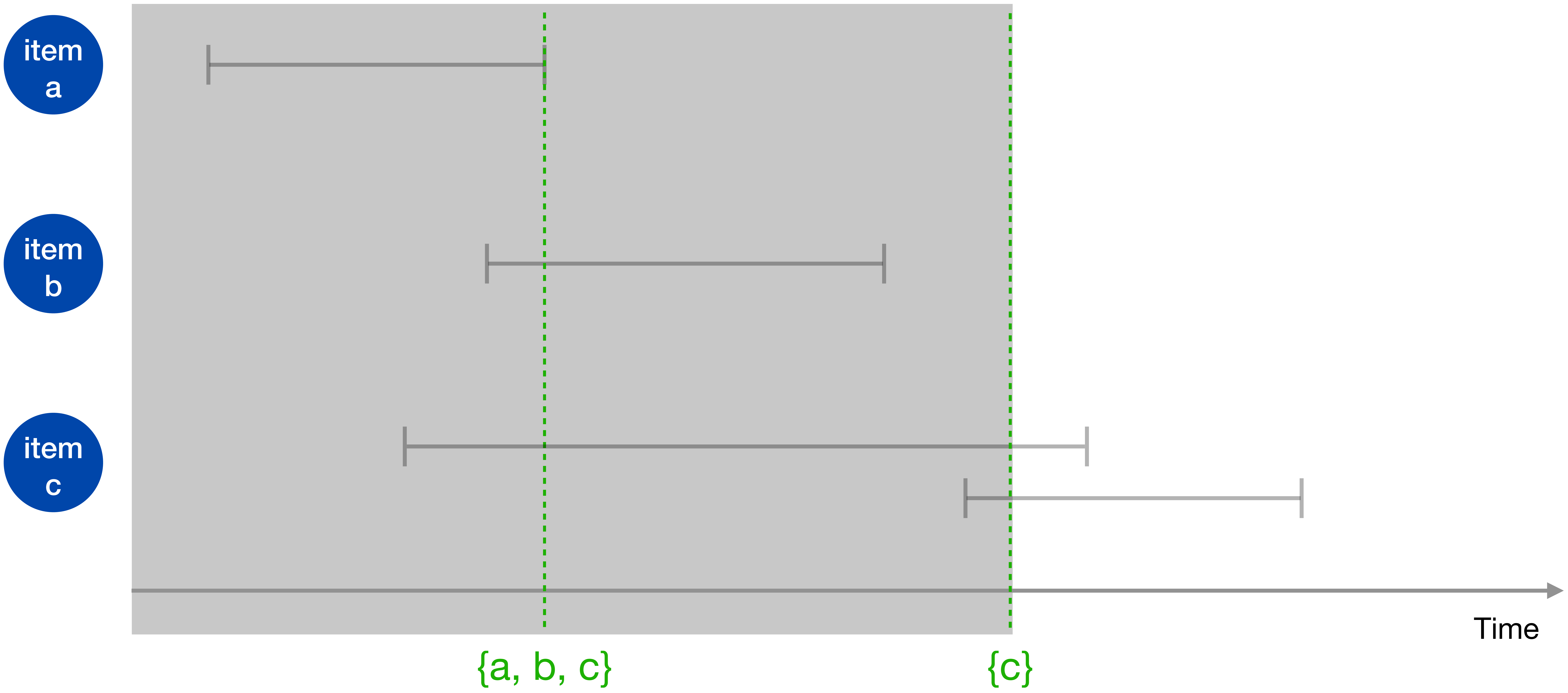


Online Setting



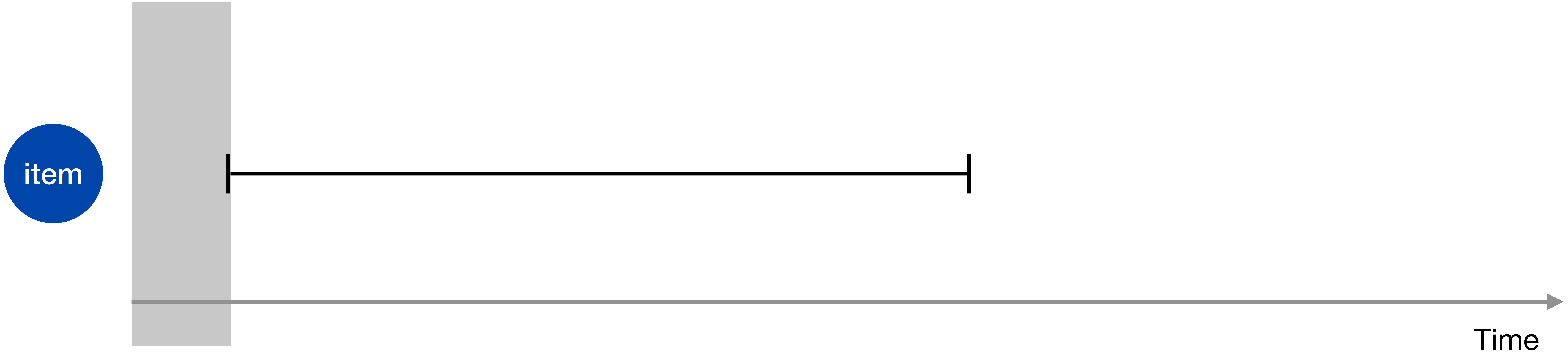
Measuring Performance

- OPT := cost of offline optimal solution
- ALG := cost of online algorithm

$$\textbf{Competitive ratio} := \frac{\textbf{ALG}}{\textbf{OPT}} \textbf{ in the worst case}$$

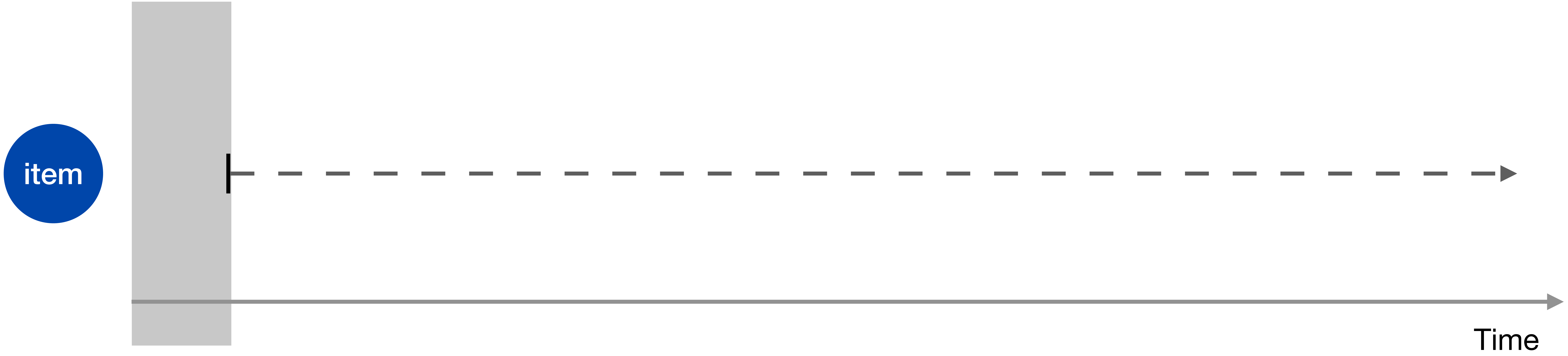
Clairvoyant Online Setting

- When a request arrives, its deadline is revealed to the algorithm



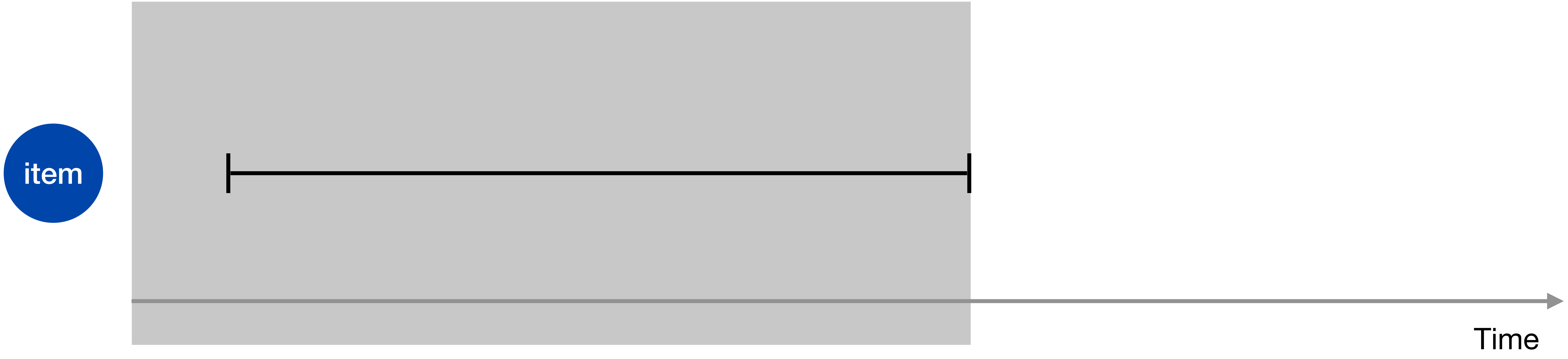
Non-clairvoyant Online Setting

- When a request arrives, its deadline is not known



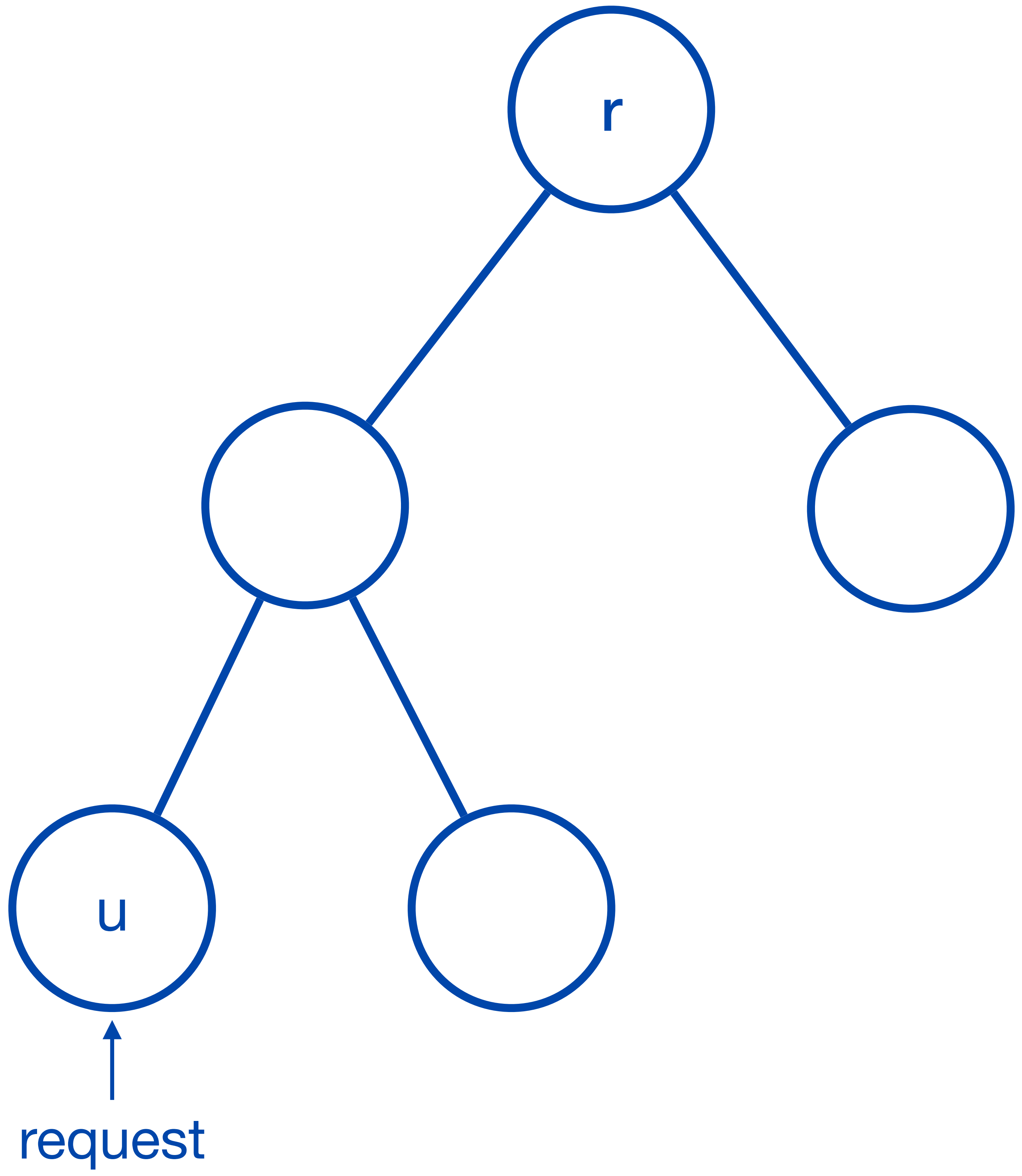
Non-clairvoyant Online Setting

- When a request arrives, its deadline is not known
- When a pending request reaches its deadline, the algorithm is informed and must serve it immediately



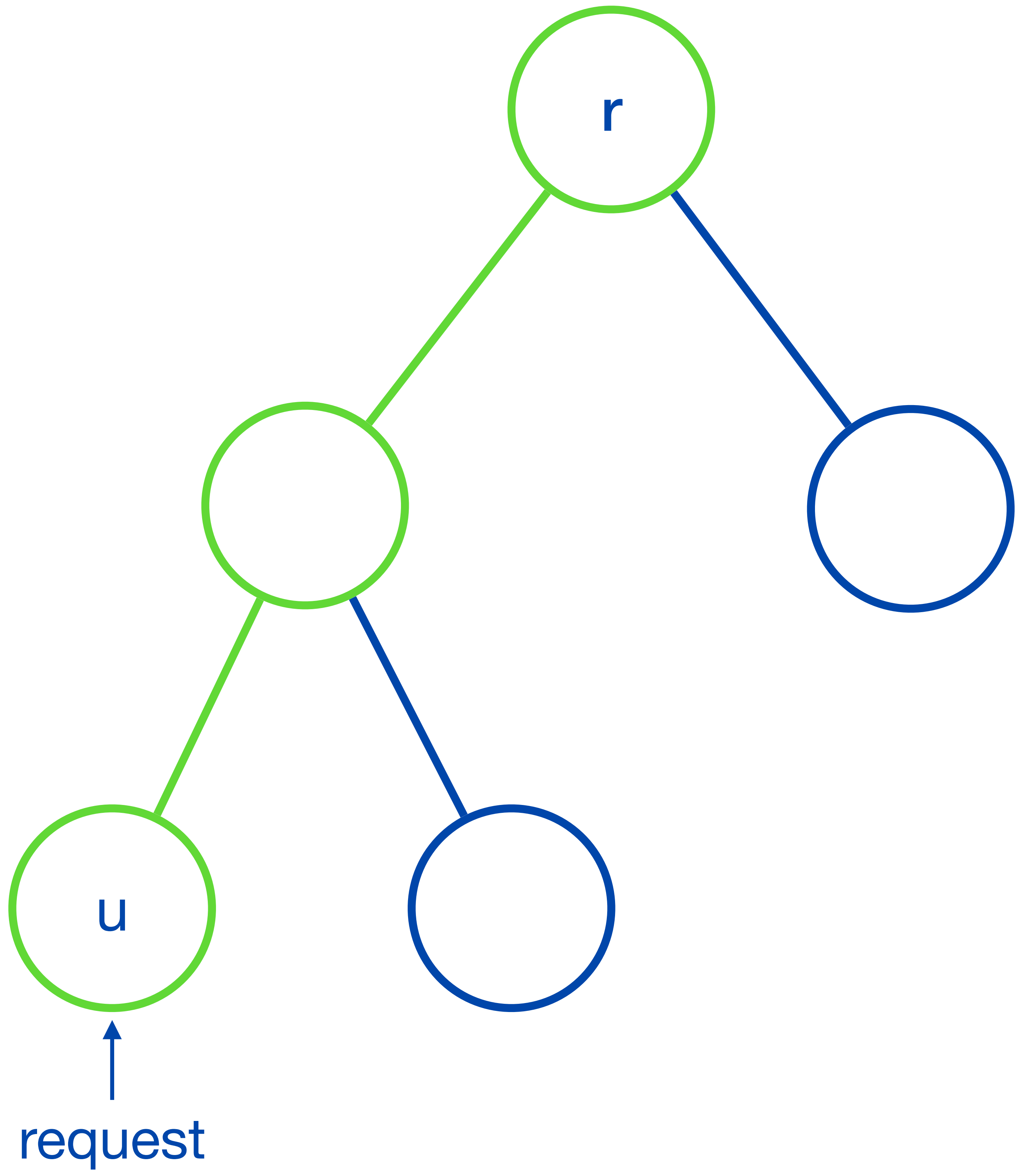
Multi-level aggregation with deadlines (MLAP)

- Given: node-weighted tree
- Each request is a node



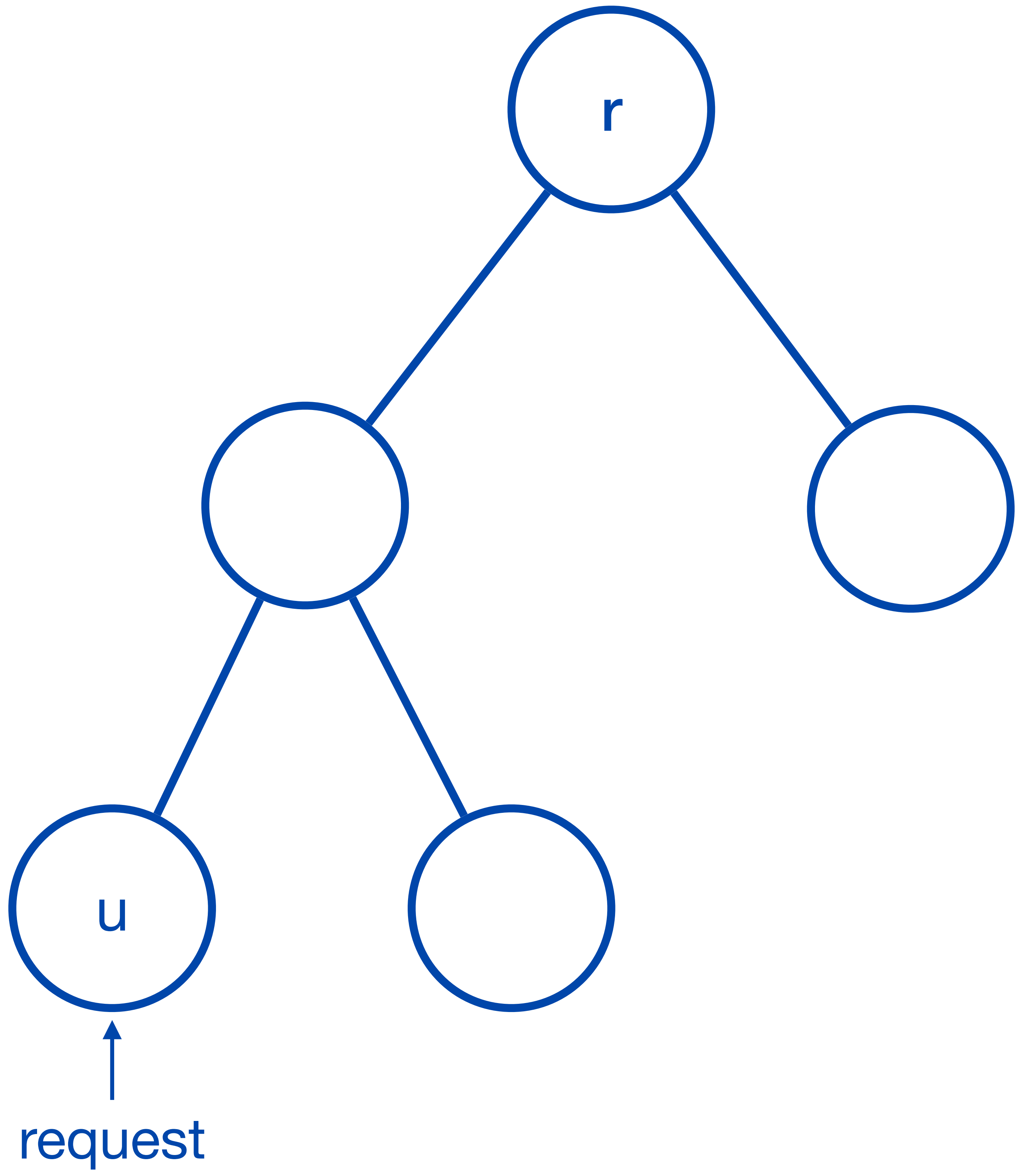
Multi-level aggregation with deadlines

- Given: node-weighted tree
- Each request is a node
- A request is served by transmitting all nodes from the root to the node it occurs on



Multi-level aggregation with deadlines

- Given: node-weighted tree
- Each request is a node
- A request is served by transmitting all nodes from the root to the node it occurs on
- JRP with deadlines is a special case where the tree has depth 1



Previous Results

	Clairvoyant	Non-clairvoyant
JRP with deadlines	2 (Bienkowski et al., 2014)	
JRP with delay	3 (Buchbinder et al., 2013)	
MLAP with deadlines	O(depth) (Buchbinder et al., 2017)	

Previous Results

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MLAP with deadlines	O(depth) (Buchbinder et al., 2017)	
Set cover with delay	O(log N) (Carrasco et al., 2018)	randomised O(log n log k) (Azar et al., 2020)

Our Results

	Clairvoyant	**Non-clairvoyant**
JRP with deadlines	2 (Bienkowski et al., 2014)	lower bound: $\Omega(\sqrt{n})$
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MLAP with deadlines	O(depth) (Buchbinder et al., 2017)	$O(\sqrt{n} + \text{depth})$
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Our Results

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Set cover with delay	O(log N) (Carrasco et al., 2018)	deterministic O(log n log k) not discussed

Lower bound

JRP with deadlines

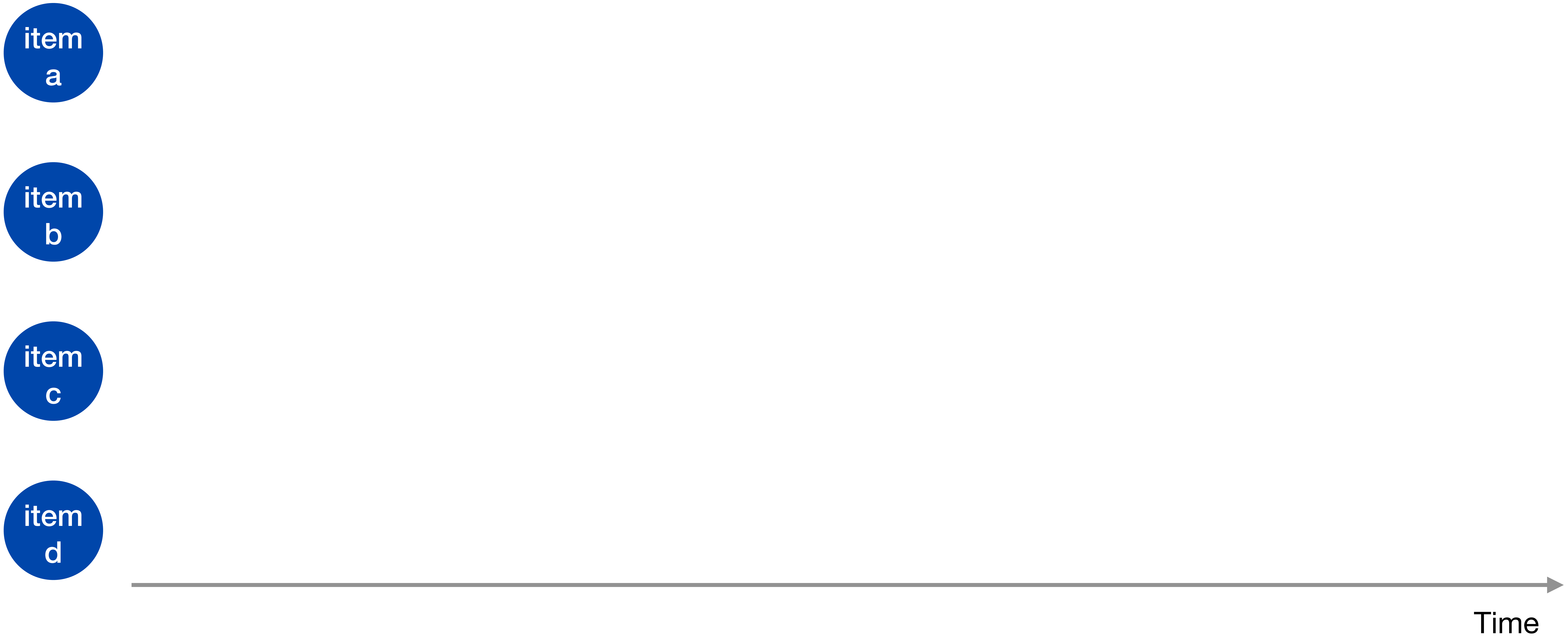
- *Theorem:* Any non-clairvoyant algorithm for JRP with deadlines is $\Omega(\sqrt{n})$ - competitive.

Lower bound

JRP with deadlines

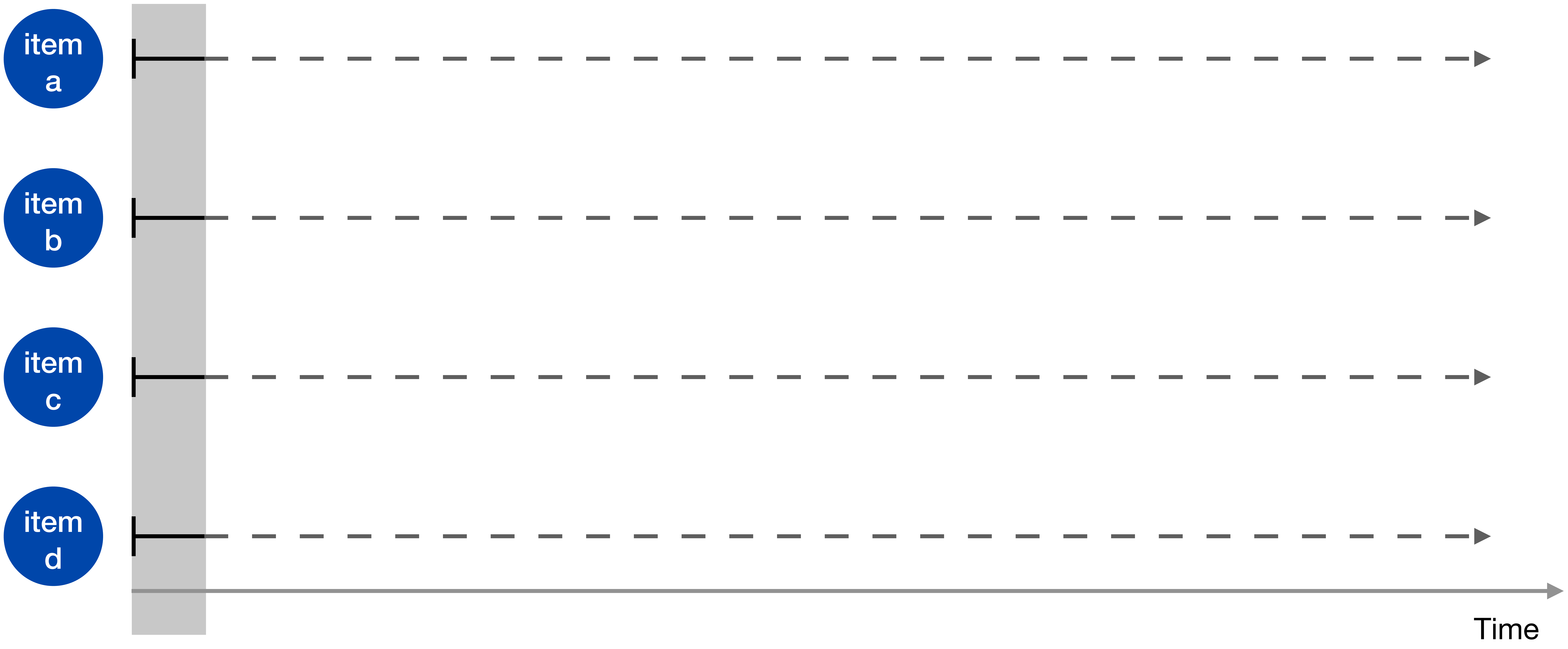
- n items
- each item costs 1
- ordering fee = $\sqrt{n}$

Lower bound



- Ordering fee = 2

Lower bound

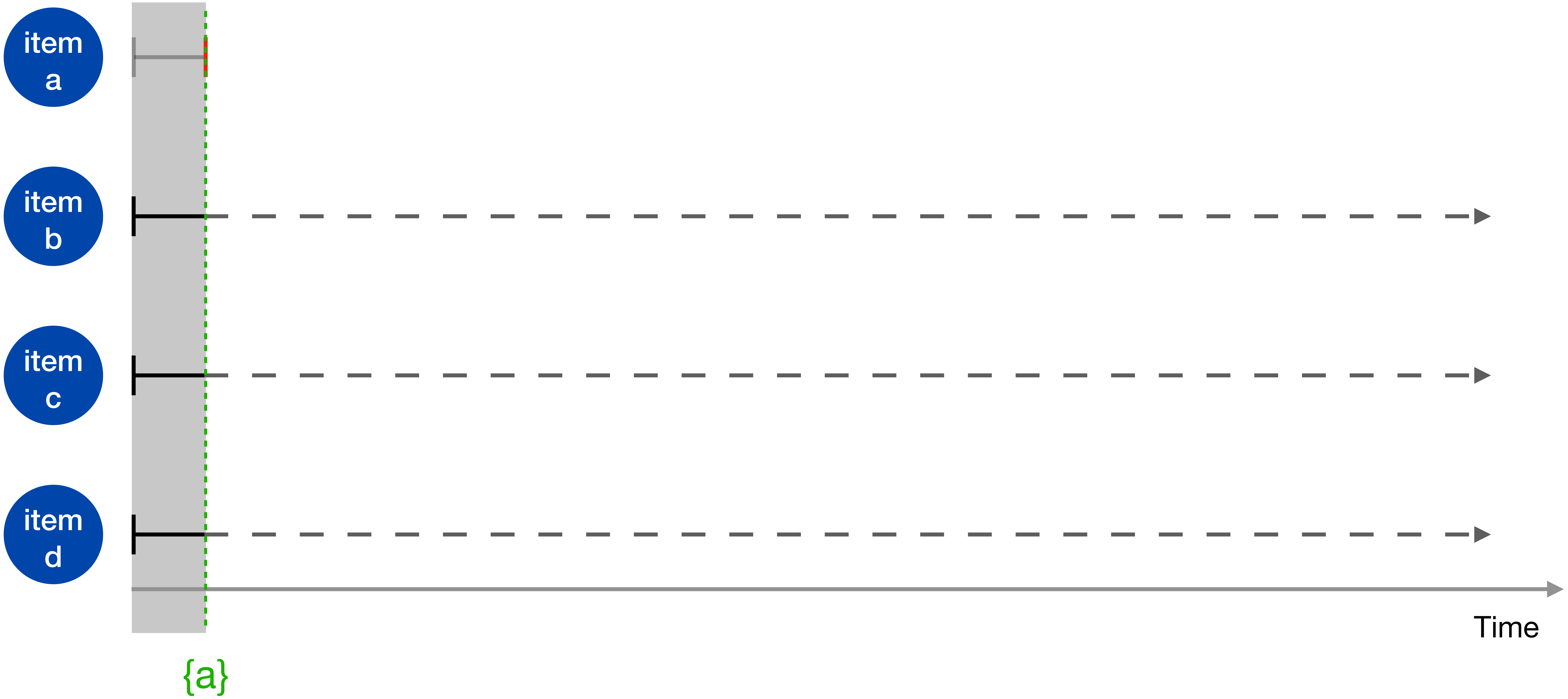


- Release requests on all items

Lower bound

- Pick a pending request and set its deadline

Lower bound



- Pick a pending request and set its deadline

Lower bound

- As long as there remains pending requests, pick one and set its deadline

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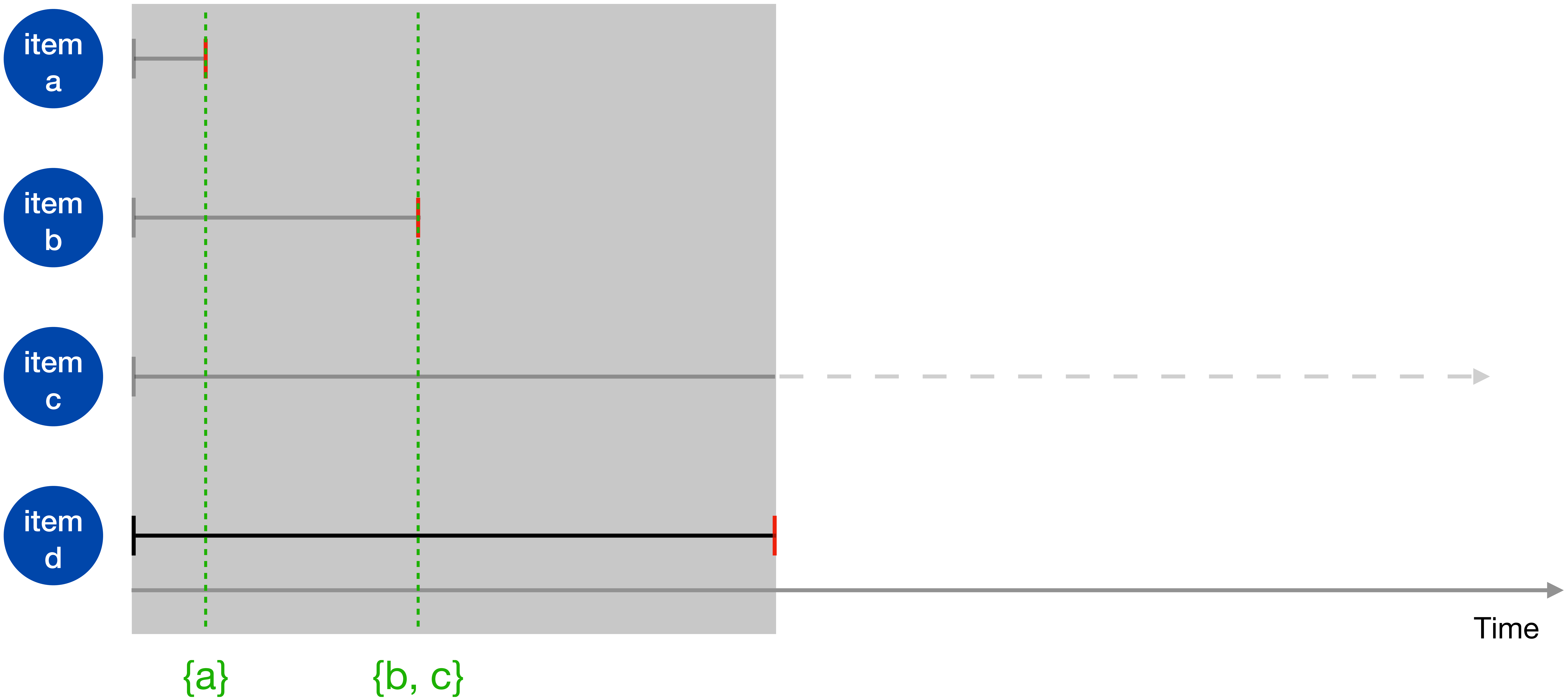

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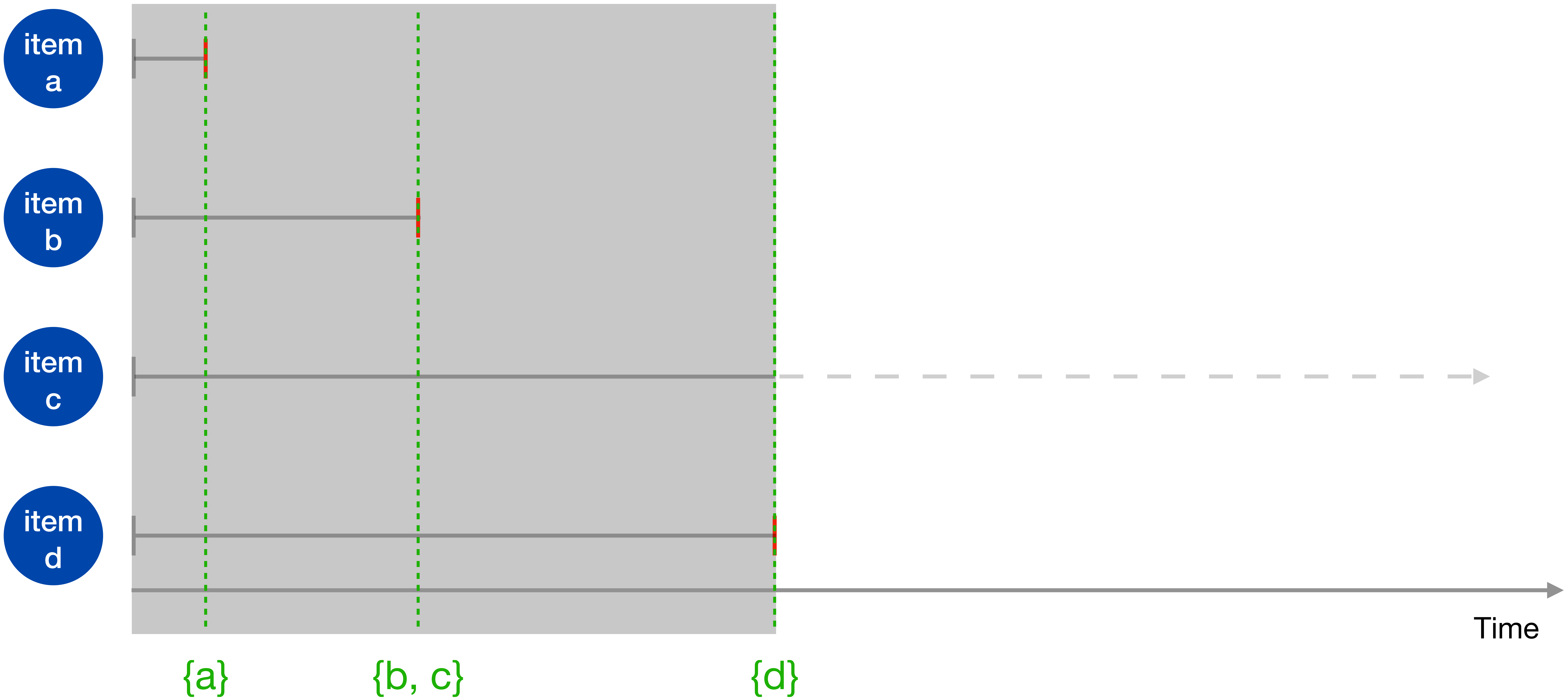
Lower bound



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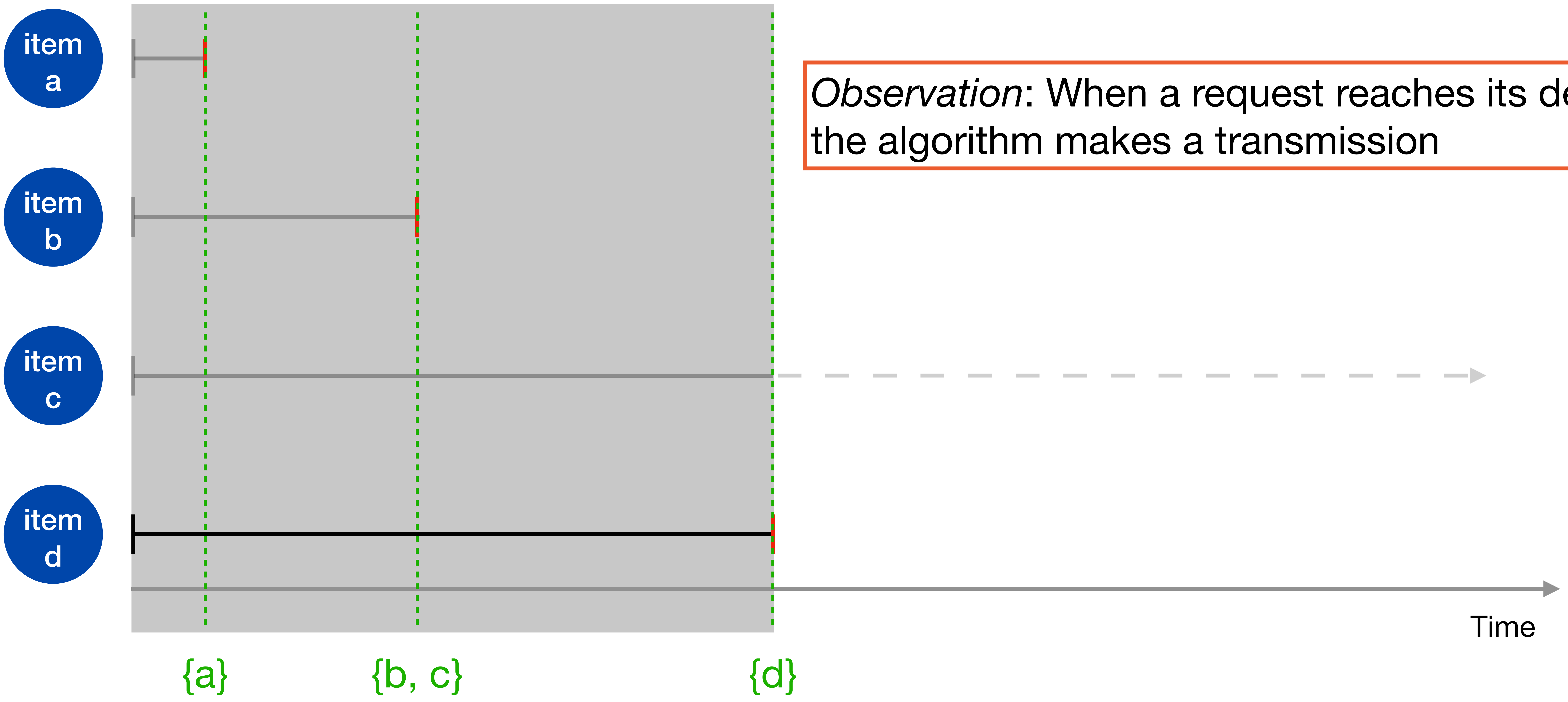
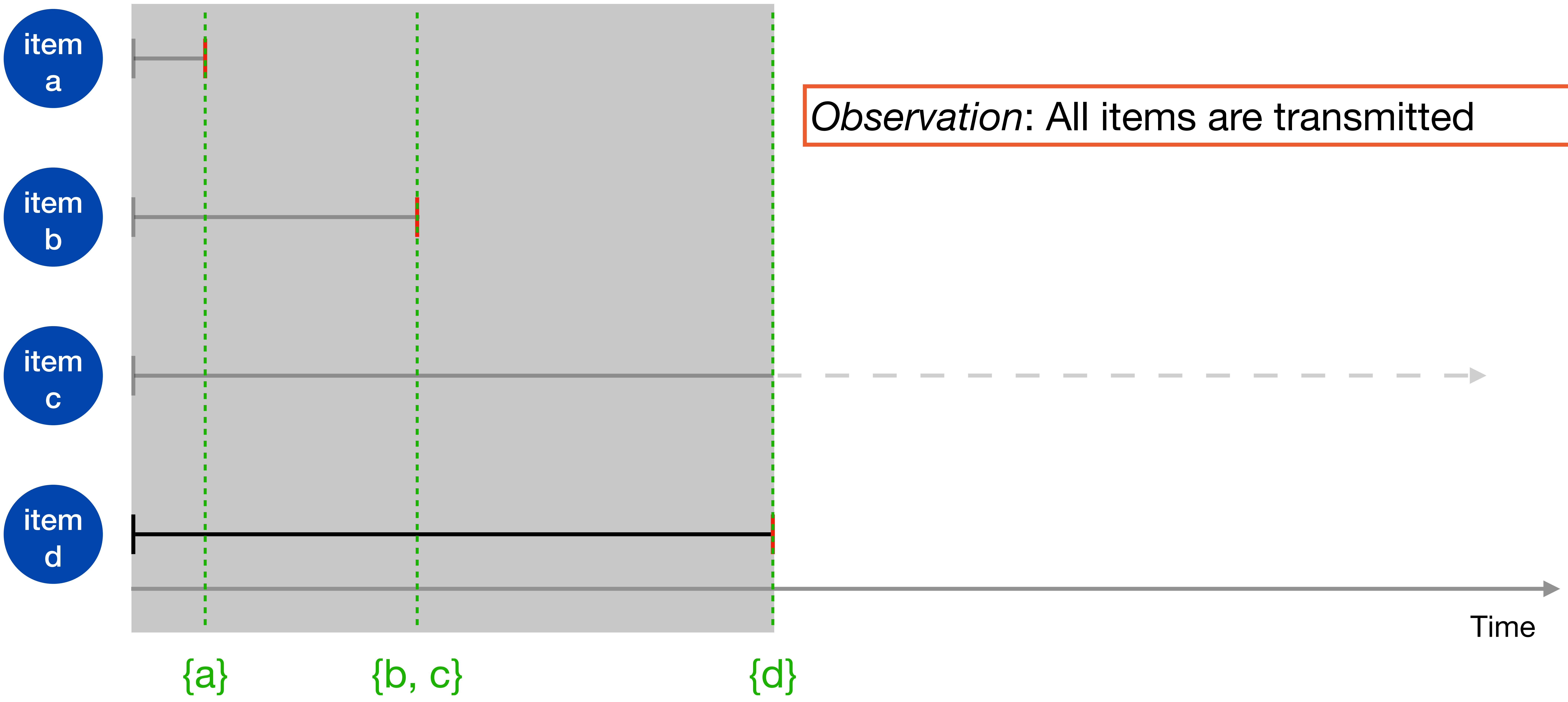
Lower bound



Observation: When a request reaches its deadline, the algorithm makes a transmission

- ALG = (#requests with finite deadlines) x $\sqrt{n}$

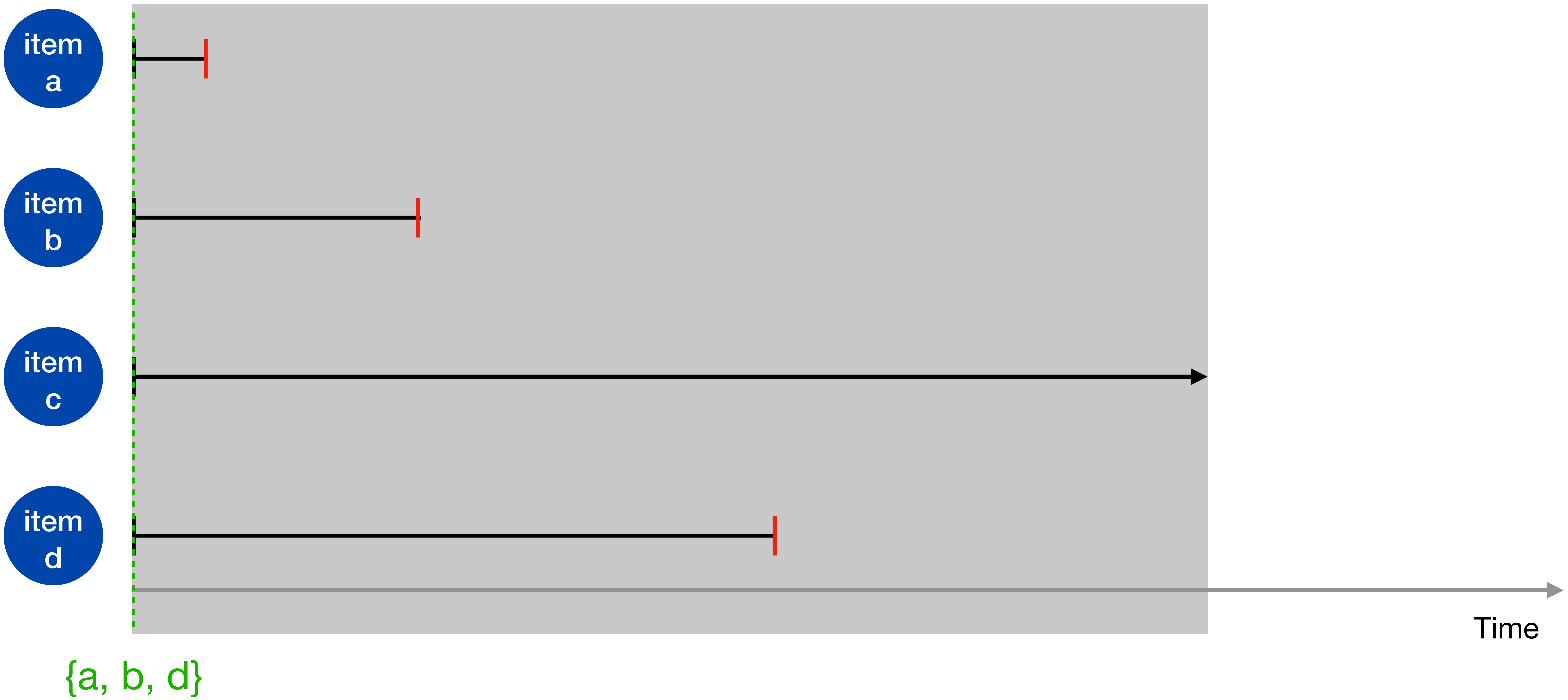
Lower bound



Observation: All items are transmitted

- ALG = (#requests with finite deadlines) x $\sqrt{n} + n$

Lower bound - Offline optimal



- OPT = $\sqrt{n}$ + (#requests with finite deadlines)

Lower bound

JRP with deadlines

- ALG = (#requests with finite deadlines) x $\sqrt{n} + n$
- OPT = $\sqrt{n}$ + (#requests with finite deadlines)

Lower bound

JRP with deadlines

- ALG = (#requests with finite deadlines) x $\sqrt{n} + n$
- OPT = $\sqrt{n}$ + (#requests with finite deadlines)

$$\text{Competitive ratio} = \frac{(\#\text{requests with finite deadlines}) \times \sqrt{n} + n}{(\#\text{requests with finite deadlines}) + \sqrt{n}}$$

Lower bound

JRP with deadlines

- ALG = (#requests with finite deadlines) x $\sqrt{n} + n$
- OPT = $\sqrt{n}$ + (#requests with finite deadlines)

$$\text{Competitive ratio} = \frac{(\#\text{requests with finite deadlines}) \times \sqrt{n} + n}{(\#\text{requests with finite deadlines}) + \sqrt{n}}$$

$$= \sqrt{n}\,\frac{(\#\text{requests with finite deadlines}) + \sqrt{n}}{(\#\text{requests with finite deadlines}) + \sqrt{n}} = \sqrt{n}$$

Insights

JRP with deadlines

- Algorithm does not know requests’ deadlines
    - serving requests in order of increasing deadlines is not possible

Insights

JRP with deadlines

- Algorithm does not know requests' deadlines
    - serving requests in order of increasing deadlines is not possible

- Prioritise requests based on the cost of the items they are on

Algorithm

JRP with deadlines

- *Theorem:* There exists a $O(\sqrt{n})$ - competitive non-clairvoyant algorithm for JRP with deadlines

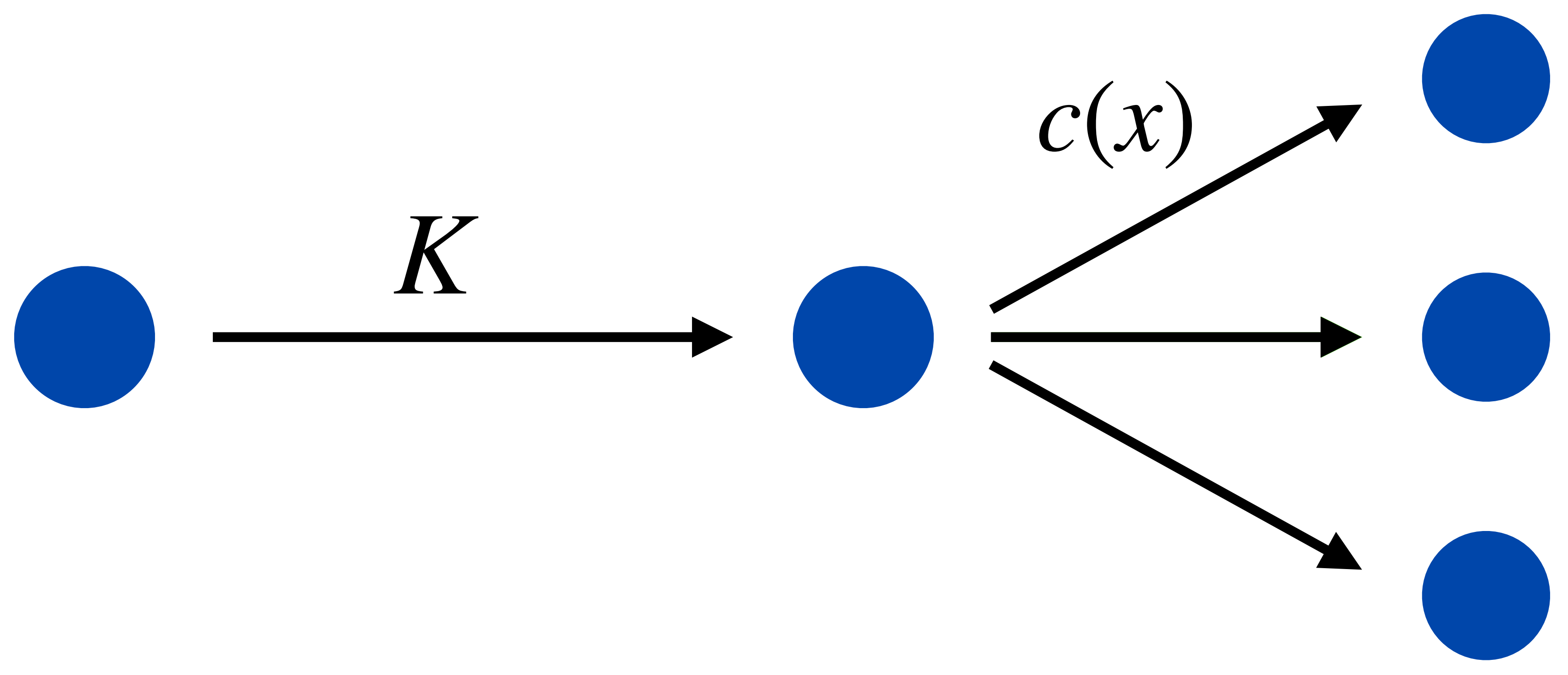
Algorithm

JRP with deadlines

- *Definition:*

$$x \text{ is } \text{cheap} \text{ if } c(x) \leq \frac{K}{\sqrt{n}}$$

$$x \text{ is } \text{expensive} \text{ if } c(x) > \frac{K}{\sqrt{n}}$$



K := ordering fee, $c(x)$:= cost of item x

Algorithm

JRP with deadlines

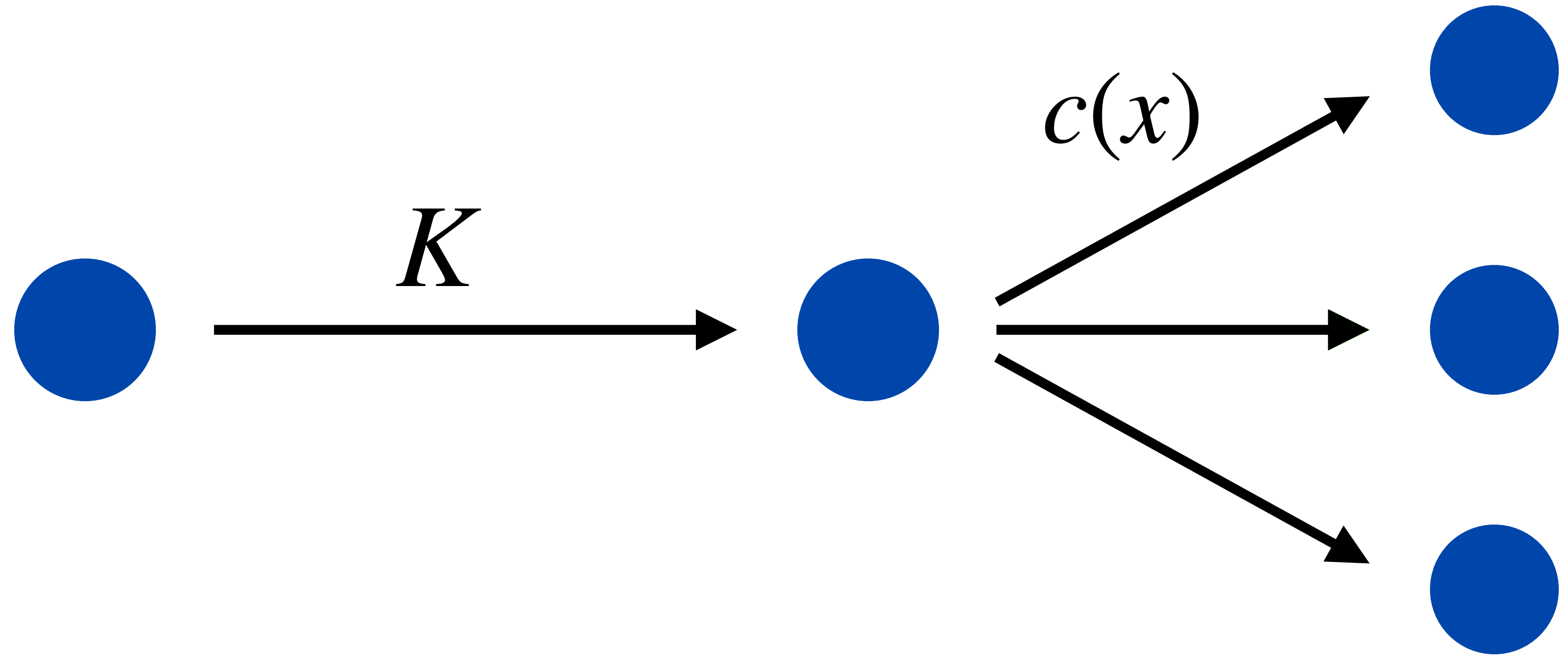
- *Definition:*

$$x \text{ is } \textcolor{orange}{\text{cheap}} \text{ if } c(x) \le \frac{K}{\sqrt{n}}$$

a request is cheap if it’s on a cheap item

$$x \text{ is } \textcolor{orange}{\text{expensive}} \text{ if } c(x) > \frac{K}{\sqrt{n}}$$

a request is expensive if it’s on an expensive item



K := ordering fee, $c(x)$:= cost of item x

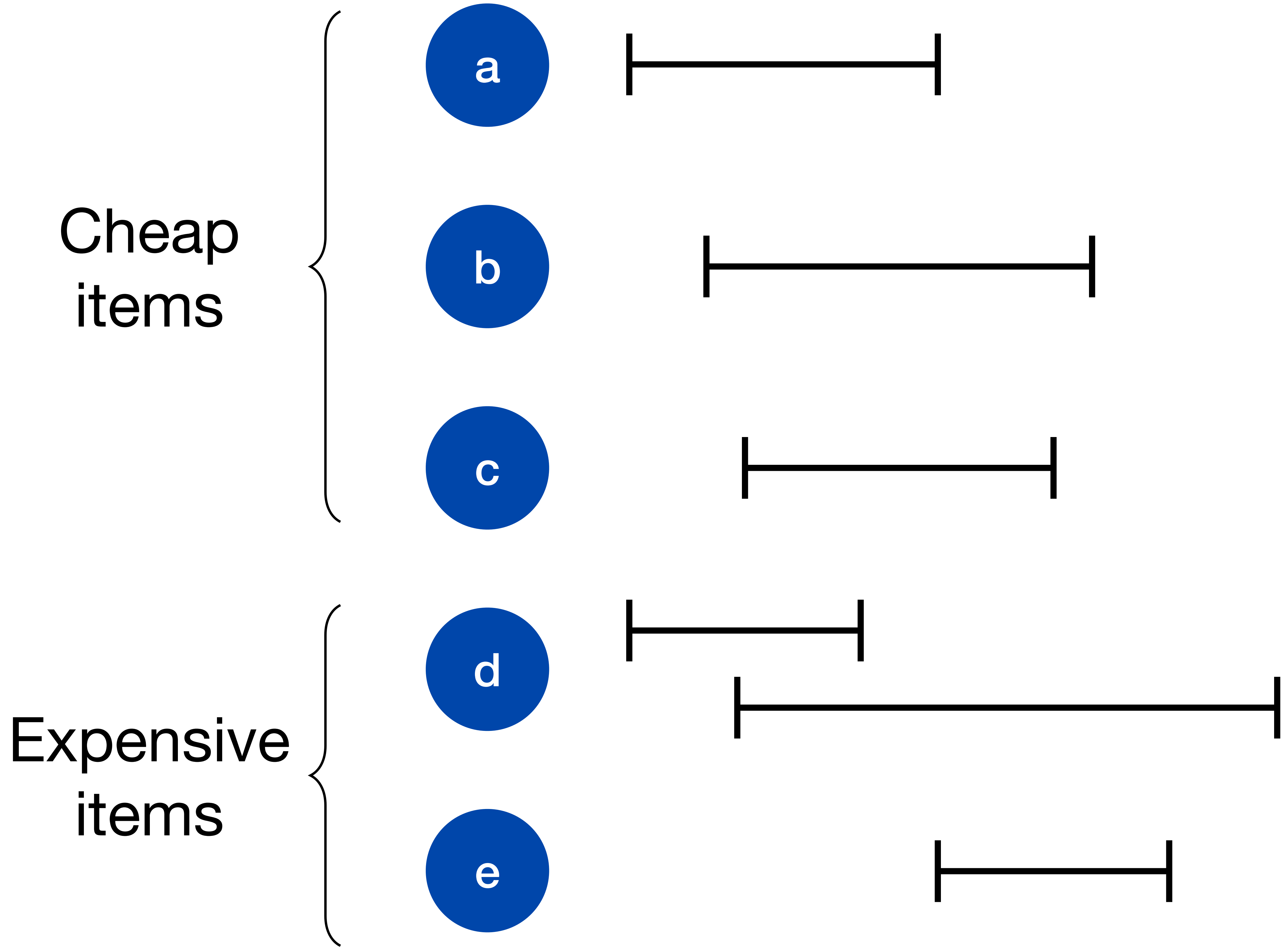
Algorithm

JRP with deadlines

UponDeadline(q):

if q is expensive:

Transmit item q is on



Algorithm

JRP with deadlines

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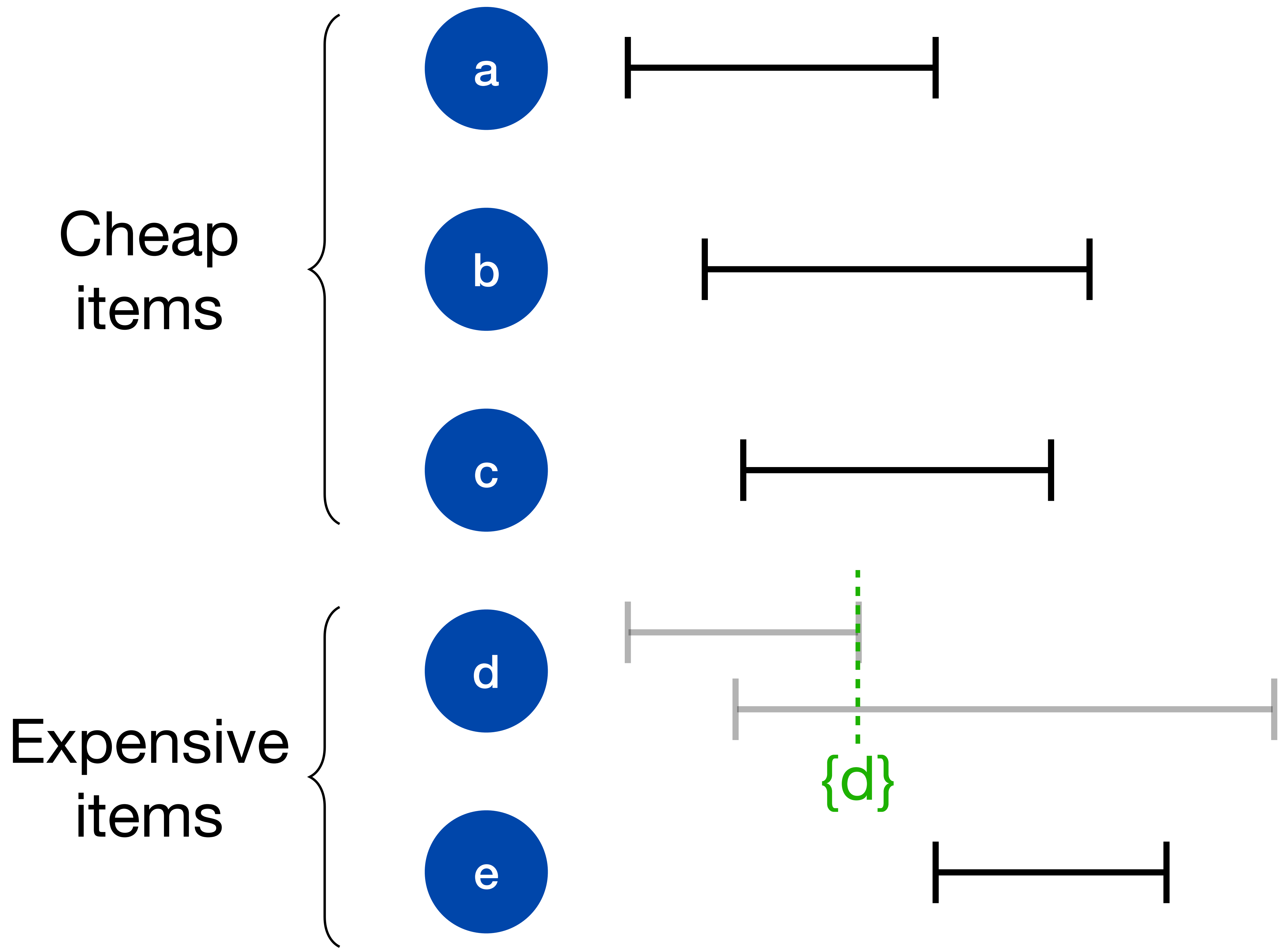


Algorithm

JRP with deadlines

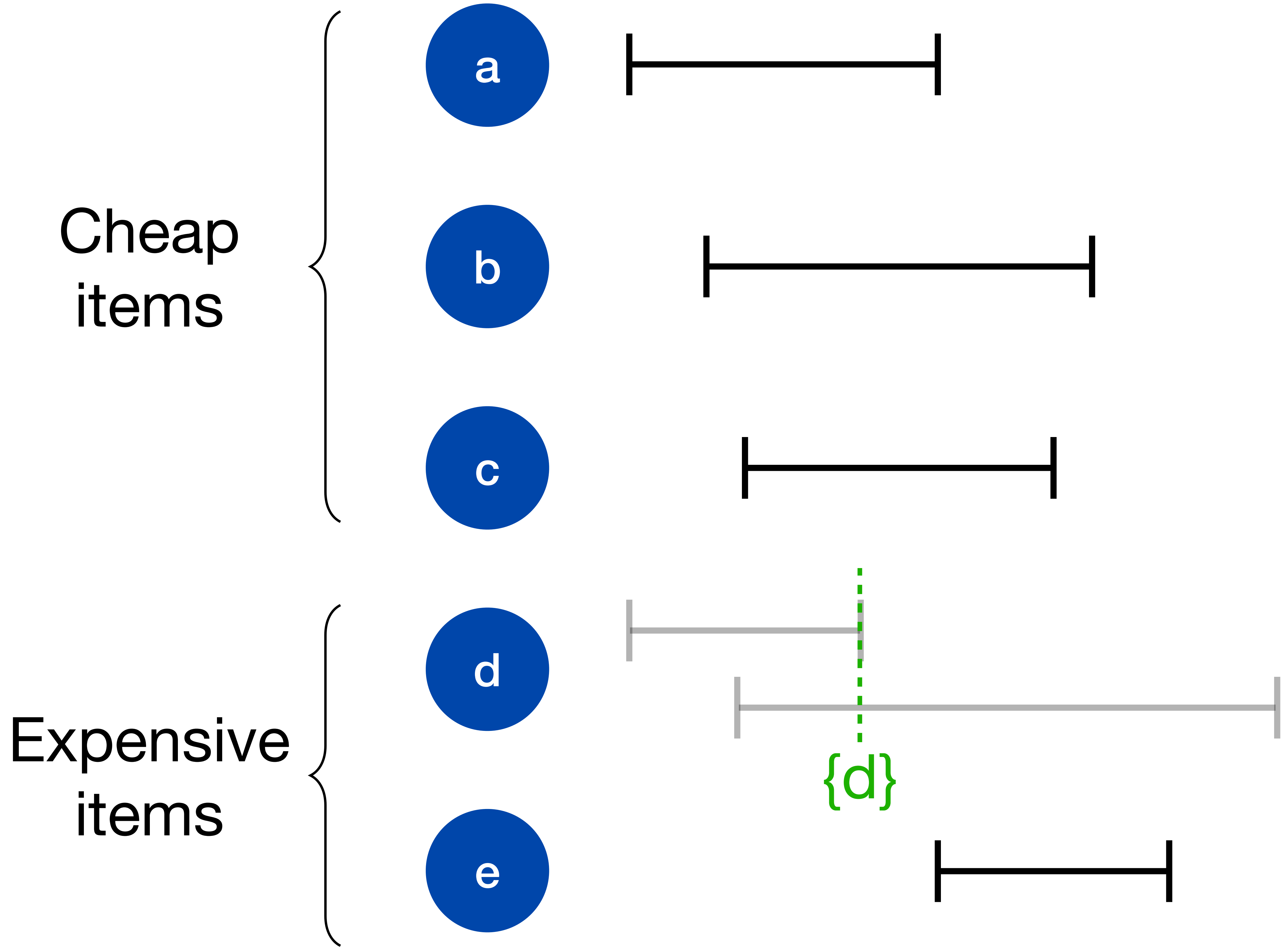
UponDeadline(q):

    if q is expensive:

        Transmit item q is on

    if q is cheap:

        Transmit all cheap items



Algorithm

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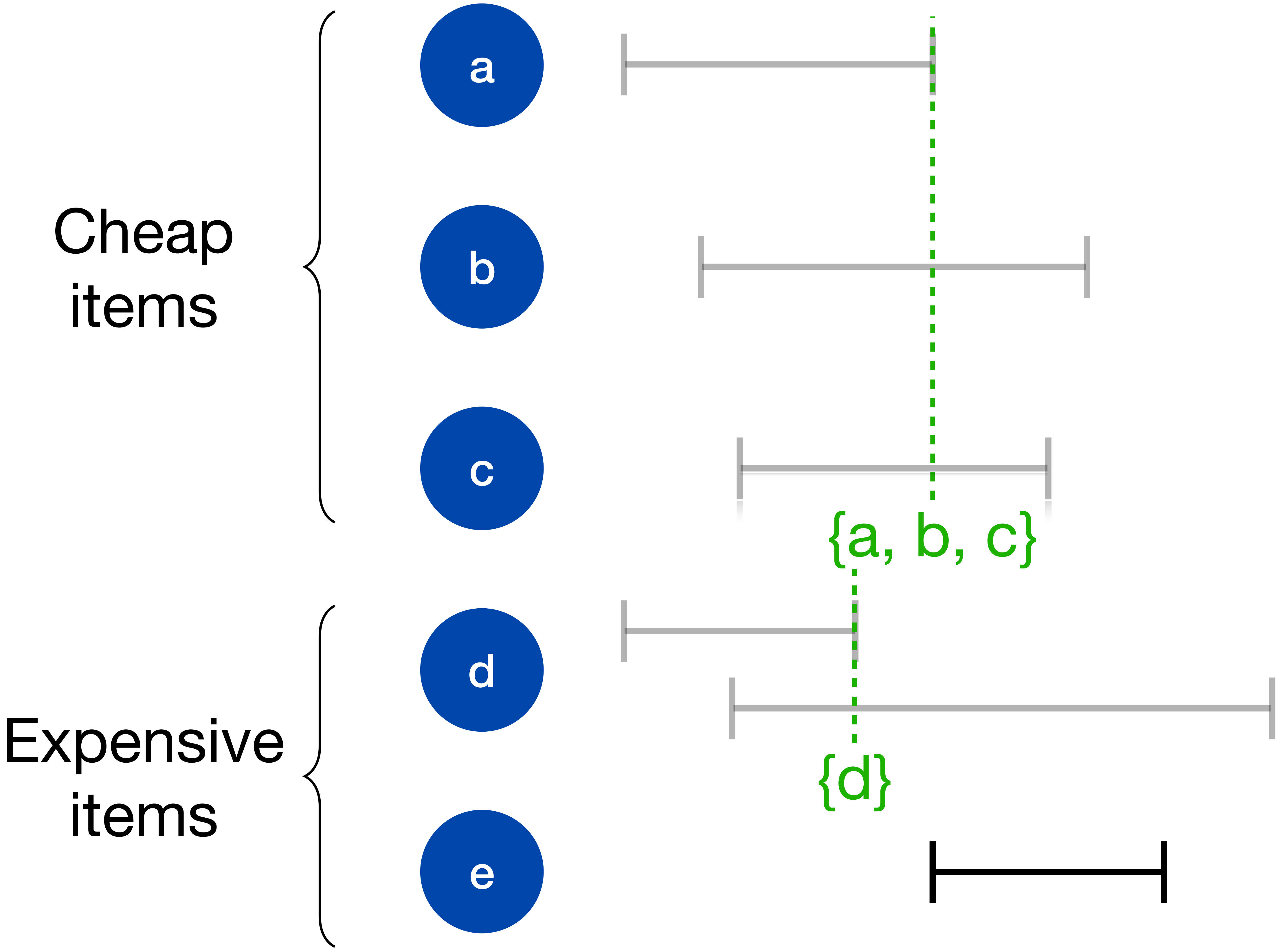

Algorithm

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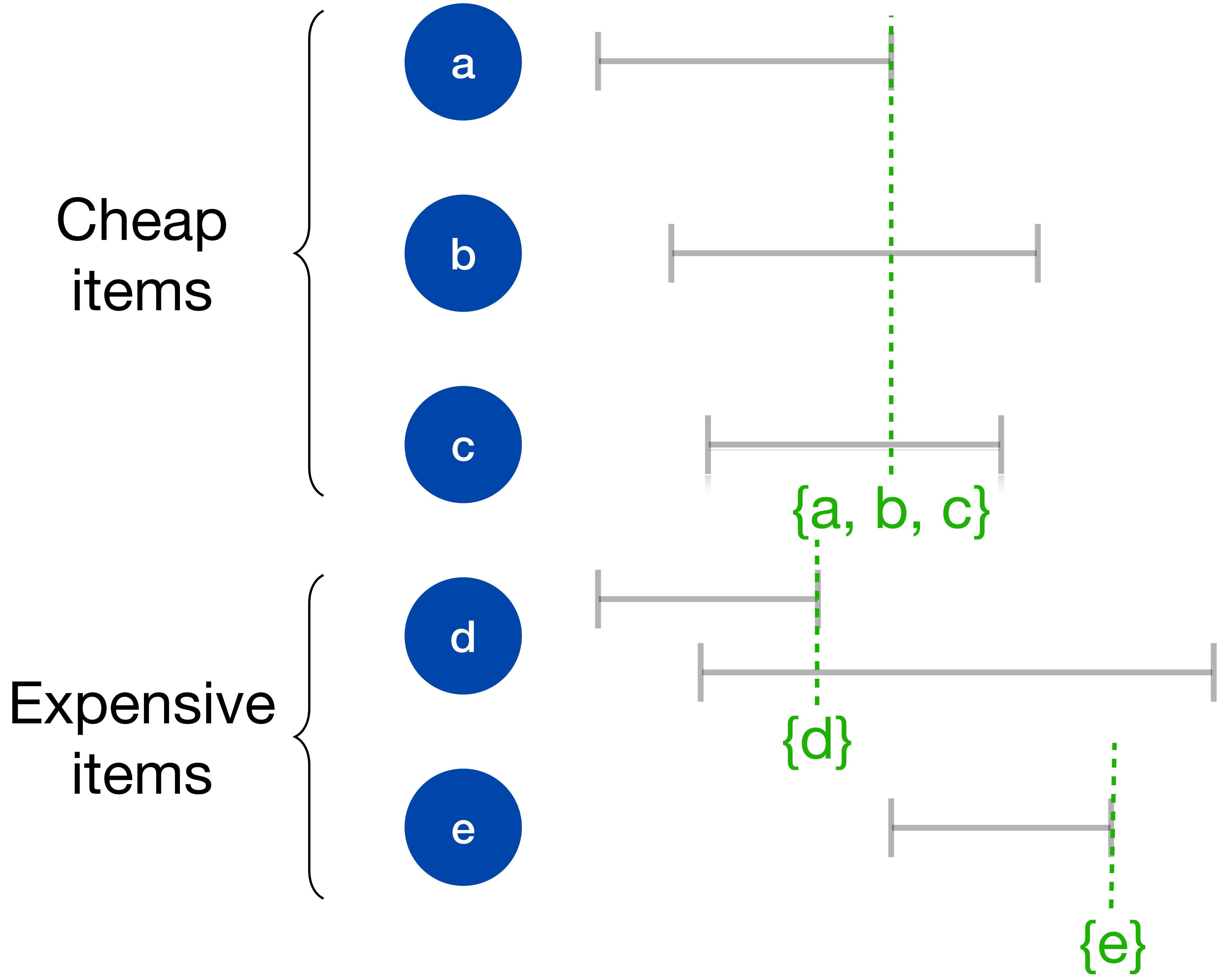


Algorithm Analysis

JRP with deadlines

A transmission is either triggered by

1. a cheap request

OR

2. an expensive request

Algorithm Analysis

JRP with deadlines - Transmissions triggered by cheap requests

- x is cheap if $c(x) \leq K/\sqrt{n}$
- There are at most n cheap items
- Cost of all cheap items is at most $\dfrac{K}{\sqrt{n}} \times n = K\sqrt{n}$

Algorithm Analysis

JRP with deadlines - Transmissions triggered by cheap requests

When a transmission is triggered by a cheap request, all cheap items are transmitted

$\boxed{\text{Cost of transmission triggered by cheap request } q}$ $\leq K + K\sqrt{n}$

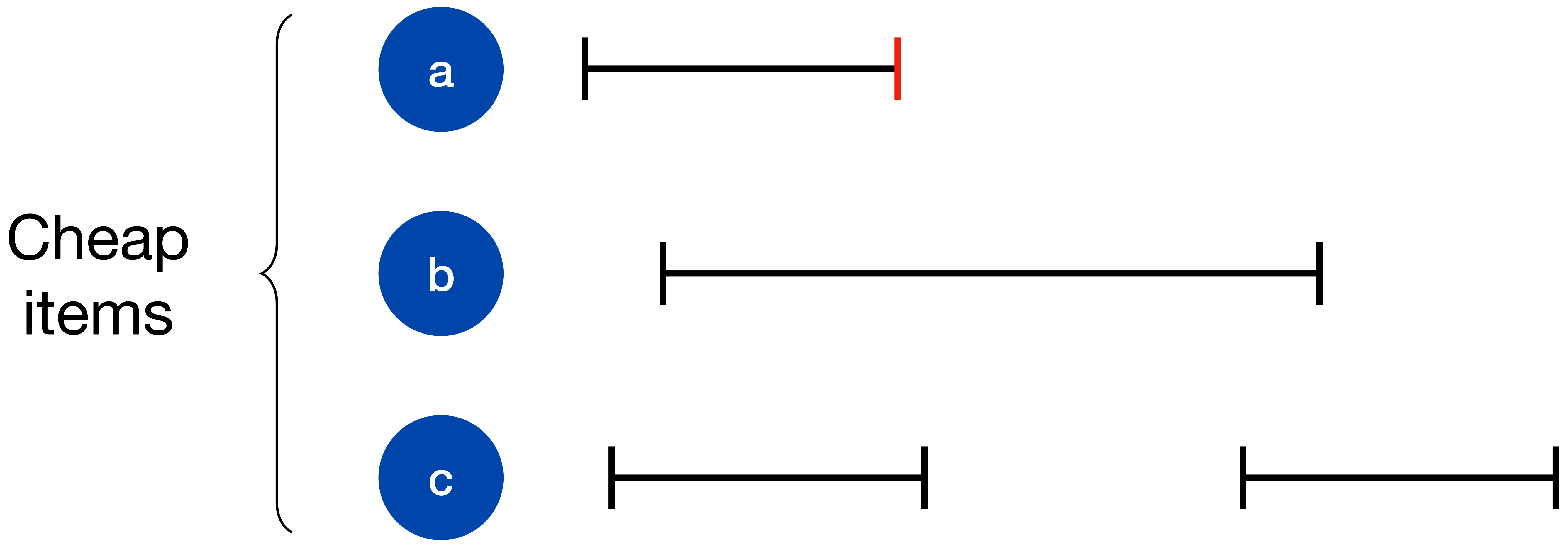
Algorithm Analysis

Charging optimal solution

Cost of transmission triggered by cheap request q	$\leq K + K\sqrt{n}$
$\downarrow K$	
Ordering fee cost of optimal transmission that served q	$= K$

- Can we charge the same optimal transmission twice?

Algorithm Analysis



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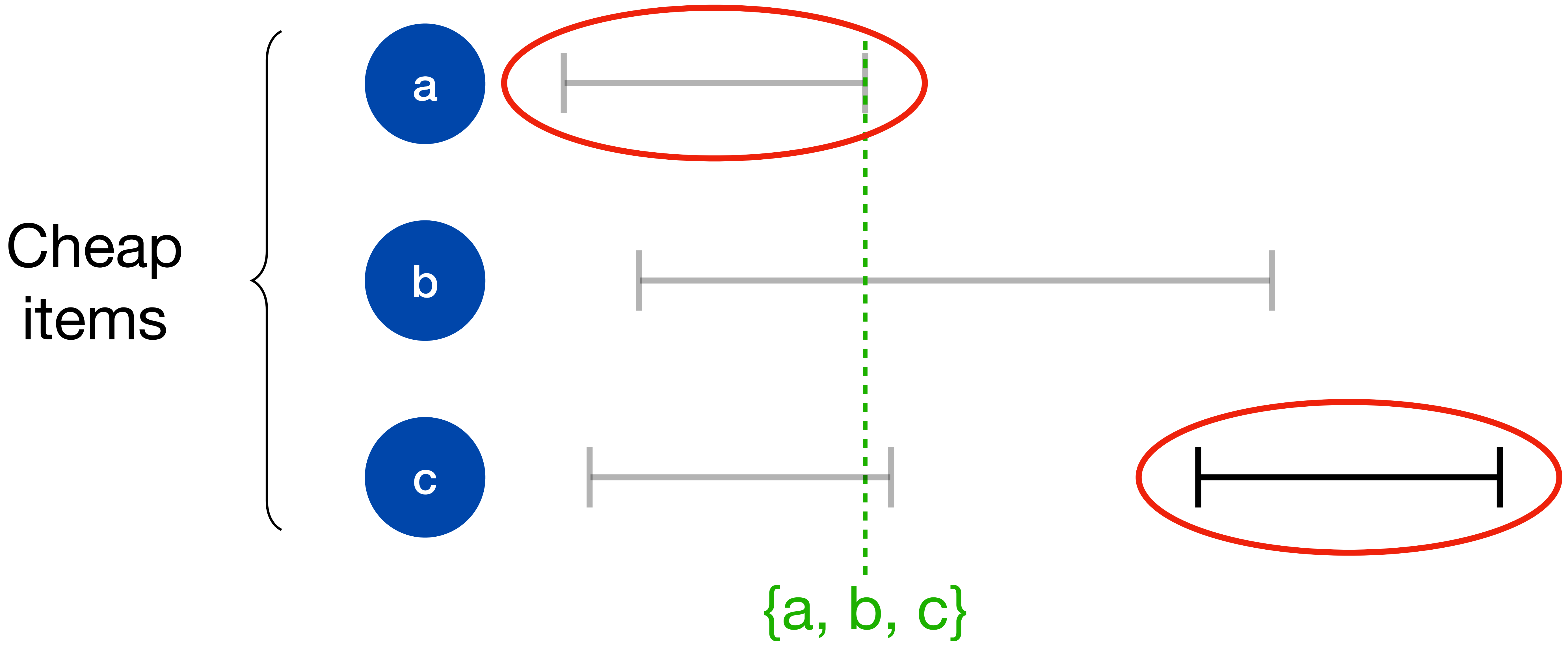
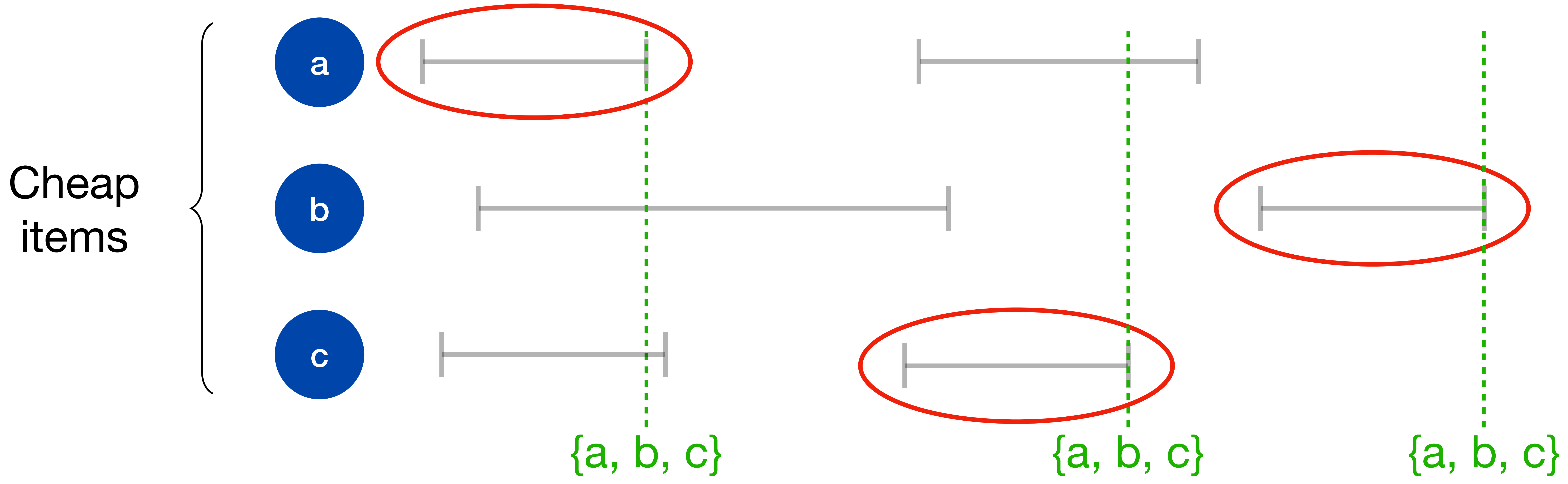
Algorithm Analysis



- Can we charge the same optimal transmission twice?

Algorithm Analysis



- Can we charge the same optimal transmission twice? **No**

Algorithm Analysis

JRP with deadlines - Transmissions triggered by expensive requests

x is expensive if $c(x) > K/\sqrt{n}$

Hence $K < \sqrt{n} \cdot c(x)$

Cost of a transmission is at most $K + c(x) < (\sqrt{n} + 1) \cdot c(x)$

Algorithm Analysis

Charging optimal solution

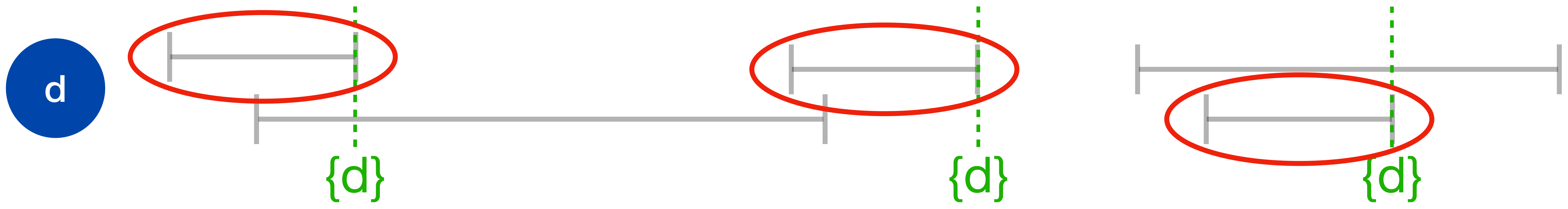
$$\boxed{\text{Cost of transmission triggered by expensive request q}} \quad \leq (\sqrt{n} + 1) \cdot c(x)$$

$$\big\downarrow c(x)$$

$$\boxed{\text{Cost of item } x \text{ in optimal transmission that served q}} \quad = c(x)$$

- Can we charge the **same item** in the same optimal transmission twice?

Algorithm Analysis



- Can we charge the **same item** in the same optimal transmission twice? **No**

Algorithm Analysis

Cost of transmission triggered by cheap request q $\leq K + K\sqrt{n}$

$(\sqrt{n} + 1)$ factor

Ordering fee cost of optimal transmission that served q $= K$

Cost of transmission triggered by expensive request q $\leq (\sqrt{n} + 1) \cdot c(x)$

$(\sqrt{n} + 1)$ factor

Cost of item x in optimal transmission that served q $= c(x)$

Competitive ratio = $\sqrt{\mathbf{n}} + 1$

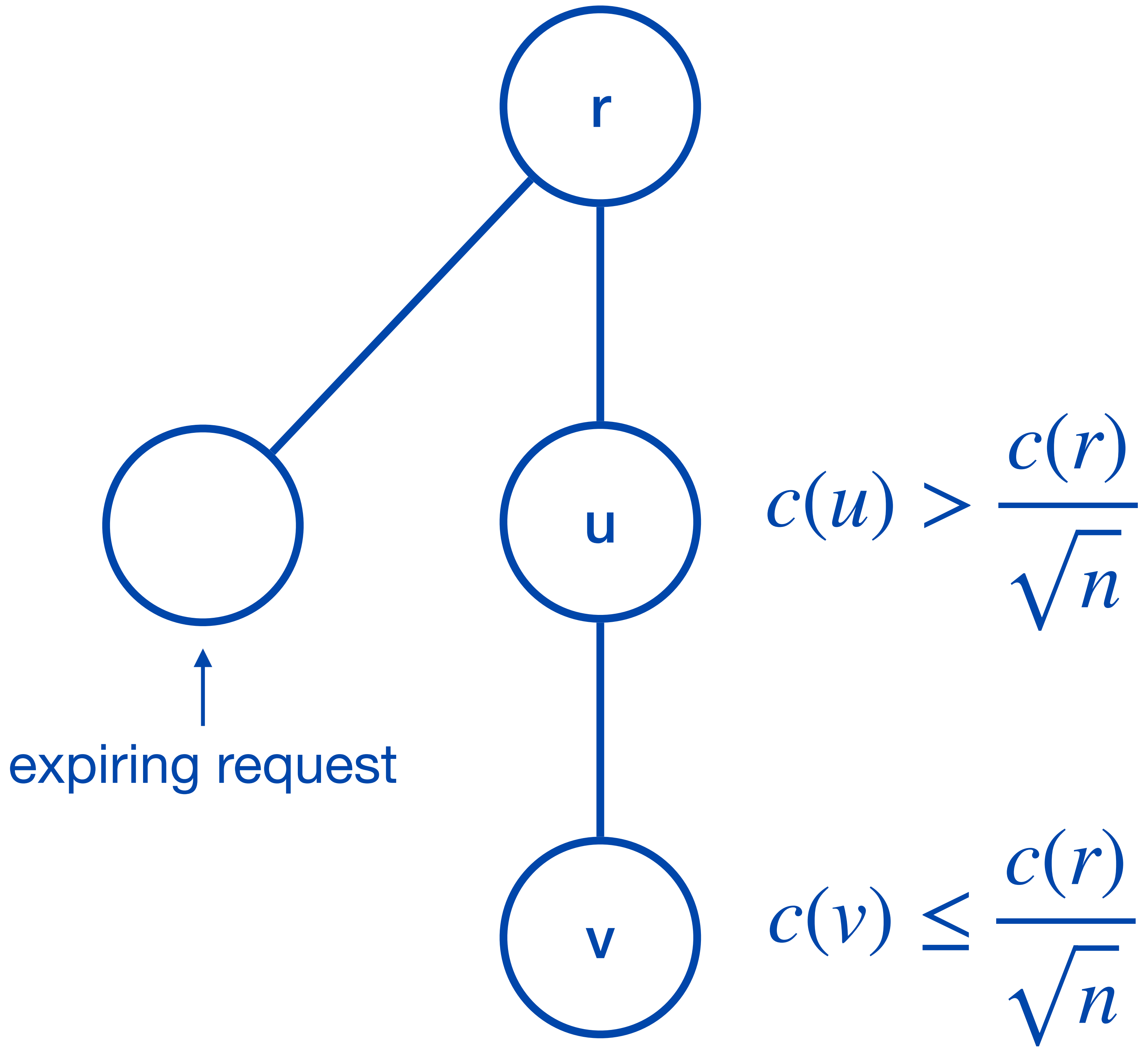
Algorithm

JRP with deadlines

- Can be generalised to the delay case, giving $O(\sqrt{n})$-competitive algorithm

Multi-level aggregation with deadlines

- Use a similar idea as in JRP
- What to do if we must go through an expensive node to reach a cheap node?
- Key idea: **Fractionally** pay ahead for nodes we can't transmit yet
- Theorem: There exists a $O(\sqrt{n} + \text{depth})$ - competitive non-clairvoyant algorithm for MLAP with deadlines



Summary

	Non-clairvoyant
JRP with deadlines	lower bound: $\Omega(\sqrt{n})$ upper bound: $O(\sqrt{n})$
JRP with delay	$O(\sqrt{n})$
MLAP with deadlines	$O(\sqrt{n}$ + depth)

Technique: Divide items into cheap and expensive relative to fixed ordering cost

Open Problems

- Extend algorithm for MLAP to the delay case

Open Problems

- Extend algorithm for MLAP to the delay case
- Study more general problems in the non-clairvoyant setting
    - Steiner-tree with deadlines/delay

References

Yossi Azar, Ashish Chiplunkar, Shay Kutten, & Noam Touitou (2020). Set Cover with Delay - Clairvoyance Is Not Required. In *28th Annual European Symposium on Algorithms (ESA 2020)* (pp. 8:1–8:21). Schloss Dagstuhl–Leibniz-Zentrum für Informatik.

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