ME314 Final Project Report

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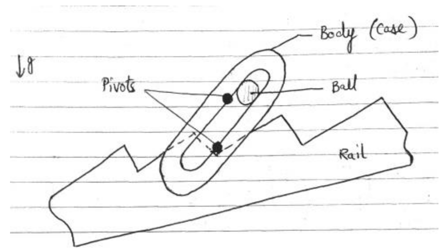
Dynamics of Tumbling Toy on an Inclined Saw-Toothed Rail



Original Proposal and Changes:

Simulating the nonlinear dynamics of the tumbling toy, which consists of a hollow wooden case with a pivot on each side, a metal ball which can roll along the long axis of the case, and an inclined saw-toothed rail on which the pivots step down. The tumbling motion is characterized by the alternating rotation of the case around one of the two pivots with the rolling of the ball from end to end of the case. The system has 4 degrees of freedom, which are the configuration variables of the simulation:

q = [xRC, yRC, θ, xCB]^T , where xRC and yRC are the tangential and normal displacement of the CoM of the case relative to the rail, and θ is the rotation of the of the case with respect to the rail. xCB is the displacement of the ball within the body. Following impacts govern the motion of the system:

- Elastic impacts of the ball with the ends of the case
- Elastic impacts of the pivots on the rail (in the original proposal I had wrongly stated plastic impacts of the pivots, but they should be elastic since they do not remain stuck on the surface).

The assumptions are:

- Friction between the pivots and the rail in negligible, hence it is ignored
- The ball is considered a point mass moving without friction within the case

An additional change is that the pivots are modeled as triangles, instead of circles as in the original proposal (to make impacts less cumbersome to handle).

Drawing of the modeled system:

Transformation from World to Rail Frame:

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Transformation from Rail to Case Frame:

$$\begin{pmatrix} \text{Cos}[\theta[t]] & -\text{Sin}[\theta[t]] & 0 & \text{xRC}[t] \\ \text{Sin}[\theta[t]] & \text{Cos}[\theta[t]] & 0 & \text{yRC}[t] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Transformation from Case to Ball Frame:

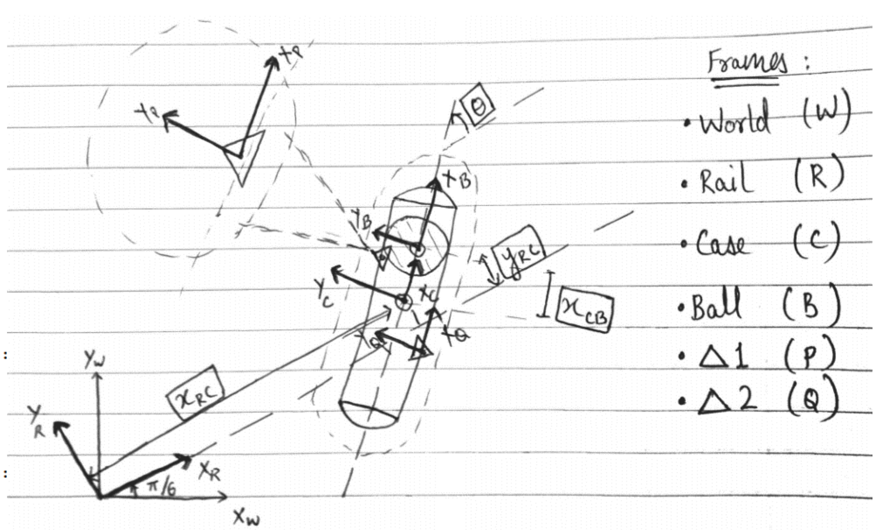
$$\begin{pmatrix} 1 & 0 & 0 & \text{xCB}[t] \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Transformation from Case to Pivot 1 Frame:

$$\begin{pmatrix} 1 & 0 & 0 & 0.0105 \\ 0 & 1 & 0 & 0.002 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Transformation from Case to Pivot 2 Frame:

$$\begin{pmatrix} 1 & 0 & 0 & -0.0105 \\ 0 & 1 & 0 & -0.002 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



Setting up the problem:

The inertial and geometric parameters were sourced from a paper[1] which used Newton-Euler formulation for this system. The KE of the case and the ball were written in terms of their body velocities. The PE of each was found using homogeneous transformations to represent height in world frame. With the Lagrangian now available, 4 Euler-Lagrange equations (one corresponding to each state variable) were written. The RHS of each eq. was 0, since the only constraint (ball's translation confined to the case) is embedded within the transformation matrix from the case to the ball, and there are no external forces. *NDSolve* is used to numerically integrate the E-L equations, with arbitrarily chosen initial conditions. However, this integration is done piecewise, since it is stopped whenever an impact is detected (using *WhenEvent*). 8 impact conditions were checked at every instant (6 from the vertices of the two triangular pivots, and 2 from the ball hitting the case's ends). After every impact, new initial conditions for integration are found by solving the impact laws of momentum change and Hamiltonian conservation (giving 5 eqs.). It is assumed that at any instant only one impact condition from the 8 is true.

Simulation:

The simulation (often) does indicate what would realistically happen from the arbitrarily given initial conditions. Started from the right initial conditions, one does observe the case tumbling down as the ball moves to the other side. The effect of both the pivot impact as well as ball impact on the whole system's motion is clearly seen. The Hamiltonian is also mostly conserved, as is expected. Since impacts are modeled as purely elastic, their effect in changing the body's motion is more pronounced than the real case, where inelastic collisions and sliding friction subdue that effect. Additionally, since the impact surface is an approximate function, and the animation shows the real non-differentiable surface, sometimes the impact may appear to happen a bit inside or outside the rail surface.

With the current values of *MaxStepSize* and number of iterations, the code should take under a minute to run.

Issues with the simulation:

Many times, depending on the initial conditions and step size, *NDSolve* misses detecting some impacts, which has weird effects like the ball crossing through the confines of the case. Decreasing the step size improves the result, but increases computation time by a lot. For a test run with step size 1/300,000 and 30 iterations (which took probably an hour to run), impact recording accuracy for the ball improved (every impact detected), but many impacts of the pivots were missed. As a result, the overall motion was not realistic.

Tweaking the options of *NDSolve* and *WhenEvent*, or using *EventLocator* instead of *WhenEvent* might help, but I have not investigated that in depth yet.

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[1] *Leine, Campen, Glocker, "Nonlinear Dynamics and Modeling of Various Wooden Toys with Impact and Friction"