Recitation - Week 4

CSE 355: Theory of Computation

Today’s Agenda

1. Overview of Pumping Lemma
2. Some more problems
3. Examples

Pumping Lemma

- Every regular language has a pumping length *p*.
- For any string *w* in the language with $|w| \geq p$, it can be divided into three parts $w=xyz$ such that:

1. $x\ y\verb|^|i\ z \in L$, *for every* $i \geq 0$
2. $|y| > 0$
3. $|xy| \leq p$

Common Misconceptions

- **Misconception 1**: Assuming that any language with repeating patterns is regular.
- **Misconception 2**: Misinterpreting the division $w=xyz$.
- **Misconception 3**: Incorrectly applying the lemma to disprove non-regular languages.

Problem 1

Consider the alphabet $\Sigma=\{a,b\}$. Define the language:$L1'=\{w \in \Sigma^* \mid$ w contains an equal number of a′s and b′s and every prefix of w contains at least as many b′s as a′s}. Decide if the language is regular or not and prove your claim.

Solution

The language L1′is not regular. To see why, assume L1′ is regular and has a pumping length p. Consider the string *w=b^p * a^p*. This string is in L1′ because every prefix has at least as many *bs* as *as*. However, no matter how you divide w into *xyz* such that $|xy|\leq p$ and $|y|>0$, pumping y (which will contain only bs) will create a string with more *bs* than *as*, which is still in L1′, but every prefix of this string will violate the condition that every prefix must have at least as many bs as as. Thus, L1′ is not regular.

Problem 2

Consider the alphabet $\Sigma=\{0,1\}$. Define the language:$L2'=\{w \in \Sigma* \mid w$ contains the substring 101 exactly once$\}$. Decide if the language is regular or not and prove your claim.

Solution

The language L2′ is regular. We can construct a DFA that recognizes L2′. The DFA would have to ensure that it reads the substring "101" exactly once and does not allow a second occurrence of "101". This can be achieved by designing states that track the progress of reading "101" and move to a dead state if a second "101" is detected. Since it is possible to create such a DFA, L2′ is regular.

Problem 3

Let $\Sigma=\{a,b,c\}$ and define the language: $L3'=\{w \in \Sigma* \mid$ w is of the form a^m * b^n * c^m where m=n}. Decide if the language is regular or not and prove your claim.

Solution

The language L3′ is not regular. We can use the Pumping Lemma to prove this. Assume L3′ is regular with a pumping length p. Consider the string w=a^p* b^p*c^p. This string is in L3′ because m=n=p. However, no matter how you divide w into xyz such that $|xy|\leq p$ and $|y|>0$, pumping y will disrupt the balance between the number of as, bs, and cs, leading to a string not in L3′. Therefore, L3′ is not regular.

Questions!!