CLASS : T.E. ENTE **SUBJECT: DIP**

EXPT. NO. : 4 **DATE :**

TITLE : PERFORM HISTOGRAM EQUALIZATION ON AN IMAGE

CO 1:	Apply the fundamentals of digital image processing to perform various image-enhancement and Image segmentation operations on gray scale image.

AIM:

1. To plot the Histogram of an image (Dark, Light, Low contrast, High contrast)
2. To Perform the Histogram equalization on a Low Contrast Image.

SOFTWARES REQUIRED: Jupyter Notebook or Google Colaboratory.

THEORY:

4.1 Basics of Histogram.

The histogram of an image is a plot of the number of occurrences of gray levels in the image against gray level values. Histograms are the basis for numerous spatial domain processing techniques. It plays an important role in enhancement of perceived brightness and contrast of an image. It specifies the number of pixels having each gray level, but gives no hint as to where those pixels are located within the image. The histogram of an N × M image is defined as the percentage of pixels within the image at a given gray level:

$$h_i = \frac{n_i}{MN} \qquad for\ 0 \le i \le G_{\max}$$

where n_i is the number of pixels at gray level i,

NM is the total number of pixels within the image and

G_{max} is the maximum gray level value of the image.

4.1.1 Properties of Histogram.

1) The histogram is unique for any particular image, but the reverse is not true.
2) The sum of each normalized histogram value over the range of gray levels present within an image equals 1.

$$\sum_{i=0}^{G_{\max}} h_i = 1$$

3) The average gray level of an image in terms of histogram is:

$$avg = \sum_{i=0}^{G_{\max}} i \cdot h_i$$

   And standard deviation of the gray levels of an image in terms of its histogram is given as:

$$std = \sqrt{\sum_{i=0}^{G_{\max}} i^2 \cdot h_i - avg^2}$$

4) The histogram of a dark image will be clustered towards the lower gray levels whereas; the histogram of bright images will be clustered towards higher gray levels.
5) For a low-contrast image, the histogram will not be spread equally that is the histogram will be narrow. For a high-contrast image, the histogram will have an equal spread in the gray level.

4.2 Histogram Modelling.

The histogram of an image represents the relative frequency of occurrence of the various gray levels in the image. Histogram modeling techniques modify an image so that the histogram has a desired shape. This is useful in stretching the low contrast levels of images with narrow histograms. Histogram modeling has been found to be a powerful technique for image enhancement.

4.2.1 Histogram Equalization.

The histogram technique that is used to enhance the brightness and contrast of an image is histogram equalization. The goal of histogram equalization is to distribute the gray levels within an image so that every gray level is equally likely to occur. In other words, histogram equalization takes an image's histogram and produces a new image with a histogram that is uniformly distributed. Histogram equalization will increase the brightness and contrast of a dark and low contrast image, making features observable that were not visible in the original image. Since histogram equalization distributes an image's gray levels uniformly about the range of gray levels, all images will have approximately the same brightness and contrast, hence allowing images to be compared equally without a bias due to perceived contrast and brightness differences.

4.2.1 Histogram Matching.

The method used to generate a processed image that has a specified histogram is called histogram matching or histogram specification. Histogram equalization automatically determines a transfer function that seeks to produce an output image that has a uniform histogram. Sometimes it is useful to specify the shape of the histogram that we wish the processed image to have. The process of histogram matching takes the concept of histogram equalization one step further, allowing for the specification of the output gray level histogram. Given a desired histogram d_i, the first step is to produce a mapping function $D(d_k)$ that histogram equalizes the desired histogram:

$$D(d_k) = \sum_{i=0}^{k} d_i \qquad for\, 0 \le k \le G_{\max}$$

The next step is to histogram equalize the input image using above equation.

$$b_k = M(a_k) = G_{\max} \sum_{i=0}^{k} h_i \qquad for\, 0 \le k \le G_{\max}$$

Since, the desired histogram and the input image have been equalized, producing approximately uniformly distributed histograms, the output graylevels of the histogram equalized input image can be equated to the mapping function that was used to get the desired histogram:

$b_k = D(d_k)$

The new graylevel mapping function that maps the equalized image to the desired histogram is the inverse of the above equation:

$d_k = D^{-1}(b_k)$

4.3 Histogram equalization derivation:

Step 1:

s = T(r) where $0 \leq r \leq 1$

T(r) satisfies,

(a) T(r) is single-valued and monotonically increasing function in the interval $0 \leq r \leq 1$

(b) $0 \leq T(r) \leq 1$ for $0 \leq r \leq 1$

(C) The inverse transformation from s back to r is

$\boldsymbol{r = T^{-1}(s)}$ $\boldsymbol{0 \leq s \leq 1}$

Step 2:

Let ,

- $p_r(r)$ denote the PDF of random variable r
- $p_s(s)$ denote the PDF of random variable s

If $p_r(r)$ and $T(r)$ are known and $T^{-1}(s)$ satisfies inverse condition then $p_s(s)$ can be obtained using a formula :

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

The PDF of the transformed variable s is determined by the gray-level PDF of the input image and by the chosen transformation function

Step 3:

A transformation function is a cumulative distribution function (CDF) of random variable r:

$$s = T(r) = \int_0^r p_r(w)dw$$

Where w is a dummy variable of integration.

Step 4:

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = \frac{d}{dr}\left[\int_0^r p_r(w)dw\right] = p_r(r)$$

$$\Rightarrow$$

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right| = p_r(r)\left|\frac{1}{p_r(r)}\right| = 1 \quad \text{where } 0 \le s \le 1$$

4.4 Algorithm:

4.5 Conclusion:

4.6 References:

i. Gonzalez R, Woods R, “Digital image processing”, Pearson Prentice Hall, 2008.

ii. Gonzalez R, Woods R, Steven E, “Digital Image Processing Using MATLAB®”, McGraw Hill Education, 2010.

iii. Jayaraman S, Esakkirajan S and Veerakumar T, “Digital Image Processing” Tata McGraw Hill, 2010

iv. Joshi, Madhuri A. “Digital Image Processing: an algorithm approach”, PHI Learning Pvt. Ltd., 2006.

v. Pictures taken from: http://www.imageprocessingplace.com/root_files_V3/image_databases.html

(Course Teacher)