BRAC UNIVERSITY

MAT 110

MATH I
Differential Calculus and Co-ordinate Geometry

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Inspiring Excellence

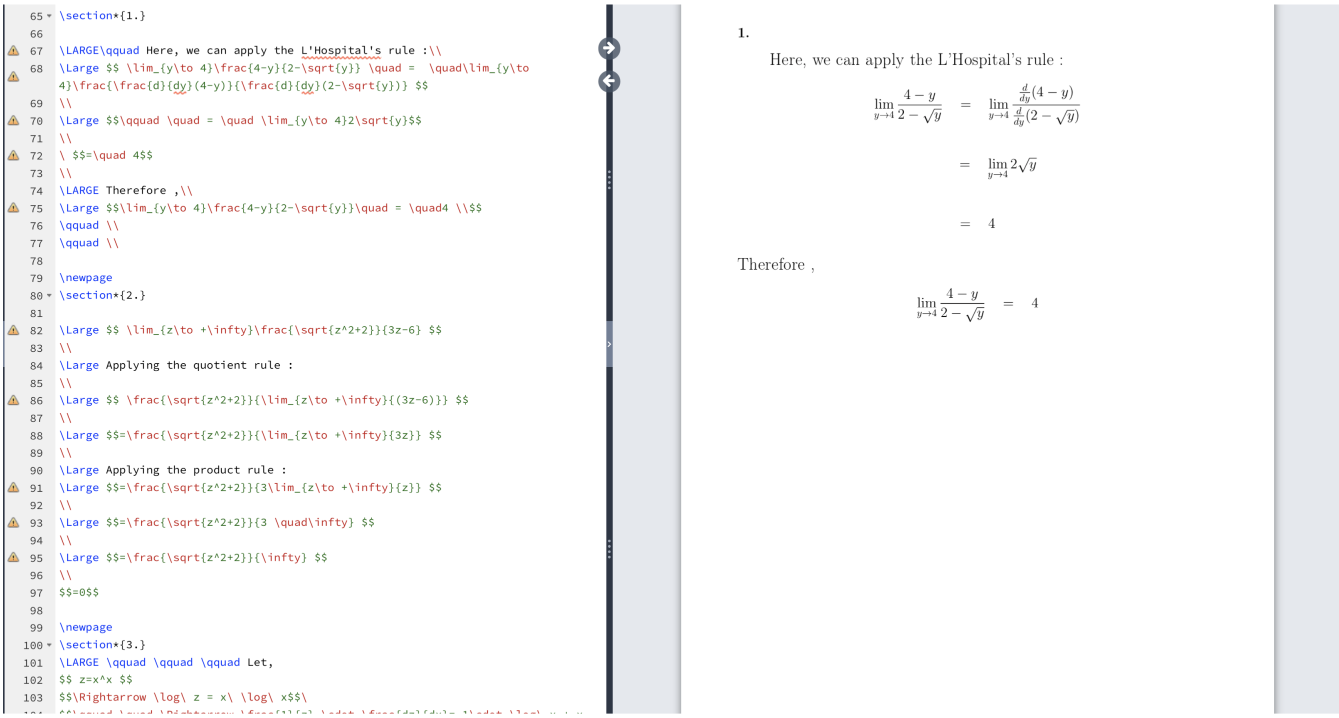

Figure 1: Latex Screenshot

1.

Here, we can apply the L'Hospital's rule :

$$\begin{aligned}\lim_{y\to 4}\frac{4-y}{2-\sqrt{y}} &= \lim_{y\to 4}\frac{\frac{d}{dy}(4-y)}{\frac{d}{dy}(2-\sqrt{y})}\\ &= \lim_{y\to 4} 2\sqrt{y}\\ &= 4\end{aligned}$$

Therefore ,

$$\lim_{y\to 4}\frac{4-y}{2-\sqrt{y}} = 4$$

2.

$$\lim_{z \to +\infty} \frac{\sqrt{z^2+2}}{3z-6}$$

Applying the quotient rule :

$$\frac{\sqrt{z^2+2}}{\lim_{z \to +\infty}(3z-6)}$$

$$= \frac{\sqrt{z^2+2}}{\lim_{z \to +\infty} 3z}$$

Applying the product rule :

$$= \frac{\sqrt{z^2+2}}{3\lim_{z \to +\infty} z}$$

$$= \frac{\sqrt{z^2+2}}{3 \quad \infty}$$

$$= \frac{\sqrt{z^2+2}}{\infty}$$

$$= 0$$

3.

Let,

$$z = x^x$$

$$\Rightarrow \log\ z = x\ \log\ x$$

$$\Rightarrow \frac{1}{z} \cdot \frac{dz}{dx} = 1 \cdot \log\ x + x \cdot \frac{1}{x}$$

$$\Rightarrow \frac{dz}{dx} = x^x(1 + \log\ x)$$

$$y = x^{x^x} = x^z$$

$$\Rightarrow \log\ y = z \cdot \log x$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \frac{dz}{dx} \cdot \log x + z \cdot \frac{1}{x}$$

$$= x^x(1 + \log x) \cdot \log x + \frac{x^x}{x}$$

$$= x^x(1 + \log x) \cdot \log x + x^x - 1$$

$$\therefore \qquad \frac{dy}{dx} = x^{x^x}\left[x^x(1+\log x)\cdot\log x + x^x - 1\right]$$

4.

$$x = a$$

$$\lim_{x \to a^-} f'(x) = \lim_{x \to a^+} f'(x)$$

$x = 0$

$$\lim_{x \to 0^-} f'(x) = 0$$

$$\lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} \cos x = 1$$

$$\lim_{x \to 0^-} f'(x) \neq \lim_{x \to 0^+} f'(x)$$

Answer: Function is not Differentiable.

5.

$$\frac{d}{dx}\ln\Big(\cos(x)\sin(x)\Big)$$

$$= \ln\Big(\cos(x)\Big)\left\{\frac{d}{dx}\sin(x)\right\} + \sin(x)\left\{\frac{d}{dx}\Big(\ln\cos(x)\Big)\right\}$$

$$= \ln\Big(\cos(x)\Big)\cos(x) + \sin(x)\left\{\frac{1}{\cos(x)}\Big(\sin(x)\Big)\right\}$$

$$= \frac{\ln\Big(\cos(x)\Big)\Big(\cos^2(x) - \sin^2(x)\Big)}{\cos(x)}$$

6.

$$y = x^2(\sin^{-1} x)^3$$

$$\frac{dy}{dx} = x^2 \cdot 3\left\{(\sin x)^{-1}\right\}^2 \cdot \frac{1}{\sqrt{1-x^2}} + 2x \cdot \left\{(\sin x)^{-1}\right\}^3$$

$$\frac{dy}{dx} = \frac{3x^2\left\{(\sin x)^{-1}\right\}^2}{\sqrt{1-x^2}} + 2x\left\{(\sin x)^{-1}\right\}^3$$