EE2703: Assignment 9

Tanay Dixit
EE19B123

1st May

Introduction

In this assignment, we analyse DFTs of signals using the `numpy.fft` package.

To check how accurate the package is, we shall take the DFT of a sequence of random numbers, then take it's IDFT and compare the two, as to how close they are. This is done using the following code

```
1    x=np.random.rand(100)
2    X=fft(x)
3    y=ifft(X)
4    c_[x,y]
5    maxError = max(np.abs(y-x))
6    print('Magnitude of maximum error between actual and computed values of the
     random sequence:', maxError)
7
```

The error is of order 10^{-15} which shows `numpy.fft` module is very accurate.

DFT of signals

We make use of the following python code to calculate DFT of various functions

```
1    def dft(func,N=512,steps = 513, r=4*np.pi, phase_limit=1e-3, xlim=40, w_lim=64,
     ylimit=None):
2        t = np.linspace(-r,r,steps)[:-1]
3        y= func_dict[func](t)
4        Y = fftshift(fft(y))/N
5        w = np.linspace(-w_lim,w_lim,steps)[:-1]
6
7        customplot(w,Y,
8        func =func,
9        path ='imgs/Q'+func,
10       xlim = xlim ,
11       ylimit = ylimit,
12       phase_limit = phase_limit)
13       return Y,w
```

We calculate the DFT of $f(t)$ by the method mentioned in the above section. $sin(5t)$ can be written as :-

$$sin(5t) = \frac{1}{2j}(e^{5jt} - e^{-5jt}) \tag{1}$$

We plot the phase and magnitude of the DFT and the following is obtained for $sin(5t)$:
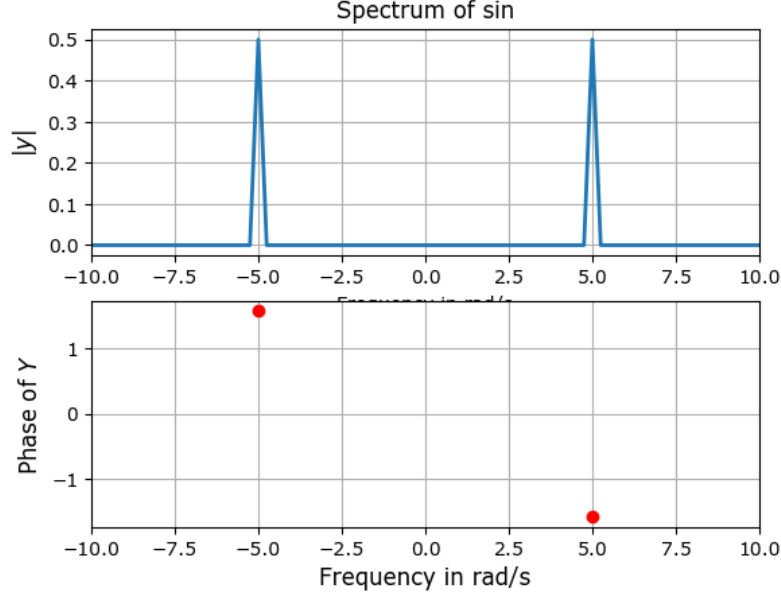
Figure 1: Spectrum of $sin(5t)$

The frequencies present in the DFT of $sin(5t)$ are $\omega = \pm 5 \ rad/sec$, and the phase associated with them is $\phi = \pm\frac{\pi}{2} \ rad/sec$ respectively.

Amplitude Modulation

We have,

$$(1 + 0.1cos(t))cos(10t) = \frac{1}{2}(e^{10jt} + e^{-10jt}) + 0.1 \cdot \frac{1}{2} \cdot \frac{1}{2}(e^{11jt} + e^{-11jt} + e^{9jt} + e^{-9jt}) \quad (2)$$

Writing $(1+0.1cos(t))cos(10t)$ in a different form as shown in (2), we observe that the frequencies present in the signal are $\omega = \pm 10 \ rad/sec$, $\omega = \pm 11 \ rad/sec$ and $\omega = \pm 9 \ rad/sec$. Thus we expect the DFT also to have non-zero magnitudes only at these frequencies.
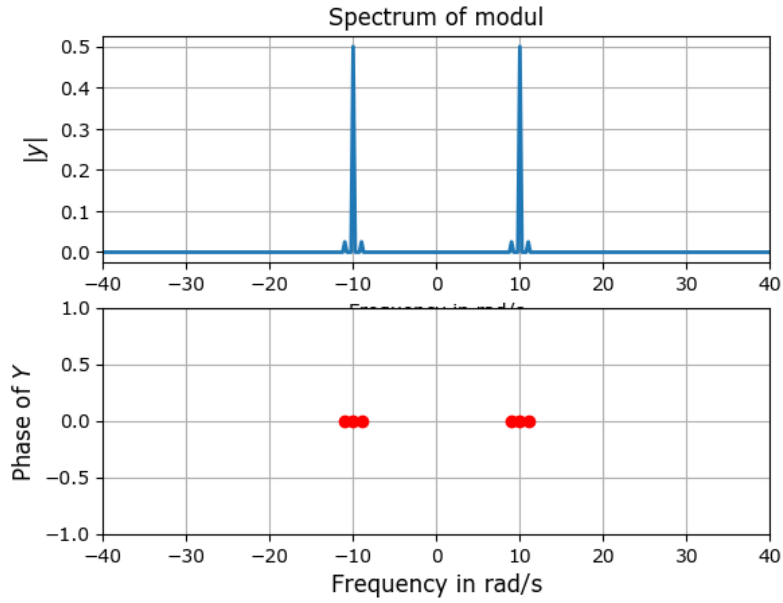
On plotting the DFT we get:

Figure 2: Amplitude modulated signal

Spectra of functions

$sin^3(t)$ and $cos^3(t)$ can be written as :

$$sin^3(t) = \frac{3}{4}sin(t) - \frac{1}{4}sin(3t) \tag{3}$$

$$cos^3(t) = \frac{3}{4}cos(t) + \frac{1}{4}cos(3t) \tag{4}$$

So, we expect peaks $\omega = \pm 1 \ rad/sec$ and $\omega = \pm 3 \ rad/sec$.

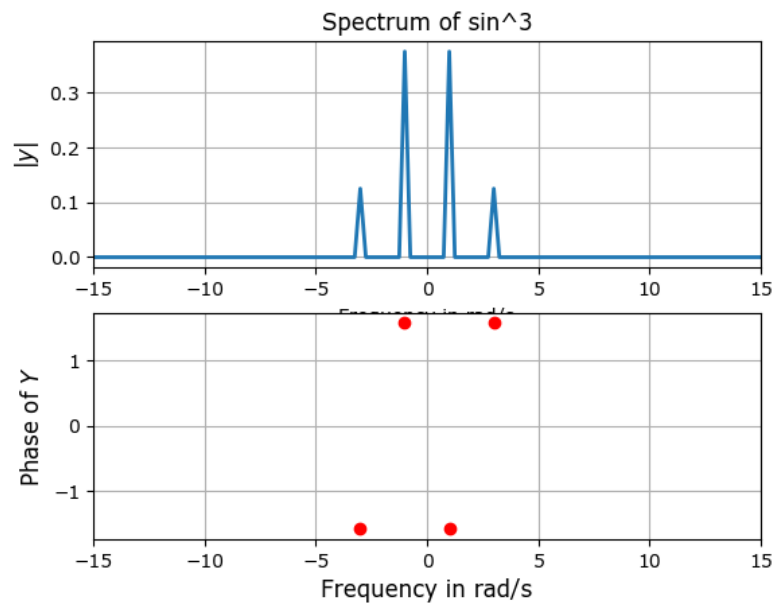## DFT Spectrum of $sin^3(t)$

:

3

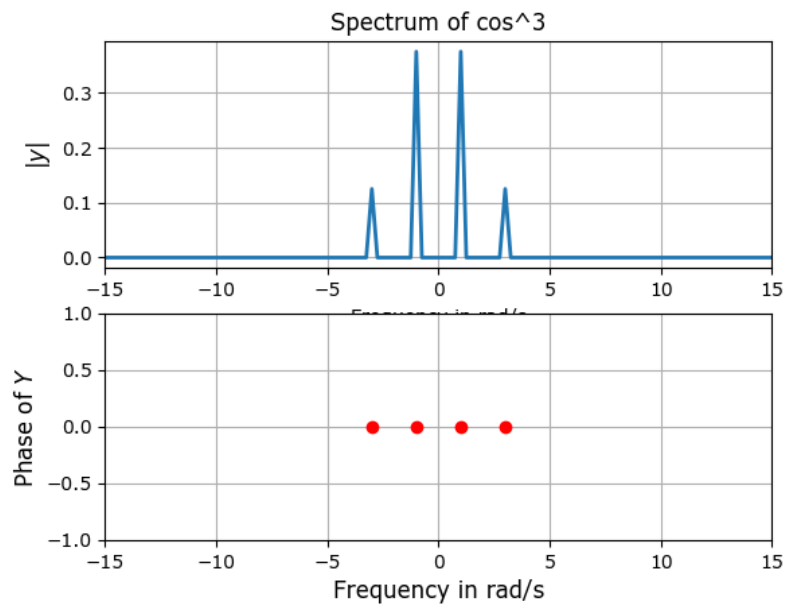Figure 3: Spectrum of $sin^3(t)$

DFT Spectrum of $cos^3(t)$

:

Figure 4: Spectrum of $cos^3(t)$

Frequency Modulation
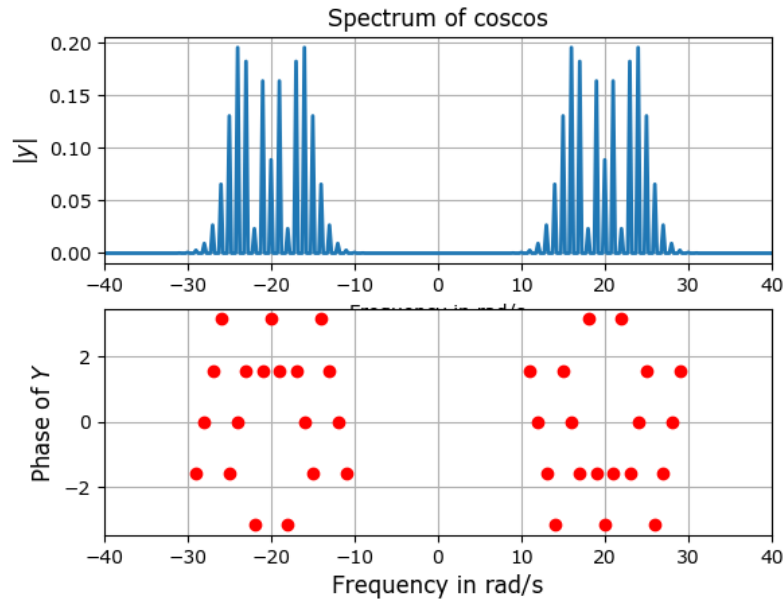
The DFT of $cos(20t + 5cos(t))$ can be seen below:

Figure 5: Frequency Modulated Signal

When we compare this result with that of the Amplitude Modulation as seen in Fig (2), we see that there are more side bands, and some of them have even higher energy than $\omega = \pm 20\ rad/sec$.

DFT of a Gaussian

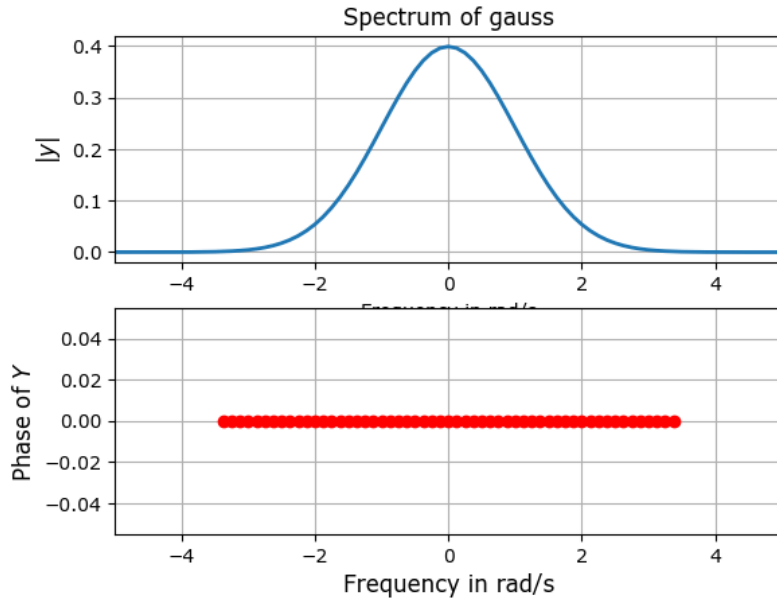The DFT of a gaussian is also a gaussian, as shown below:

Figure 6: Gaussian Spectrum

Conclusion

- We test the accuracy of the fft module and show that it's very accurate.

- We found out the DFT's of various sinusoids signals and AM, FM signals

- We the observe that for a sufficiently large window and sampling rate the DFT approximated the CTFT of the Gaussian.This is because the magnitude of the Gaussian quickly approaches 0 for large values of time and thus there is lesser frequency domain aliasing due to windowing.

- Sampling after windowing is done so that the DFT can be calculated using the Fast Fourier Transform. This is then a sampled version of the DTFT of the sampled time domain signal. With sufficiently large sampling rates, this approximates the CTFT of the original time domain signal.