EE2703: Assignment 10

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EE19B123

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Introduction

In this assignment,we find the Fourier Transforms of non periodic functions.As seen in one of the previous assignments these functions have a discontinuity when periodically extended. The discontinuity causes fourier components in frequencies other than the sinusoids frequency which decay as $\frac{1}{\omega}$, due to Gibbs phenomenon. We solve this problem using a hamming window.We use this windowed transform to analyse signals known to contain a sinusoid of unknown frequencies and extract its phase and frequency. We also perform a sliding DFT on a chirped signal and plot the results

Given Examples

We try to reproduce the results mentioned in the assignment: Spectrum of $sin(\sqrt{2}t)$ is given below

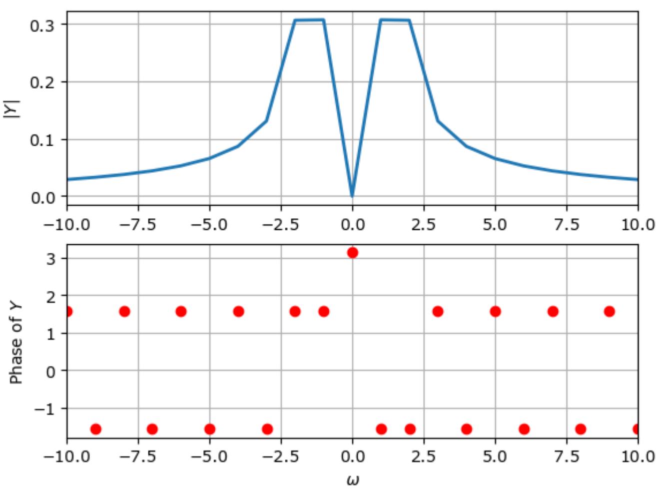


Figure 1: Spectrum of $\sin(\sqrt{2}t)$

Original function for which we want the DFT:

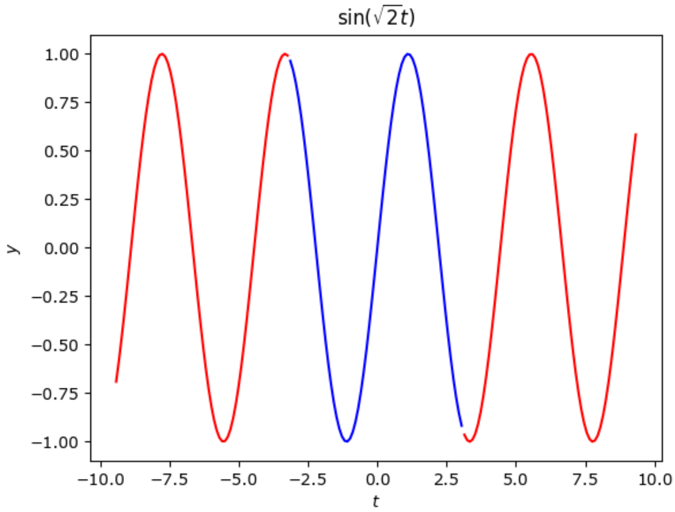


Figure 2: $\sin(\sqrt{2}t)$

Since the DFT is computed over a finite time interval, We actually plotted the DFT for this function

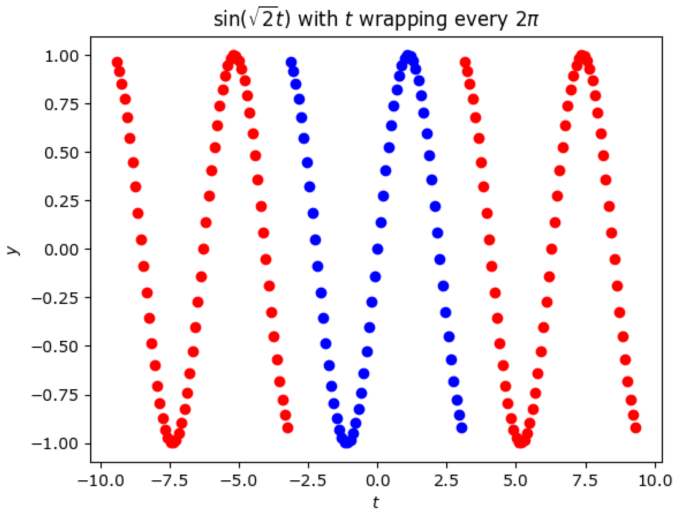


Figure 3: Spectrum of $\sin(\sqrt{2}t)$

These discontinuities lead to non harmonic components in the FFT which decay as $\frac{1}{\omega}$. To confirm this, we plot the spectrum of the periodic ramp below:

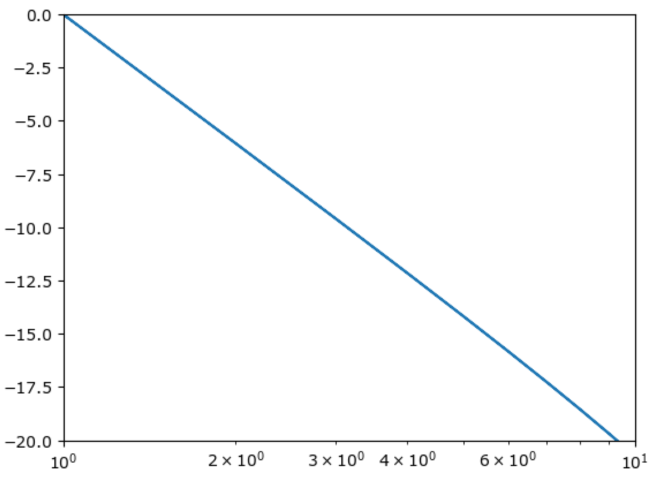


Figure 4: Spectrum of $\sin(\sqrt{2}t)$

Hamming Window

The hamming window removes discontinuities by attenuating the high frequency components that cause the discontinuities.
The hamming window function is as follows:

$$x[n] = 0.54 + 0.46cos(\frac{2\pi n}{N-1}) \tag{1}$$

We now multiply our signal with the hamming window and periodically extend it. Note that now the discontinuities nearly vanish
The following is the spectrum after applying the hamming window:

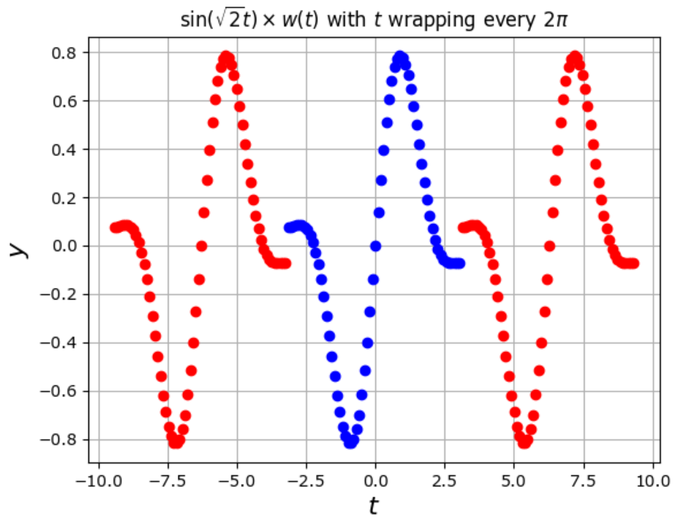


Figure 5: Spectrum of $\sin(\sqrt{2}t) * w(t)$

The spectrum that is obtained with a time period 2π is given below:

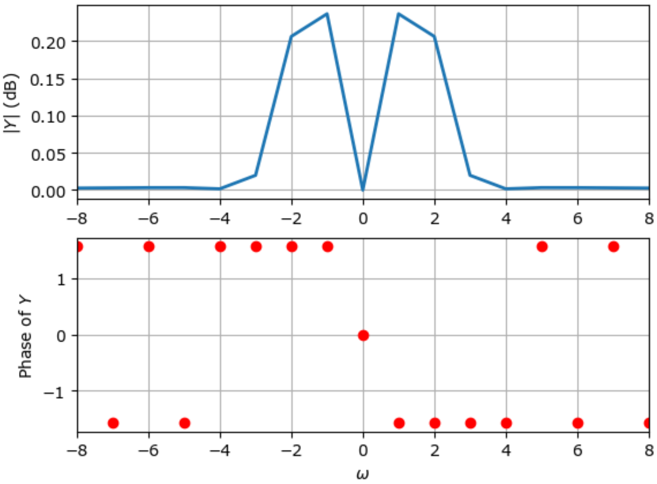


Figure 6: Spectrum of $\sin(\sqrt{2}t) * w(t)$

The spectrum that is obtained with a time period 8π has a slightly sharper peak(as expected) and is given below:

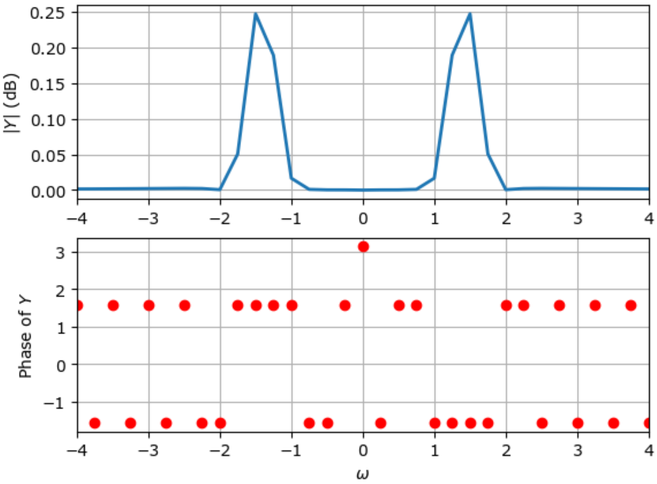


Figure 7: Spectrum of $\sin(\sqrt{2}t) * w(t)$

Assignment questions

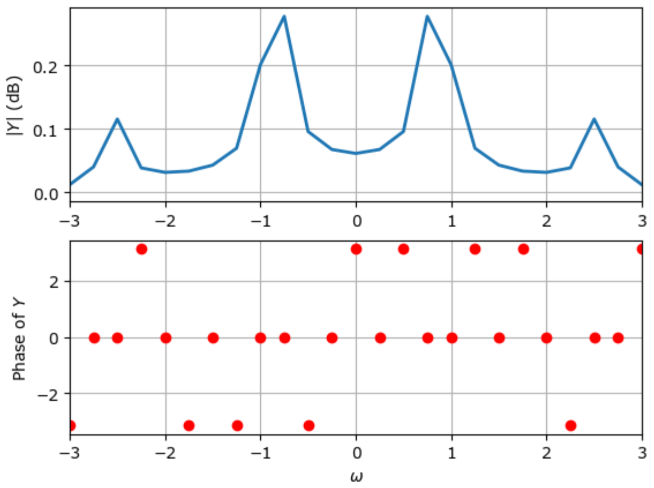
Fourier Transform Function

We use the following functions to calculate all the FFTs in this assignments:

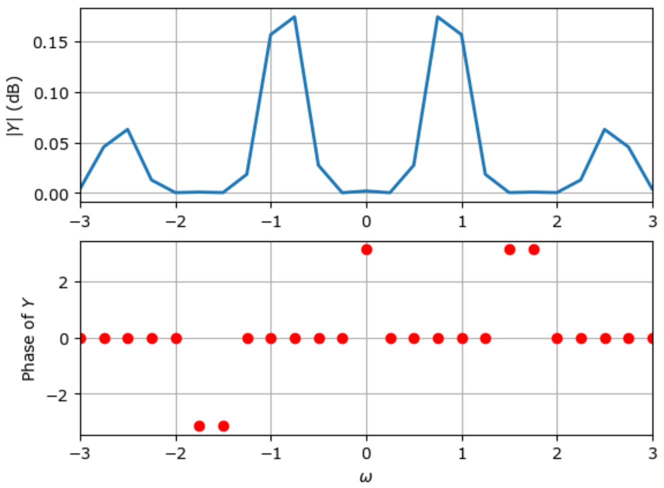
```python
def FFTCalci(f,N,plotting = True, **kwargs):

    if  not kwargs.get('use_new_t'):
        t=np.linspace(-kwargs['tlim'],kwargs['tlim'],N+1)[:-1]
    else:
        t = kwargs['time']

    dt=t[1]-t[0]
    fMax=1/dt
    y = f(t)
    if kwargs.get('windowing'):
        m=np.arange(N)
        window=fftshift(0.54+0.46*np.cos(2*np.pi*m/N))
        y = y*window

    y[0]=0
    y=fftshift(y)
    Y=fftshift(fft(y))/float(N)
    w=np.linspace(-np.pi*fMax,np.pi*fMax,N+1)[:-1]

    magnitude = np.abs(Y)
    phase = np.angle(Y)
    if plotting:
        plotter(w,magnitude,phase, **kwargs)
    return w,Y
```

Fourier Transforms of functions with/without Hamming window

In this question, we shall plot the FFT of $cos^3(0.86t)$
The FFT without the hamming Window:



The FFT with the hamming Window:



One can notice on adding windowing it has sharped the peaks compared to previous plots, we notice that a lot of the energy is stored in frequencies that aren't a part of the signal. After windowing, these frequencies are attenuated and hence the peaks are sharper in the windowed function. It is still not an impulse because the convolution with the Fourier transform of the windowed function smears out the peak

Question 3

We need to estimate ω and δ for a signal $\cos(\omega t + \delta)$ for 128 samples between $[-\pi, \pi)$. We estimate omega using a weighted average. We have to extract the digital spectrum of the signal and find the two peaks at $\pm\omega_0$, and estimate ω and δ.

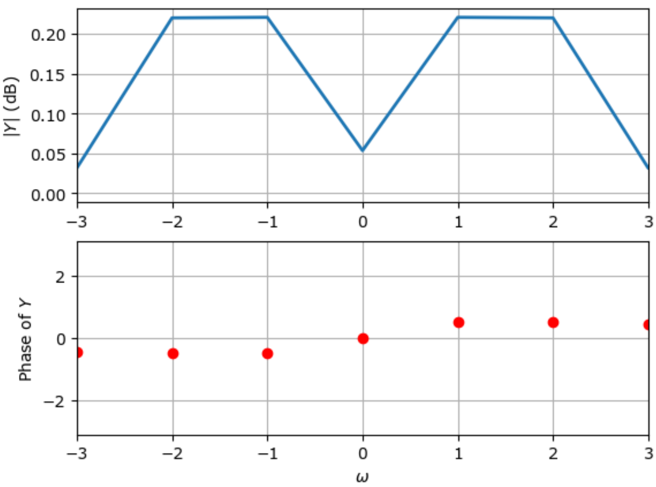


Figure 8: Fourier transform of $cos(1.5t + 0.5)$

We estimate omega by performing a Mean average of ω over the magnitude of $|Y(j\omega)|$.

For delta we consider a widow on each half of ω (split into positive and negative values) and extract their mean slope. The intuition behind this is that, a circular shift in the time domain of a sequence results in the linear phase of the spectra.

Question 4

We repeat the exact same process as question 3 but with noise added to the original signal. For true value of ω and δ = 1.5 and 0.5 respectively we get the following estimates:
omega = 1.5163179648582414
delta = 0.506776265719626 in no noise case
omega = 2.128915947064576
delta = 0.5146782553206393 in the noisy case

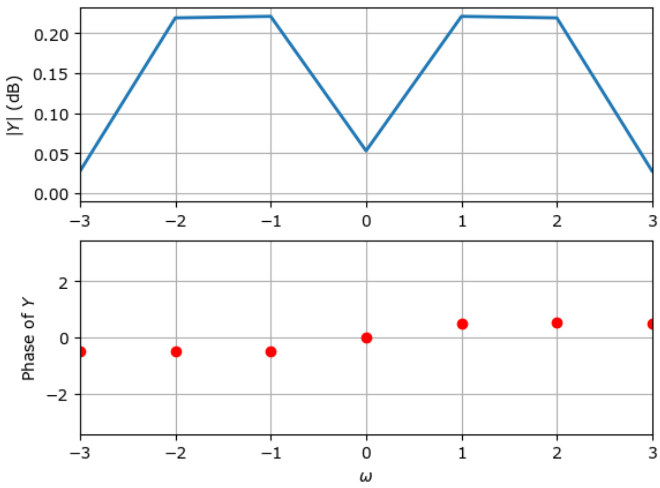


Figure 9: Fourier transform of noise + $cos(1.5t + 0.5)$

Question 5

In this question we analyze a chirp signal which is an FM signal where frequency varies linearly with time. A chirp signal we shall consider is given by

$$f(t) = cos(16t(1.5 + \frac{t}{2\pi})) \tag{2}$$

The FFT of the chirp is given by: We note that the frequency response is spread between 5-50 rad/s. A large section of this range apears due to Gibbs phenomenon. On windowing, only frequencies between 16 and 32 rad/s remain.

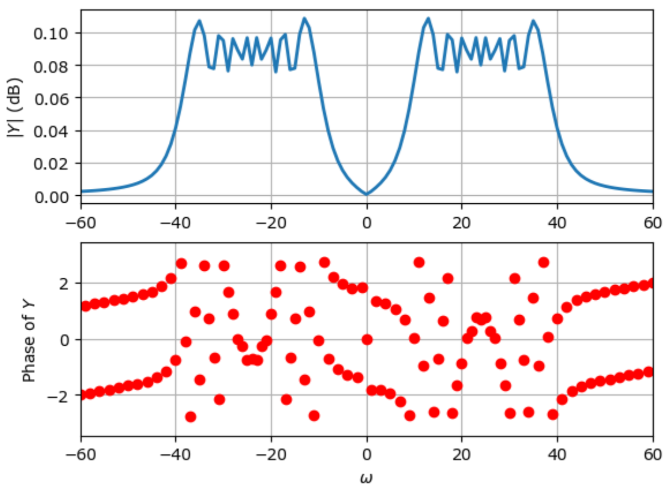


Figure 10: Chirp function fourier transform

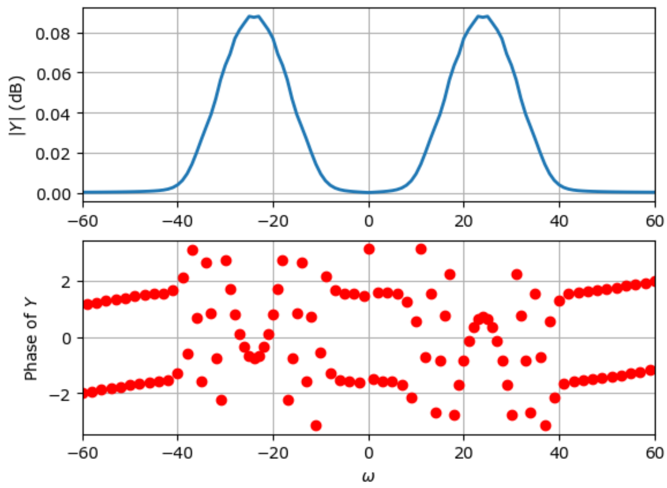


Figure 11: Chirp function fourier transform, windowed

Question 6

For the same chirped signal, we break the 1024 vector into pieces that are 64 samples wide. Extract the DFT of each and store as a column in a 2D array. Then plot the array as a surface plot to show how the frequency of the signal varies with time.This is new. So far we worked either in time or in frequency. But this is a "time- frequency" plot, where we get localized DFTs and show how the spectrum evolves in time. We do this for both phase and magnitude. Let us explore their surface plots.

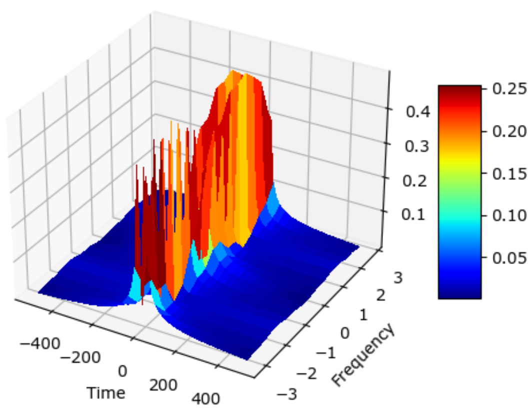


Figure 12: Chopped Chirp function, Mag of Fourier transform—

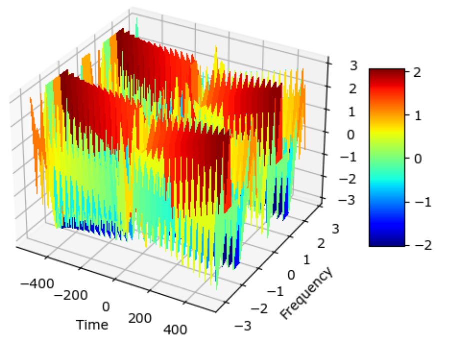


Figure 13: Chopped Chirp function, Phase of Fourier transform

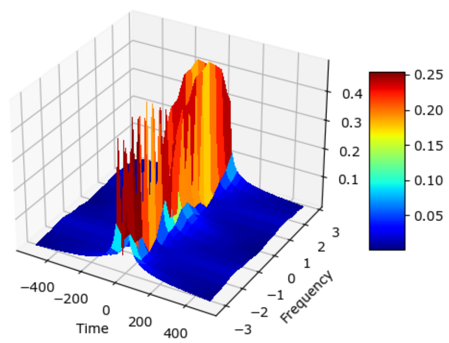


Figure 14: Windowed Chopped Chirp function,Mag of Fourier transform—

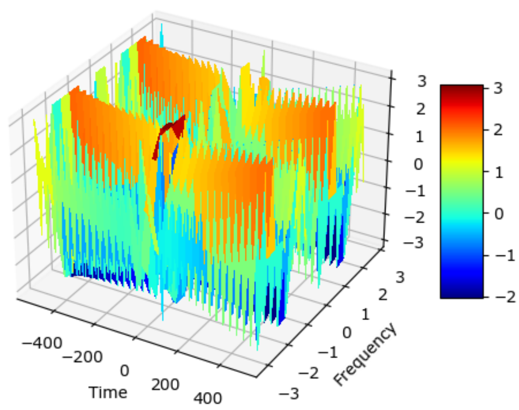


Figure 15: Windowed Chopped Chirp function, Phase of Fourier transform

Conclusion

- In this assignment we study the requirement of windowing in the case of non-periodic series in DFT's. In particular this is to remove the effect of Gibbs phenomena owing to the discontinuous nature of the series realised by a discrete fourier transform.

- The last question addresses the time varying spectra for a chirped signal, where we plot fourier spectra for different time slices of a signal. We noted the case of sparse number of slices and hence took more closely spaced slices.

- By performing localised DFTS at different time instants, we obtained a time-frequency plot which helped to analyse signals with varying frequencies in time.