STATISTICS WORKSHEET-7

1. A die is thrown 1402 times. The frequencies for the outcomes 1, 2, 3, 4, 5 and 6 are given in the following table:

Outcome: 1, 2, 3, 4, 5, 6

Frequency: 400, 300, 157, 180, 175, 190

Find the probability of getting 6 as outcome:

Answer: b) 0.135

The total number of trials (n) is given as:

n = 400 + 300 + 157 + 180 + 175 + 190 = 1402

The probability of getting 6 as outcome is given by:

P(6) = Number of times 6 appears / Total number of trials

P(6) = 190 / 1402

P(6) = 0.135

Therefore, the probability of getting 6 as outcome is 0.135

2. A telephone directory page has 400 telephone numbers. The frequency distribution of their unit place digit (for example, in the number 25827689, the unit place digit is 9 is given in table below:

Outcome: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Frequency: 44, 52, 44, 44, 40, 20, 28, 56, 32, 40

What will be the probability of getting a digit with unit place digit odd number that is 1, 3,5,7,9?

a) 0.67

b) 0.60

c) 0.45

d) 0.53

Answer: (d) 0.53

The total frequency of unit place digits with odd numbers is:

Frequency of 1 + Frequency of 3 + Frequency of 5 + Frequency of 7 + Frequency of 9 = 52 + 44 + 20 + 56 + 32 = 204

Therefore, the probability of getting a digit with unit place digit odd number that is 1, 3, 5, 7, 9 is:

Probability = Frequency of odd digits / Total frequency

Probability = 204 / 400

Probability = 0.51

Therefore, the answer is not among the options given. However, the closest option is (d) 0.53.

3. A tyre manufacturing company which keeps a record of the distance covered before a tyre needed to be replaced. The table below shows the results of 1100 cases.

Distance(miles): <4000, 4000-9000, 9001-14000, >14000

Frequency: 20, 260, 375, 445

If we buy a new tyre of this company, what is the probability that the tyre will last more than 9000 miles?

Answer: (c) 0.745

The total number of cases is 1100, and the frequency of tyres that lasted more than 9000 miles is the sum of tyres that lasted 9001-14000 miles and >14000 miles, which is 375 + 445 = 820.

Therefore, the probability that a tyre will last more than 9000 miles is 820/1100 = 0.745.

So, the answer is (c) 0.745.

4. Please refer to the case and table given in the question No. 3 and determine what is the probability that if we buy a new tyre then it will last in the interval [4000-14000] miles?

Answer: b) 0.577

Probability of a tyre lasting between 4000 and 14000 miles can be calculated by adding the frequency of the corresponding intervals and dividing it by the total frequency:

Probability = (260 + 375) / 1100 = 0.577

5. We have a box containing cards numbered from 0 to 9. We draw a card

randomly from the box. If it is told to you that the card drawn is greater than 4 what is the probability that the card is odd?

Answer: c) 0.6

Cards greater than 4 are: 5, 6, 7, 8, and 9. Out of these, three cards (5, 7, 9) are odd. Therefore, the probability of drawing an odd card given that the card is greater than 4 is:

Probability = 3/5 = 0.6

6. We have a box containing cards numbered from 1 to 8. We draw a card randomly from the box. If it is told to you that the card drawn is less than 4 what is the probability that the card is even?

Answer: (a) 0.33

If it is told that the card drawn is less than 4, then there are only three possible outcomes: 1, 2, or 3.

Out of these three outcomes, only one is even, which is 2.

Therefore, the probability that the card is even, given that it is less than 4, is 1/3.

7. A die is thrown twice and the sum of the numbers appearing is observed to be 7. What is the conditional probability that the number 6 has appeared at least on one of the die?

Answer: a) 0.45

To get a sum of 7 on two dice, we can have the following combinations: (1,6), (2,5), (3,4), (4,3), (5,2), and (6,1).

Out of these combinations, only two have a 6 on at least one of the dice, i.e., (1,6) and (6,1).

Therefore, the conditional probability of getting a 6 on at least one of the die given that the sum is 7 is:

Probability = 2/6 = 1/3

8. Consider the experiment of tossing a coin. If the coin shows tail, toss it again but if it shows head, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 4' given that 'there is at least one Head'.

Answer: (a) 0.1

Let A be the event that 'the die shows a number greater than 4' and B be the event that 'there is at least one Head'. We need to find P(A|B).

We can use the Bayes' theorem to calculate the conditional probability:

P(A|B) = P(A and B) / P(B)

To find P(A and B), we can use the law of total probability:

P(A and B) = P(A and B and H) + P(A and B and T)

where H is the event that the first coin toss results in a Head and T is the event that it results in a Tail.

Since the probability of getting Head on the first toss is 1/2 and the probability of getting Tail is 1/2, we have:

P(A and B and H) = P(H) * P(die shows number > 4 | H) = (1/2) * 0 = 0

P(A and B and T) = P(T) * P(coin shows Tail | T) * P(die shows number > 4 | T and Tail) = (1/2) * (1/2) * (2/3) = 1/6

Therefore,

P(A and B) = 1/6

To find P(B), we can use the law of total probability again:

P(B) = P(H) * P(B | H) + P(T) * P(B | T) = (1/2) * 1 + (1/2) * P(B | T)

Since we know that if the coin shows Tail, we toss it again and the probability of getting Head is 1/2, we have:

P(B | T) = P(H) = 1/2

Therefore,

P(B) = 3/4

Now we can calculate the conditional probability:

P(A|B) = P(A and B) / P(B) = (1/6) / (3/4) = 1/8

9. There are three persons Evan, Ross and Michelle. These people lined up randomly for a picture. What is the probability of Ross being at one of the ends of the line?

Answer: (a) 0.66

Ross can be at either end of the line, so there are two possible favorable outcomes out of three possible positions for Ross.

Therefore, the probability of Ross being at one of the ends of the line is 2/3.

10. Let us make an assumption that each born child is equally likely to be a boy or

a girl. Now suppose, if a family has two children, what is the conditional probability that both are girls given that at least one of them is a girl?

Answer: (d) 0.26

Let A be the event that 'both children are girls' and B be the event that 'at least one of them is a girl'. We need to find P(A|B).

We can use the Bayes' theorem to calculate the conditional probability:

P(A|B) = P(A and B) / P(B)

To find P(A and B), we can use the fact that if at least one of the children is a girl, there are three equally likely possibilities: BB, BG, and GB. Out of these, only one corresponds to both children being girls. Therefore,

P(A and B) = 1/3

To find P(B), we can use the law of total probability:

P(B) = P(BB) + P(BG) + P(GB) = 1/4 + 1/2 + 1/2 = 3/4

Therefore,

P(A|B) = P(A and B) / P(B) = (1/3) / (3/4) = 4/9

11. Consider the same case as in the question no. 10. It is given that elder child is a boy. What is the conditional probability that both children are boys?

Answer: (a) 0.33

Let A be the event that 'both children are boys' and B be the event that 'the elder child is a boy'. We need to find P(A|B).

We can use the Bayes' theorem to calculate the conditional probability:

P(A|B) = P(A and B) / P(B)

To find P(A and B), we can use the fact that if the elder child is a boy, there are two equally likely possibilities: BB and BG. Out of these, only one corresponds to both children being boys. Therefore,

P(A and B) = 1/2

To find P(B), we can use the law of total probability:

P(B) = P(BB) + P(BG) = 1/4 + 1/2 = 3/4

Therefore,

P(A|B) = P(A and B) / P(B) = (1/2) / (3/4) = 2/3

12. We toss a coin. If we get head, we toss a coin again and if we get tail we throw a die. What is the probability of

getting a number greater than 4 on die?

Answer: (c) 0.78

Let A be the event that 'getting a number greater than 4 on die' and B be the event that 'the first coin toss results in a Tail'. We need to find P(A|B).

If the first coin toss results in a Tail, we toss the coin again with probability 1/2 and throw a die with probability 1/2. Therefore,

P(A and B) = P(T) * (1/2) * P(A | T and coin toss #2) + P(T) * (1/2) * P(A | T and die throw) = (1/2) * (1/2) * (2/3) + (1/2) * (1/2) * (2/6) = 1/6

To find P(B), we can use the law of total probability:

P(B) = P(H) * P(B | H) + P(T) * P(B | T) = (1/2) * 0 + (1/2) * (1/2) = 1/4

Therefore,

P(A|B) = P(A and B) / P(B) = (1/6) / (1/4) = 2/3

13. We toss a coin. If we get head, we toss a coin again and if we get tail we throw a die. What is the probability of getting an odd number on die?

Answer:

14. Suppose we throw two dice together. What is the conditional probability of getting sum of two numbers found on the two die after throwing is less than 4, provided that the two numbers found on the two die are different?

Answer:

15. A box contains three coins: two regular coins and one fake two-headed coin, you pick a coin at random and toss it. What is the probability that it lands heads up?

Answer: (c) ½

There are three coins in the box, and each has an equal chance of being picked at random. Let's consider the probability of getting heads up for each coin:

The two regular coins have a probability of 1/2 of landing heads up.

The fake two-headed coin has a probability of 1 of landing heads up.

Therefore, the probability of getting heads up after picking a coin at random and tossing it can be calculated as the weighted average of the probabilities of getting heads up for each coin, with weights given by the probability of picking each coin:

$P(\text{heads up}) = (2/3) \times (1/2) + (1/3) \times 1$

$P(\text{heads up}) = 2/6 + 1/3$

$P(\text{heads up}) = 1/2$