13/8/19

Number System

Number systems are of 4 types

1. Decimal number system [Base-10] → 0 to 9 [For counting]
2. Binary number system [Base-2] → 0, 1 [For machine]
3. Hexadecimal number system [Base-16] → 0-9, A-F } → processors.
4. Octal [Base-8] → 0 to 7

Number systems

↓ positional ↓ Non-positional

positional

Eg:- Decimal $[\ldots 10^3\, 10^2\, 10^1\, 10^0 \,;\, 10^{-1}\, 10^{-2}\, 10^{-3}]$ ← Radix point

Binary

Octal

Hexadecimal

Non-positional

Eg:- Gray codes

Example:- $(2572 \cdot 25)_{10}$

$= 2\times10^3 + 5\times10^2 + 7\times10^1 + 2\times10^0 + 2\times10^{-1} + 5\times10^{-2}$

Binary - $2^2\ 2^1\ 2^0 \cdot 2^{-1} \cdot 2^{-2} \ldots$

Eg:- $(10110)_2$

$1\times2^4 + 0\times2^3 + 1\times2^2 + 1\times2^1 + 0\times2^0$

$(1011\cdot101)_2$

$= 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0 + 1\times2^{-1} + 0\times2^{-2} + 1\times2^{-3}$

Octal - $\ldots 8^2\ 8^1\ 8^0 \cdot 8^{-1}\ 8^{-2} \ldots$

Eg:- $(176)_8$

$= 1\times8^2 + 7\times8^1 + 6\times8^0$

Hexadecimal (16)

$\ldots\ 16^1\ 16^0 \cdot 16^{-1}\ 16^{-2} \ldots$

Eg:- $(AF)_{16}$

$= A\times16^1 + F\times16^0$

$(CA\cdot CC)_{16}$

$= C\times16^1 + A\times16^0 + C\times16^{-1} + C\times16^{-2}$

$(1\,0\,1\,0\,1)_2 = (21)_{10}$

(place values $2^4\ 2^3\ 2^2\ 2^1\ 2^0$; MSB on the left, LSB on the right)

$(10100)_2 = (20)_{10}$

$(00101)_2 = (5)_{10}$

Conversions

1. Decimal to (any base)

i. $(8)_{10} \rightarrow (8)_2$

$$\begin{array}{r|l l} 2 & 8 & \\ 2 & 4 & 0 \\ 2 & 2 & 0 \\ & 1 & 0 \end{array}$$

$(8)_{10} \rightarrow (1000)_2$

ii. $(8)_{10} \rightarrow (20)_4$

$$\begin{array}{r|l l} 4 & 8 & \\ 4 & 2 & 0 \\ & & 2 \end{array}$$

iii. $(8)_{10} \rightarrow (12)_6$

$$\begin{array}{r|l l} 6 & 8 & \\ 6 & 1 & 2 \\ & 1 & \end{array}$$

iv. $(8)_{10} \rightarrow (10)_8$

$$\begin{array}{r|l l} 8 & 8 & \\ 8 & 1 & 0 \\ & 1 & \end{array}$$

v. $(52)_{10} - (110100)_2$

$$\begin{array}{r|l l} 2 & 52 & \\ 2 & 26 & 0 \\ 2 & 13 & 0 \\ 2 & 6 & 1 \\ 2 & 3 & 0 \\ 2 & 1 & 1 \\ & 1 & \end{array}$$

vi. $(0.125)_{10} \rightarrow (001)_2$

$0.125 \times 2 = 0.25 \quad 0$

$0.25 \times 2 = 0.5 \quad 0$

$0.5 \times 2 = 1.00 \quad 1$

$0 \times 2 =$

vii. $(59.125)_{10} = (111011.125)_2$

$59.125 \times 2 = 118.25 \quad 1$

$118.25 \times 2 = 236.5 \quad 2$

$236.5 \times 2 =$

$$\begin{array}{r|l l} 2 & 59 & \\ 2 & 29 & 1 \\ 2 & 14 & 1 \\ 2 & 7 & 0 \\ 2 & 3 & 1 \\ 2 & 1 & 1 \\ & 1 & \end{array}$$

$0.125 \times 2 = 0.25 \quad 0$

$0.25 \times 2 = 0.5 \quad 0$

$0.5 \times 2 = 1.00 \quad 1$

$(59.125)_{10} = (111011.001)_2$

viii. $(149)_{10} = (225)_8$

$$\begin{array}{r|l l} 8 & 149 & \\ 8 & 18 & 5 \\ 8 & 2 & 2 \\ & 2 & \end{array}$$

ix) $(32)_{10} = [100000]_2$

$$\begin{array}{r|l l} 2 & 32 & \\ 2 & 16 & 0 \\ 2 & 8 & 0 \\ 2 & 4 & 0 \\ 2 & 2 & 0 \\ 2 & 1 & 0 \\ & 1 & \end{array}$$

x. $(16)_{10} = (10000)_2$

2	16	
2	8	0
2	4	0
2	2	0
2	1	0
	1	

xi. $(8)_{10} = (1000)_2$

2	8	
2	4	0
2	2	0
2	1	0
	1	

xii. $(378.93)_{10} = (101111010.1110)_2$

2	378	
2	189	0
2	94	1
2	47	0
2	23	1
2	11	1
2	5	1
2	2	1
2	1	0
	1	

~~0.93 x 2 = 1.86 1~~
~~0.89 x 2 = 3.72 3~~
~~0.3 x 2 = 0.6 1~~
~~0 x 2 =~~

0.93 x 2 = 1.86 → 1
0.86 x 2 = 1.72 → 1
0.72 x 2 = 1.44 → 1
0.44 x 2 = 0.88 → 0

16-8-19

1. $(105.15)_{10} = (1101001.0010)_2$

2	105	
2	52	1
2	26	0
2	13	0
2	6	1
2	3	0
2	1	1
	1	

0.15 x 2 = 0.30 → 0
0.30 x 2 = 0.60 → 0
0.60 x 2 = 1.20 → 1
0.20 x 2 = 0.40 → 0

2. $(378.93)_{10} = (572.734)_8$

8	378	
8	47	2
8	5	7
	5	

0.93 x 8 = 7.44 → 7
0.44 x 8 = 3.52 → 3
0.88 x 8 = 4.76 → 4
0.76 x 2 = 1.52 → 1

3. $(2598.675)_{10} = (10\,2\,6.10\,12)_{16} \Rightarrow (A26.AC)$

16	2598	
16	162	6
16	10	2
	10	

0.675 x 16 = 10.8 → 10
0.8 x 16 = 12.8 → 12
0.8 x 16 = 12.8 → 12

$\cdot (11011.101)_2 = (26.625)_{10}$

$2^4 2^3 2^2 2^1 2^0 2^{-1} 2^{-2} 2^{-3}$

$= 16 + 8 + 2 + 0.5 + 0.125 + 1$

$= 27.625$

5. $(237)_8 \rightarrow (159)_{10}$

$= 2 \times 8^2 + 3 \times 8^1 + 7 \times 8^0$

$= 32 + 24 + 7$

$= 2 \times 64 + 3 \times 8 + 7$

$= 159$

6. $(1102)_4 = (82)_{10}$

$= 1 \times 4^3 + 1 \times 4^2 + 0 \times 4^1 + 2 \times 4^0$

$= 64 + 16 + 2$

$= 82$

7. $(4057.06)_8$

$= 4 \times 8^3 + 0 \times 8^2 + 5 \times 8^1 + 7 \times 8^0 + 0 \times 8^{-1} + 6 \times 8^{-2}$

$= 2048 + 40 + 7 + 0.09375$

$= (2095.09375)_{10}$

8. $(A0F9.0EB)_{16}$

$= 10 \times 16^3 + 0 \times 16^2 + 15 \times 16^1 + 9 \times 16^0 + 0 \times 16^{-1} + 14 \times 16^{-2} + 11 \times 16^{-3}$

$= 40960 + 240 + 144 + 0.0546 + 2.685 \times 10^{-3}$

$= (412089.057)_{10}$

Octal to Binary :-

1. $(427)_8 = (100\ 010\ 111)_2$

2. $(123)_8 = (001\ 010\ 011)_2$

Binary to octal :-

1. $\underbrace{010}_{3}\ \underbrace{001}_{1}\ \underbrace{011}_{3} = (313)_8$

2.

$(236)_8 = (010\ 011\ 110)_2$

$(236.67)_8 = (010011110.110111)_2$

$(\underbrace{010}_{2}\underbrace{111}_{7}.\underbrace{101}_{5}\underbrace{100}_{4})_2 = (27.54)_8$

$(\underbrace{011}_{3}\underbrace{111}_{7}\underbrace{011}_{3}.\underbrace{011}_{3}\underbrace{110}_{6})_2 = (373.36)_8$

$(\underbrace{010}_{2}\underbrace{111}_{7}.\underbrace{001}_{1}\underbrace{000}_{0})_2 = (27.10)_8$

Hexa to Binary:-

$(1F \cdot 2A)_{16} = (00011111 \cdot 00101010)_2$

$(ABC \cdot B)_{16} = (101010111100 \cdot 1101)_2$

$(21 \cdot 1D)_{16} = (00100001 \cdot 00011101)_2$

Binary to Hexa:-

$(0110\ 1011 \cdot 0010)_2 = (6B \cdot 2)_{16}$

$(0101 \cdot 0100)_2 = (5 \cdot 4)_{16}$

0000

$(0001\ 0110\ 1101 \cdot 1010\ 1011\ 0)_2 = (16D \cdot AB0)_{16}$

Octal to Hexa:-

$(37)_8 \rightarrow (\quad)_{16}$

$\downarrow$

~~$(00011\ 0111)_2 \rightarrow (06$~~

$0(0011111)_2 = (1F)_{16}$

$(103 \cdot 27)_8 - [001000011 \cdot 010111100]_2 - [043 \cdot 5C]_{16}$

$(217 \cdot 76)_8 - [010001111 \cdot 11111000]_2$

$= [08F \cdot F8]_{16}$

Hexa to octal:-

$(F8)_{16} \rightarrow (011111000)_2 = [370]_8$

$(FD \cdot DE)_{16} = [011111101 \cdot 110111100]_2 = [375 \cdot 674]_8$

$(DB4F)_{16} - [001101101101001111]_2 = [155517]_8$

Binary Numbers:-

- Binary Numbers
  - unsigned
  - signed
    - sign & magnitude
    - 1's complement
    - 2's complement

2's complement

Ex:-

1. 0110 → 1000 + 11 = 1010 (or) 0110 → 1010

{ After seeing '1' we complement each bit. }

2. 1010 →(2's) 0110

3. 100101 → 011011

→ Sign and Magnitude Representation:-

$+11_{10} \rightarrow 0\,1011$ (signed, Magnitude)

$0101 \rightarrow +5_{10}$

$1101 \rightarrow -5_{10}$

{ For '+' → 0; For '-' → 1 }

$-11_{10} \rightarrow 11011$

$(\underbrace{0}\,\underbrace{1111})_2 = \cancel{+547}_{10}$

$= 1\times2^0 + 1\times2^1 + 1\times2^2 + 1\times2^3 + 0\times2^4$

$= 1 + 2 + 4 + 8 + 0$

$\cancel{= 47_{10}} = -15_{10}$

8 4 2 1
0 1 0 1 1

* 10101 →(2's) $= -01011$ $= -11_{10}$

ii) 10101 →(1's) $= -01011$ $= -10_{10}$

iii) $+11_{10} \rightarrow 01011$

iv) $-11_{10} \rightarrow 11011$

1's complement:-

$+11_{10} \rightarrow 01011$ →(1's) 10100

$-11_{10} \rightarrow 11011 \rightarrow 10100$ →(1's) 01011

1)

$+7_{10} \rightarrow 01110$

$-7_{10} \rightarrow 1111$

1's

$+7_{10} \rightarrow 0111$

$-7_{10} \rightarrow 0111$

$\downarrow$ 1's

1000

2's

$+7_{10} \rightarrow 0111$

$-7_{10} \rightarrow 0111$

$\downarrow$ 2's

1001

2)

-45 in 2's

$+45_{10} \rightarrow 00101101$

$-45_{10} \rightarrow 00101101$

00101101

$\downarrow$ 2's

11010011

3) $(-83.375)_{10} = (1010011.011)_2$

2	83	
2	41	1
2	20	1
2	10	0
2	5	0
2	2	1
2	1	0
	1	

$0.375 \times 2 = 0.750$ — 0

$0.75 \times 2 = 1.50$ — 1

$0.50 \times 2 = 1.0$ — 1

1010011.011

$\downarrow$ 2's

0101100.101

2's $\rightarrow$ 0101100.101

4) $\overset{16\,8\,4\,2\,1}{11001} \rightarrow 25_{10}$ - unsigned

-9_{10} - signed & magnitude.

5) $\overset{8\,4\,2\,1}{1101} \rightarrow 13_{10}$ - unsigned

$-5_{10} \rightarrow$ signed & magnitude.

1's

1101

$\downarrow$ 1's

-0010

$= -2_{10}$

2's

1101

$\downarrow$ 2's

-0011

$= -3_{10}$

Complements of Numbers:-

* complements is mainly used for digital components to simply the subtraction.

$A - B \Rightarrow A + (-B)$

$A +$ complement (B)

It is of two types.

1. Dimminshed Radix complement[(r-1)s complement]
2. Radix complement[(r)s complement]

(r-1) s complement [9's complement]:-

① $r^n - N - 1$

Ex:- $(39)_{10}$

$r = 10 \quad r - 1 = 9$

$n = 2$

$N = 39$

9's complement $= 10^2 - 39 - 1$

$= 100 - 40$

$= 60$

② subtract the given digit with 9's.

Ex:-

$$\begin{array}{r} 99 \\ -\ 39 \\ \hline 60 \end{array}$$

(r)s complement [10's complement]

① $r^n - N$

Ex:- $N = 39$

$r = 10$

$n = 2$

$= 10^2 - 39$

$= 100 - 39$

$= 61$

② Subtract the given number with 9's and add '1' to the result.

Ex:-

$$\begin{array}{r} 99 \\ 39 \\ \hline 60+ \\ 1 \\ \hline 61 \end{array}$$

Examples:-

$\cancel{(360)_8}$

$\cancel{r-1 \Rightarrow 8-1 = 7}$

$n = 2$

$N = 360$

$\Rightarrow 8^{3} \cancel{2}\ 360 - 1$

$= 512 - 361$

$= 151$

512

$= 5 \times 8^2 + 1 \times 8^1 + 2 \times 8^0$

$= 5 \times 64 + 8 + 2$

$= 320 + 10$

Examples:-

$(360)_8$

$= r^n - N - 1$

$= 8^3 - (360)_8 - (1)_8$

$= (512)_{10} - (360)_8 - (1)$

$= (1000)_8 - (360)_8 - 1$

$= 1000 - (360)_8 - 1$

$= 777 - 360$

$= 417$

$\quad + 1$

$\quad 420$

1000 − 1 = 777

0 … 7 } 10

10 … 77 } 100

100 … 777 } 1000

1000

ii) $(370)_{11}$

$\Rightarrow n = 3 \quad r - 1 = 11 - 1 = 10$

$= r^n - N - 1$

$= (10^3)_{10} - (370)_{11} - (1)_{11}$

$= (1000)_{11} - (370)_{11} - 1$

$= AAA - 370$

$= 73A$

$11^3 - 1331$

11 | 1331
11 | 121 0
11 | 11 0
11 | 1 0
1

1

AAA − 370 = 73A

73A + 1 = 740

1000 − 1 = AAA

0 … 9 } 10

A … AA } 100

AA … AAA } 1000

iii) $(r-1)s$

Ex:- $(37)_{10}$ Base-10 0-9

99 − 37 = 62

95 + 3

38

$(r)s$

$(37)_{10}$

62 + 1 = 63

99 − 60 = 39

9 − 7 = 2, 2 + 1 = 3

$(37)_{10}$ − $(7)_{10}$

99 − 37 = 62, 62 + 1 = 63

9 − 7 = 2, 2 + 1 = 3

23-8-19

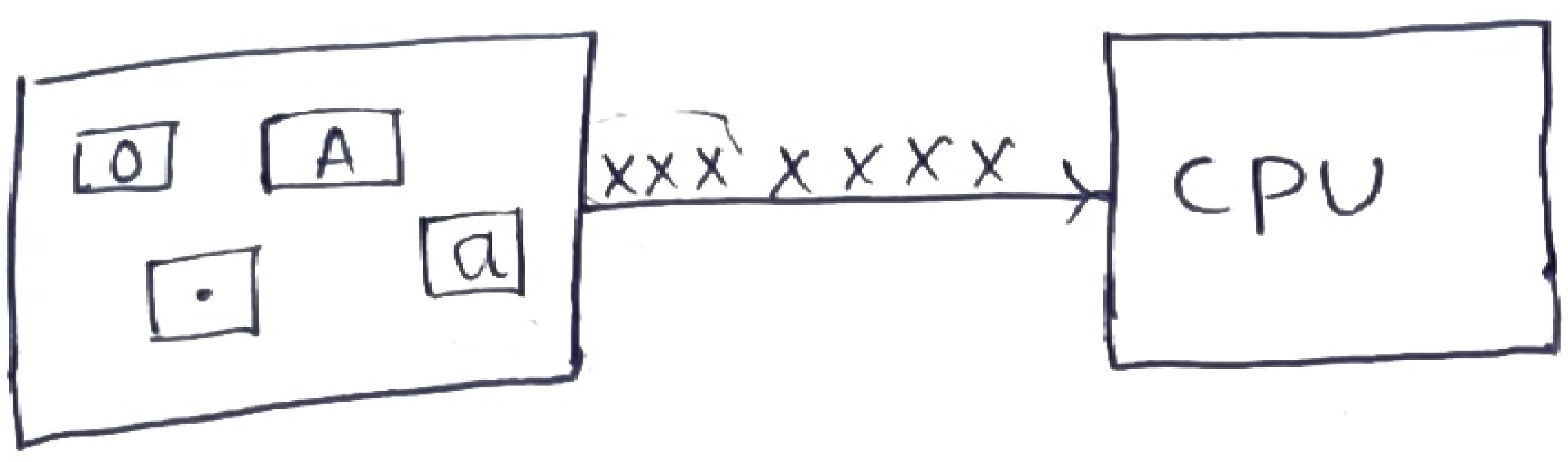
Binary Codes:-

It is of two types → Ex:- ASCII code

1. Alphanumeric codes
2. Numeric codes → Weighted codes [Ex - 8421 BCD]; → Non-weighted codes. [Ex - Gray code, Excess -3 code]

ASCII

*. It is a 7-bit & it has 2^7 combinations codes = 128



'0' → $(30)_{16}$ → 0110000

'A' → $(41)_{16}$ → $\underbrace{100\ 0001}_{\text{ASCII code for 'A'.}}$

Weighted Codes:-

BCD → Binary coded Decimal

BCD is a 4-bit code & we get 2^4 combinations = 16.

Dec	BCD
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

*. From 10 to 15 those numbers are invalid.

*. 1 nibble = 4 bits

*. 1 byte = 8 bits

$(10)_{BCD}$ → (00010000).

$(324)_{BCD}$ → (0011 0010 0100)

$(23)_{BCD}$ → (0010 0011)

BCD Addition :-

A	B	sum	carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

38 +
98
136.

⇒
1 1 1
00111000 +
10011000
10010000 +
01100110
←1] 00010110

0001 00110110 − 136.

57 + ⇒ 0101 0111
26 ⇒ 0010 0110
1 1 1
01111101
0110
1
10000011 = (83) BCD

341 + ⇒ 0011 0100 0001
659 ⇒ 0110 0101 1001
1 1 1 1
1001 1001 1010
0110
1 1 1 1
1001 1010 0000
0110
1 1
1010 0000 0000
0110
0001 0000 0000 0000 = (1000) BCD

24-8-19

Non-Weighted codes

Excess - 3 code :-

0 + 3 = 3
0011

Excess

Dec	BCD	Excess - 3
0	0000	0011
1	0001	0100
2	0010	0101
3	0011	0110
4	0100	0111
5	0101	1000
6	0110	1001
7	0111	1010
8	1000	1011
9	1001	1100

{ Add '3' for every BCD then we get excess - 3 }

→ valid Excess - 3 codes.

0000
0001
0010
1101
1110
1111

→ Invalid Excess - 3 codes.

0	0	0	→ XOR
0	1	1	
1	0	1	
1	1	0	

② Gray Codes:-

MSB LSB

Binary ⇒ 1⊕1⊕0⊕1

↓

Gray code → 1011

Binary → b_3 b_2 b_1 b_0

Gray → g_3 g_2 g_1 g_0

MSB

1⊕0⊕1⊕0

↓

1111 → Gray code

$b_3 \leftarrow g_3$

$g_2 = b_3 \oplus b_2$

$g_1 = b_2 \oplus b_1$

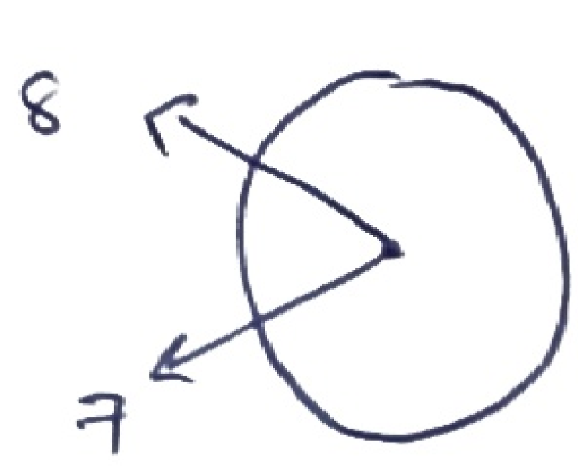
$g_0 = b_1 \oplus b_0$

Binary	Gray code
0⊕0⊕0	000
001	001
010	011
011	010
100	110
101	111
110	101
111	100

*. In Gray code there is a unit distance cod

4-bit shaft encodes

*. It is better to use Gray codes than binary numbers for higher numbers. Because, in binary we have high chances of error.



Binary		Gray
0111	→ 7 −	0100
1000	→ 8 −	1100
1001	→ 9 −	1101

*. So, better to use gray codes high number representation

*. K-Maps are used to minimise the expression

In K-Maps we use gray codes.

* So, Gray codes are ~~used~~ to minimise the expression.

A \ BC	00	01	11	10
0	(0)	1 (1)	(3)	(2)
1	(4)	(5)	(7)	(6)

B	C
0	1
1	0

Gray to Binary conversions:-

MSB

Gray => $g_3\ g_2\ g_1\ g_0$

Binary => $b_3\ b_2\ b_1\ b_0$

MSB LSB

Gray => 0 1 0 0

↓ ↓

Binary => 0 1 1 1

$b_3 = g_3$

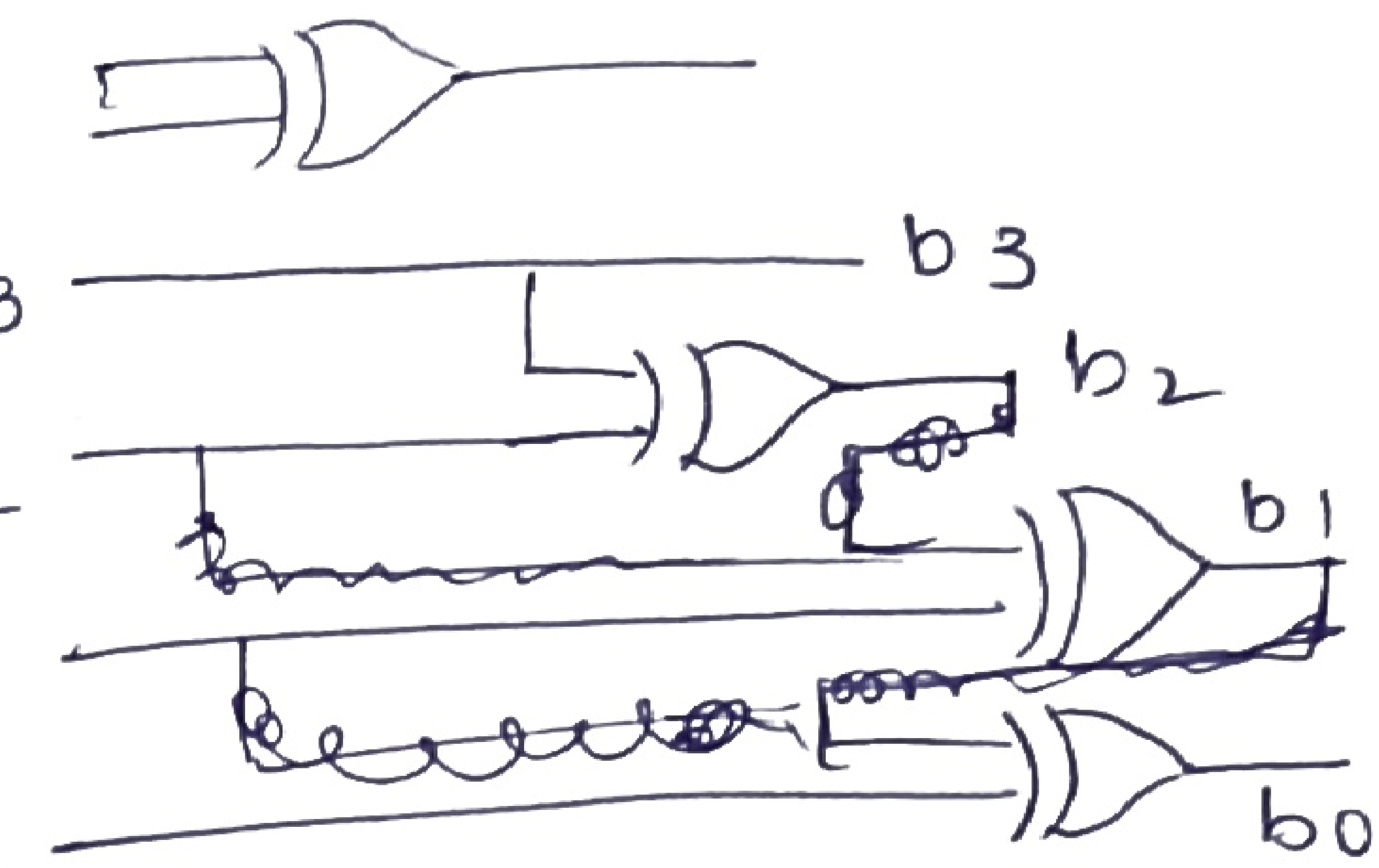
$b_2 = b_3 \oplus g_2$

$b_1 = b_2 \oplus g_1$

$b_0 = b_1 \oplus g_0$

Gray	Binary
0 0 0	0 0 0
0 0 1	0 0 1
0 1 0	~~0 1 1~~
0 1 1	0 1 0
1 0 0	1 1 1
1 0 1	1 1 0
1 1 0	1 0 0
1 1 1	1 0 1

Gray to Binary



code

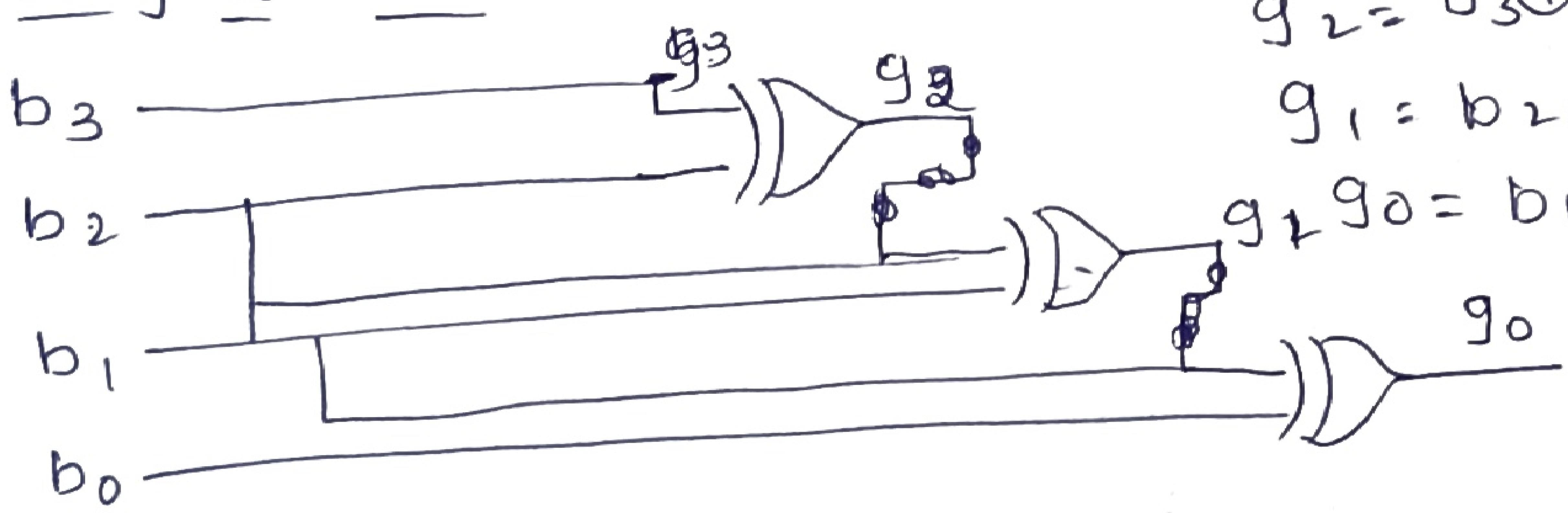
Binary to Gray

$g_3 = b_3$

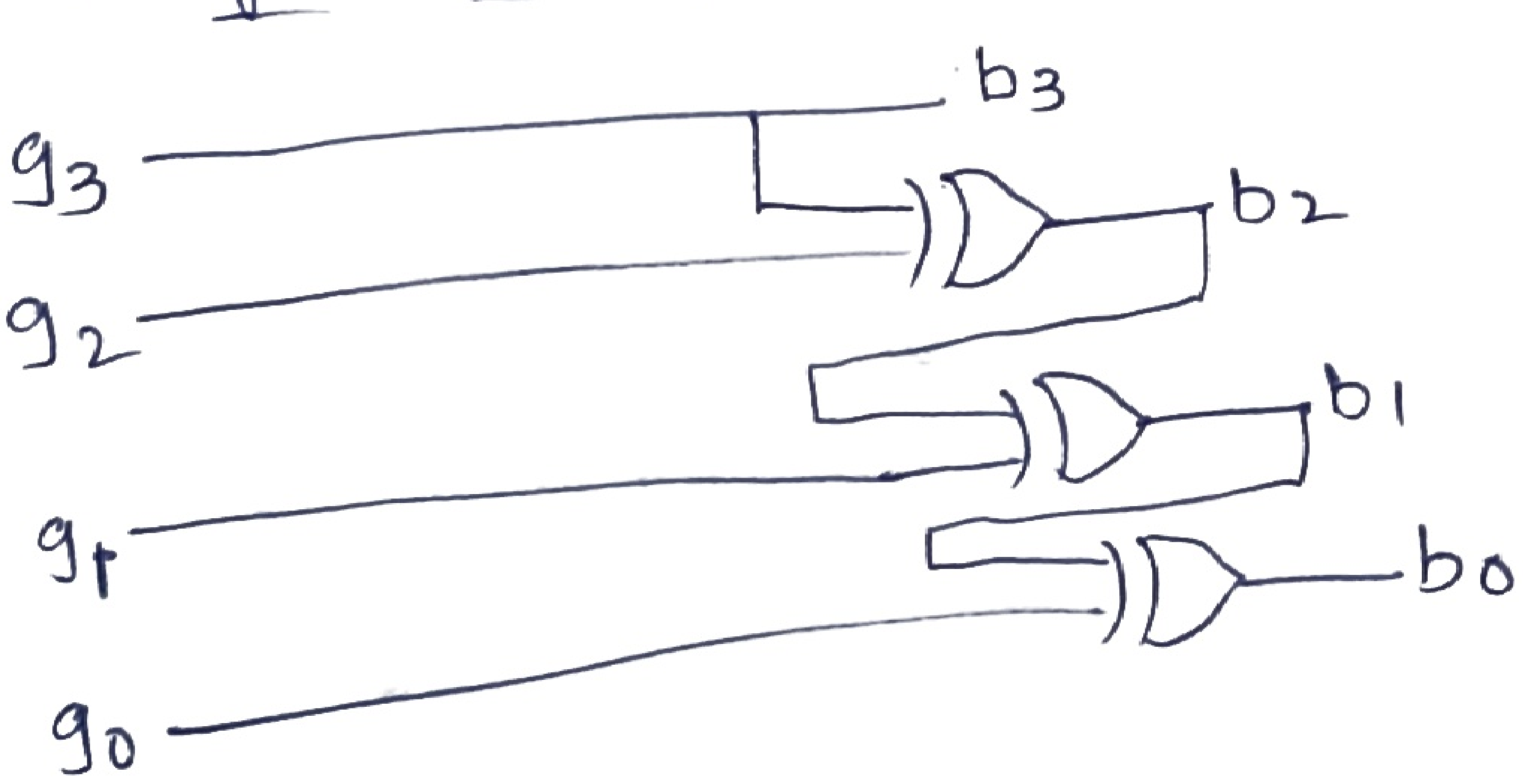
$g_2 = b_3 \oplus b_2$

$g_1 = b_2 \oplus b_1$

$g_0 = b_1 \oplus b_0$



Gray to Binary



n

Binary Multiplication:-

101 x 110

$$\begin{array}{r} 000 \\ 1010 \\ 10100 \\ \hline 11110 \\ \hline \end{array}$$

Subtraction Using Complements:-

= A − B

⇒ A + (−B)

= A + complement of B

Ex:-

A = $(225)_{10}$, B = $(155)_{10}$ [A > B]

A − B ⇒ 225 + (9's complement of 155)

= 225 + 844

= ①069

$$\begin{array}{r} 999 \\ 155 \\ \hline \text{9's} \rightarrow 844 + \\ 4 \\ \hline 844 \\ 225 \\ \hline 9 \end{array}$$

$$\Rightarrow \begin{array}{r} ①069 + \\ ①1 \\ \hline 070 \end{array}$$

(i) A > B

⇒ A + (comp of B)

*. If carry is present neglect it and add it to result.

(ii) A < B

*. If their is no carry then again complement should be done to the result.

Ex:-

A > B

A B

$(39)_{10} - (07)_{10}$

$(39)_{10}$ + (9's of 07)

= 39 + 92

= ①31 +

1

$\overline{\quad 32 \quad}$

$$\begin{array}{r} 99 \\ 07 \\ \hline 92 \\ 39 \\ \hline 131 \end{array}$$

A < B

$(07)_{10} - (39)_{10}$

= 07 + (60)

= 67 ⇒ 99 − 67 = −32

= −32

$$\begin{array}{r} 99 \\ 39 \\ \hline 60 \end{array}$$

$$\begin{array}{r} 99 \\ -67 \\ \hline -32 \end{array}$$

A = 155 , B = 225

= A + (9's complement of B)

= 155 + 999 − 225 [A < B]

$$\begin{array}{r} 999 \\ -\ 225 \\ \hline 774 \\ +\ 155 \\ \hline 929 \end{array} \Rightarrow \begin{array}{r} 999 \\ -\ 929 \\ \hline -70 \end{array}$$

$$\begin{array}{r} 225 \\ -\ 155 \\ \hline 70 \end{array}$$

= −70

⇒ 39, 07 in 10's complement.

A = 39, B = 07

A + (9's complement of 07)

$$\begin{array}{r} 99 \\ -\ 07 \\ \hline 92 \\ +\ 1 \\ \hline 93 \end{array}$$

= 39 + 93

= ← 1 32 + 1

33

10's → 93

$$\begin{array}{r} 93 \\ +\ 39 \\ \hline 132 \end{array}$$

225 − 155

A B

= A + (10's complement of B)

= 225 + 845

= 1070

= ← 1 070

neglect

$$\begin{array}{r} 999 \\ -\ 155 \\ \hline 844 \\ +\ 1 \\ \hline 845 \\ +\ 225 \\ \hline 1070 \end{array}$$

* While doing subtraction if it is 9's complement we should add carry & if it is 10's complement we should neglect that carry.

07 − 39

A = 07 , B = 39

A + (10's complement of 39)

= 07 + 61

= 68

= −32

$$\begin{array}{r} 99 \\ -\ 39 \\ \hline 60 \\ +\ 1 \\ \hline 61 \\ +\ 7 \\ \hline 68 \end{array} \qquad \begin{array}{r} 99 \\ -\ 68 \\ \hline 31 \\ +\ 1 \\ \hline 32 \end{array}$$

$(0101)_2 - (0111)_2$ [1's complement]

$= (0101) + (1\text{'s complement of } 0111)$

$= (0101) + (1000)$

$$\begin{array}{r} 0101 \\ 1000 \\ \hline 1101 \end{array}$$

$= 1101 \rightarrow$ Again 1's complement

$= 0010 \rightarrow (-2)$

2's complement subtraction

$(0101)_2 - (0111)_2$

$= (0101) + (2\text{'s complement of } 0111)$

$$\begin{array}{r} 1000 \\ 1 \\ \hline 1001 \\ 0101 \\ \hline 1110 \end{array}$$

$= 0101 + 1001$

$= 1110 \xrightarrow{2\text{'s}} 0001$

$$\begin{array}{r} 0001 \\ 11 \\ \hline -0010 \end{array}$$

Logic Gates :-

1. NOT (7404)

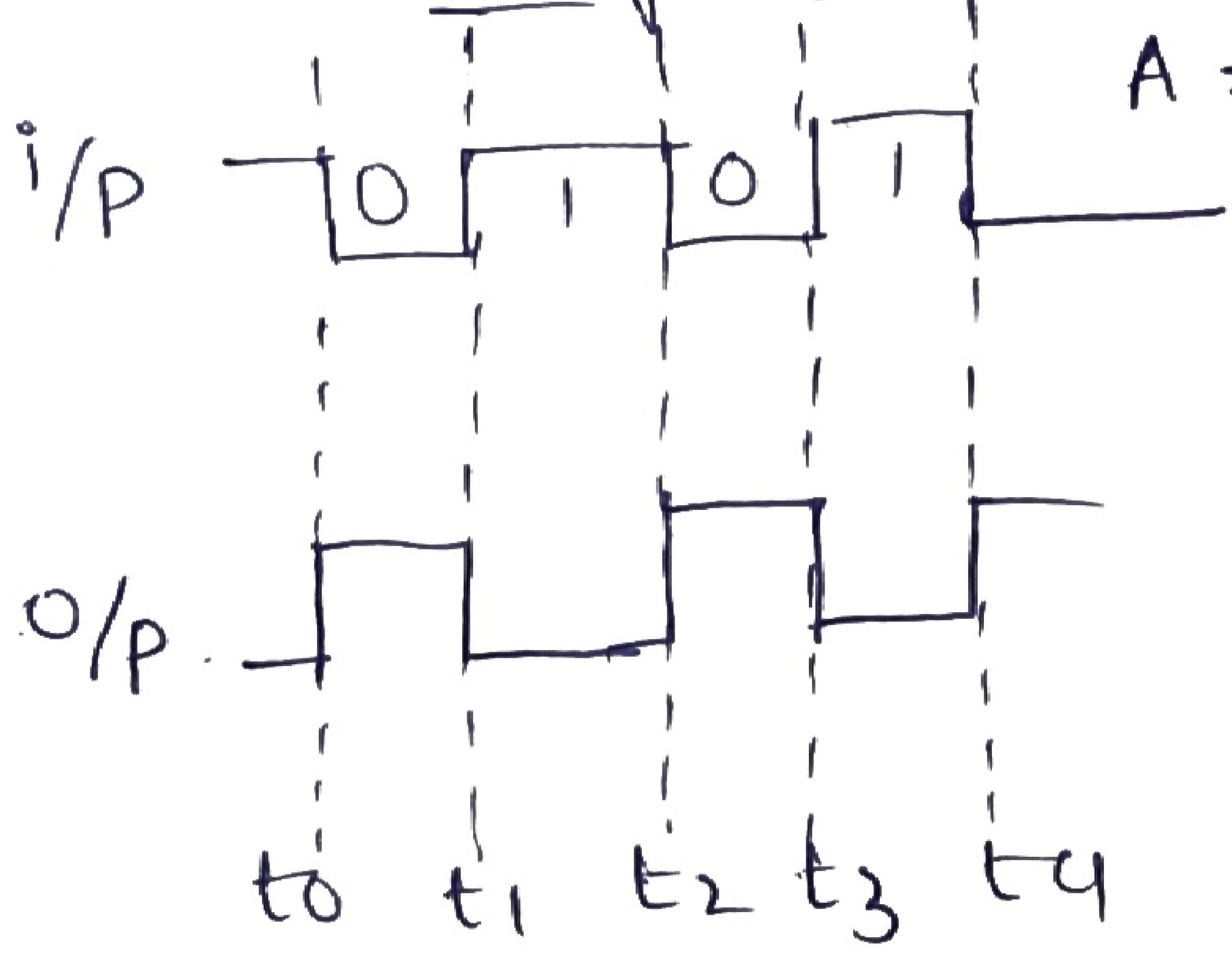
T.T

0 → 1

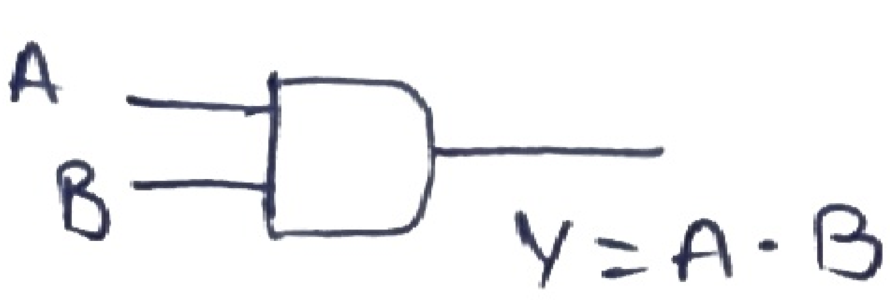
1 → 0

Timing Diagram

A = 0101



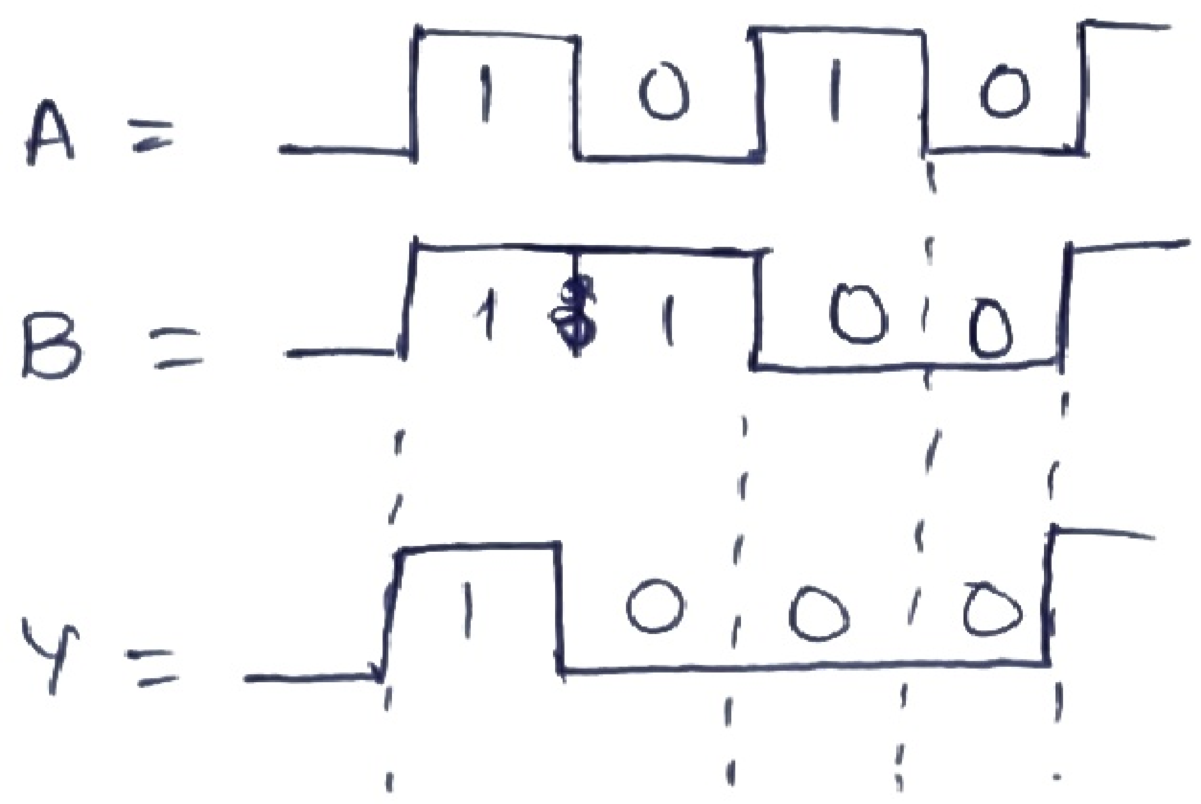
2. AND(·) [7408]



A	B	Y = A · B
0	0	0
0	1	0
1	0	0
1	1	1

T.D

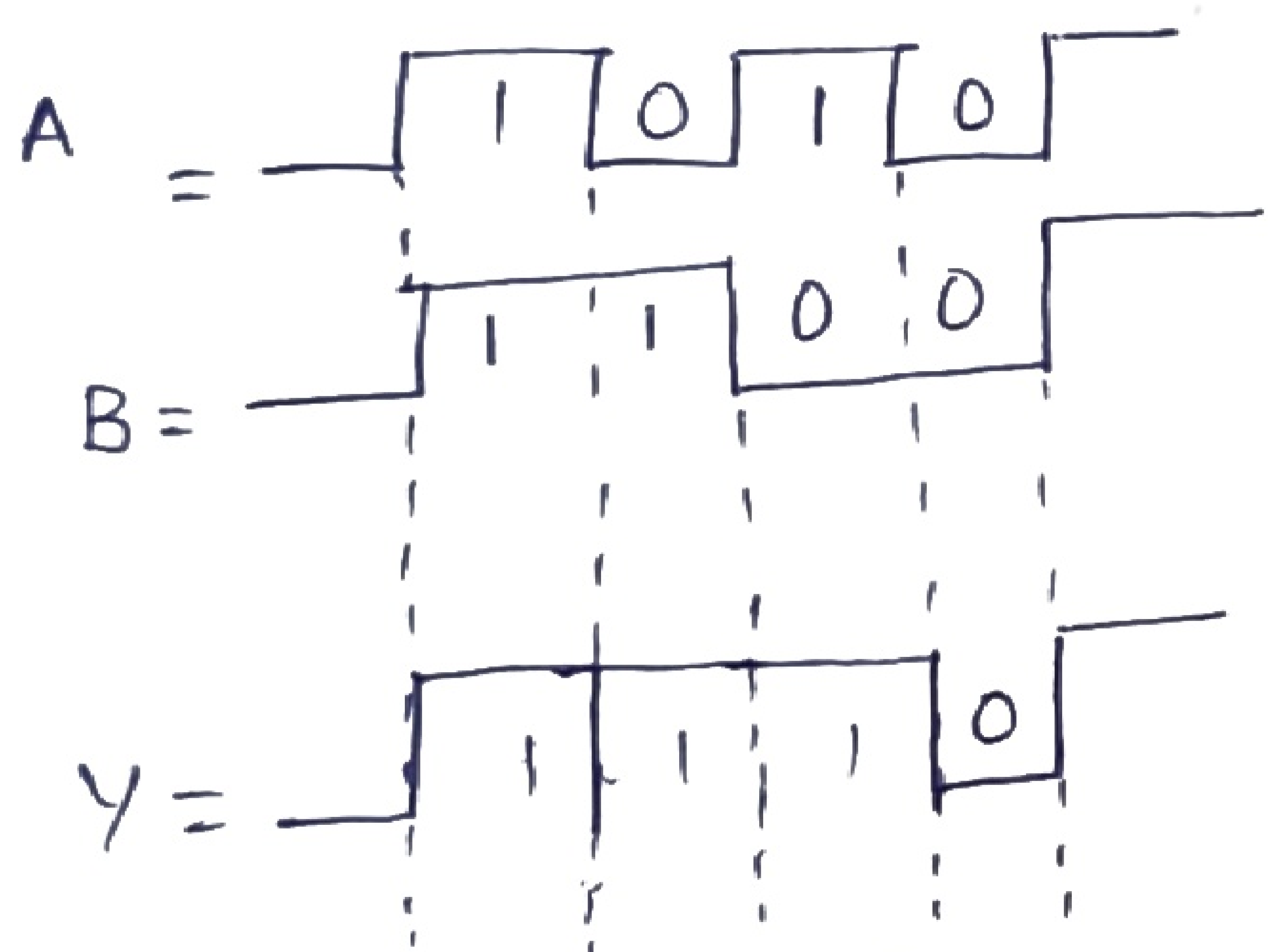
T.B A = 1010 , B = 1100



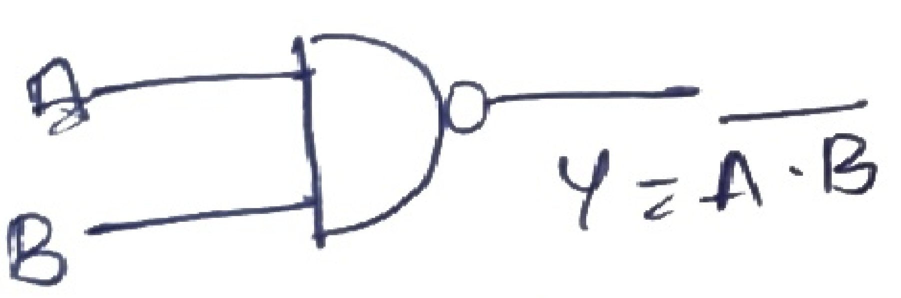
3. OR (+) [7432]

A	B	Y = A+B
0	0	0
0	1	1
1	0	1
1	1	1

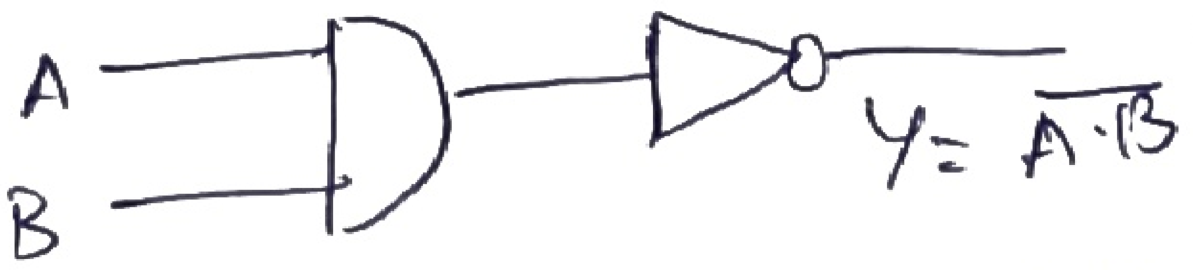
T.D A = 1010 , B = 1100



(4) NAND [7400]

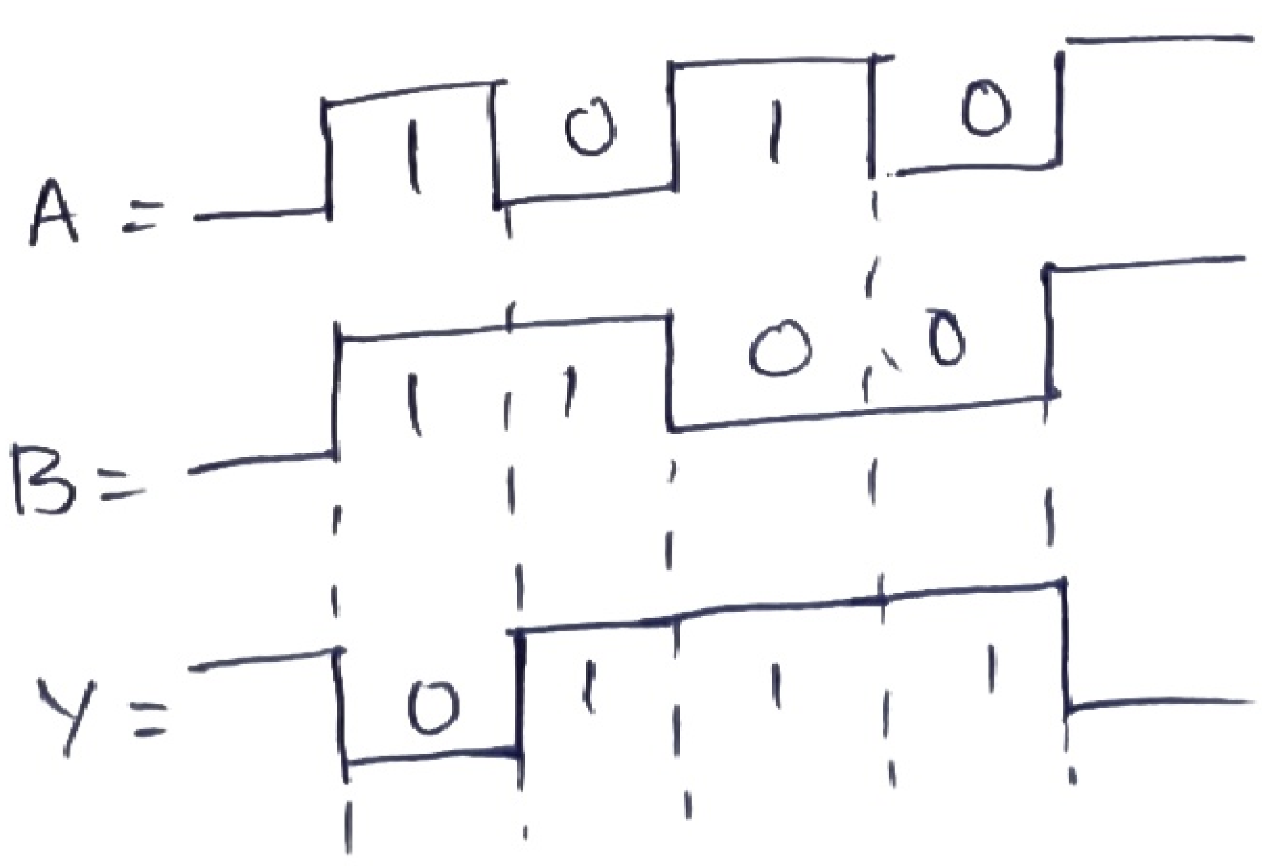


(or)

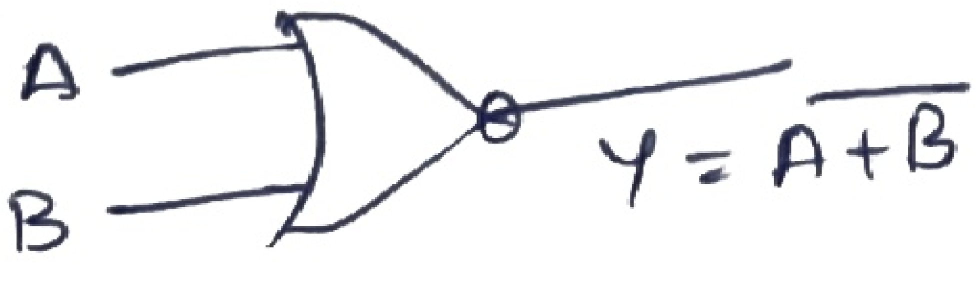


A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

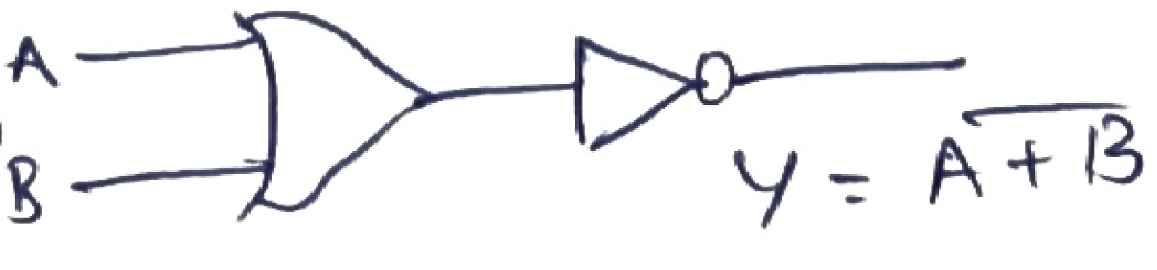
T.D A = 1010 ; B = 1100



(5) NOR [7402]

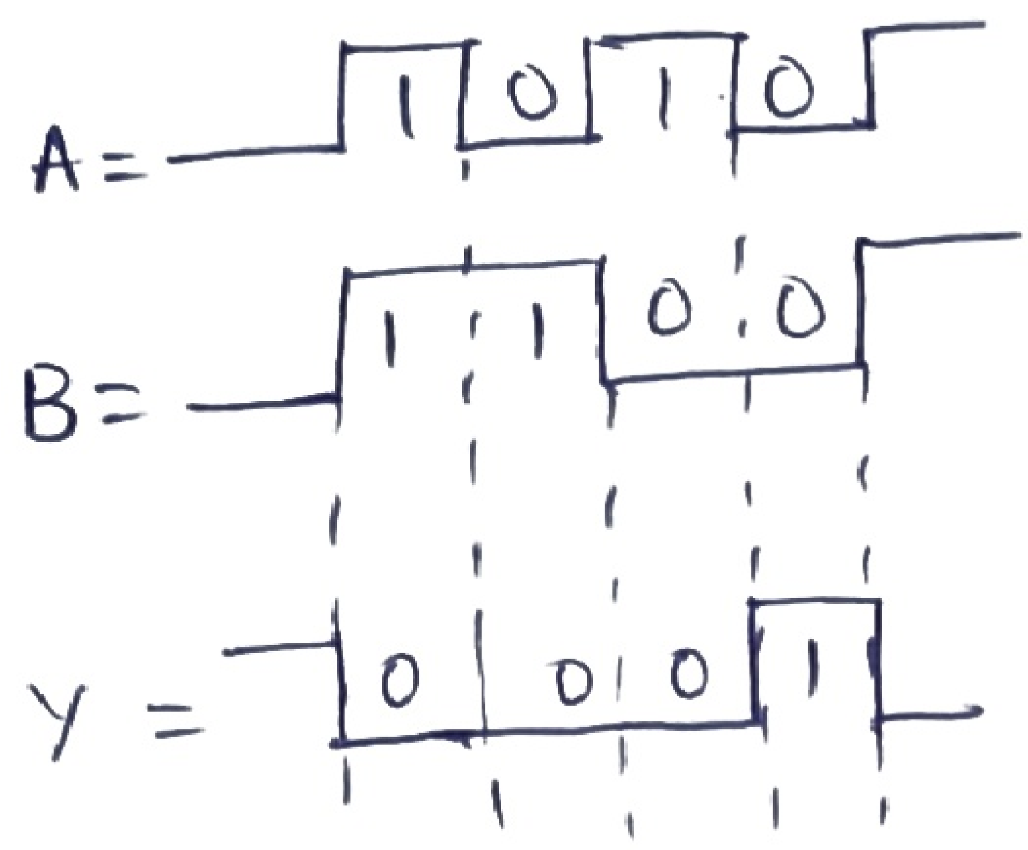


(or)



A	B	$Y = \overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

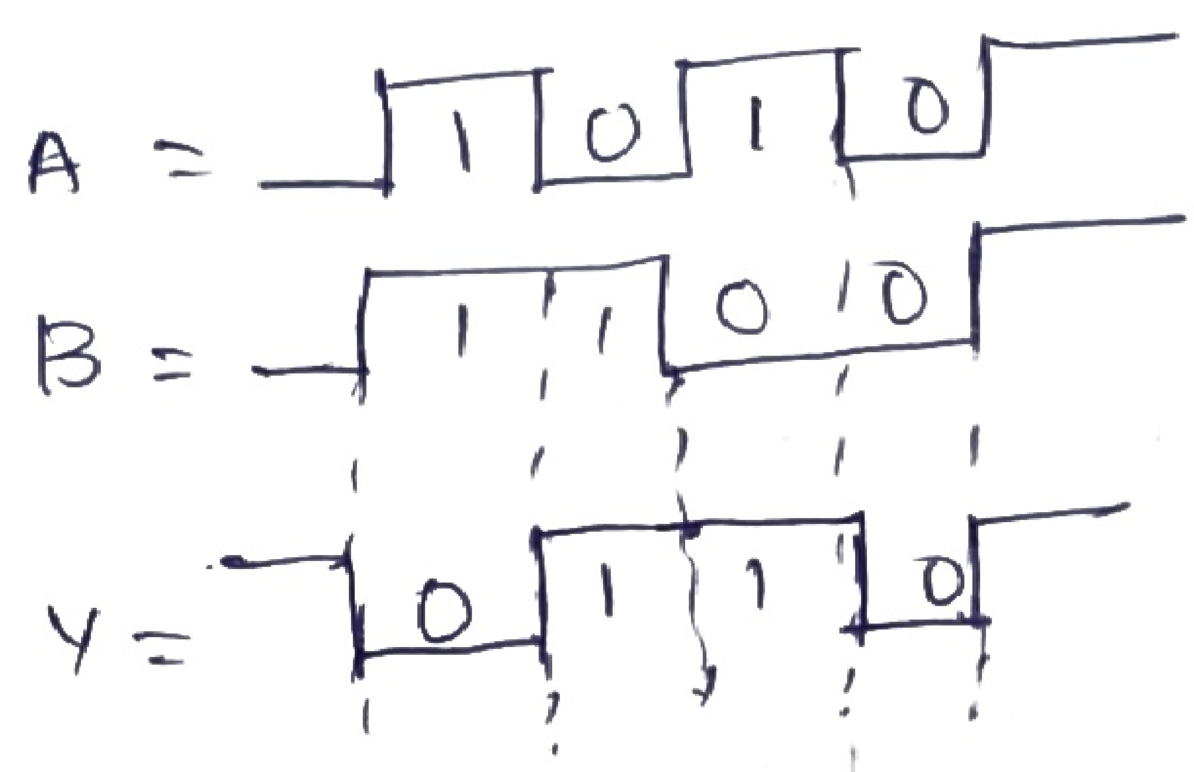
A = 1010 ; B = 1100



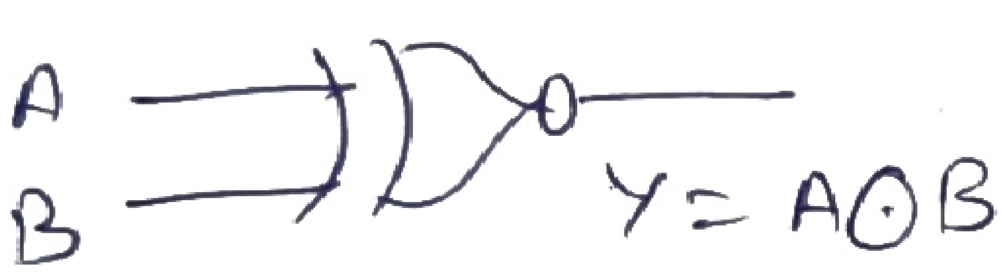
6. Ex-OR

A	B	$Y = A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

T.D A = 1010, B = 1100



7. Ex-NOR



A	B	$Y = A \odot B$
0	0	1
0	1	0
1	0	0
1	1	1

4-9-19

Boolean Algebra

① AND Law:-

$A \cdot 1 = A$

$A \cdot 0 = 0$

$A \cdot A = A$

$A \cdot \bar{A} = 0$

② OR Law:-

$A + 1 = 1$

$A + 0 = A$

$A + A = A$

$A + \bar{A} = 1$

3. Commutative Law:-

$A \cdot B = B \cdot A$

$A + B = B + A$

4. Distributive Law:-

$A \cdot (B + C) = A \cdot B + A \cdot C$

$A + (B \cdot C) = (A + B) \cdot (A + C) \rightarrow$

$(A+B) \cdot (A+C) = A \cdot A + A \cdot C + B \cdot A + B \cdot C$

$= A + AC + AB + BC$

$= A\ (1 + C + B) + BC$

$= A\ (1 + B) + BC$

$= A \cdot (1) + BC$

(ii) $P + (\bar{Q} \cdot R)$

$= (P + \bar{Q}) \cdot (P + R)$

5. Associative Law:-

$(A \cdot B) \cdot C = A \cdot (B \cdot C)$

$A = 1,\quad B = 0,\quad C = 1 \Rightarrow (1 \cdot 0) \cdot 1 = 0$

$1 \cdot (0 \cdot 1) = 1 \cdot 0 = 0$

$(A + B) + C = A + (B + C)$

Inversion Law:-

$\bar{\bar{A}} = A$

$A = 0 \Rightarrow (\bar{\bar{0}}) = \bar{\bar{0}} = \bar{1} = 0 = A$

De-Morgan's Law:-

$\overline{A \cdot B} = \bar{A} + \bar{B}$

$\overline{A + B} = \bar{A} \cdot \bar{B}$

1. $A \cdot (A + B) \Rightarrow A \cdot A + A \cdot B = A + AB \Rightarrow A\ (1 + B) = A\ (1) = A$

2. $A + \bar{A}B \Rightarrow (A + \bar{A}) \cdot (A + B)$

$= 1 \cdot (A + B)$

$= A + B$

$(A+B)\cdot(B+C)\cdot 3$

$= AB + AC + BB + BC$

$= AB + AC + B + BC$

$= B + AB + BC + AC$

$= B[\underbrace{1 + A}_{1} + C] + AC$

$= B[1+C] + AC \Rightarrow B + AC$

4. $\bar x \bar y z + xyz + \bar x y z + x \bar y z$

$= \bar x \bar y z + yz[x + \bar x] + x\bar y z$

$= \bar x \bar y z + yz + x \bar y z$

$z[\cancel{y} + \bar x \bar y + x \bar y] \Rightarrow \bar y z[x + \bar x] + yz$

$= \cancel{z}[\bar x \bar y + x\bar y] + yz$

$= \bar y z + yz$

$= z[\bar y + y]$

$= z$

5. $A+B+C+D+\overline{ABCD}$

$= A+B+C+D+\bar A+\bar B+\bar C+\bar D$

$= (A+\bar A)+(B+\bar B)+(C+\bar C)+(D+\bar D) = \underbrace{(1+1)}_{1}+\underbrace{(1+1)}_{1} = (1+1) = 1$

6. $\overline{AB+\bar C D}$

$= \overline{AB+\bar C D} = (\overline{AB})\cdot(\overline{\bar C D}) = (\overline{AB})\cdot(CD) = (\bar A + \bar B)CD$

$= \bar A CD + \bar B CD$

$F = \bar x \bar y z + x \bar y z$
001   101

x	y	z	F
~~0~~	~~0~~	~~0~~	~~0~~
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

7. $F = \overset{1}{x} + \overset{01}{\bar y z}$

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

$F = \bar x \bar y z + x \bar y \bar z + x \bar y z + x y \bar z + xyz$

$= \bar y[\bar x z + x \bar z] + x[\bar y z + y \bar z] + xyz$

$= \bar y(x+z) + x(y+z) + xyz$

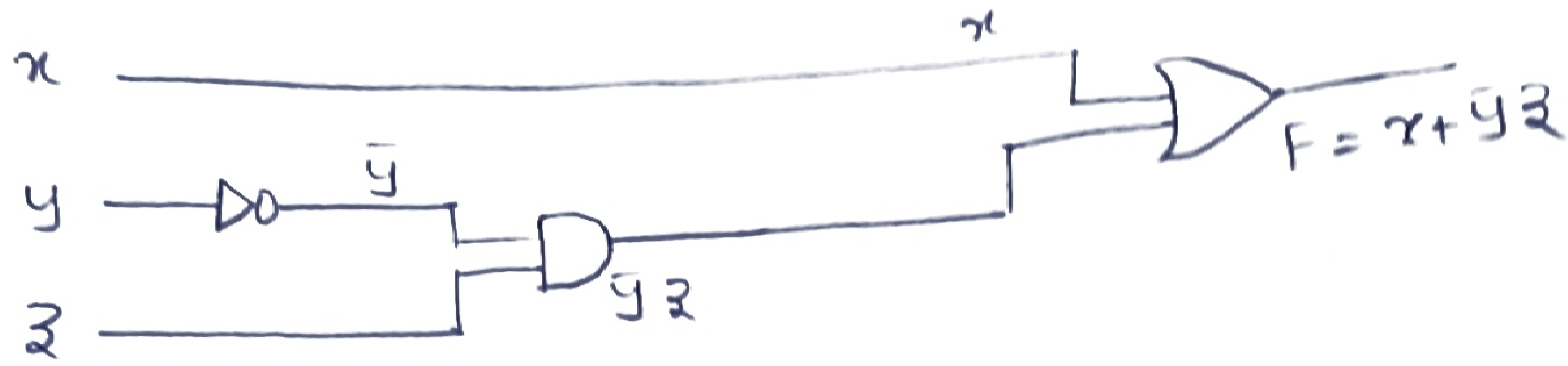
$= x\bar y + \bar y z + xy + xz + xyz$

$= x[\bar y + y] + \bar y z + xz(1+y)$

$= x + \bar y z + xz(1)$

$= x[1+z] + \bar y z$

$= x + \bar y z$



8. $F = \bar{A}C + AB + \bar{B}C + \bar{C}D$

$= C[\bar{A} + \bar{B}] + AB + \bar{C}D$

$= \underbrace{C(\overline{AB}) + AB}\, + \bar{C}D$ $\quad \{\because A + BC = (A+B)(A+C)\}$

$= (\overline{AB} + AB)(AB + C) + \bar{C}D$

$= AB + C + \bar{C}D$

$= AB + \underbrace{(C + \bar{C})}_{1}(C + D)$

$= AB + C + D$

9. $F = \underline{A \oplus B} \oplus C$

$= (\underbrace{\bar{A}B + A\bar{B}}) \oplus C$

$= (\overline{\bar{A}B + A\bar{B}})C + (\bar{A}B + A\bar{B})\bar{C}$

$\neq (\cancel{A\bar{B}} + \bar{A}B)\cancel{C} + \cancel{\bar{A}B}\bar{C} \cancel{+} A\bar{B}\bar{C}$

$\neq A\bar{B}C + \bar{A}BC + \cancel{\bar{A}B}\bar{C} + A\bar{B}\bar{C}$

$= ((\overline{\bar{A}B})\cdot(\overline{A\bar{B}}))\cdot C + \bar{A}B\bar{C} + A\bar{B}\bar{C}$

$= ((A + \bar{B})\cdot(\bar{A} + B))\cdot C + \bar{A}B\bar{C} + A\bar{B}\bar{C}$

$= (0 + AB + \bar{A}\bar{B} + 0)\cdot C + \bar{A}B\bar{C} + A\bar{B}\bar{C}$

$= ABC + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}$

10. $F = A \oplus B \oplus AB$

$= (\bar{A}B + A\bar{B}) \oplus AB$

$= (\overline{\bar{A}B + A\bar{B}})AB + (\bar{A}B + A\bar{B})\cdot\overline{AB}$

$= ((\overline{\bar{A}B})\cdot(\overline{A\bar{B}}))\,\cancel{AB} +$

$= [(A + \bar{B})\cdot(\bar{A} + B)]\cdot AB + (\bar{A}B + A\bar{B})(\bar{A} + \bar{B})$

$= [A\cdot\bar{A} + AB + \bar{A}\bar{B} + B\bar{B}]\cdot AB + \bar{A}B + 0 + 0 + A\bar{B}$

$= (AB + \bar{A}\bar{B}) \cdot AB + \bar{A}B + A\bar{B}$

$= \cancel{B}(\cancel{A}+$

$= AB + 0 + \bar{A}B + A\bar{B}$

$= B[A + \bar{A}] + A\bar{B}$

$= B + A\bar{B}$

$= A + B$

11. $F = \bar{x}y\bar{z} + \bar{x}\bar{y}z$

$\bar{F} = \overline{\bar{x}yz + \bar{x}\bar{y}z}$

$= (\overline{\bar{x}yz}) \cdot (\overline{\bar{x}\bar{y}z})$

$= \cancel{x\bar{y}z} = (x + \bar{y} + \bar{z}) \cdot (x + y + \bar{z})$

12. $F = x(\bar{y}\bar{z} + yz)$

$\bar{F} = \overline{x(\bar{y}\bar{z} + yz)}$

$= \overline{x\bar{y}z} + \overline{xyz}$ $\quad = \bar{x} + (\overline{\bar{y}\bar{z} + yz})$

$= \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z}$ $\quad = \bar{x} + (\overline{\bar{y}\bar{z}} \cdot \overline{yz})$

$= \bar{x}\bar{z}[y + \bar{y}]$ $\quad = \bar{x} + [(y+z) \cdot (\bar{y} + \bar{z})]$

$= \bar{x}\bar{z}$ $\quad = \bar{x} + y\bar{z} + \bar{y}z$

$F = \bar{x}y\bar{z} + \bar{x}\bar{y}z$

① $F_{dual} = (\bar{x} + y + \bar{z}) \cdot (\bar{x} + \bar{y} + z)$

② $\bar{F} = (x + \bar{y} + z) \cdot (x + y + \bar{z})$.

Ex:- $F = x(\bar{y}\bar{z} + yz)$

$F_{dual} = x + (\bar{y} + \bar{z}) \cdot (y + z)$

$F = \bar{X}\bar{Y} + X\bar{Y}$ (00, 10)

X	Y	F
0	0	1
0	1	0
1	0	1
1	1	0

$F = \bar{X}Y\bar{Z} + X\bar{Y}$ (Z)

010 $\quad$ 100

$= \bar{X}Y\bar{Z} + X\bar{Y}(Z + \bar{Z}) \Rightarrow \bar{X}Y\bar{Z} + X\bar{Y}Z + X\bar{Y}\bar{Z}$

010 $\quad$ 101 $\quad$ 100

x	y	z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

$$F = x\bar{y}\,(z + y\bar{z}) + \bar{z}$$

$$= x\bar{y}z + x\bar{y}y\bar{z} + \bar{z}$$

$$= x\bar{y}z + 0 + \bar{z}\,[x + \bar{x}] \Rightarrow x\bar{y}z + x\bar{z} + \bar{x}\bar{z}$$

$$= x\bar{y}z + \bar{z}$$

1 0 1

x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

x	y	z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

$F(x,y)$

(m_0)	0 0	$\bar{x}\,\bar{y}$
(m_1)	0 1	$\bar{x}\,y$
(m_2)	1 0	$x\,\bar{y}$
(m_3)	1 1	$x\,y$

→ Minterms (m_i)

variable → 1

$\bar{\text{variable}}$ → 0

$M_0 \Rightarrow (00) \quad x+y$

$M_1 \Rightarrow (01) \quad x+\bar{y}$

$M_2 \Rightarrow (10) \quad \bar{x}+y$

$M_3 \Rightarrow (11) \quad \bar{x}+\bar{y}$

$\left.\right\}$ var = 0, $\overline{\text{var}} = 1$, Max terms ($M_i$).

$F(x,y,z)$	Minterms	Maxterms
	$\bar{x}\bar{y}\bar{z}$ (m_0)	$x+y+z$ (M_0)
	⋮	⋮
	xyz (m_7)	$\bar{x}+\bar{y}+\bar{z}$ (M_7)

110 + 011

$m_6 + m_3$

$F(A, B) = AB + \bar{B}C \Rightarrow$ Sum of product (SOP)

$F(A,B,C) = (A+B)\cdot(\bar{B}+C) \Rightarrow$ product of sum (POS)

$F(A,B,C) = AB\bar{C} + \bar{A}BC \Rightarrow$ Standard SOP

$= m_6 + m_3 \Rightarrow \Sigma m(3,6)$ //

$F(A,B,C) = (A+B+\bar{C})\cdot(\bar{A}+B+C) \Rightarrow$ standard POS

$= M_1 \cdot M_4 \Rightarrow \Pi M(1,4)$ $\left\{ \begin{matrix} 001 & 100 \\ M_1 & M_4 \end{matrix} \right\}$

Converting SOP (or) POS into standard forms:-

① $F(A,B,C) = A + BC$ [SOP]

$= A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC + \bar{A}BC + ABC.$

A	BC	
1	00	$\Rightarrow m_4$
1	01	$\Rightarrow m_5$
1	10	$\Rightarrow m_6$
1	11	$\Rightarrow m_7$

$= m_4 + m_5 + m_6 + m_7 + m_3$

$= m_3 + m_4 + m_5 + m_6 + m_7$

$= \Sigma m(3,4,5,6,7)$

A	B	C	
0	1	1	$\Rightarrow m_3$
1	1	1	$\Rightarrow m_7$

② $F(A,B) = \bar{A} + \bar{B}$

$= \bar{A}(1) + \bar{B}(1)$

$= \bar{A}(B+\bar{B}) + \bar{B}(A+\bar{A})$

$= \bar{A}B + \bar{A}\bar{B} + A\bar{B} + \bar{A}\bar{B} \Rightarrow \underset{01}{\bar{A}B} + \underset{10}{A\bar{B}} + \underset{00}{\bar{A}\bar{B}}$

2 1

10 −

$= m_0 + m_1 + m_2$

$= \Sigma m(0,1,2)$

A	B	C	D	
1	1	0	0	$\Rightarrow m_{12}$
1	1	1	0	$\Rightarrow m_{14}$
1	1	1	1	$\Rightarrow m_{15}$

③ $F(A,B,C,D) = A + B\bar{C} + AB\bar{D} + ABCD$

$= \cancel{A(1)(1)(1)} + B\bar{C}\cancel{(1)(1)} \cancel{+ AB\bar{D}(1) + ABCD}$

$= A\cancel{(B+\bar{B})}$

$= m_8 + m_9 + m_{10} + m_{11} + m_{12} + m_{13} + m_{14} + m_{15} + m_4 + m_5$

$= \Sigma m(4,5,8,9,10,11,12,13,14,15)$

A (8)	B (4)	C (2)	D (1)	
1	0	0	0	$\Rightarrow m_8$
1	0	0	1	$\Rightarrow m_9$
1	0	1	0	$\Rightarrow m_{10}$
1	0	1	1	$\Rightarrow m_{11}$
1	1	0	0	$\Rightarrow m_{12}$
1	1	0	1	$\Rightarrow m_{13}$
1	1	1	0	$\Rightarrow m_{14}$
1	1	1	1	$\Rightarrow m_{15}$

A (8)	B (4)	C (2)	D (1)	
0	1	0	0	$\Rightarrow m_4$
0	1	0	1	$\Rightarrow m_5$
1	1	0	0	$\Rightarrow m_{12}$
1	1	0	1	$\Rightarrow m_{13}$

Convert POS into standard POS:-

① $F(A,B,C) = (A+B)\cdot(A+C)$

$= (A+B+C)\cdot(A+B+\bar{C})\cdot(A+\bar{B}+C)\cdot(A+B+C)$

$\cancel{= \Pi M(5,6,7)}$

A (4)	B (2)	C (1)	
$\cancel{0}$	$\cancel{0}$	0	$\Rightarrow m_6$
$\cancel{0}$	$\cancel{0}$	1	$\Rightarrow m_7$

$\bar{A}$	$\bar{B}$	C	
0	0	1	$\rightarrow M_1$
0	0	0	$\rightarrow M_0$

A (4)	B (2)	C (1)	
$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	$\Rightarrow m_5$
$\cancel{0}$	1	$\cancel{0}$	$\Rightarrow M_7$

$= M_0 \cdot M_1 \cdot M_2$

$= \Pi M(0,1,2)$

→ It can be known as canonical POS form.

A	$\bar{B}$	$\underline{C}$	
0	1	0	$- M_2$
0	0	0	$- M_0$

② $F = (A+\bar{B})\cdot(\bar{A}+C)$

$= M_2 \cdot M_3 \cdot M_4 \cdot M_6$

$= \Pi M(2,3,4,6)$

$\bar{A}$	B	$\underline{C}$	
0	1	0	$= M_2$
0	1	1	$= M_3$

$\bar{A}$	B	$\underline{C}$	
1	0	0	$= M_4$
1	1	0	$= M_6$

③ $F = A + \bar{B}CD$

$= (A+\bar{B})\cdot(A+CD) \Rightarrow (A+\bar{B})\cdot(A+C)\cdot(A+D)$

~~A B C D~~

$= (A + AC + A\bar{B} + \bar{B}C)\cdot(A+D)$

$= \cancel{A + AD + AC} \Rightarrow A + AC + A\bar{B} + A\bar{B}C + AD + ACD + A\bar{B}D + \bar{B}CD$

~~A B C D~~

0 X 1

$= \cancel{(A+\bar{B}) \cdot (AA + AD + AC} + CD)$

(3) $F = A + \bar{B}CD \Rightarrow SOP$

$= (A+\bar{B}) \cdot (A + CD)$

$= (A+\bar{B}) \cdot (A+C) \cdot (A+D)$

$= (A+\bar{B}) \cdot [AA + AD + AC + CD]$

$= \cancel{(A+\bar{B})}\ AAA + AAD + AAC + ACD + A\bar{B} + A\bar{B}D + A\bar{B}C + \bar{B}CD$

$= A \cdot [1 + D + C + CD + \bar{B} + \bar{B}D + \bar{B}C] + \bar{B}CD$

$= A + \bar{B}CD.$

(4) $F = xy + \bar{x}z$

$= xy(1) + \bar{x}z(1)$

$= \Sigma m(1,3,6,7)$

4	2	1	
x	y	z	
1	1	0	$- m_6$
1	1	1	$- m_7$

x	y	z	
0	0	1	$- m_1$
0	1	1	$- m_3$

$= xy(z+\bar{z}) + \bar{x}z(y+\bar{y})$

$= \underset{111}{xyz} + \underset{110}{xy\bar{z}} + \underset{011}{\bar{x}yz} + \underset{001}{\bar{x}\bar{y}z}$

$= m_7 + m_6 + m_3 + m_1 \Rightarrow \Sigma m(1,3,6,7)$

converting to pos.

$= xy + \bar{x}z$

$= (xy + \bar{x}) \cdot (xy + z)$

$= (x+\bar{x}) \cdot (\bar{x}+y) \cdot (x+z) \cdot (y+z)$

$= 1 \cdot (\bar{x}+y) \cdot (x+z) \cdot (y+z)$

$= M_4 \cdot M_5 \cdot M_0 \cdot M_2$

$= \Pi M(0,2,4,5)$

x	y	z	
1	0	0	$= M_4$
1	0	1	$= M_5$

x	y	z	
0	0	0	$= M_0$
0	1	0	$= M_2$

x	y	z	
0	0	0	$= M_0$
1	0	0	$= M_4$

⑤ $F = \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$ → canonical SOP form

$\quad$ 001 $\quad$ 100 $\quad$ 101 $\quad$ 110 $\quad$ 111

$= m_1 + m_4 + m_5 + m_6 + m_7$

$= \Sigma m(1,4,5,6,7)$

converting to pos form

$F \Rightarrow \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC.$

$= \bar{A}\bar{B}C + A\bar{B}[C+\bar{C}] + AB[C+\bar{C}]$

$= \bar{A}\bar{B}C + A\bar{B} + AB.$

$= \bar{A}\bar{B}C + A[B+\bar{B}] \Rightarrow \bar{A}\bar{B}C + A$

$= (A+\bar{A})\cdot(A+\bar{B})\cdot(A+C)$

$= 1\cdot(A+\bar{B})\cdot(A+C)$

$= (A+\bar{B})\cdot(A+C)$ → canonical pos form

$= M0 \cdot M2 \cdot M3$

$= \pi M(0,2,3)$

A	B	C	
0	0	0	$= M_0$
0	1	0	$= M_2$

A	B	C	
0	1	0	$= M2$
0	1	1	$= M3$

Tutorial

① A positional number system is having symbols 0, 1, 2, N, M ⓘ Find the 12th number in this system.
ⓘⓘ Find the minimum decimal value of $(21D)_x$.

0	N 0
1	N 1
2	N 2
N	N N
M	N M
1 0	M 0
1 1	M 1
1 2	M 2
1 N	M N
1 M	M M
2 0	
2 1 → 12th number	
2 2	
2 N	
2 M	

(0, 1, 2, N, M: 5)

Min dec $(21D)_x$

$2\,1\,D$ → x^2, x^1, x^0

$= (2x^2 + 1\cdot x + D\cdot x^0)_{10}$

$= (2x^2 + x + 13)_{10}$

Min Base $= 14$

$x = 14$

$\Rightarrow (2(14)^2 + 14 + 13)_{10}$

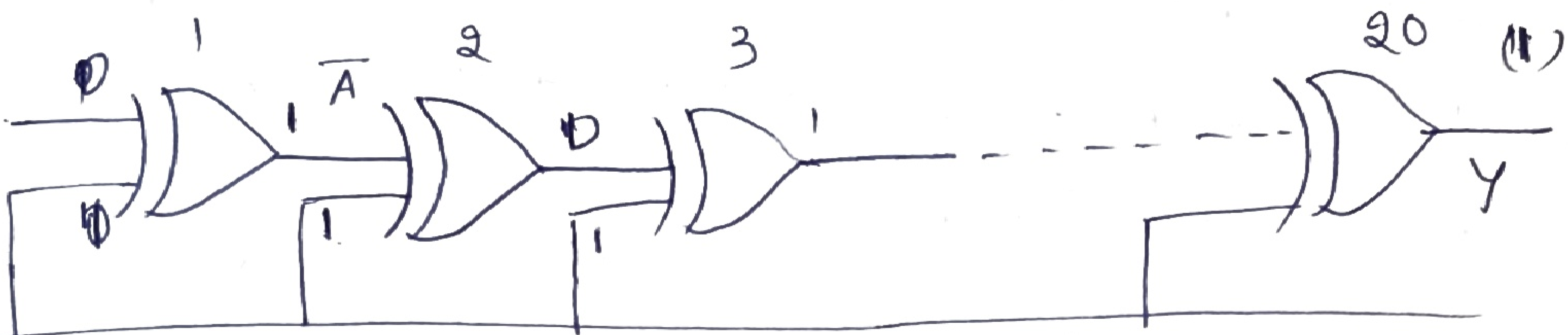
$= (419)_{10}$

(2) The output y could be zero only when inputs A & B are 0. Suggest a suitable gate to be used here.

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

$Y = A + B$

∴ OR gate should be used.



$1 \oplus A$

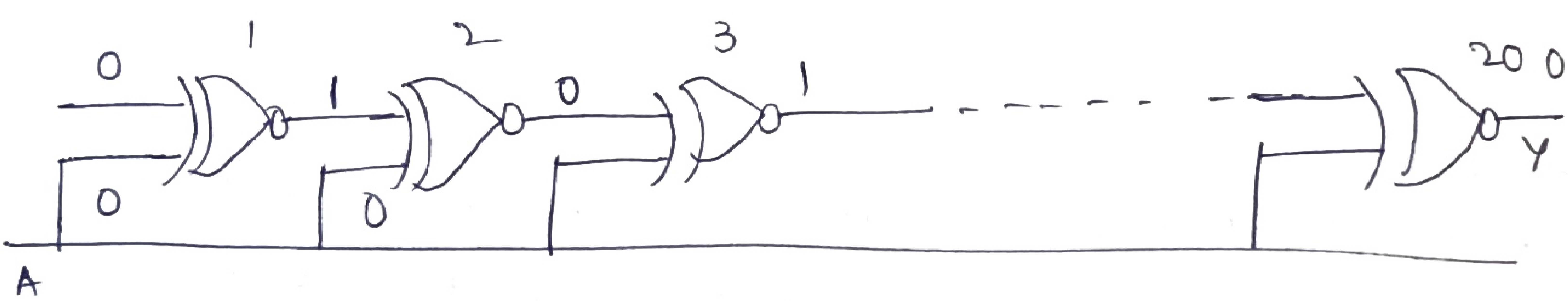
$0 \oplus 0 = 1$

$1 \oplus 1 = 0$

$\bar{A} \oplus A$

$1 \oplus 0 = 1$

$1 \oplus 1 = 0$



$0 \odot A$

$0 \odot 0 = 1$

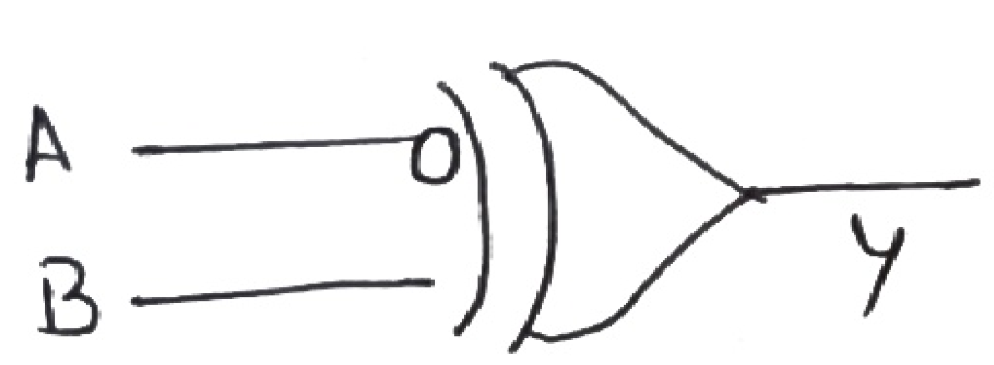
$0 \odot 1 = 0$

$\bar{A} \odot A$

$1 \odot 1 = 1$

$1 \odot 0 = 0$

(3) Find output.



$Y = \bar{A} \odot B$

A	$\bar{A}$	B	$\bar{A} \odot B$
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0

$Y = \bar{A}\bar{B} + AB$

$Y = \bar{\bar{A}}\bar{B} + \bar{A}B$

$= A\bar{B} + \bar{A}B$

$\boxed{Y = A \oplus B}$

(4) find output.

Ex-OR

$Y = \bar{A}B + A\bar{B}$

$Y = \bar{A} \oplus \bar{B}$

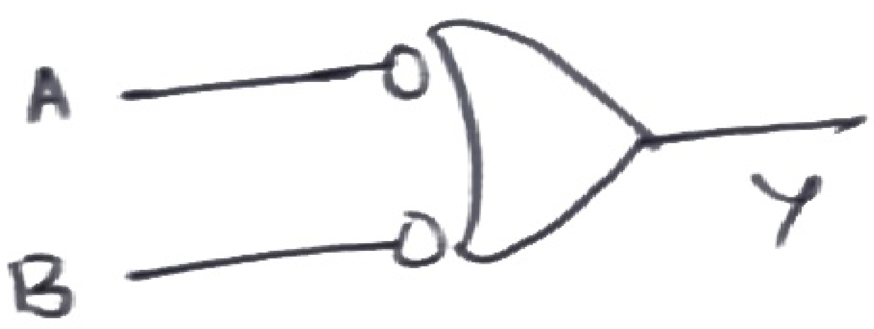
$\downarrow$

Bubbled Ex-OR Gate. $= \bar{\bar{A}}\,\bar{B} + \bar{A}\,\bar{\bar{B}}$

$= A\bar{B} + \bar{A}B$

$\boxed{Y = A \oplus B}$

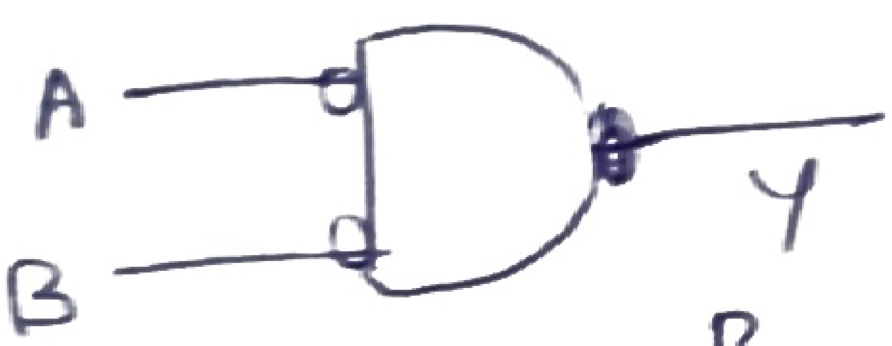
(5) find output



$Y = A + B$

$Y = \bar{A} + \bar{B}$

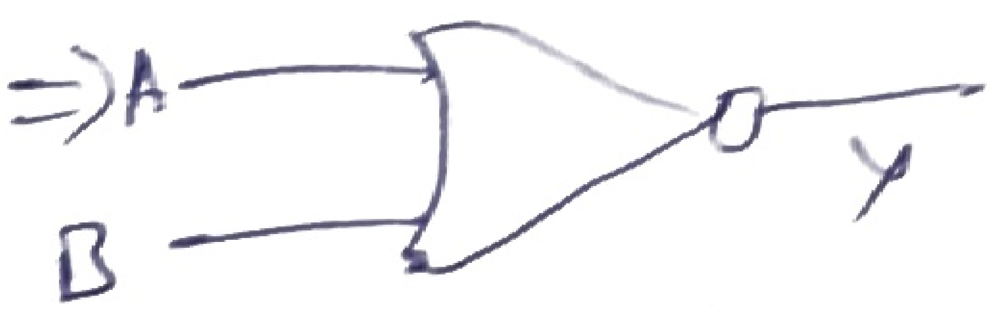
(6)

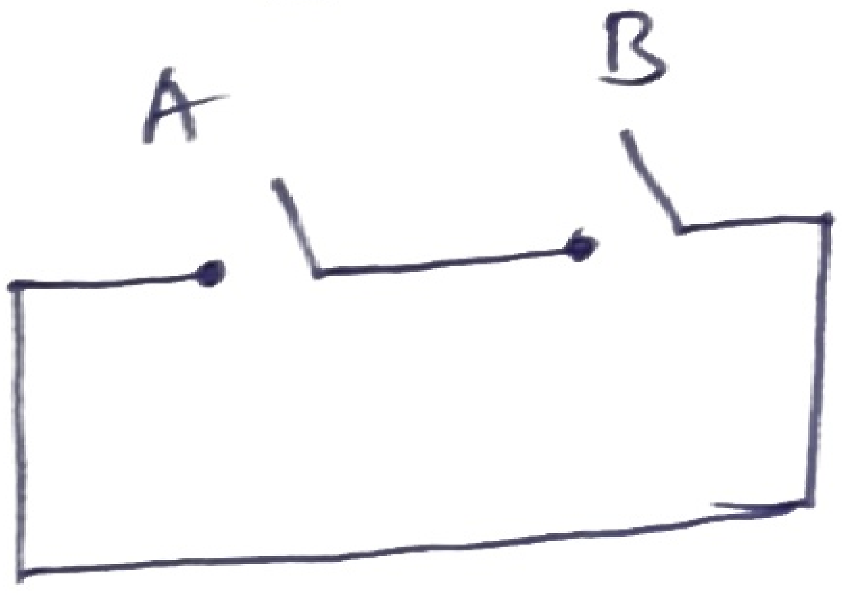


$Y = \bar{A} \cdot \bar{B}$

$= \overline{A + B}$

$\Rightarrow$





when both switches are 'on' then only we get the output. so, it is AND gate.

① Boolean laws

② Karnaugh Map (K-Map)

③ Quine-McClusky (QM) Method.

K-Map for 2-variable function:-

A	B	
$\bar{A}$	$\bar{B}$	$\to m_0$
$\bar{A}$	B	$\to m_1$
A	$\bar{B}$	$\to m_2$
A	B	$\to m_3$

A \ B	$0(\bar{B})$	$1(B)$
$(\bar{A})\ 0$	$\bar{A}\bar{B}$ m_0	$\bar{A}B$ m_1
$(A)\ 1$	$A\bar{B}$ m_2	AB m_3

B \ A	$(\bar{A})\ 0$	$1\ (A)$
$(\bar{B})\ (0)$	$\bar{A}\bar{B}$ m_0	$A\bar{B}$ m_2
$(B)\ (1)$	$\bar{A}B$ m_1	AB m_3

For 3-variable function:-

A \ BC	$(\bar{B}\bar{C})$ 00	$(\bar{B}C)$ 01	(BC) 11	$(B\bar{C})$ 10
$(\bar{A})\ 0$	$\bar{A}\bar{B}\bar{C}$ (m_0)	$\bar{A}\bar{B}C$ (m_1)	$\bar{A}BC$ (m_3)	$\bar{A}B\bar{C}$ (m_2)
$(A)\ 1$	$A\bar{B}\bar{C}$ (m_4)	$A\bar{B}C$ (m_5)	ABC (m_7)	$AB\bar{C}$ (m_6)

For 4-variable function:-

AB \ CD	$(\bar{C}\bar{D})$ 00	$(\bar{C}D)$ 01	(CD) 11	$(C\bar{D})$ 10
$(\bar{A}\bar{B})$ 00	$\bar{A}\bar{B}\bar{C}\bar{D}$ (m_0)	$\bar{A}\bar{B}\bar{C}D$ (m_1)	$\bar{A}\bar{B}CD$ (m_3)	$\bar{A}\bar{B}C\bar{D}$ (m_2)
$(\bar{A}B)$ 01	$\bar{A}B\bar{C}\bar{D}$ (m_4)	$\bar{A}B\bar{C}D$ (m_5)	$\bar{A}BCD$ (m_7)	$\bar{A}BC\bar{D}$ (m_6)
(AB) 11	$AB\bar{C}\bar{D}$ (m_{12})	$AB\bar{C}D$ (m_{13})	$ABCD$ (m_{15})	$ABC\bar{D}$ (m_{14})
$(A\bar{B})$ 10	$A\bar{B}\bar{C}\bar{D}$ (m_8)	$A\bar{B}\bar{C}D$ (m_9)	$A\bar{B}CD$ (m_{11})	$A\bar{B}C\bar{D}$ (m_{10})

① $F(A,B) = \Sigma m(0,1,2)$

A \ B	$(\bar{B})\ 0$	$1\ (B)$
$(\bar{A})\ 0$	1	1
$(A)\ 1$	1	

$\to \bar{A}$

$\downarrow \bar{B}$

$\therefore F = \bar{A} + \bar{B}$

By Boolean law,

$= \bar{A}\bar{B} + \bar{A}B + A\bar{B}$

$= \bar{A}[\bar{B} + B] + A\bar{B}$

$= \bar{A} + A\bar{B} \Rightarrow (\bar{A}+A) + (\bar{A}+\bar{B})$

$= 1 + (\bar{A}+\bar{B})$

$= \bar{A} + \bar{B}$

$F(A,B) = \Sigma m(0,1,3)$

A \ B	0 ($\bar{B}$)	1 (B)
($\bar{A}$) 0	1	1
(A) 1		1

$\rightarrow \bar{A}$

$\rightarrow B$

$\boxed{F = \bar{A} + B}$

By Boolean law,

$F = \bar{A}\bar{B} + \bar{A}B + AB \Rightarrow \bar{A}[\bar{B} + B] + A'B \Rightarrow \bar{A} + AB \Rightarrow (\bar{A} + A) + (\bar{A} + B)$

$= \bar{A} + B.$

$F(A,B) = \Sigma m(1,3)$

A \ B	0 ($\bar{B}$)	1 (B)
($\bar{A}$) 0	0	1
(A) 1		1

$\rightarrow B$

$\bar{A}B + AB$
$B[\bar{A} + A] = B$

By B.L,

$F = \bar{A}B + AB$

$= B[\bar{A} + A] = B.$

$F(A,B,C) = \Sigma m(0,2,3,4,5,6)$

A \ BC	($\bar{B}\bar{C}$) 00	($\bar{B}C$) 01	(BC) 11	($B\bar{C}$) 10
($\bar{A}$) 0	1	0	1	1
(A) 1	1	1	0	1

$\rightarrow \bar{A}B$ $\quad \bar{C} \quad A\bar{B}$

$A\bar{B}\bar{C} + A\bar{B}C$
$A\bar{B}[\bar{C} + C]$
$\bar{A}BC + \bar{A}B\bar{C}$
$\bar{A}B\bar{C} + A\bar{B}\bar{C}$

$\boxed{F = \bar{C} + A\bar{B} + \bar{A}B}$

$F = \bar{A}\bar{B}\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C}$

$= AB[\bar{C} + \bar{C}] + \bar{B}\bar{C}[\bar{A} + A] + \bar{A}B\bar{C} + A\bar{B}C$

$= AB + \bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}C$

Rules to simplify K-map:

- At the time of grouping the adjacent cells containing ones always use the maximum possible group.
- All the cells containing ones must be covered atleast once's in the any group.
- At the time of grouping don't care (x) values can be taken as '1'. All don't care values need not be grouped. The only need to include don't cares is the assistant making a bigger group other wise can be ignored.

A

u

② $F = \Sigma m(0, 2, 8, 10, 4, 6)$

AB \ CD	00	01	11	10
00	1 (0)	0 (1)	0 (3)	1 (2)
01	1 (4)	0 (5)	0 (7)	1 (6)
~~1Φ~~	0 (12)	0 (13)	0 (15)	0 (14)
10	1 (8)	0 (9)	0 (11)	1 (10)

$\bar{A}\bar{D}$

$\bar{B}\bar{D}$

$Y = \bar{A}\bar{D} + \bar{B}\bar{D}$

② Reduce $F = \Sigma m(2, 3, 6, 7, 8, 10, 11, 13, 14, 15)$

	AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$	00	0 (0)	0 (1)	1 (3)	1 (2)
$\bar{A}B$	01	0 (4)	0 (5)	1 (7)	1 (6)
AB	11	0 (12)	1 (13)	1 (15)	1 (14)
$A\bar{B}$	10	1 (8)	0 (9)	1 (11)	1 (10)

$Y = A\bar{B}\bar{D} + ABD + C$

C

ABD

$A\bar{B}\bar{D}$

Ā

③ $F = \Sigma m(1, 2, - - - - -, 15)$

	AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$	00	0 (0)	1 (1)	1 (3)	1 (2)
$\bar{A}B$	01	1 (4)	1 (5)	1 (7)	1 (6)
AB	11	1 (12)	1 (13)	1 (15)	1 (14)
$A\bar{B}$	10	1 (8)	1 (9)	1 (11)	1 (10)

B

C

~~AC~~

A D

$F = A + B + C + D$

④ $F(A, B, C) = \Pi M(0, 1, 2, 5, 6, 7)$

BC → / A ↓	00 ($B+C$)	01 ($B+\bar{C}$)	11 ($\bar{B}+\bar{C}$)	10 ($\bar{B}+C$)
A 0	0 (0)	0 (1)	ϕ (3)	0 (2)
$\bar{A}$ 1	ϕ (4)	0 (5)	0 (7)	0 (6)

$\rightarrow \bar{B}+C$

$B+\bar{C}$ $\bar{C}$ $\bar{B}$

BC → / A ↓	00 ($B+C$)	01 ($B+\bar{C}$)	11 ($\bar{B}+\bar{C}$)	10 ($\bar{B}+C$)
A 0	0 (0)	0 (1)	1 (3)	0 (2)
$\bar{A}$ 1	1 (4)	0 (5)	0 (7)	0 (6)

$\rightarrow \bar{B}+C$

$A+B$ $\bar{A}+\bar{C}$

$$F = (A+B)\cdot(\bar{A}+\bar{C})\cdot(\bar{B}+C)$$

5) $F = \pi M(0,1,2,3,6,7,13,15)$

CD → / AB ↓	00 ($C+D$)	01 ($C+\bar{D}$)	11 ($\bar{C}+\bar{D}$)	10 ($\bar{C}+D$)
$A+B$ 00	0 (0)	0 (1)	0 (3)	0 (2)
$A+\bar{B}$ 01	1 (4)	1 (5)	0 (7)	0 (6)
$\bar{A}+\bar{B}$ 11	1 (12)	0 (13)	0 (15)	1 (14)
$\bar{A}+B$ 10	1 (8)	1 (9)	1 (11)	1 (10)

$\rightarrow A+\bar{C}$

$A+B+C$ $\bar{A}+\bar{B}+\bar{D}$

$$F = (A+B+C)\cdot(\bar{A}+\bar{B}+\bar{D})\cdot(A+\bar{C})$$

6) $F(A,B,C,D) = \bar{A}B\bar{D} + BCD + \bar{A}B\bar{C}D + CD.$

$\bar{A}B\bar{D}$: 0 1 0; BCD: 1 1 1; $\bar{A}B\bar{C}D$: 0 1 0 1; CD: 1 1

A	B	C	D	
0	1	1	1	$\Rightarrow m_7$
1	1	1	1	$\Rightarrow m_{15}$

$\bar{A}$ (8)	B (4)	$\bar{C}$ (2)	$\bar{D}$ (1)	
0	1	0	0	$\Rightarrow m_4$
0	1	0	1	$\Rightarrow m_5$

$\Rightarrow m_7 + m_{15} + m_4 + m_5 + m_3 + m_7 + m_{11}$

$\bar{A}$ (8)	B (4)	$\bar{C}$ (2)	D (1)	
0	1	0	1	$\Rightarrow m_5$

A (8)	B (4)	C (2)	D (1)	
0	0	1	1	$\rightarrow m_3$
0	1	1	1	$\rightarrow m_7$
1	0	1	1	$\rightarrow m_{11}$
1	1	1	1	$\rightarrow m_{15}$

$\Rightarrow m_3 + m_4 + m_5 + m_7 + m_{11} + m_{15}$

$= \Sigma m(3,4,5,6,7,11,15)$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0 (0)	0 (1)	1 (3)	0 (2)
$\bar{A}B$ 01	1 (4)	1 (5)	1 (7)	1 (6)
AB 11	0 (12)	0 (13)	1 (15)	0 (14)
$A\bar{B}$ 10	0 (8)	0 (9)	1 (11)	0 (10)

→ $\bar{A}\bar{B}D$

→ $\bar{A}BD$

$\bar{A}B\bar{C}$ CD

$Y = \bar{A}B + CD$

⑦ $F = \Sigma m(0,2,6,8) + d(1,3,4,10)$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1 (0)	1 (1)	1 (3)	1 (2)
$\bar{A}B$ 01	1 (4)	(5)	(7)	1 (6)
AB 11	(12)	(13)	(15)	(14)
$A\bar{B}$ 10	1 (8)	(9)	(11)	1 (10)

$\bar{A}\bar{B}$ $\bar{A}$ $\bar{A}$ $\bar{D}$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1 (0)	X (1)	X (3)	1 (2)
$\bar{A}B$ 01	X (4)	(5)	(7)	1 (6)
AB 11	(12)	(13)	(15)	(14)
$A\bar{B}$ 10	1 (8)	(9)	(11)	X (10)

$\bar{A}\bar{D}$

$\bar{B}\bar{D}$ $Y = \bar{A}\bar{D} + \bar{B}\bar{D}$

⑧ $F = \Sigma m(1,3,6,9,11,14) + d(4,12,10)$

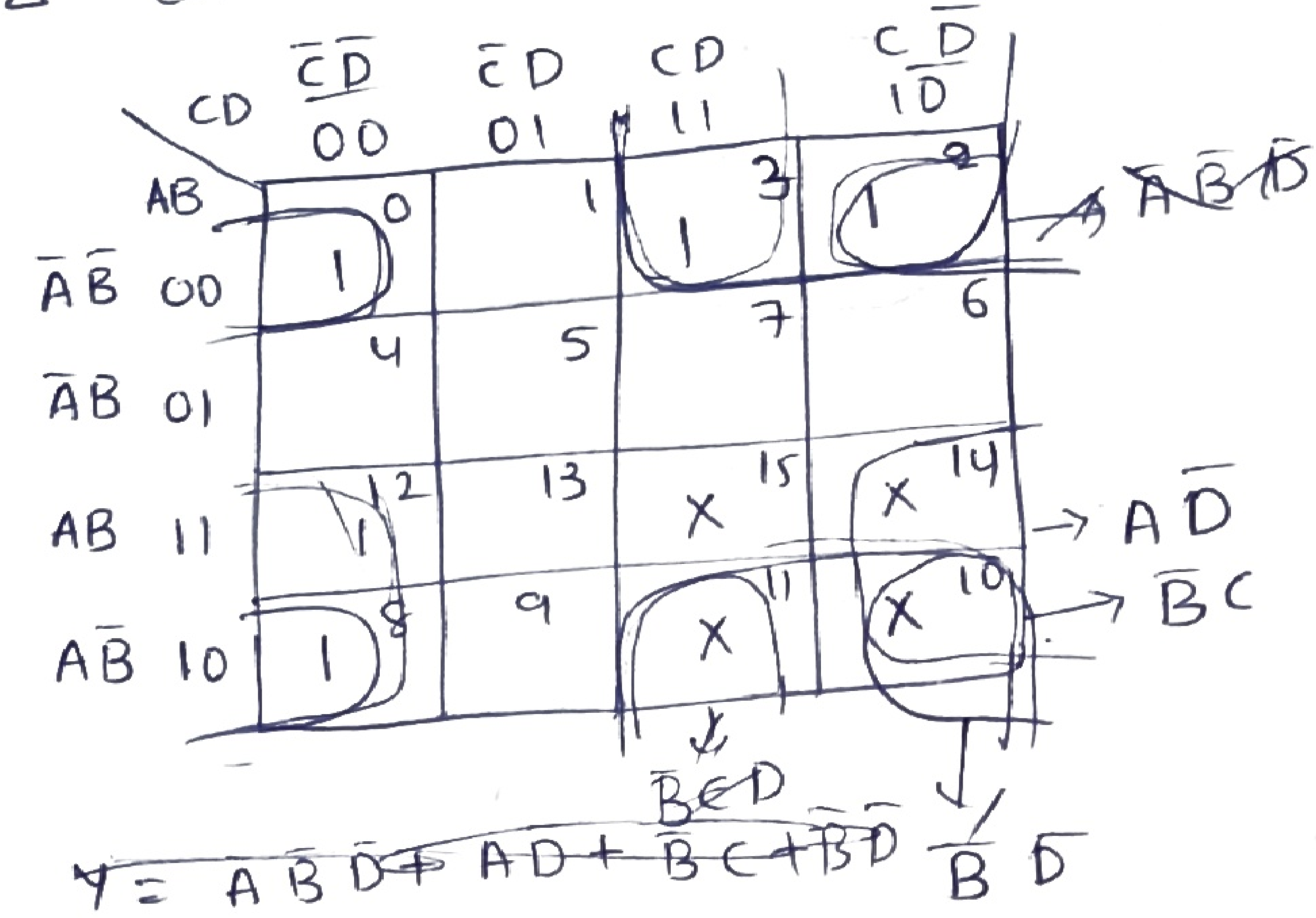
AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	(0)	1 (1)	1 (3)	(2)
$\bar{A}B$ 01	X (4)	(5)	(7)	1 (6)
AB 11	X (12)	(13)	(15)	1 (14)
$A\bar{B}$ 10	(8)	1 (9)	1 (11)	X (10)

$Y = \bar{B}D + B\bar{D}$

→ $B\bar{D}$

$\bar{B}D$

④ $F = \Sigma m(0,2,3,8,12) + d(10,11,14,15)$



$Y = A\bar{B}\bar{D} \cancel{+ A\bar{D} + \bar{B}C + \bar{B}\bar{D}}$ $\bar{B}\bar{D}$

$F = A\bar{D} + \bar{B}C + \bar{B}\bar{D}$

QM Method:

Step-1:- Tabulate and arrange the min-terms and don't cares and group them according to the number of one's.

Step-2:- compare the nth group and (n+1)th group and obtain the matched pair minterms. matched pair is the pair of minterms that differ only by 1-bit.

Step-3:- Compare the nth group with (n+1)th group and again check for the matched pair and continue this till no grouping is possible.

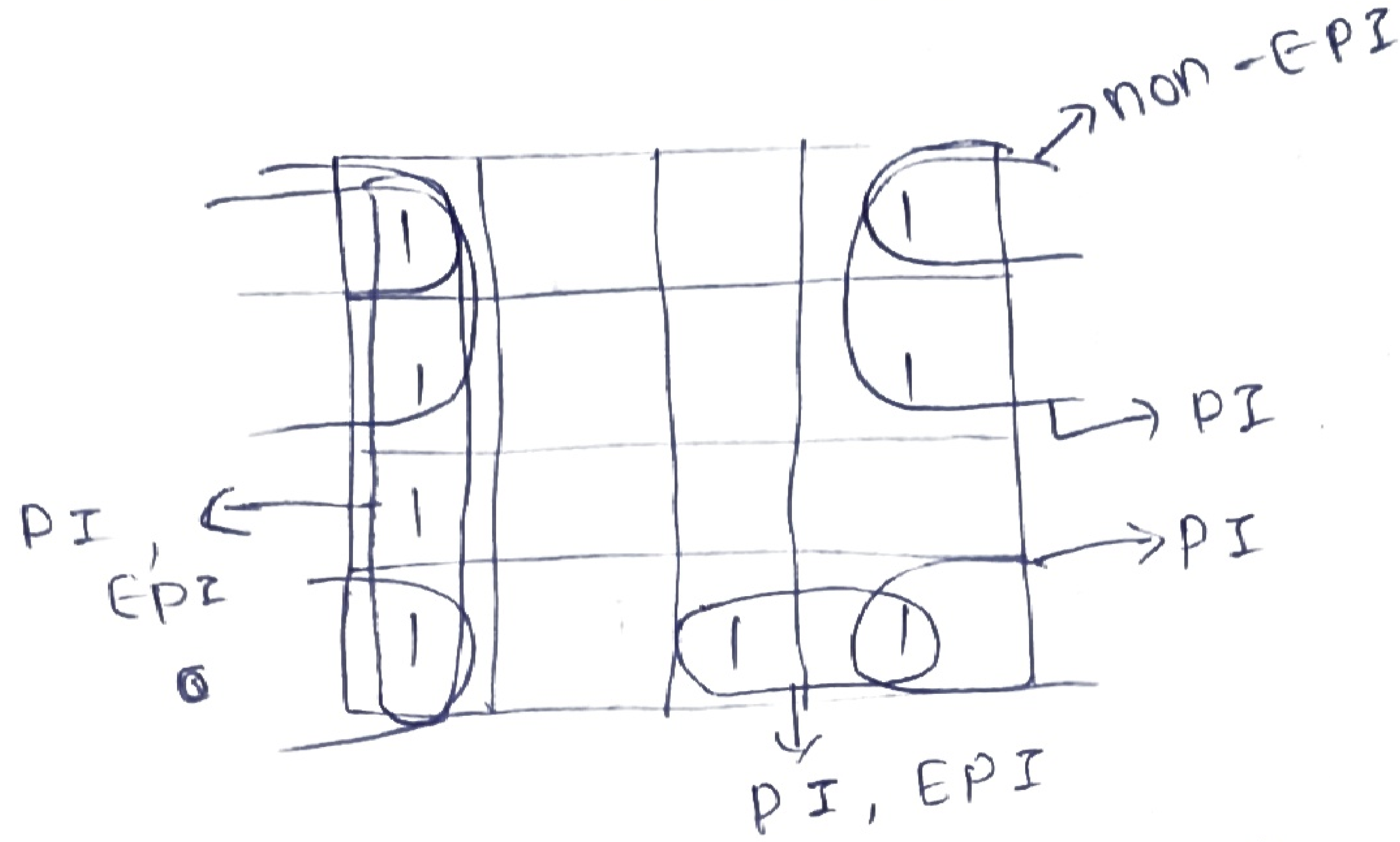
Step-4:- Generate the prime implicant (PI) table and obtain the essential prime implicants (EPI) and hence the simplified expression.

* Implicant → is the set of the all adjacent minterms. octat, quad, pair.

Ex:- Group of 16 min terms,

Prime implicants is the implicant which is not a subset of another implicant.

* Essential prime implicant (EPI) is PI which contains atleast one min-term that is not covered by other PI's.



1) Simplify $F = \Sigma m(0,1,3,7,8,9,11,15)$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1 (0)	1 (1)	1 (3)	(2)
$\bar{A}B$ 01	(4)	(5)	1 (7)	(6)
AB 11	(12)	(13)	1 (15)	(14)
$A\bar{B}$ 10	1 (8)	1 (9)	1 (11)	(10)

$\rightarrow CD$

$\rightarrow \bar{B}\bar{C}$

$F = \bar{B}\bar{C} + CD$

(I)

0 - 0000
1 - 0001
3 - 0011
7 - 0111
8 - 1000
9 - 1001
11 - 1011
15 - 1111

Group	Minterms	Bin ABCD
0	m_0	0000
1	m_1 m_8	0001 1000
2	m_3 m_9	0011 1001
3	m_7 m_{11}	0111 1011
4	m_{15}	1111

(II)

Grp	Matched pair minterms	Bin ABCD
0	$m_0 - m_1$ $m_0 - m_8$	000- -000
1	$m_1 - m_3$ $m_1 - m_9$ $m_8 - m_9$	00-1 -001 100-

	m3 - m7	0 - 11
	m3 - m11	- 011
	m9 - m11	10 - 1
3	m7 - m15	- 111
	m11 - m15	1 - 11

III.

Group	Matched Pair	Bin ABCD
0	m0 - m1 - m8 - m9	- 00 - ⇒ $\bar{B}\bar{C}$
	m0 - m8 - m1 - m9	- 00 - ⇒ $\bar{B}\bar{C}$
1	m1 - m3 - m9 - m11	- 0 - 1 ⇒ $\bar{B}D$
	m1 - m9 - m3 - m11	- 0 - 1 ⇒ $\bar{B}D$
2	m3 - m7 - m11 - m15	- - 11 ⇒ CD
	m3 - m11 - m7 - m15	- - 11 ⇒ CD

~~m9~~ - m

IV.

PI	0	1	3	7	8	9	11	15
EPI ← $\bar{B}\bar{C}$	X	X			X	X		
$\bar{B}D$		X	X			X	X	
EPI ← CD			X	X			X	X

$F = \bar{B}\bar{C} + CD$

2) Simplify by QM-Method $F = \Sigma m(0,1,2,5,6,7,8,9,10,14)$.

Group	Minterms	Bin ABCD
0 - 0000	7 - 0111	14 - 1110
1 - 0001	8 - 1000	6 - 0110
2 - 0010	9 - 1001	
5 - 0101	10 - 1010	

Group	Minterms	Bin ABCD
0	m_0	0000
1	m_1	0001
	m_2	0010
	m_8	1000
2	m_5	0101
	m_9	1001
	m_{10}	1010
	m_6	0110
3	m_7	0111
	m_{14}	1110

II.

Group	Matched pair minterms	Bin ABCD
0	$m_0 - m_1$	000–
	$m_0 - m_2$	00–0
	$m_0 - m_8$	–000
1	$m_1 - m_5$	0–01
	$m_1 - m_9$	–001
	$m_2 - m_6$	0–10
	$m_2 - m_{10}$	–010
	$m_8 - m_9$	100–
	$m_8 - m_{10}$	10–0
2	$m_5 - m_7$	01–1 $\Rightarrow \bar{A}B D$ (not involved in matching pair)
	$m_{10} - m_{14}$	1–10
	$m_6 - m_7$	011–
	$m_6 - m_{14}$	–110

III.

Group	Matched Pair	Bin ABCD
0	$m_0 - m_1 - m_8 - m_9$	–00– $\Rightarrow \bar{B}\bar{C}$
	$m_0 - m_2 - m_8 - m_{10}$	–0–0 $\Rightarrow \bar{B}\bar{D}$
	$m_0 - m_8 - m_1 - m_9$	–00– $\Rightarrow \bar{B}\bar{C}$
	$m_0 - m_8 - m_2 - m_{10}$	–0–0 $\Rightarrow \bar{B}\bar{D}$

1

$m_2 - m_6 - m_{10} - m_{14}$ $\quad - - 1 0 \Rightarrow C\bar{D}$

$m_2 - m_{10} - m_6 - m_{14}$ $\quad - - 1 0 \Rightarrow C\bar{D}$

v.

PI	0	1	2	5	6	7	8	9	10	14
							X	X	X	
$\bar{B}\bar{C}$	X	X	X							
									X	X → EPI
$\bar{B}\bar{D}$			X		X					
$\bar{B}\bar{D}$										

IV.

PI	0	1	2	5	6	7	8	9	10	14
EPI – $\bar{B}\bar{C}$	X	X					X	X		
$\bar{B}\bar{D}$	X		X				X		X	
EPI – $C\bar{D}$			X		X				X	X

$F = \bar{B}\bar{C} + C\bar{D}$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1 (0)	1 (1)	(3)	1 (2)
$\bar{A}B$ 01	(4)	1 (5)	1 (7)	1 (6)
AB 11	(12)	(13)	(15)	1 (14)
$A\bar{B}$ 10	1 (8)	1 (9)	(11)	1 (10)

$\to C\bar{D}$

$F = \bar{B}\bar{C} + C\bar{D}$

$\bar{B}\bar{C}$ $\quad \bar{A}BD$