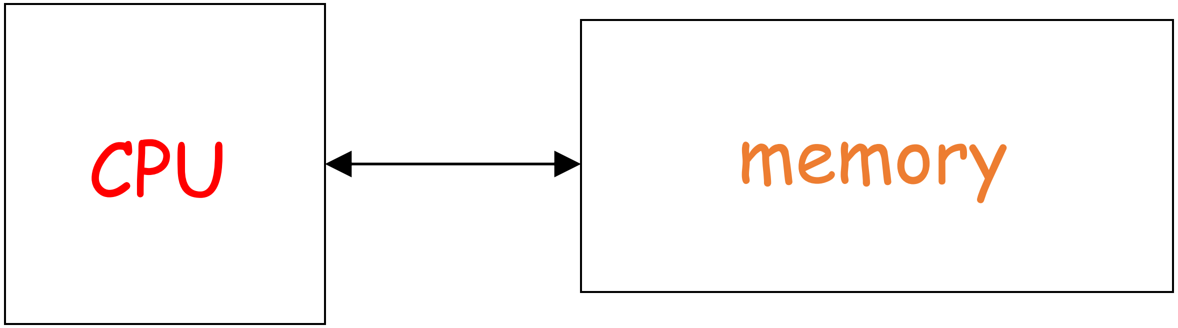
Module-1

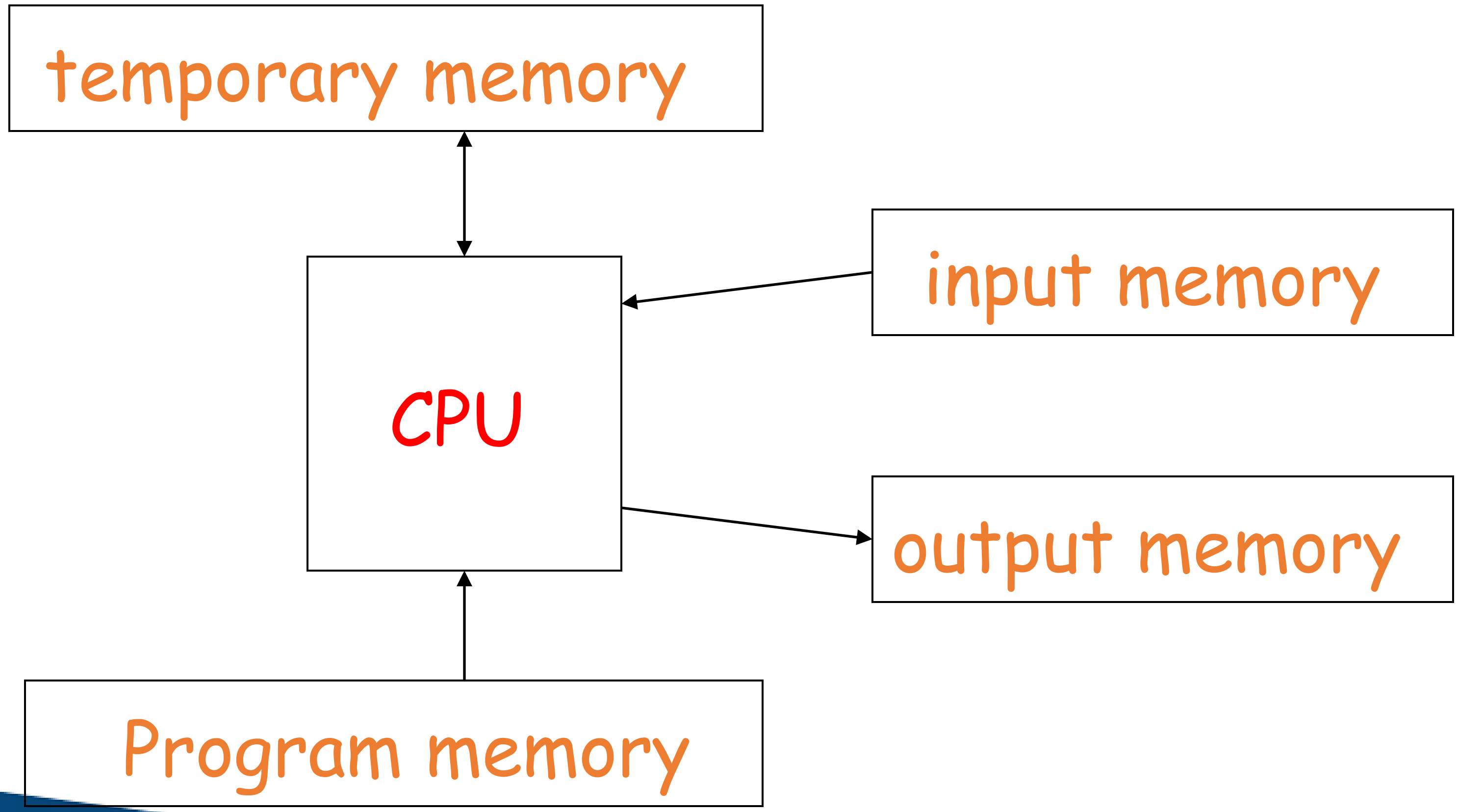
Introduction to Finite Automata

Private University Estd. in Karnataka State by Act No. 41 of 2013

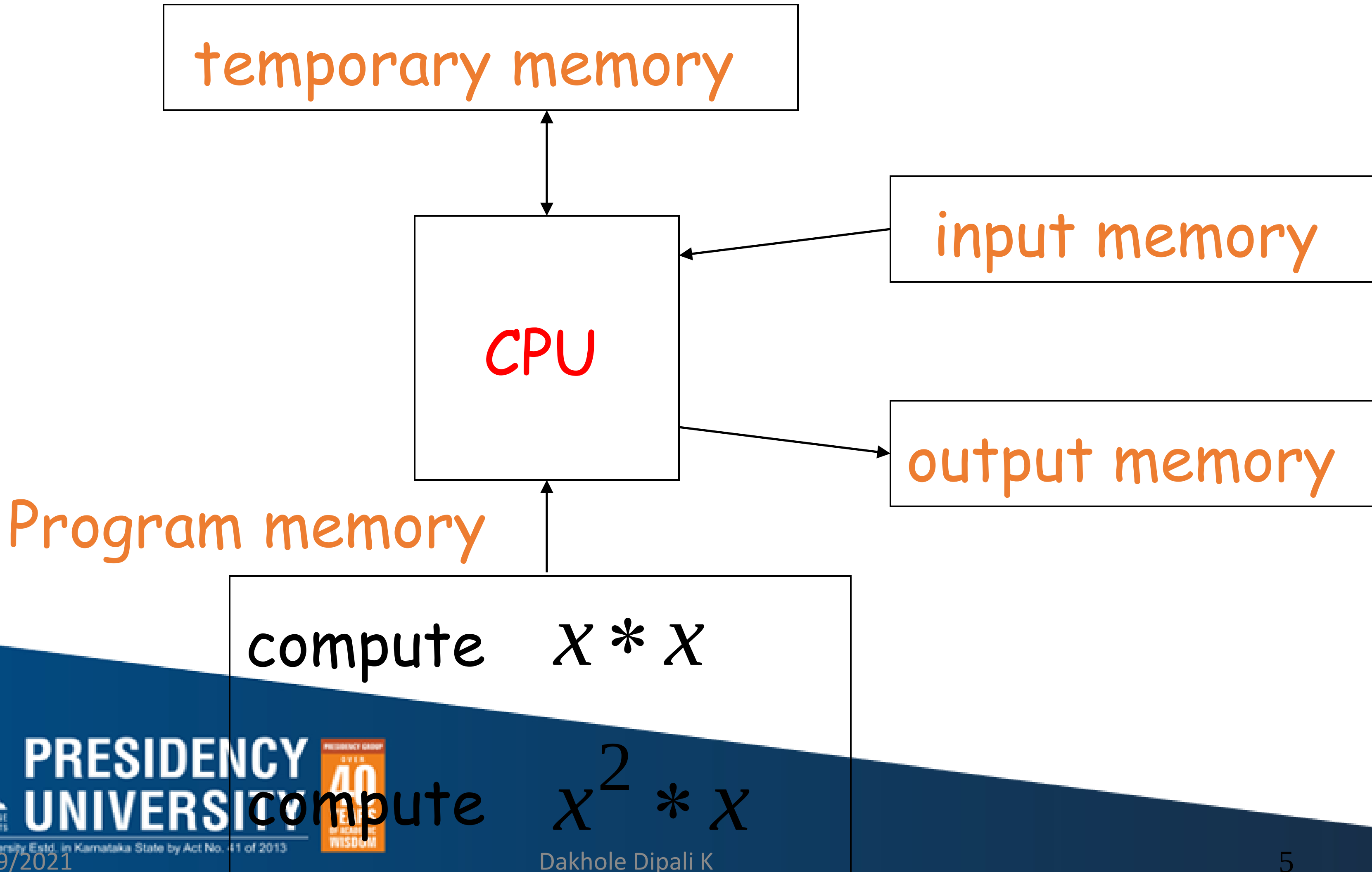
Models of Computation

Computation

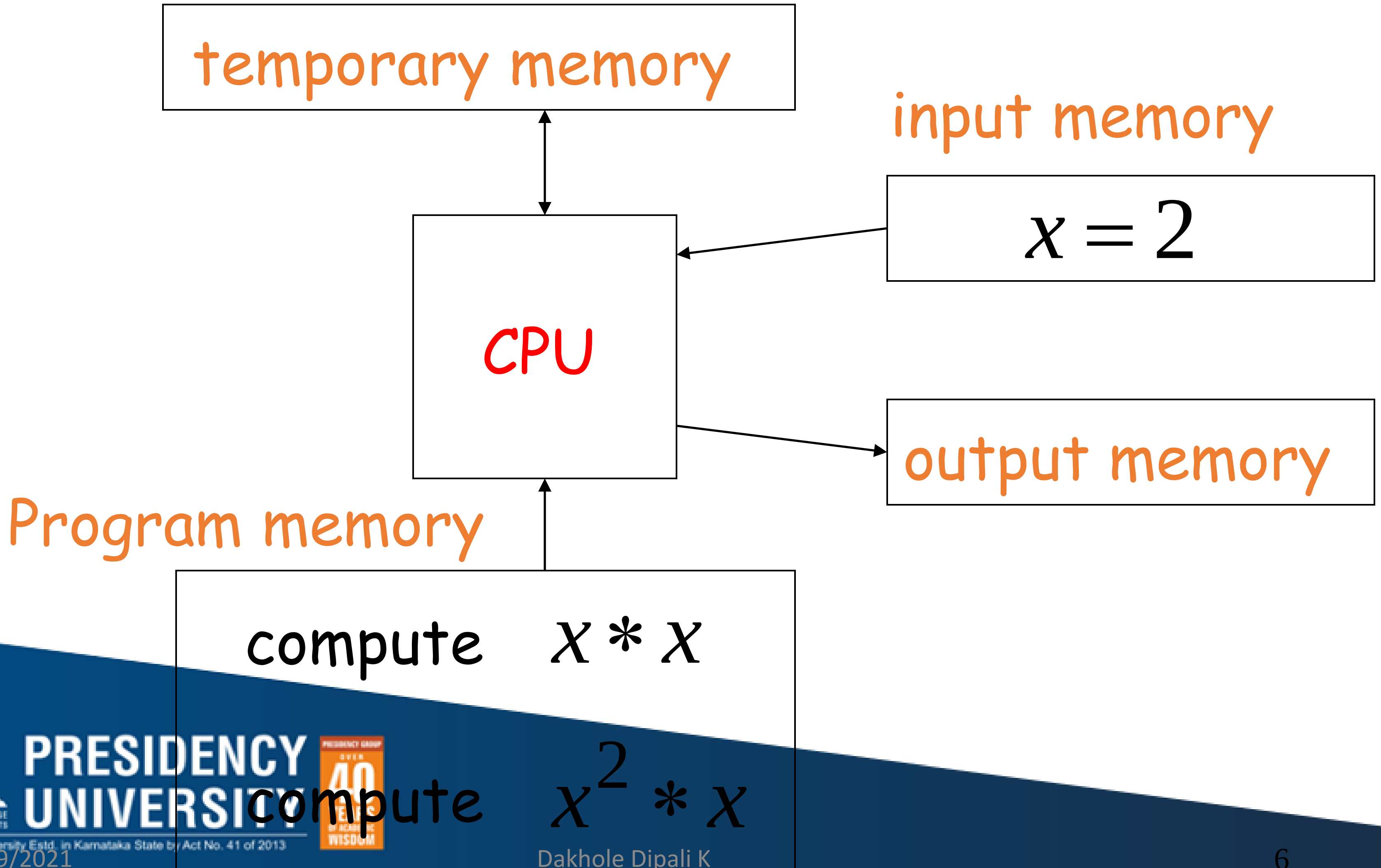



temporary memory
input memory
CPU
output memory
Program memory

Example: $f(x) = x^3$

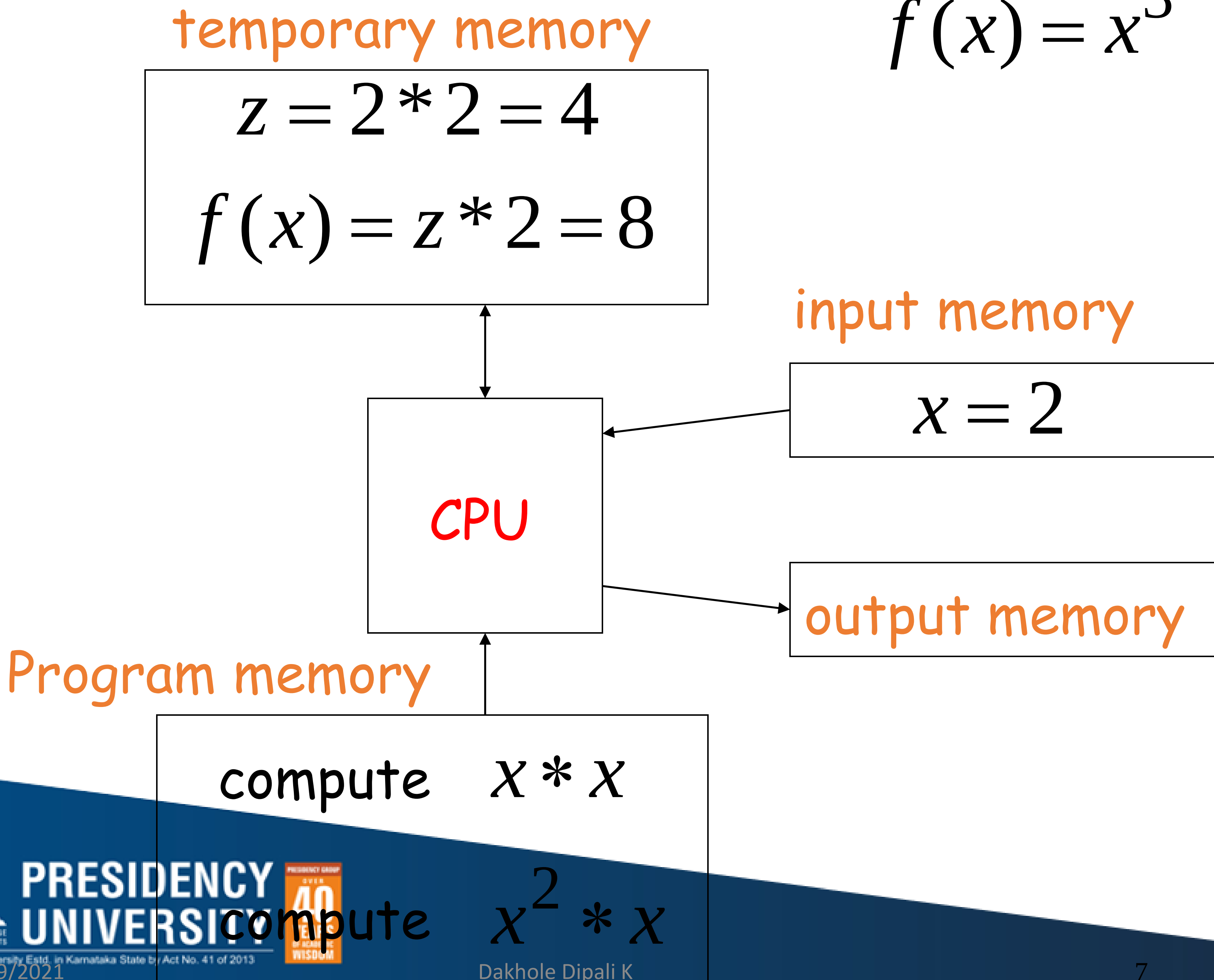


$$f(x) = x^3$$



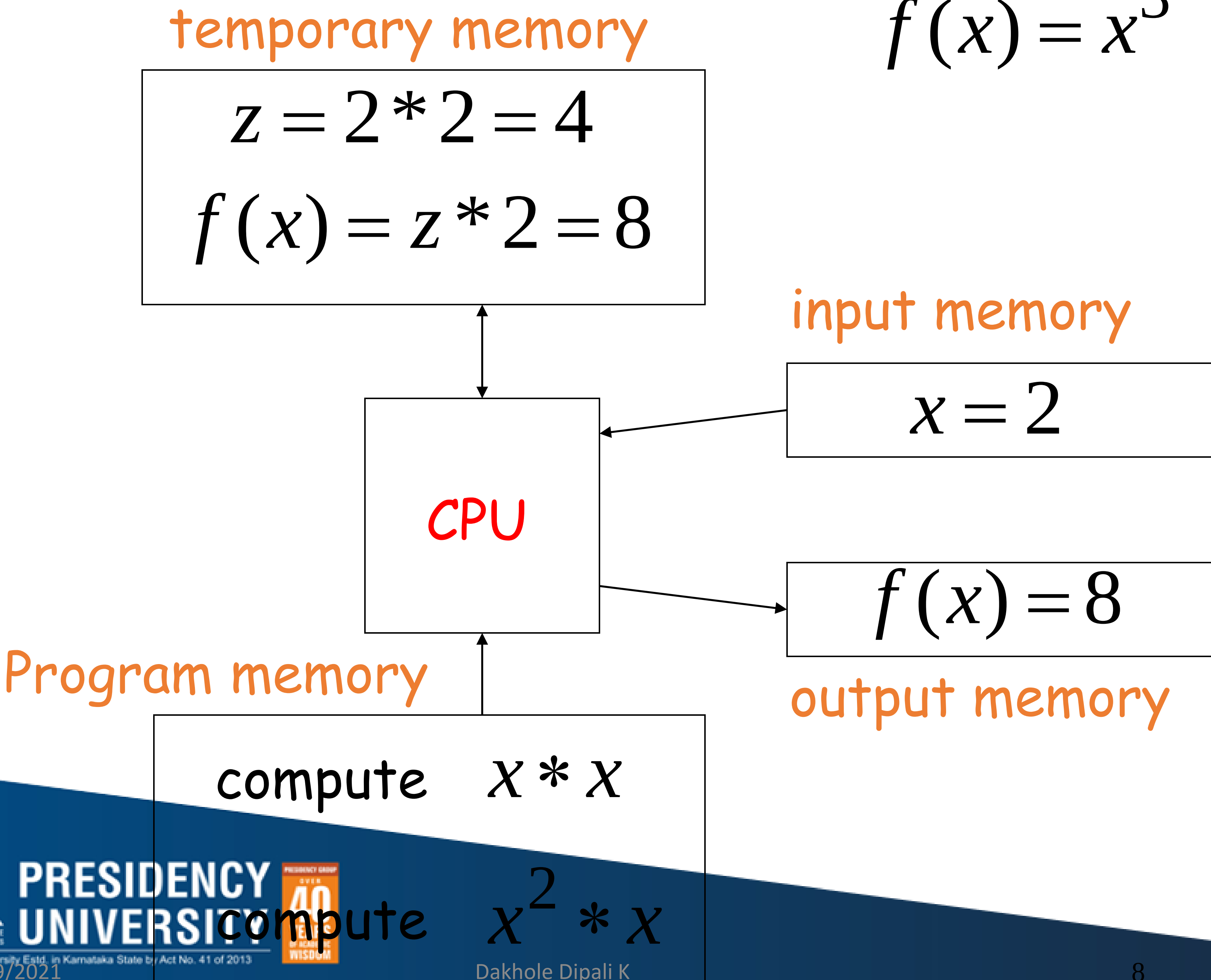
temporary memory
input memory
$x = 2$
CPU
output memory
Program memory
compute $x * x$
compute $x^2 * x$

$$f(x) = x^3$$



temporary memory
z = 2 * 2 = 4
f(x) = z * 2 = 8
input memory
x = 2
CPU
output memory
Program memory
compute x * x
compute x^2 * x

$$f(x) = x^3$$

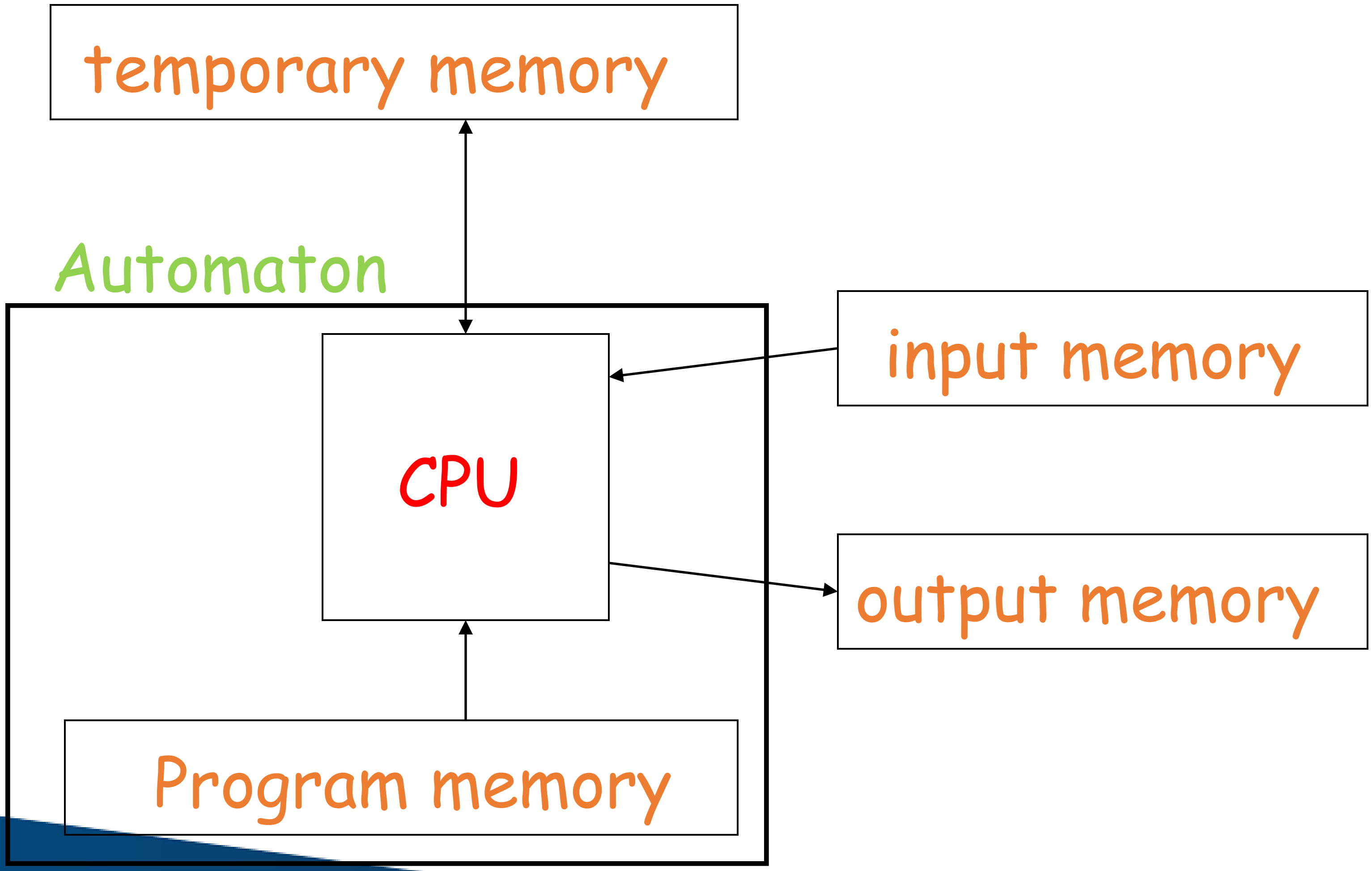

temporary memory
$z = 2*2 = 4$
$f(x) = z*2 = 8$
input memory
$x = 2$
CPU
$f(x) = 8$
output memory
Program memory
compute $x*x$
compute $x^2 * x$

Automata theory is the study of abstract machines and automata.

The computational problems can be solved using them.

It is a theory in theoretical computer science and discrete mathematics

Automaton



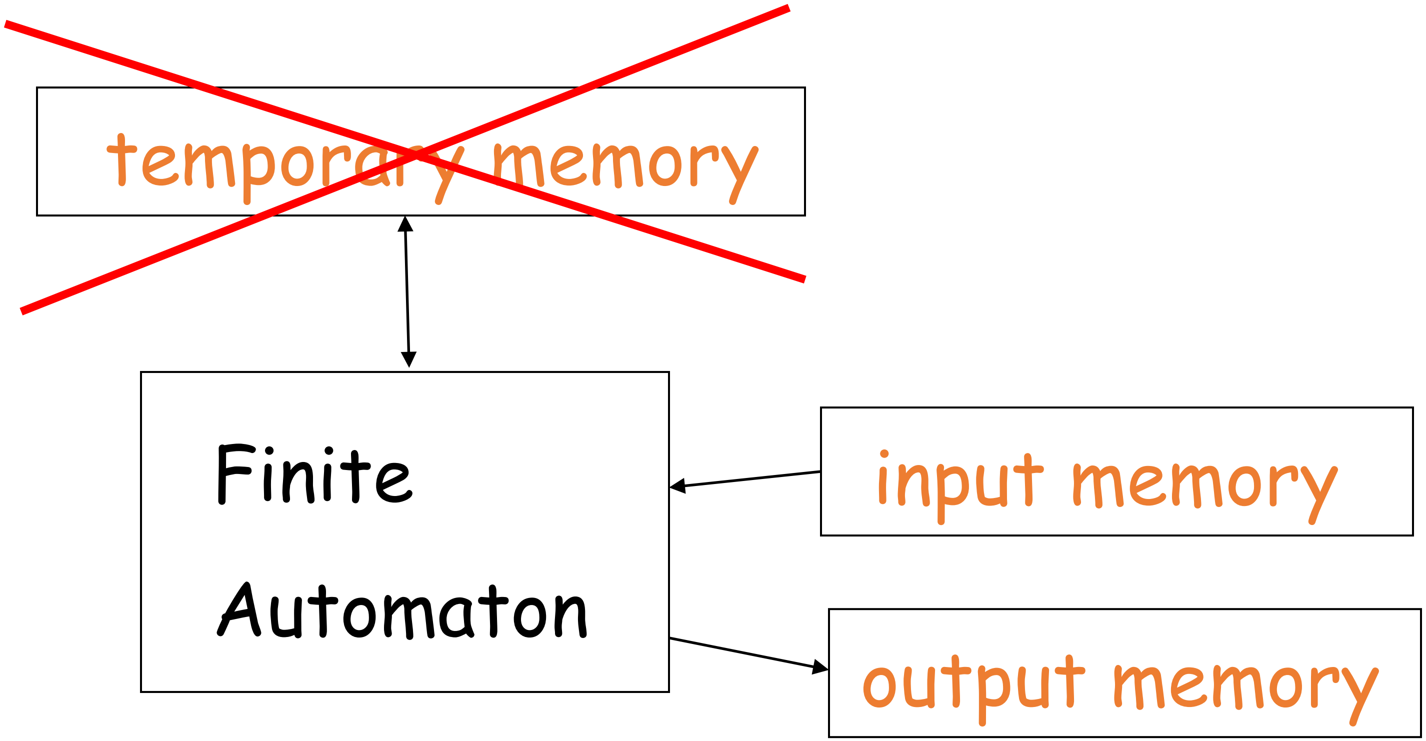
temporary memory
Automaton
CPU
input memory
output memory
Program memory

Different Kinds of Automata

Automata are distinguished by the temporary memory

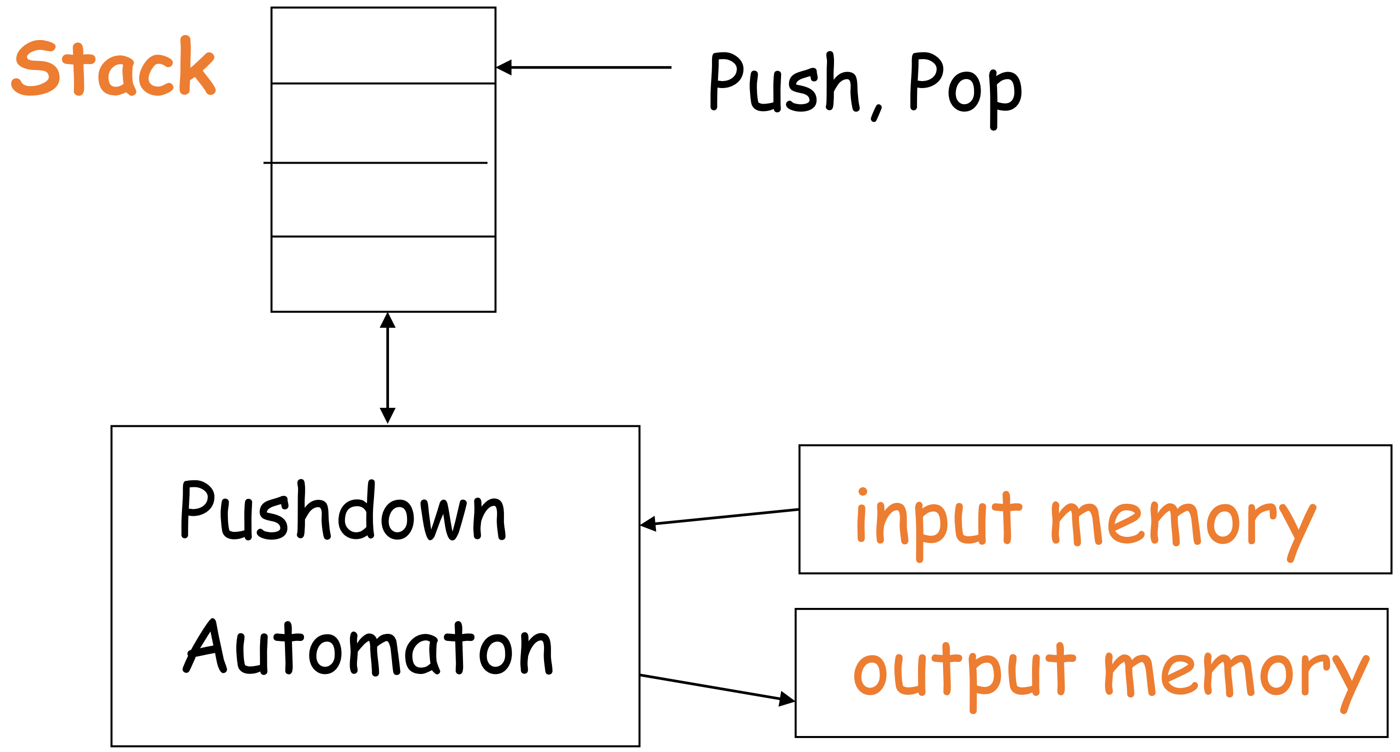
- **Finite Automata:** no temporary memory
- **Pushdown Automata:** stack
- **Turing Machines:** random access memory

Finite Automaton



Example: Vending Machines (small computing power)

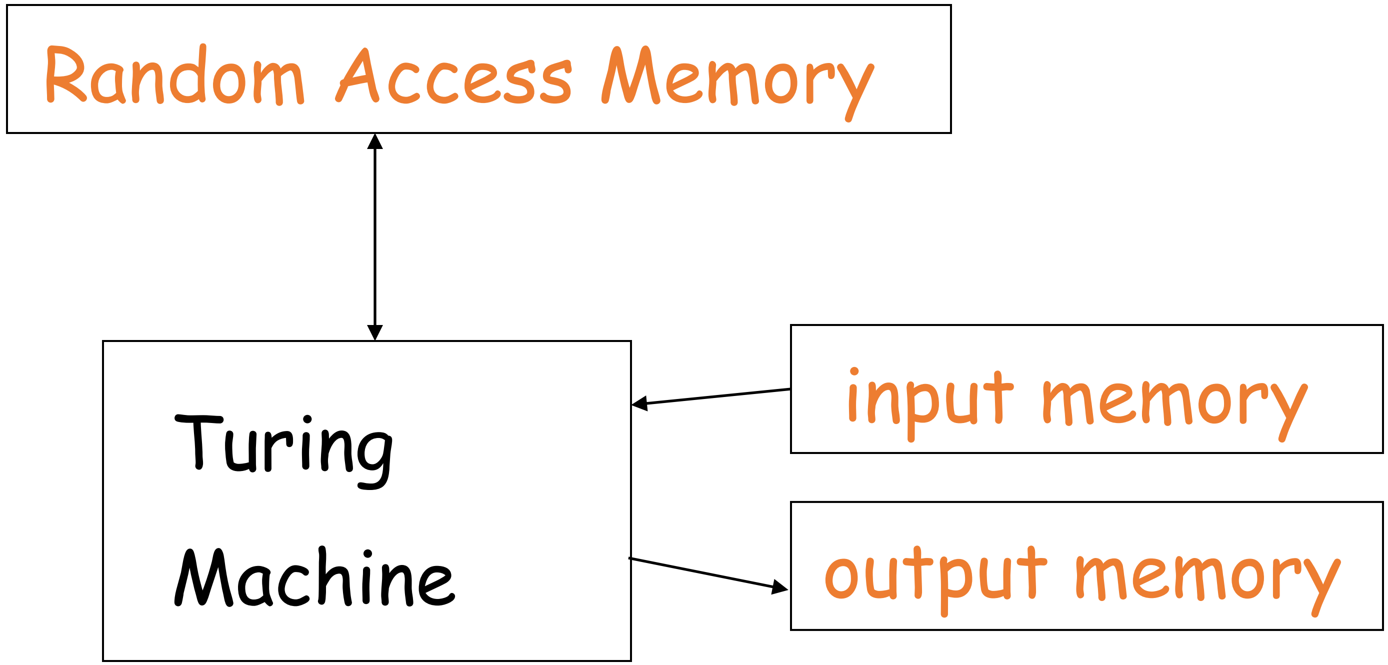
Pushdown Automaton



Example: Compilers for Programming Languages

(medium computing power)

Turing Machine



Examples: Any Algorithm (highest computing power)

Power of Automata

Finite Automata $<$ Pushdown Automata $<$ Turing Machine

Less power $\longrightarrow$ More power

Solve more computational problems

Applications of finite automata

- For the designing of lexical analysis of a compiler which breaks the input text into logical units like identifiers, keywords etc.
- For recognizing the pattern using regular expressions.
- For the designing of the combination and sequential circuits.
- Software for designing and checking the behavior of digital circuits.
- Used in text editors.
- For the implementation of spell checkers.
- Software for scanning large bodies of text like web pages to find occurrence of words, phrases and other patterns.
- Software to verify all types that have finite number of distinct states such as communications protocols for secure exchange of information.

Applications of Push Down Automata

- For designing the parsing phase of a compiler (Syntax Analysis).
- For implementation of stack applications.
- For evaluating the arithmetic expressions.
- For solving the Tower of Hanoi Problem.

Applications of Turing machine

- For solving any recursively enumerable problem.
- For understanding complexity theory.
- For implementation of neural networks.
- For implementation of Robotics Applications.
- For implementation of artificial intelligence.

General Concepts of Automata Theory

- Symbol
- Alphabet
- Strings
- Empty Strings
- Length of the string
- Power of an Alphabet
- Concatenation of two strings
- Languages

Alphabets and Strings

- Symbol: a, b, 1 ,2, 3, 0, etc.

- We will use small alphabets: $\Sigma = \{a, b\}$

- Strings

$$a$$
$$ab$$
$$abba$$
$$baba$$
$$aaabbbaabab$$

$$u = ab$$
$$v = bbbaaa$$
$$w = abba$$

String Operations

$$w = a_1 a_2 \cdots a_n \qquad abba$$

$$v = b_1 b_2 \cdots b_m \qquad bbbaaa$$

1. Concatenation

$$wv = a_1 a_2 \cdots a_n b_1 b_2 \cdots b_m \qquad abbabbbaaa$$

2. Reverse

$$w = a_1 a_2 \cdots a_n$$

$$abab aaabbb$$

$$w^R = a_n \cdots a_2 a_1$$

$$bbbaaababa$$

3. String Length

$$w = a_1 a_2 \cdots a_n$$

Length: $|w| = n$

Examples:

$$|abba| = 4$$

$$|aa| = 2$$

$$|a| = 1$$

4. Length of Concatenation

$$|uv| = |u| + |v|$$

- Example:

$$u = aab, \quad |u| = 3$$
$$v = abaab, \quad |v| = 5$$

$$|uv| = |aababaab| = 8$$
$$|uv| = |u| + |v| = 3 + 5 = 8$$

5. Empty String

- A string with no letters: λ or ϵ

- Observations: $|\lambda| = 0$

$$\lambda w = w\lambda = w$$

$$\lambda abba = abba\lambda = abba$$

6. Substring

- Substring of string:
  - a subsequence of consecutive characters

String	Substring
$abbab$	ab
$\underline{ab}bab$	$abba$
$\underline{abba}b$	b
$ab\underline{b}ab$	$bbab$

7. Prefix and Suffix

$$abbab$$

Prefixes	Suffixes
λ	$abbab$
a	$bbab$
ab	bab
abb	ab
$abba$	b
$abbab$	λ

$$w = uv$$

prefix

suffix

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8. Another Operation

$$w^n = \underbrace{ww \cdots w}_{n}$$

- Example: $(abba)^2 = abbaabba$

- Definition: $w^0 = \lambda$

$$(abba)^0 = \lambda$$

9. The * Operation

Σ^* the set of all possible strings from Σ alphabet

$$\Sigma = \{a, b\}$$

$$\Sigma^* = \{\lambda, a, b, aa, ab, ba, bb, aaa, aab, \ldots\}$$

10. The + Operation

Σ^+ : the set of all possible strings from alphabet Σ except λ

$$\Sigma = \{a, b\}$$

$$\Sigma^* = \{\lambda, a, b, aa, ab, ba, bb, aaa, aab, \ldots\}$$

$$\boxed{\Sigma^+ = \Sigma^* - \lambda}$$

$$\Sigma^+ = \{a, b, aa, ab, ba, bb, aaa, aab, \ldots\}$$

Languages

- A language is a set of strings

- String: A sequence of letters
  - Examples: "cat", "dog", "house", …
  - Defined over an alphabet:

$$\Sigma = \{a, b, c, \ldots, z\}$$

- A language is any subset of Σ^*

- Example:

$$\Sigma = \{a, b\}$$

$$\Sigma^* = \{\lambda, a, b, aa, ab, ba, bb, aaa, \ldots\}$$

- Languages:

$$\{\lambda\}$$

$$\{a, aa, aab\}$$

$$\{\lambda, abba, baba, aa, ab, aaaaaa\}$$

Note that

Sets $\varnothing = \{\ \} \neq \{\lambda\}$

Set size $|\{\ \}| = |\varnothing| = 0$

Set size $|\{\lambda\}| = 1$

String length $|\lambda| = 0$

Another Example

An infinite language

$$L = \{a^n b^n : n \geq 0\}$$

$$\left.\begin{array}{l} \lambda \\ ab \\ aabb \\ aaaaabbbbb \end{array}\right\} \in L \qquad abb \notin L$$

Operations on Languages

1. The usual set operations

$$\{a, ab, aaaa\} \cup \{bb, ab\} = \{a, ab, bb, aaaa\}$$

$$\{a, ab, aaaa\} \cap \{bb, ab\} = \{ab\}$$

$$\{a, ab, aaaa\} - \{bb, ab\} = \{a, aaaa\}$$

Complement: $\overline{L} = \Sigma^* - L$

$$\overline{\{a, ba\}} = \{\lambda, b, aa, ab, bb, aaa, \ldots\}$$

2. Reverse

Definition:
$$L^R = \{w^R : w \in L\}$$

Examples:
$$\{ab, aab, baba\}^R = \{ba, baa, abab\}$$

$$L = \{a^n b^n : n \geq 0\}$$

$$L^R = \{b^n a^n : n \geq 0\}$$

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3.Concatenation

- Definition: $L_1L_2 = \{xy : x \in L_1, y \in L_2\}$

- Example: $\{a, ab, ba\}\{b, aa\}$

$$= \{ab, aaa, abb, abaa, bab, baaa\}$$

4. Another Operation

- Definition: $$L^n = \underbrace{LL \cdots L}_{n}$$

$$\{a,b\}^3 = \{a,b\}\{a,b\}\{a,b\} =$$
$$\{aaa, aab, aba, abb, baa, bab, bba, bbb\}$$

- Special case: $$L^0 = \{\lambda\}$$

$$\{a, bba, aaa\}^0 = \{\lambda\}$$

More Examples

$$L = \{a^n b^n : n \geq 0\}$$

$$L^2 = \{a^n b^n a^m b^m : n, m \geq 0\}$$

$$aabbaaabbb \in L^2$$

5. Star-Closure (Kleene *)

Definition: $L^* = L^0 \cup L^1 \cup L^2 \cdots$

- Example:

$$\{a, bb\}^* = \left\{ \begin{array}{l} \lambda, \\ a, bb, \\ aa, abb, bba, bbbb, \\ aaa, aabb, abba, abbbb, \ldots \end{array} \right\}$$

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6. Positive Closure

Definition: $$L^+ = L^1 \cup L^2 \cup \cdots$$
$$= L^* - \{\lambda\}$$

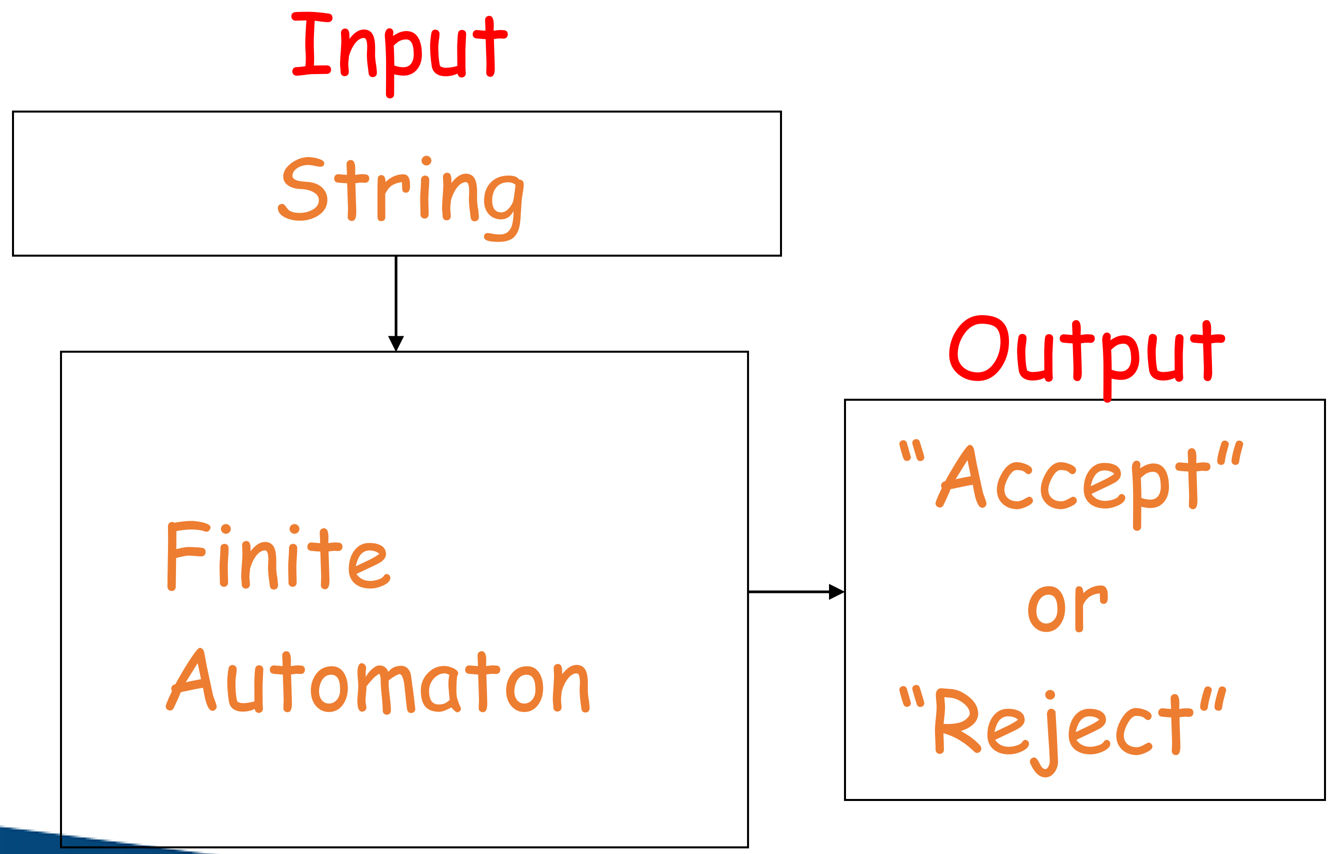
$$\{a, bb\}^+ = \left\{ \begin{array}{l} a, bb, \\ aa, abb, bba, bbbb, \\ aaa, aabb, abba, abbbb, \ldots \end{array} \right\}$$

Language Recognizers : An example of Finite Automata

An automaton is an abstract model of a digital computer.

Finite Automata(FA) is the simplest machine to recognize patterns.

Finite Automaton

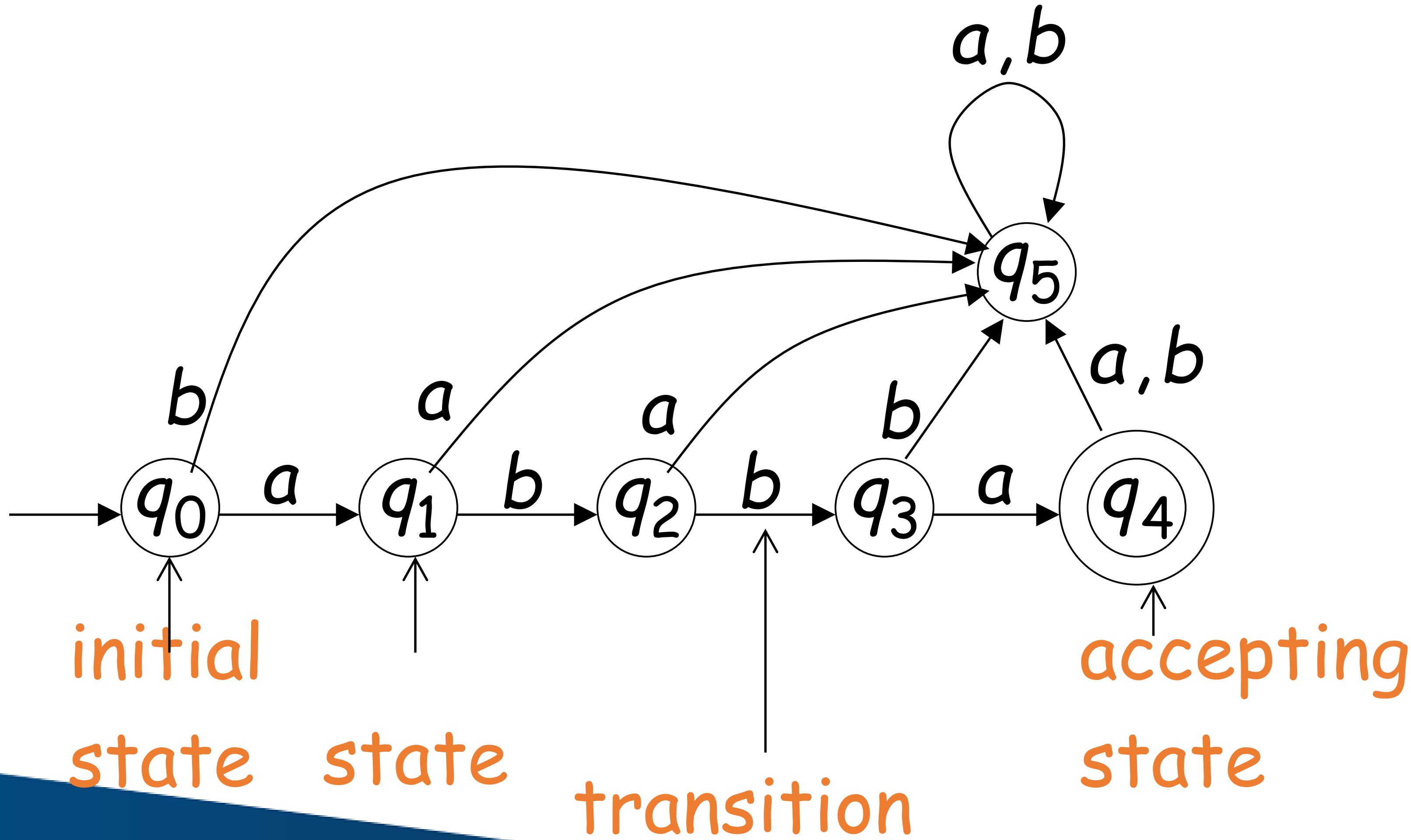


Representation of Finite Automata

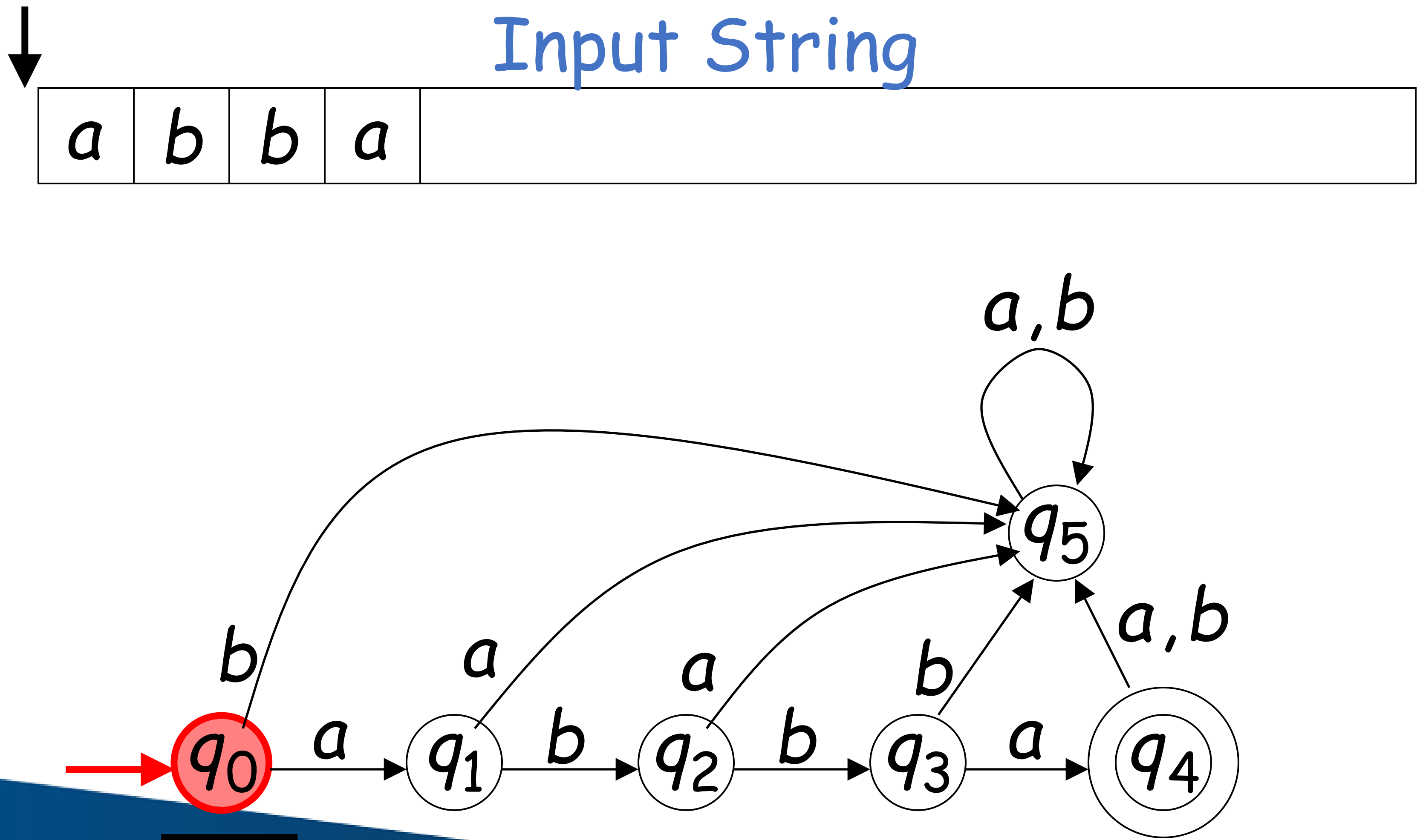
Finite Automata is represented by –

1. Transition Graph
2. Transition Table
3. Regular Expression

Transition Graph

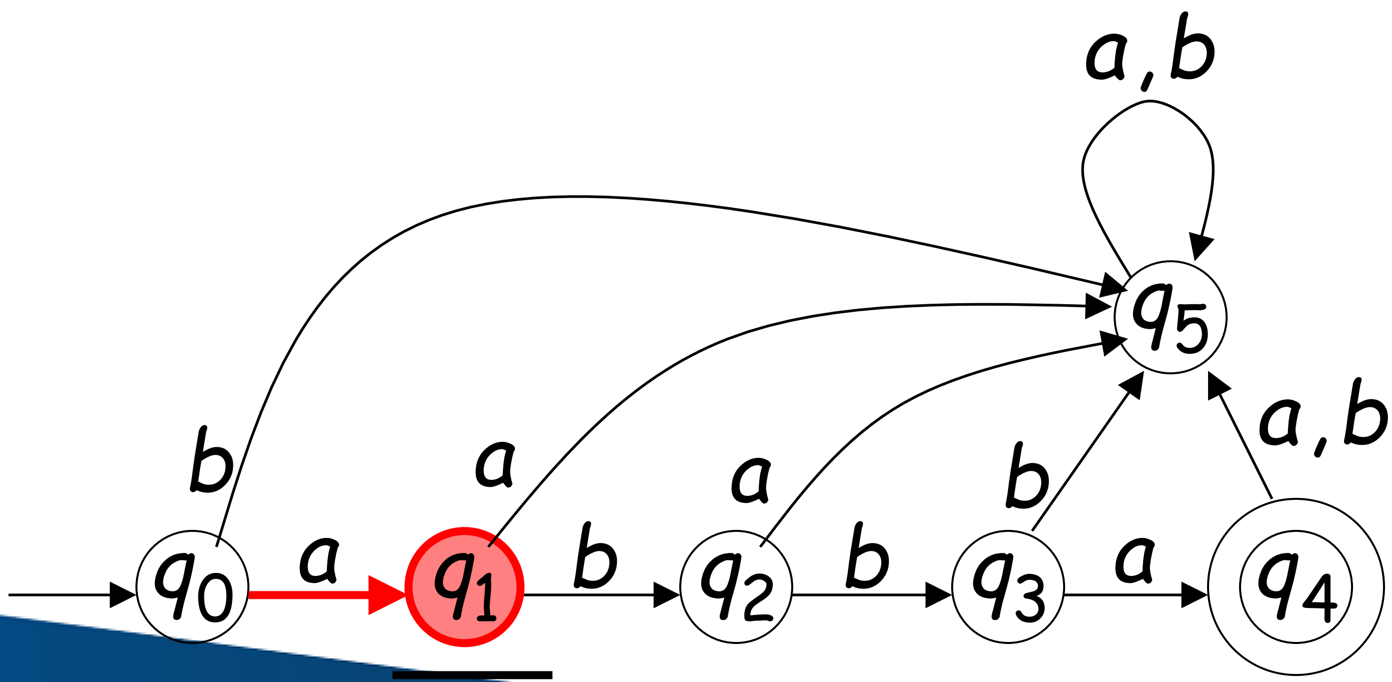


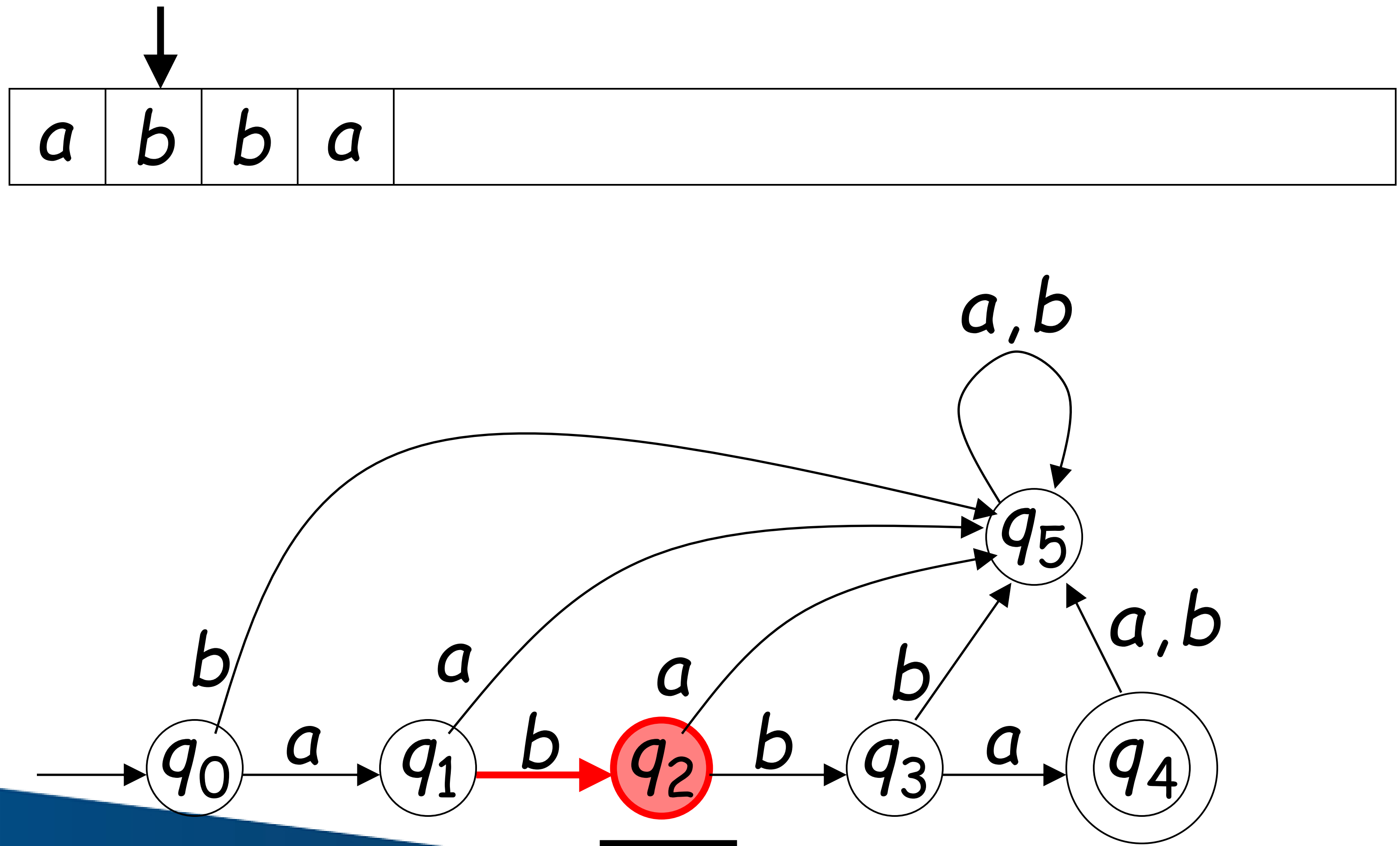
Initial Configuration

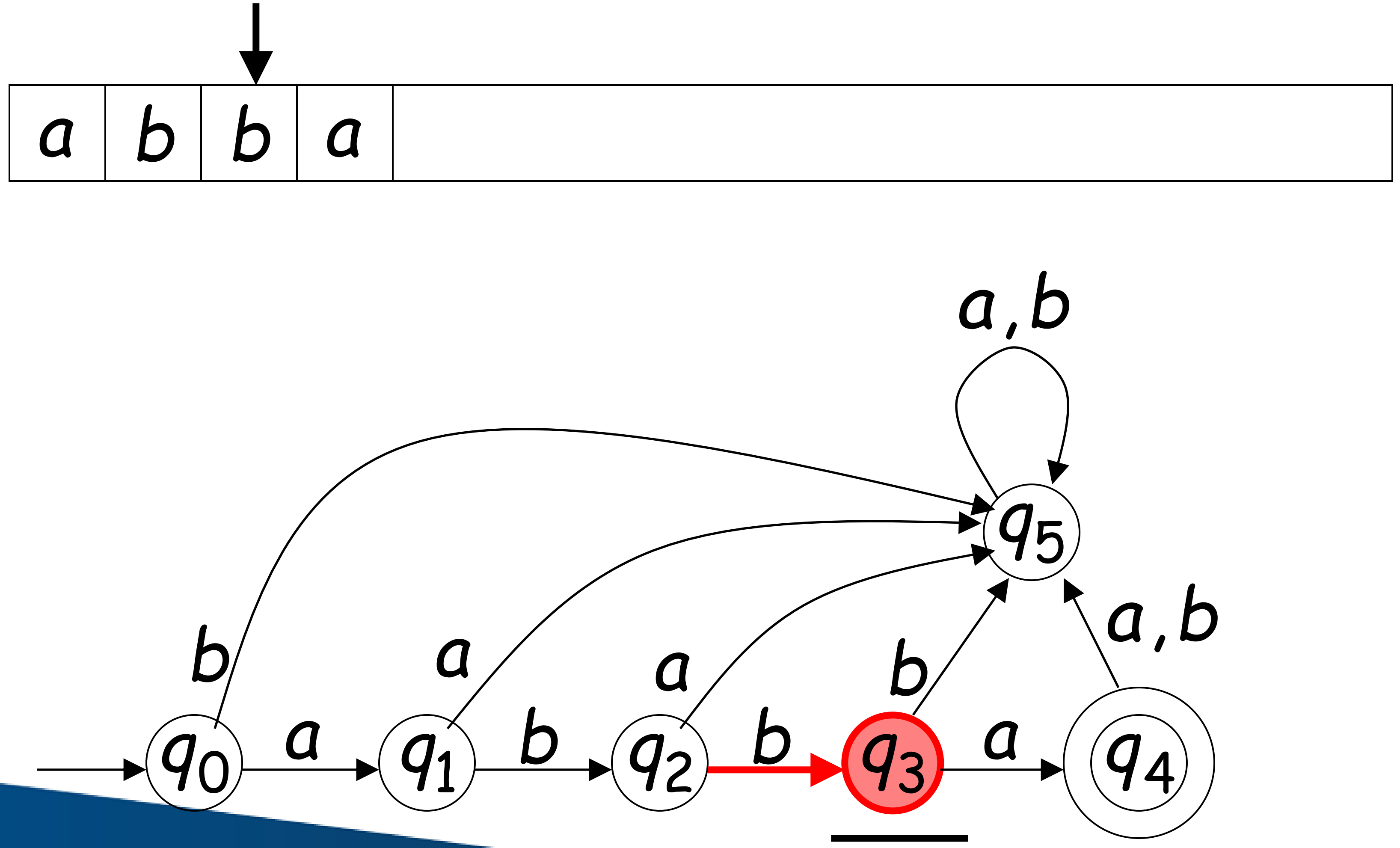


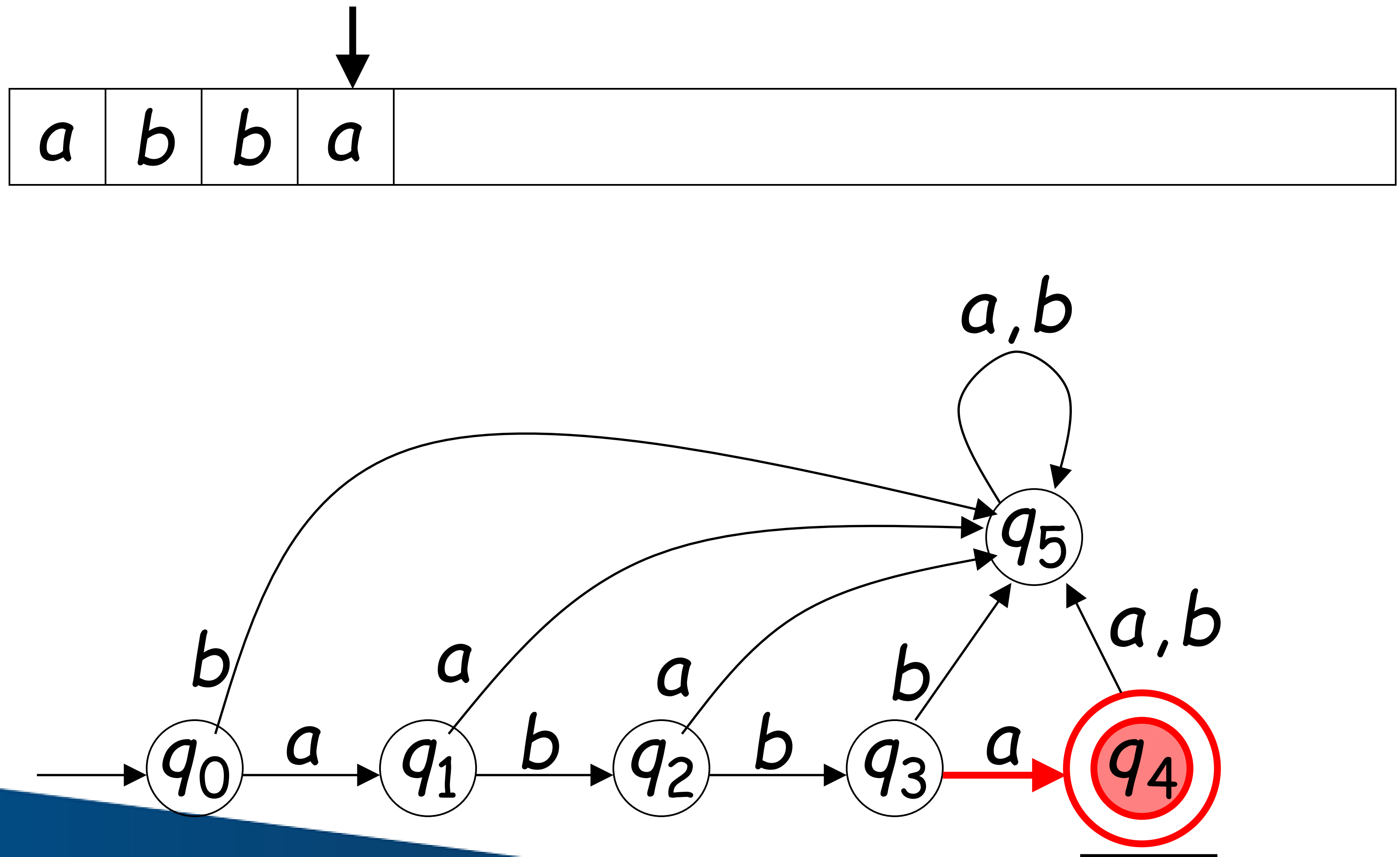
Reading the Input

a	b	b	a	

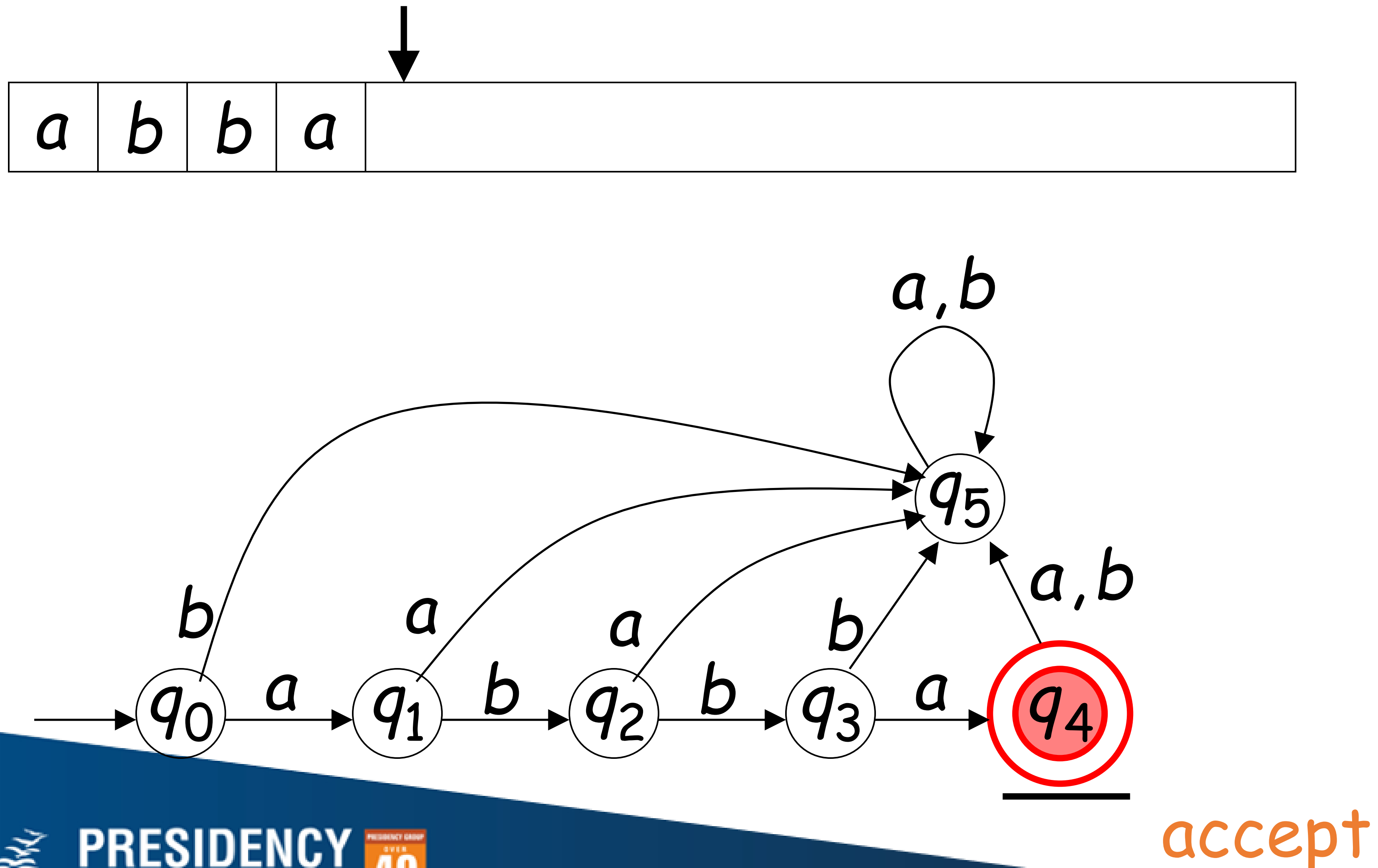



a
b
b
a
a,b
q_5
b
a
a
a,b
b
q_0
a
q_1
b
q_2
b
q_3
a
q_4


a
b
b
a
a,b
q5
b
a
a
b
a,b
q0
a
q1
b
q2
b
q3
a
q4

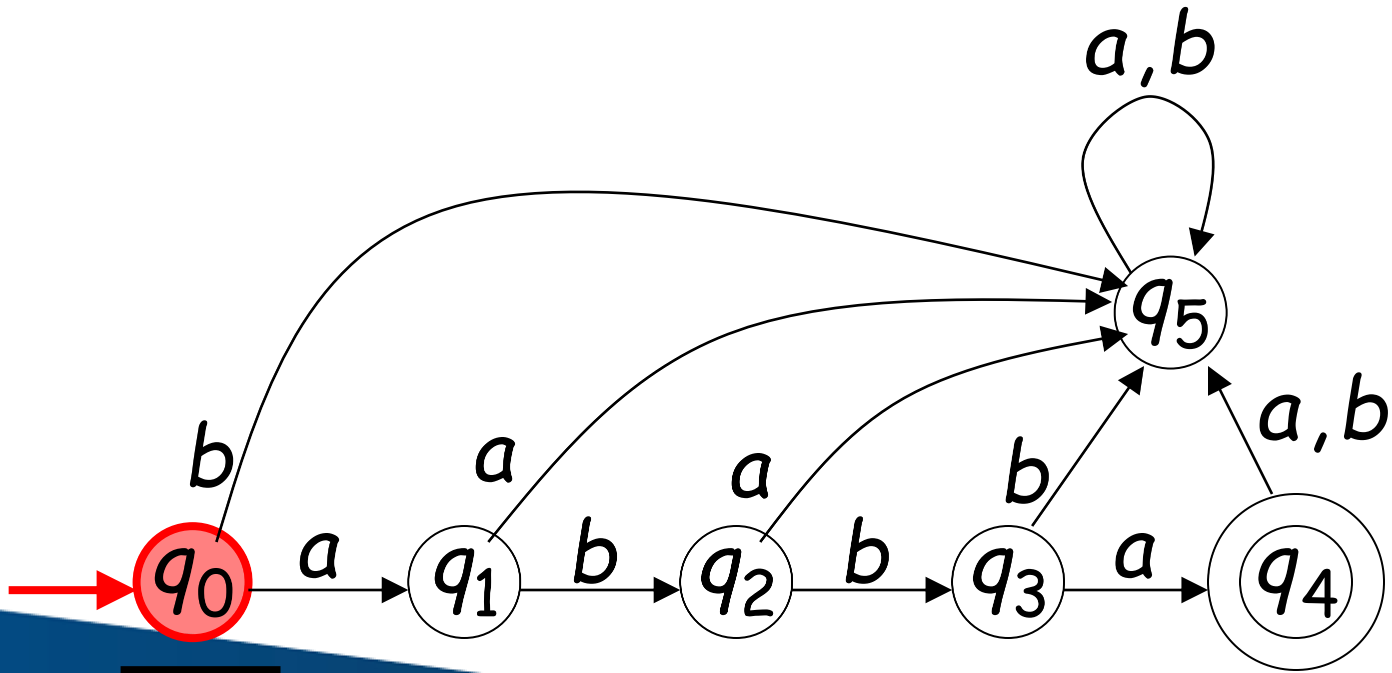

a
b
b
a
a,b
b
a
a
b
a,b
q_0
a
q_1
b
q_2
b
q_3
a
q_4
q_5

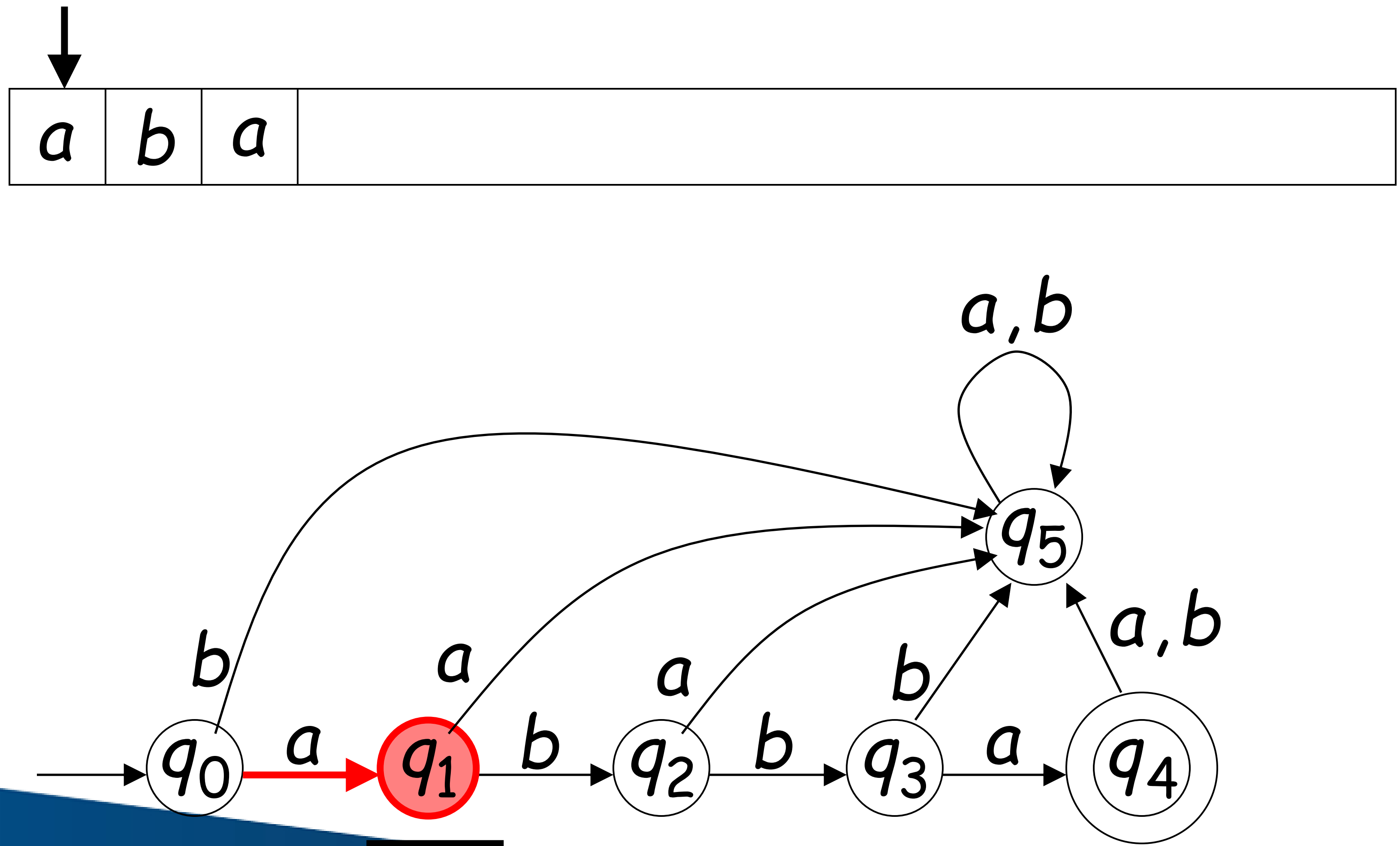
Input finished



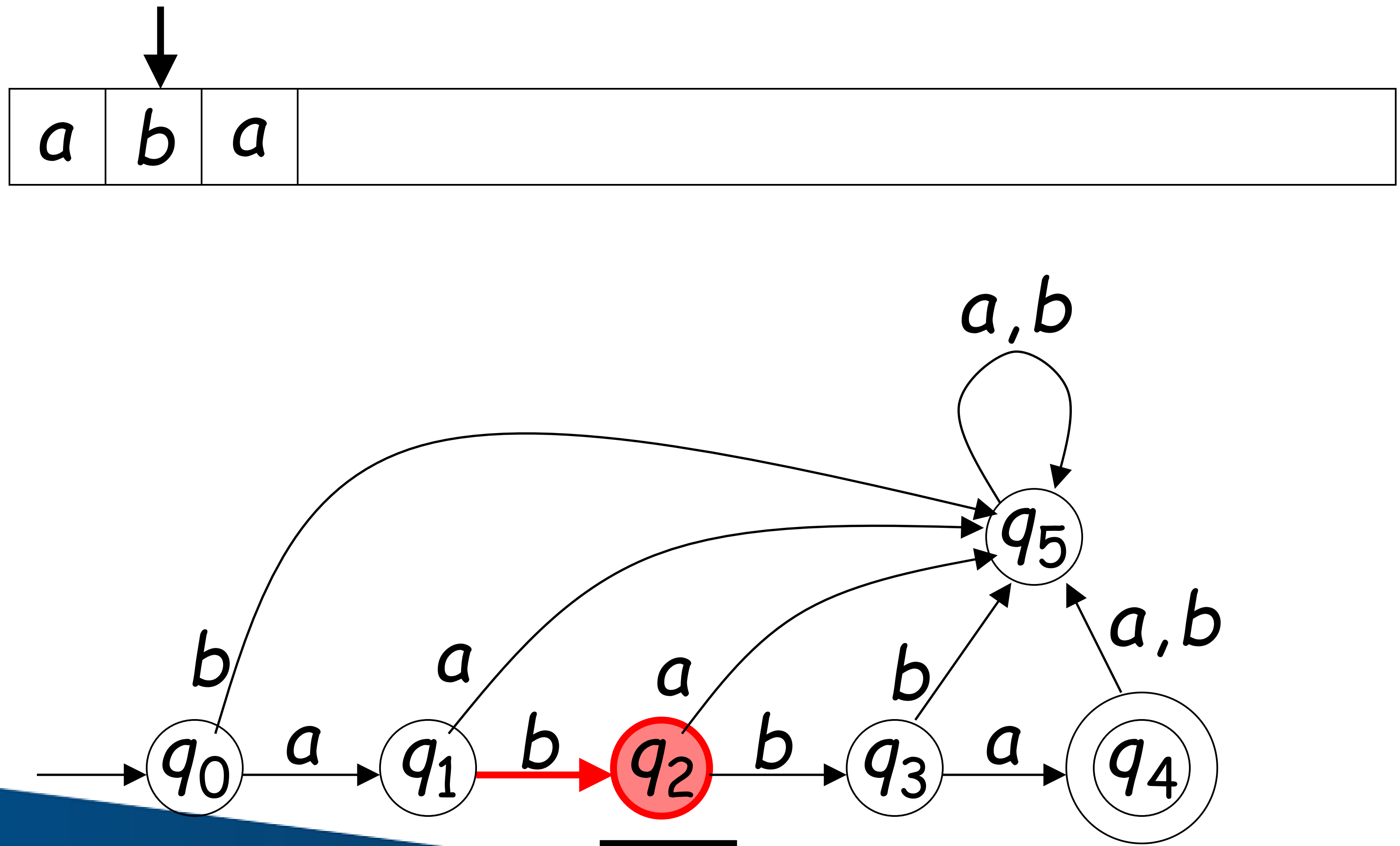
Rejection

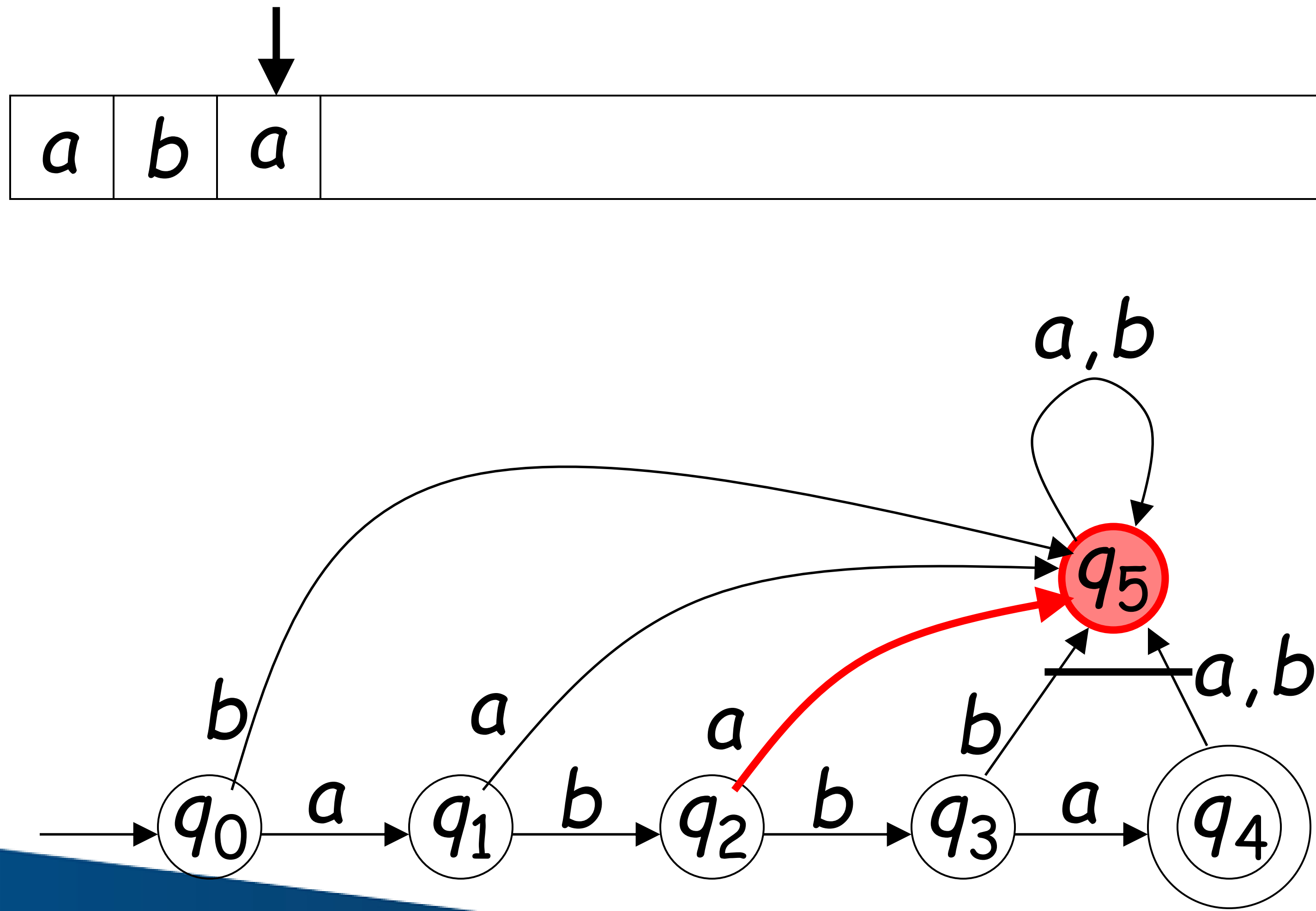
a	b	a	



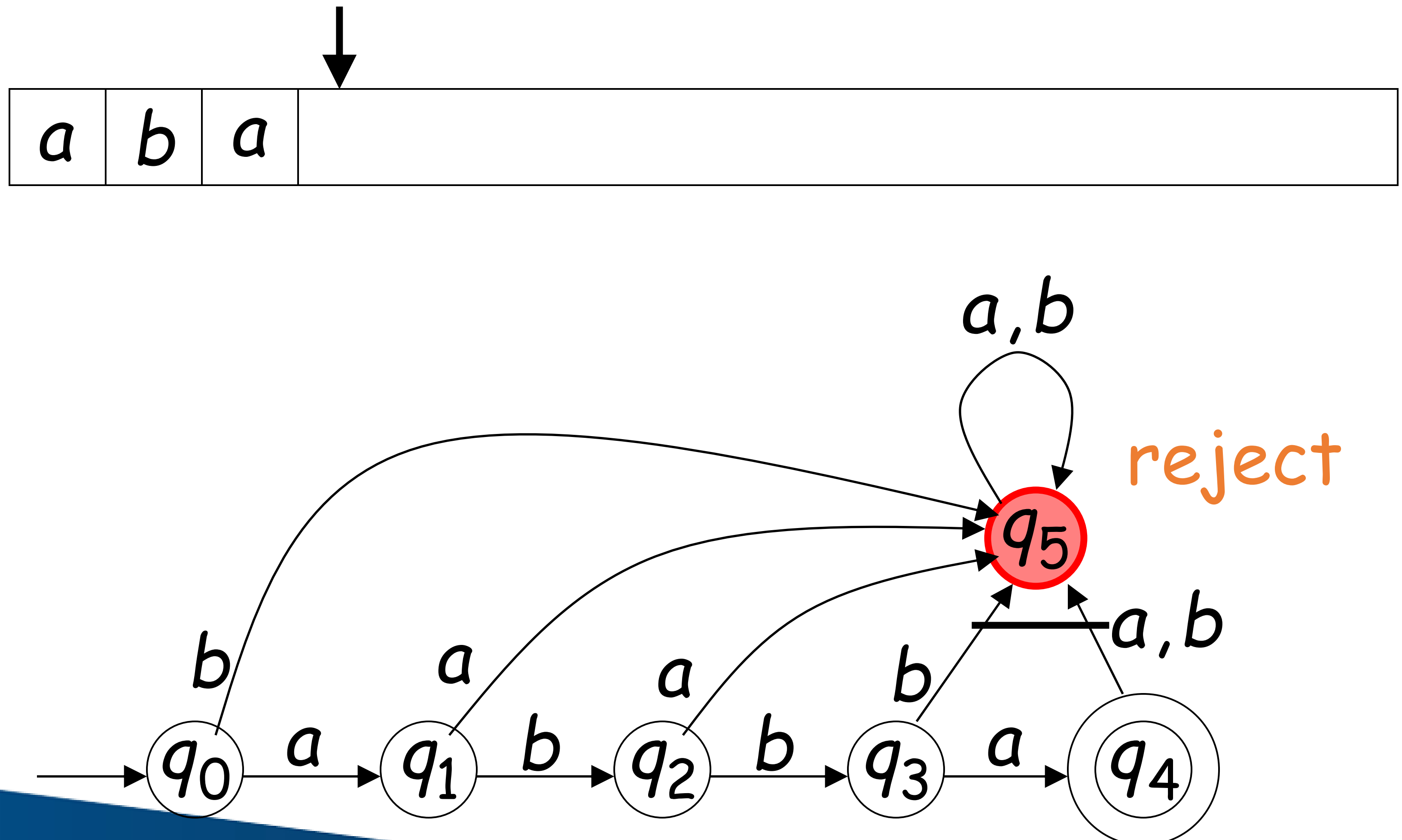


a
b
a
a,b
b
a
a
b
a
b
b
a
a,b
q_0
q_1
q_2
q_3
q_4
q_5


a
b
a
a,b
b
a
a
b
a
b
b
a
a,b
q0
q1
q2
q3
q4
q5

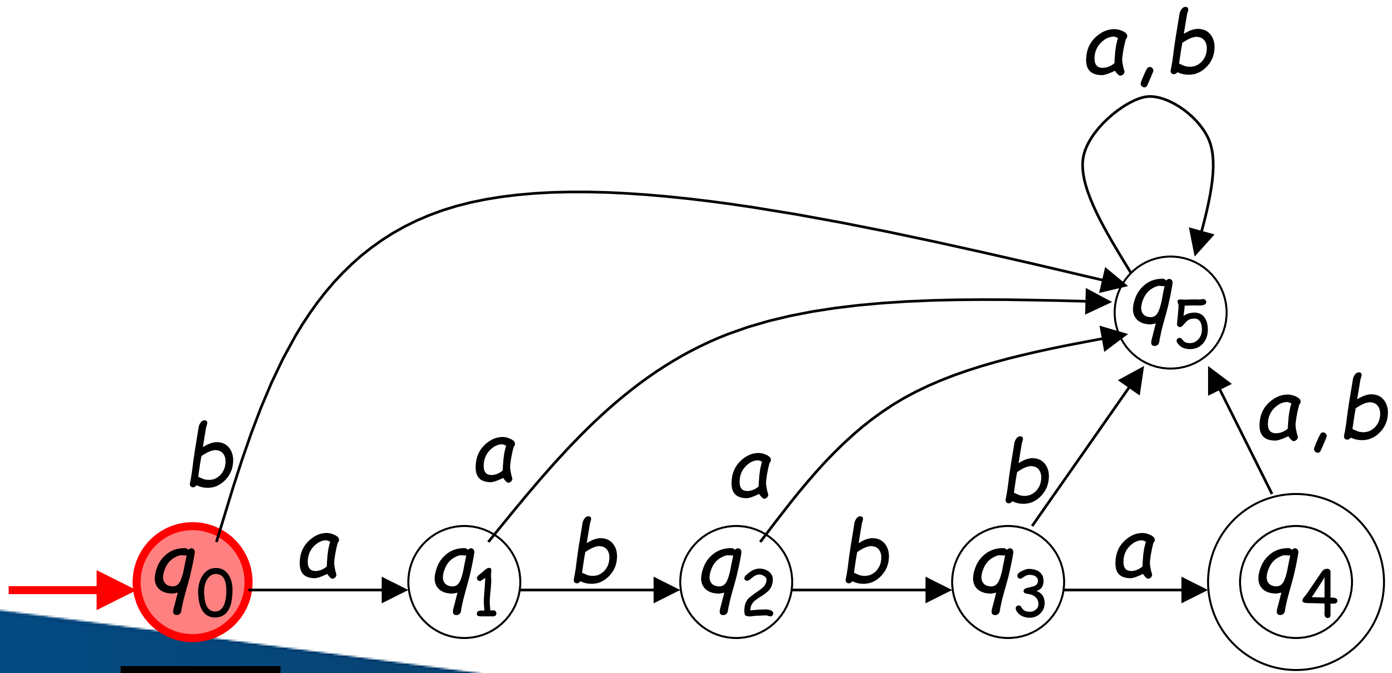

a
b
a
a,b
q5
b
a
a
b
a,b
q0
a
q1
b
q2
b
q3
a
q4

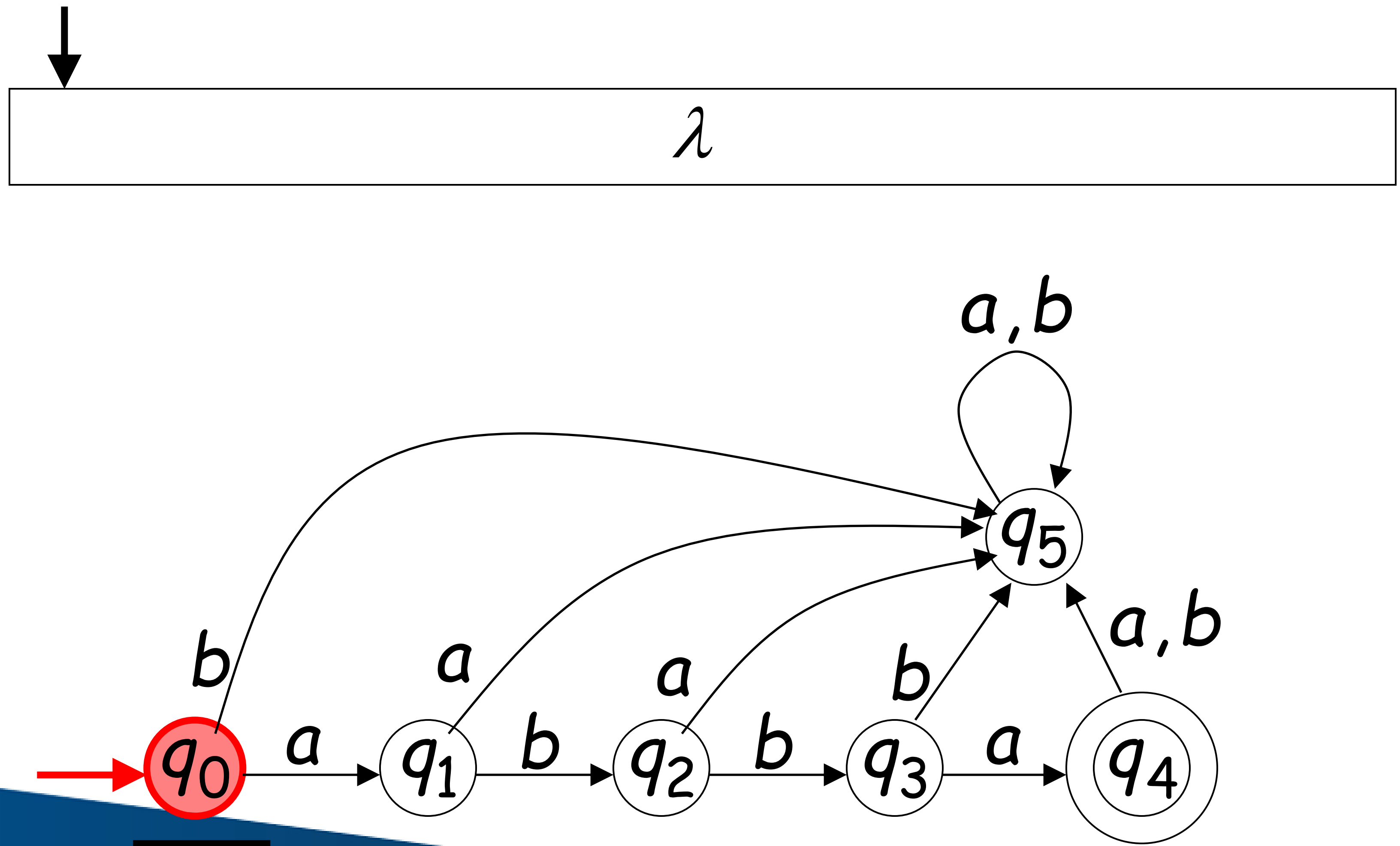
Input finished



Another Rejection

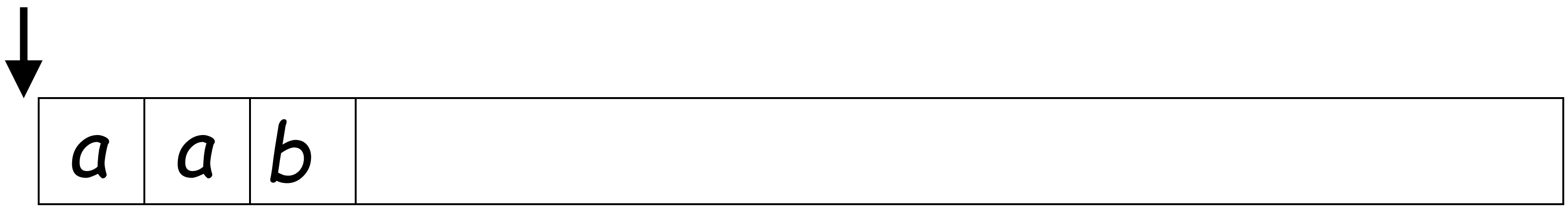
λ

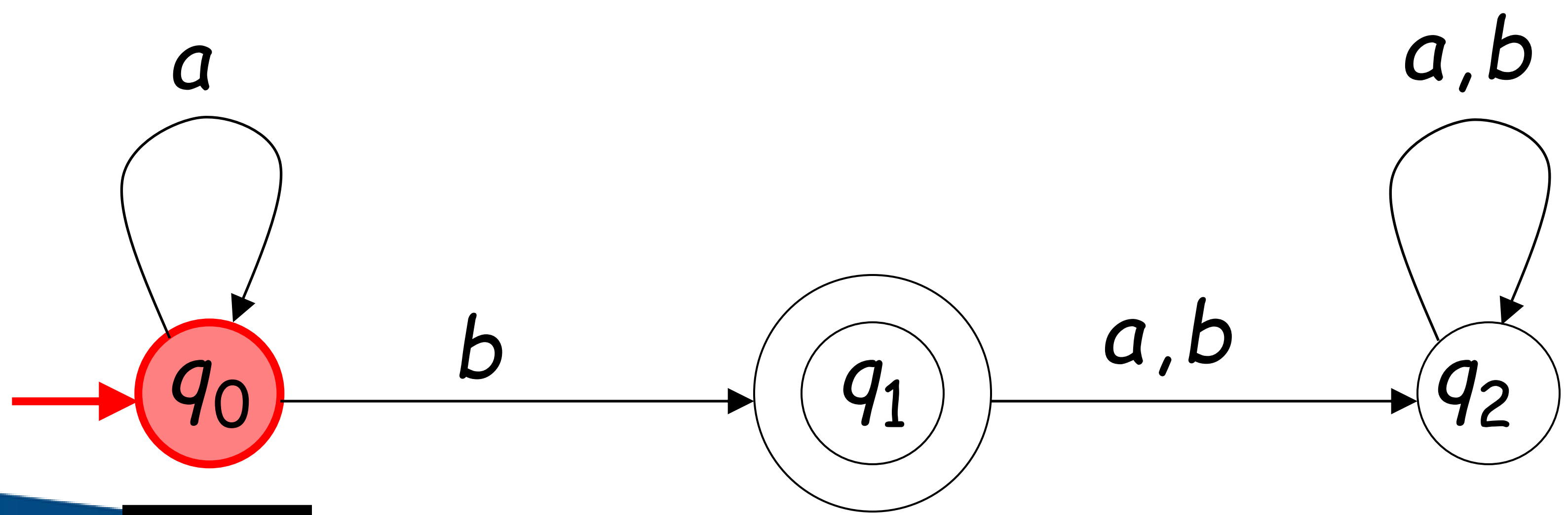


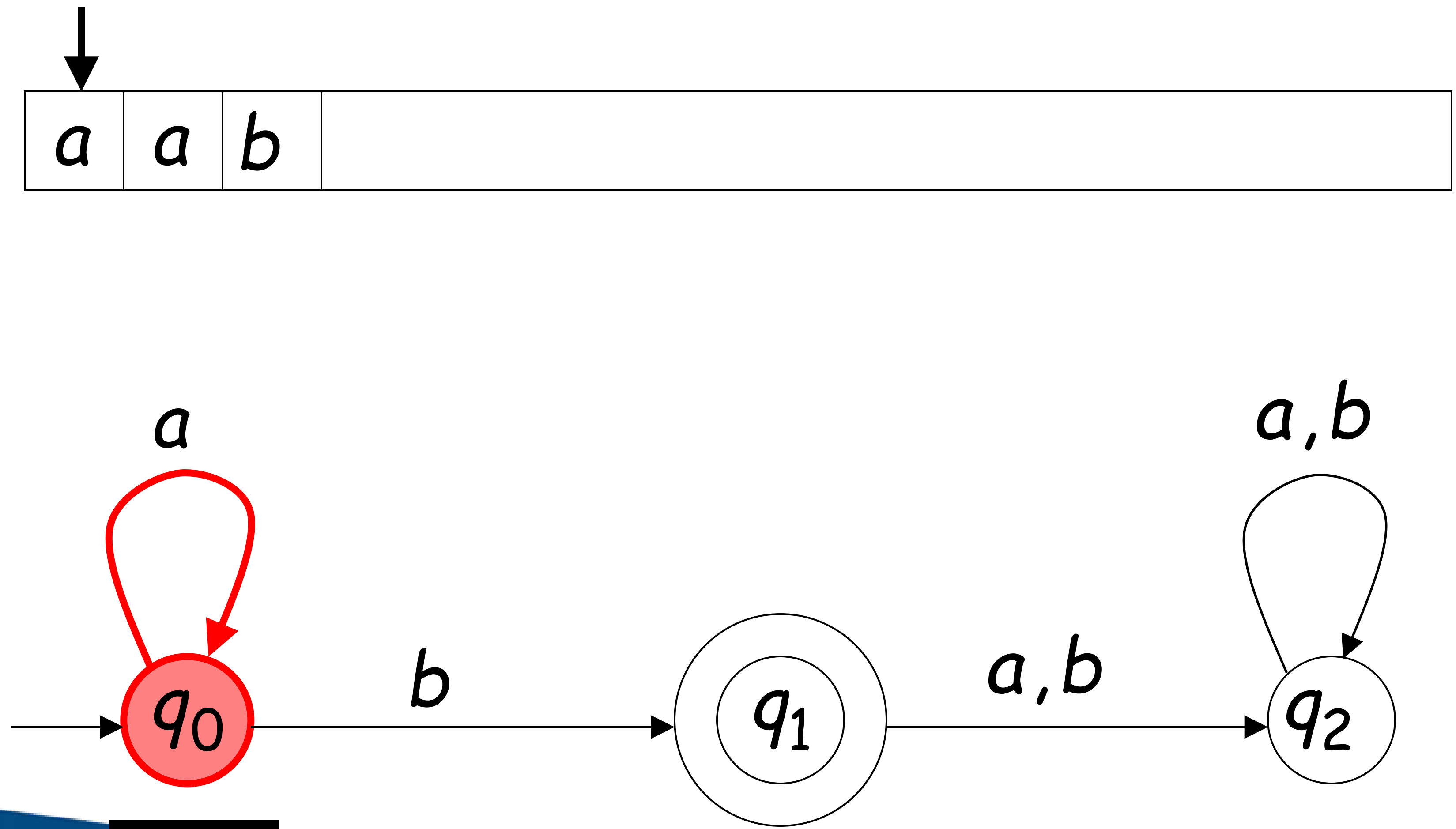


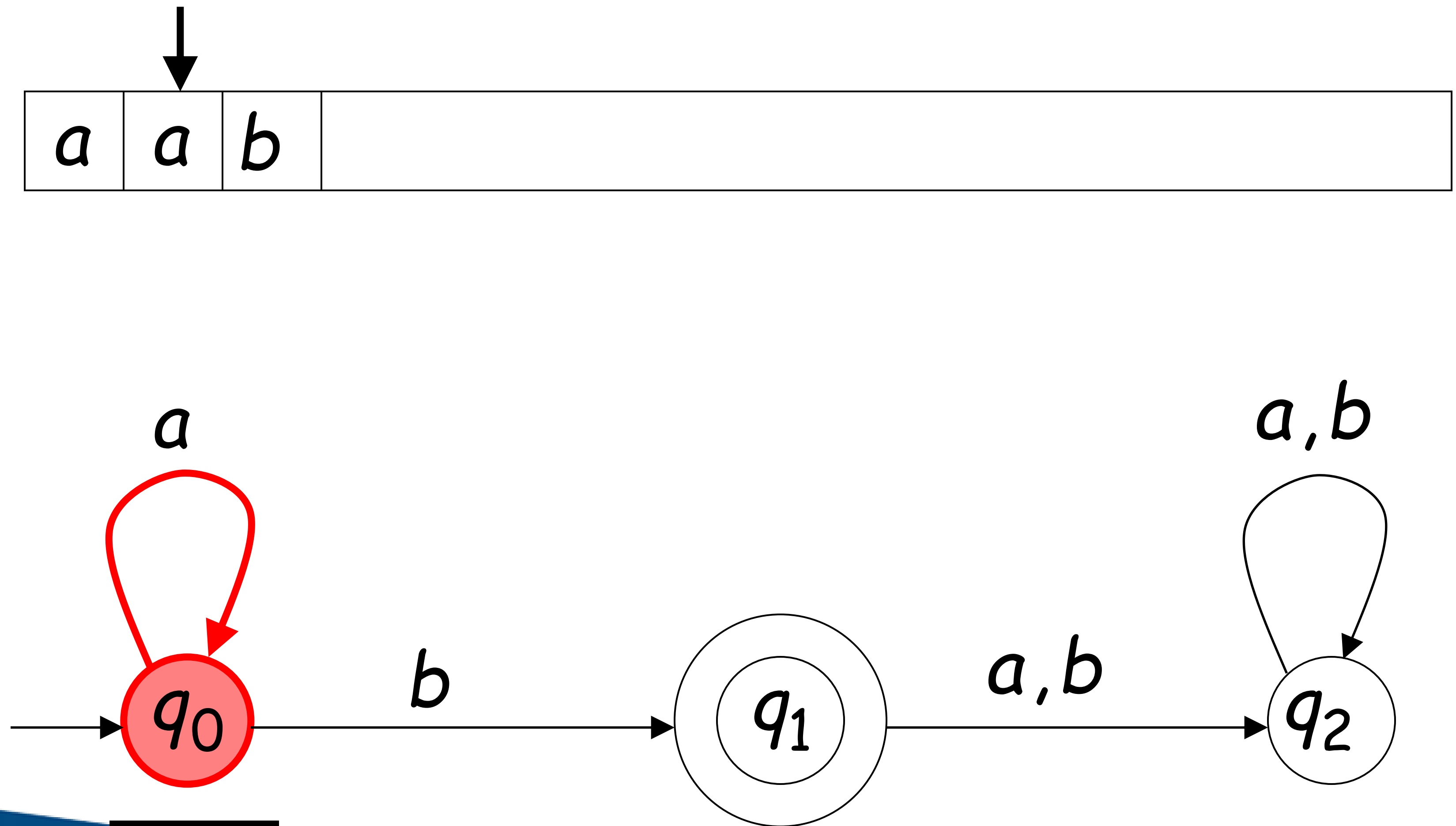
reject

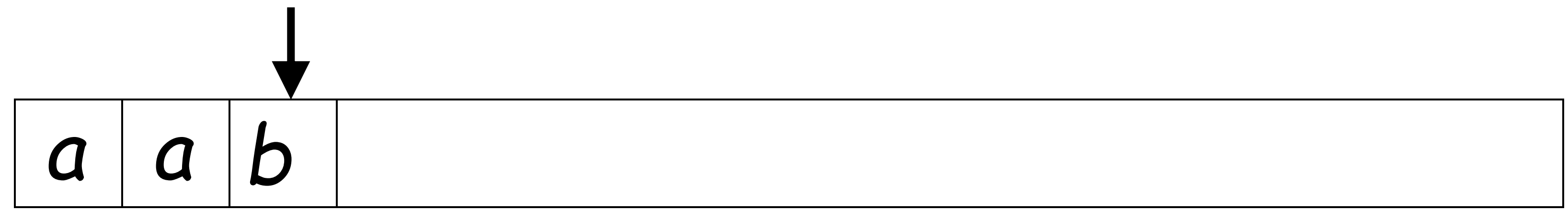
Another Example

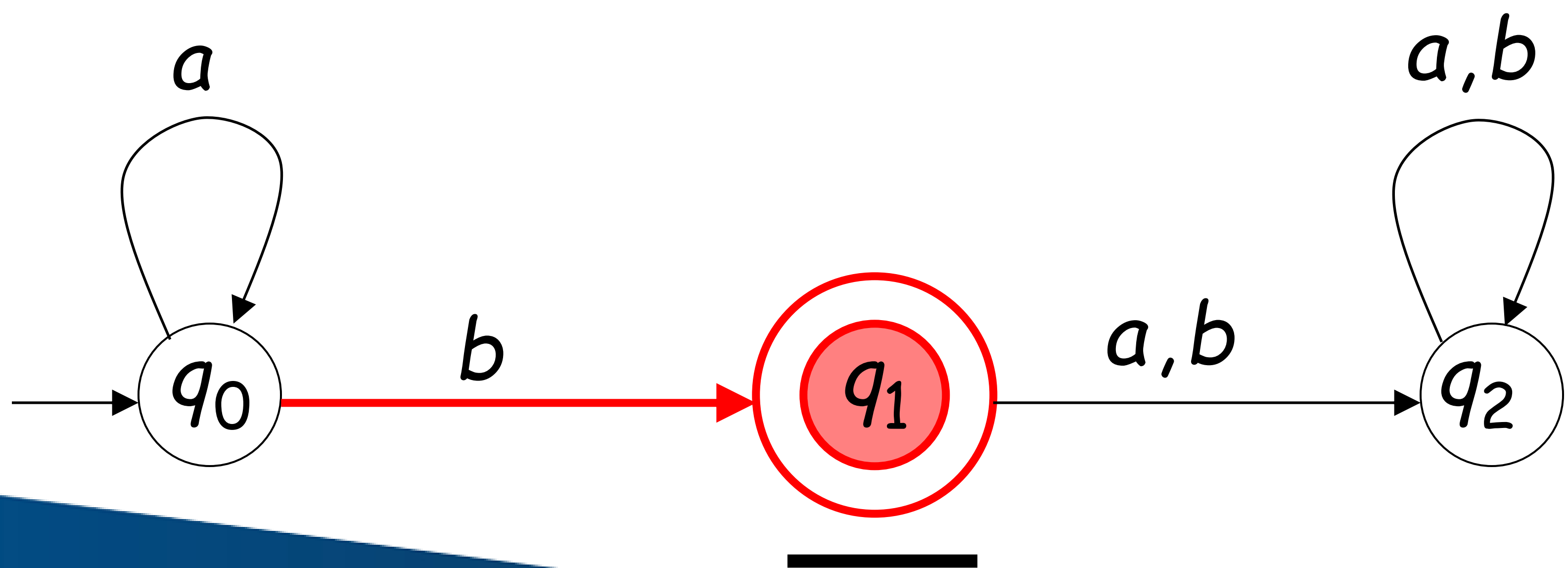




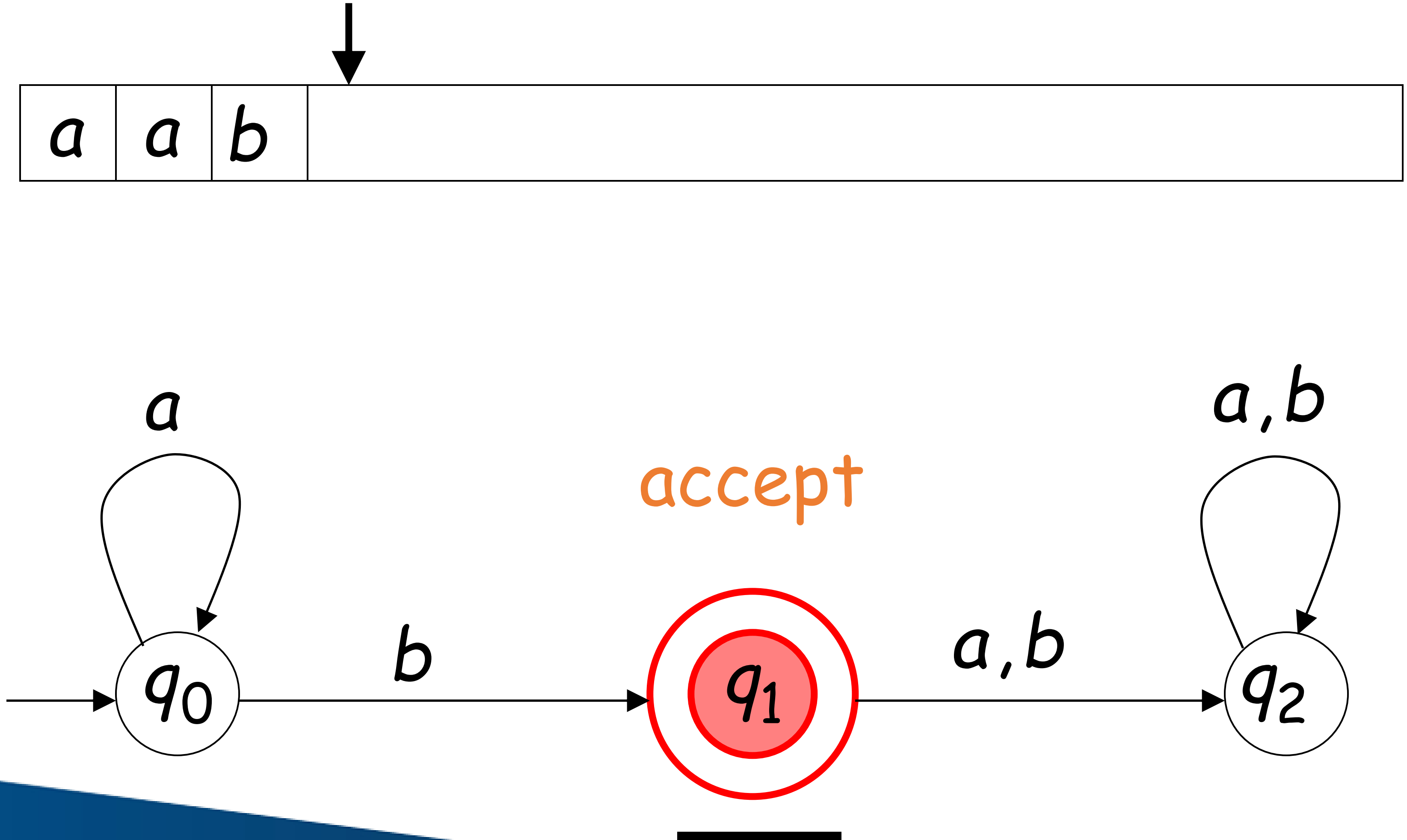

a
a
b
a
a,b
b
a,b
q_0
q_1
q_2


a
a
b
a
a,b
b
a,b
q_0
q_1
q_2

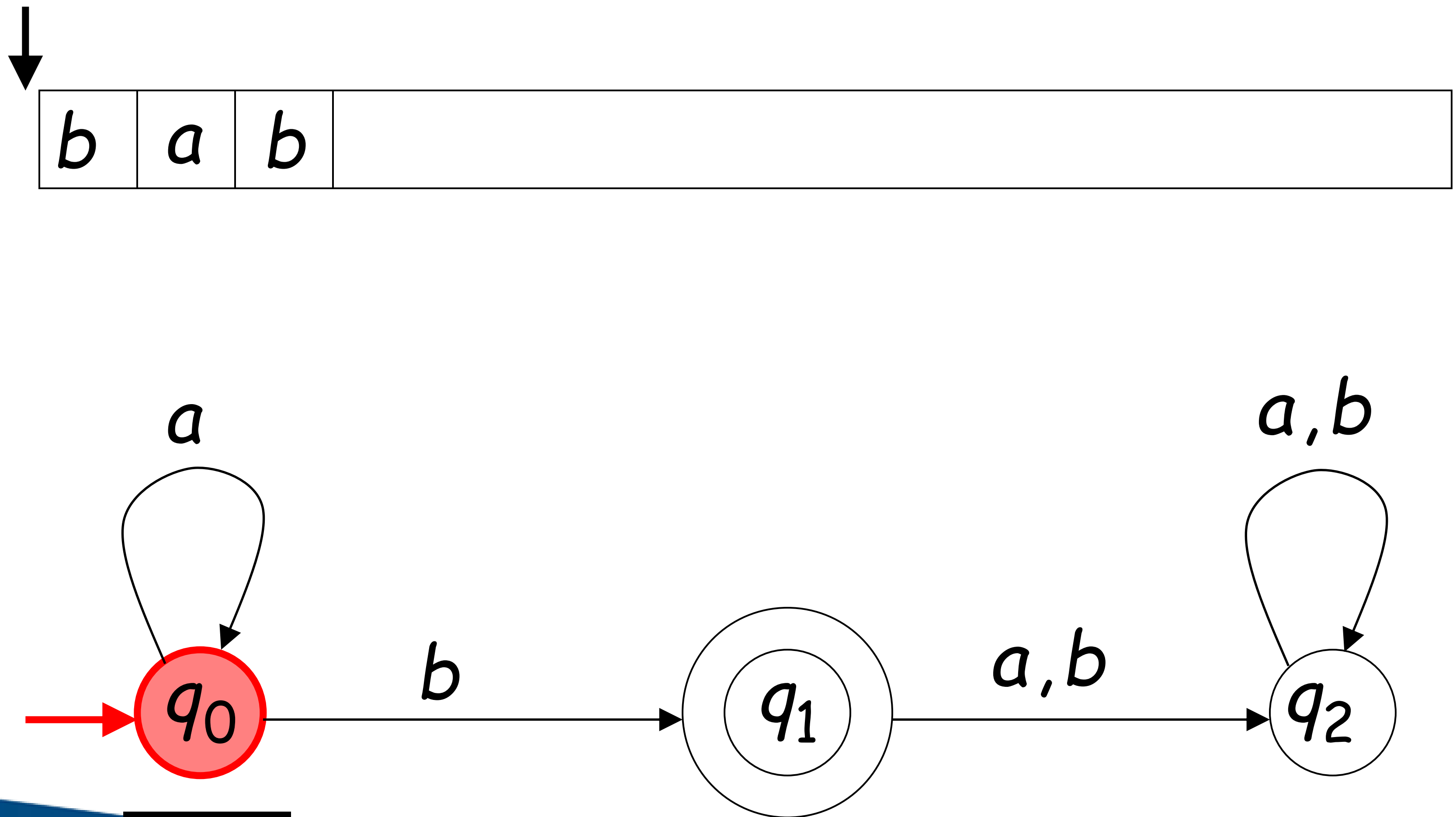

a
a
b

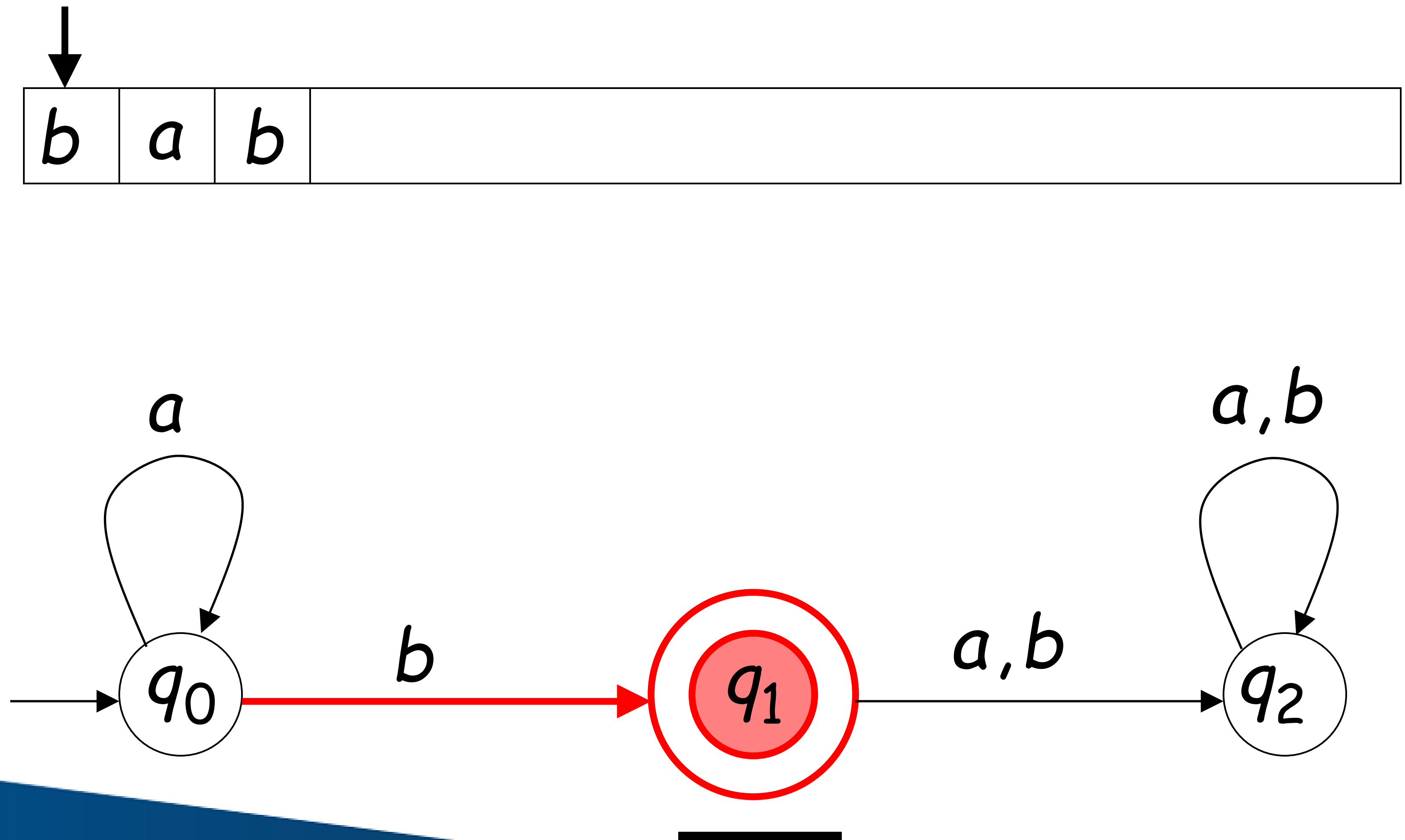

a
a,b
b
a,b
q0
q1
q2

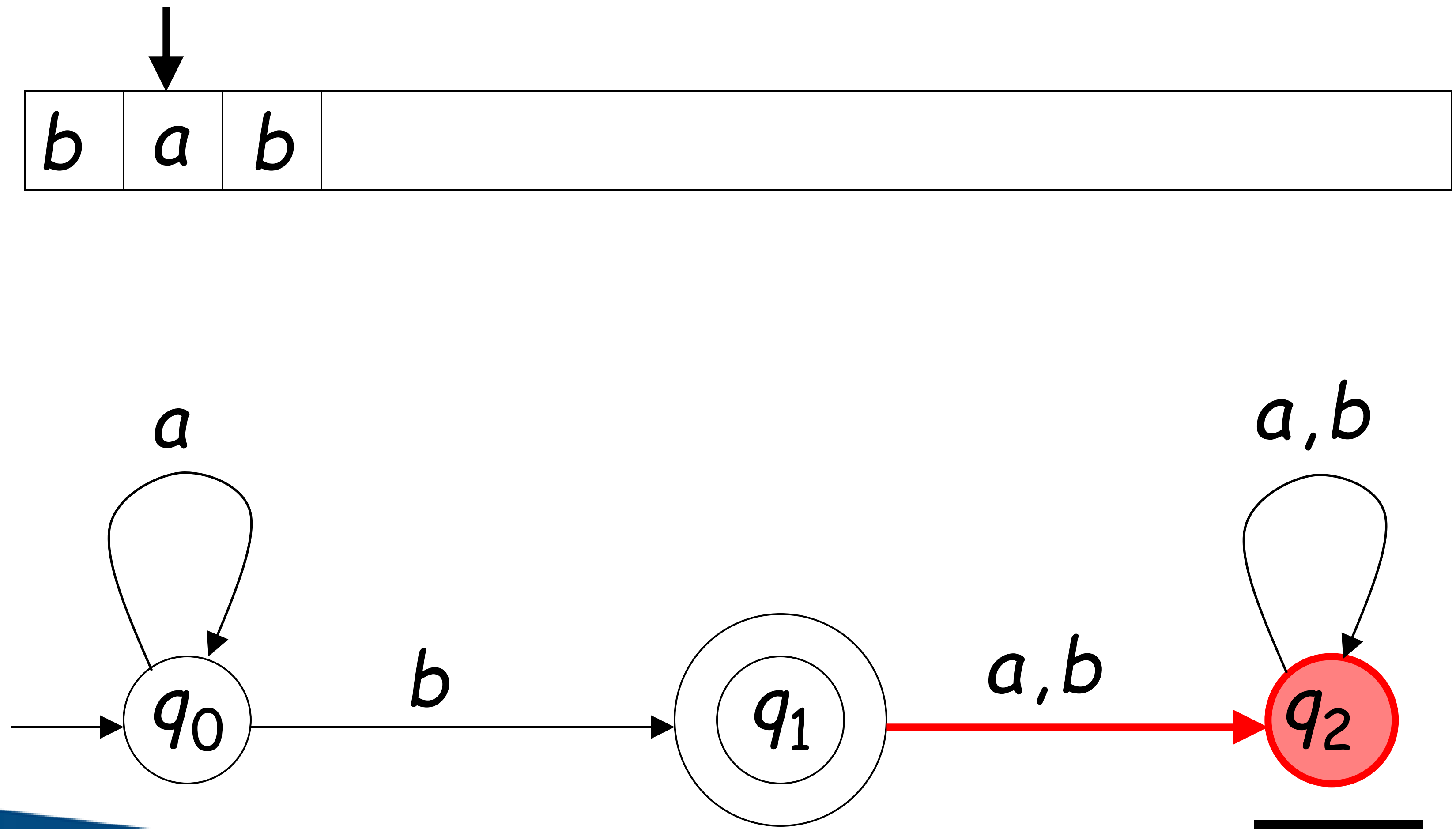
Input finished

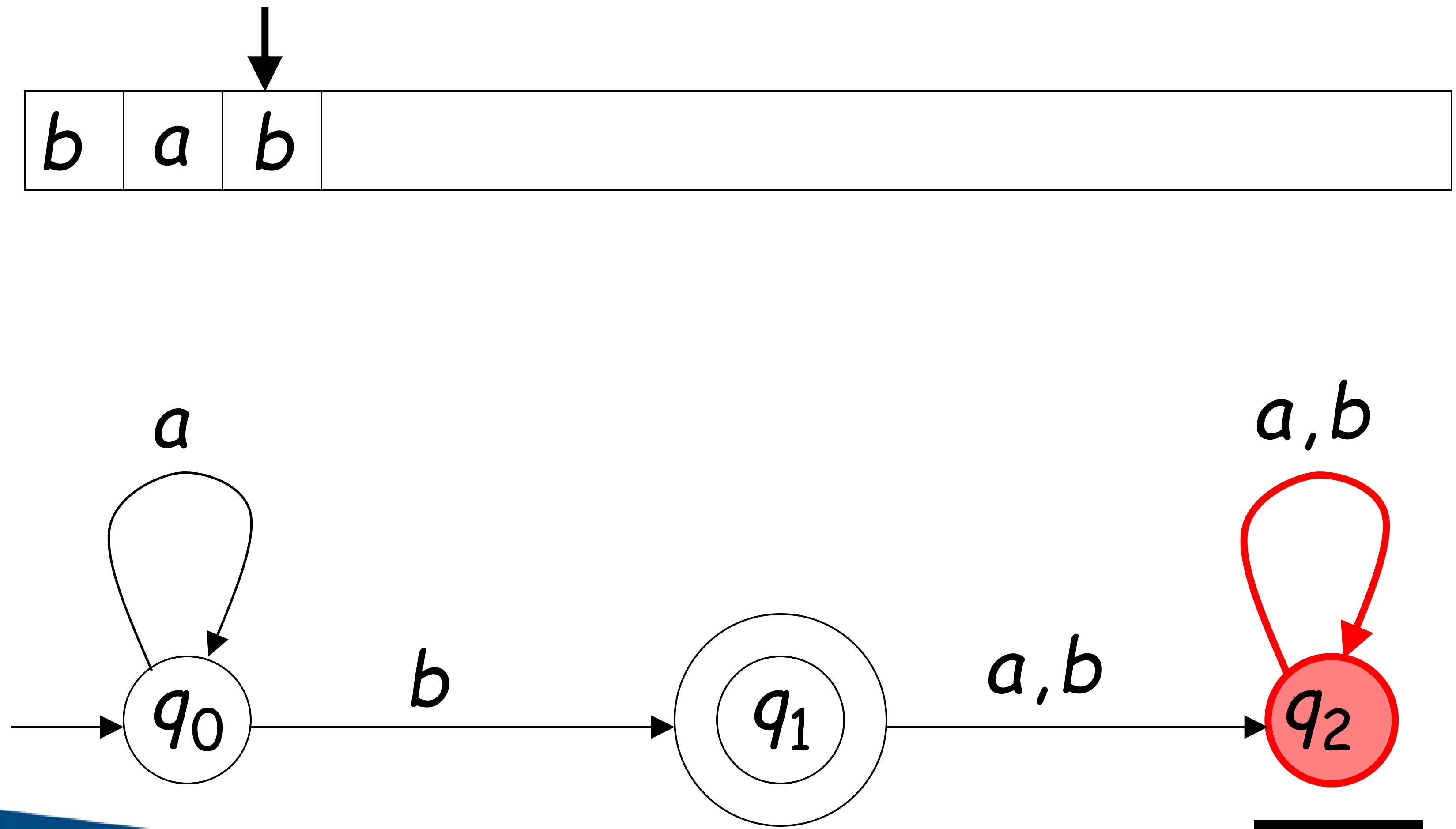


Rejection Example




b
a
b
a
a,b
b
a,b
q_0
q_1
q_2

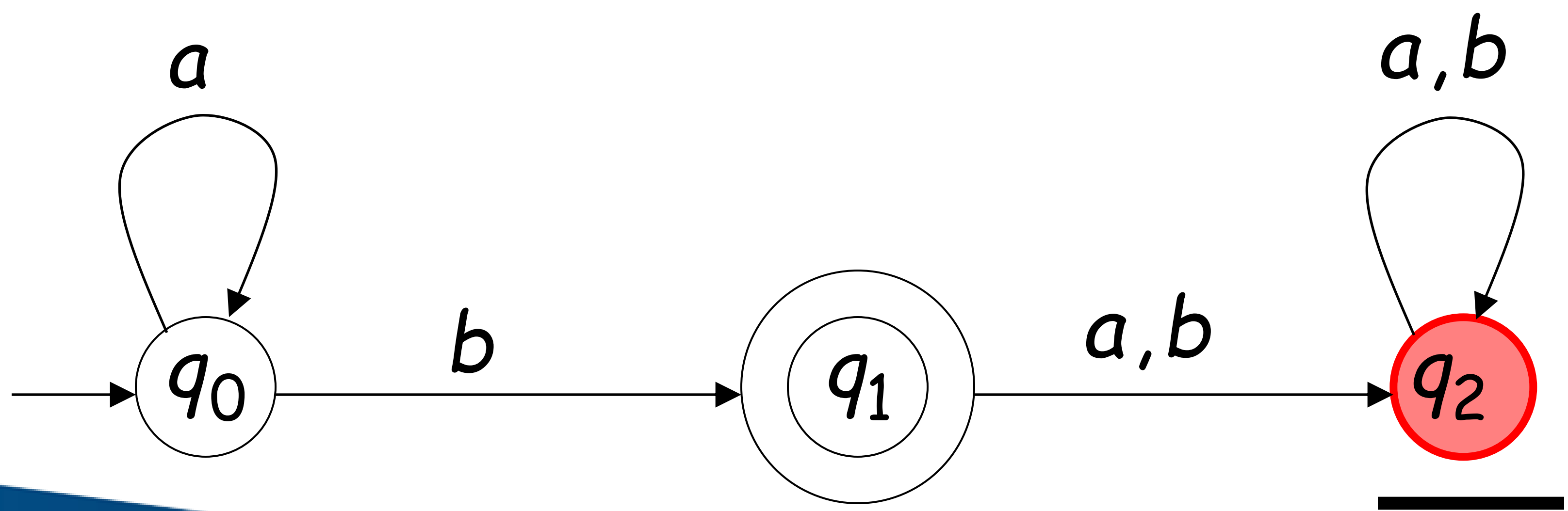

b
a
b
a
a,b
b
a,b
q_0
q_1
q_2



b
a
b
a
a,b
b
a,b
q_0
q_1
q_2

Input finished

b	a	b	



reject

Languages Accepted by FAs

Definition:

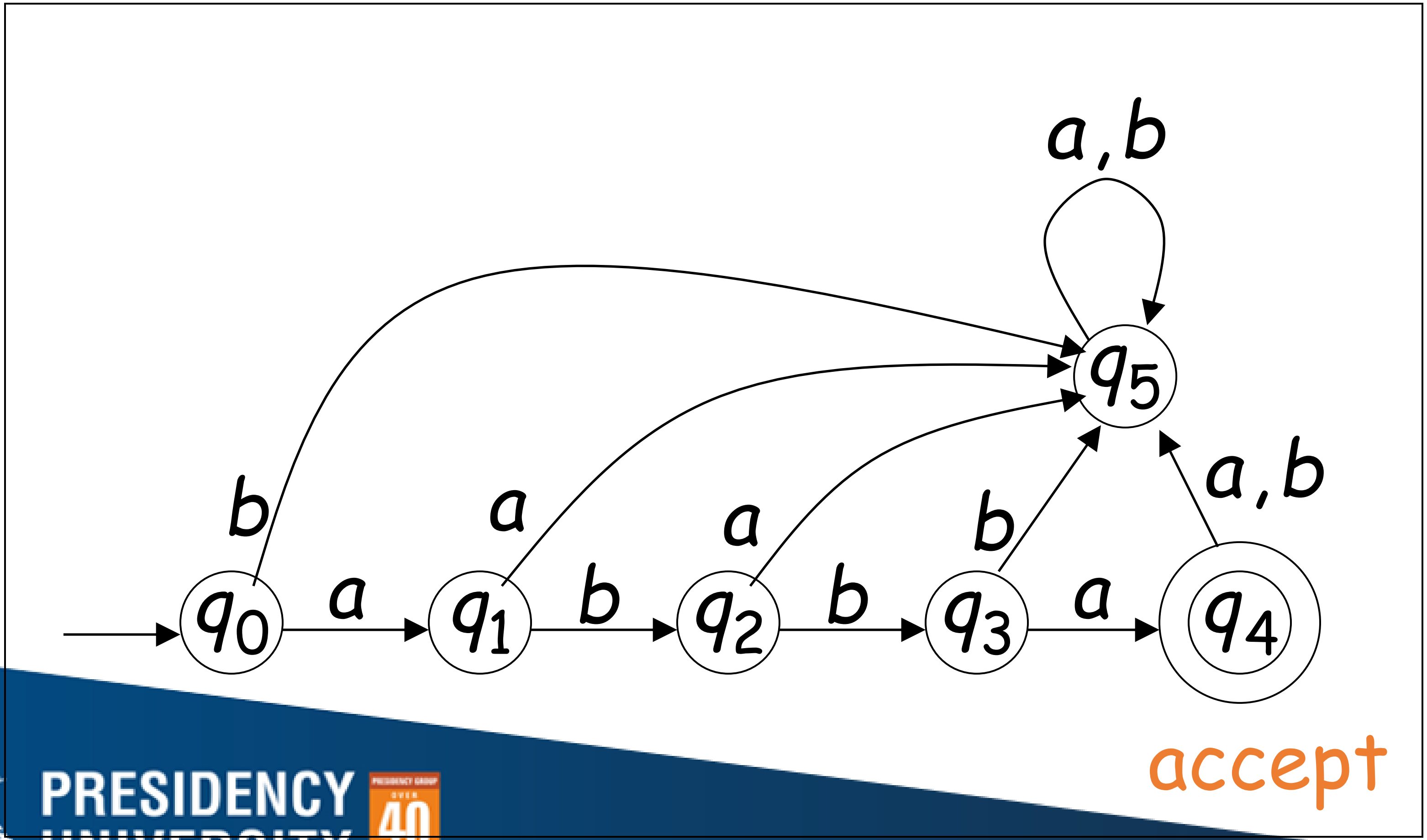
The language $L(M)$ contains all input strings accepted by M

$L(M)$ = { strings that bring M to an accepting state}

Example

$$L(M) = \{abba\}$$

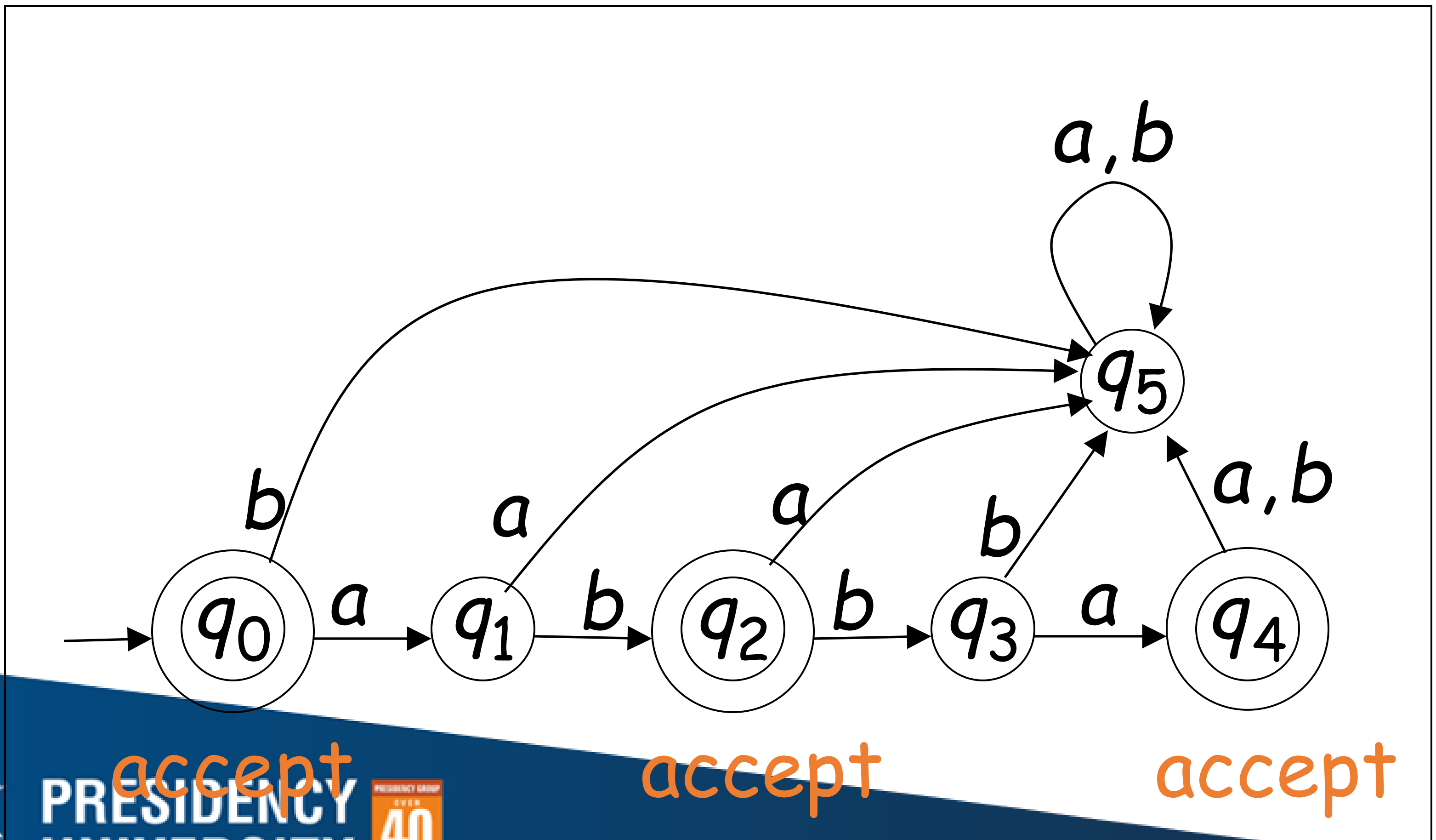
M



Example

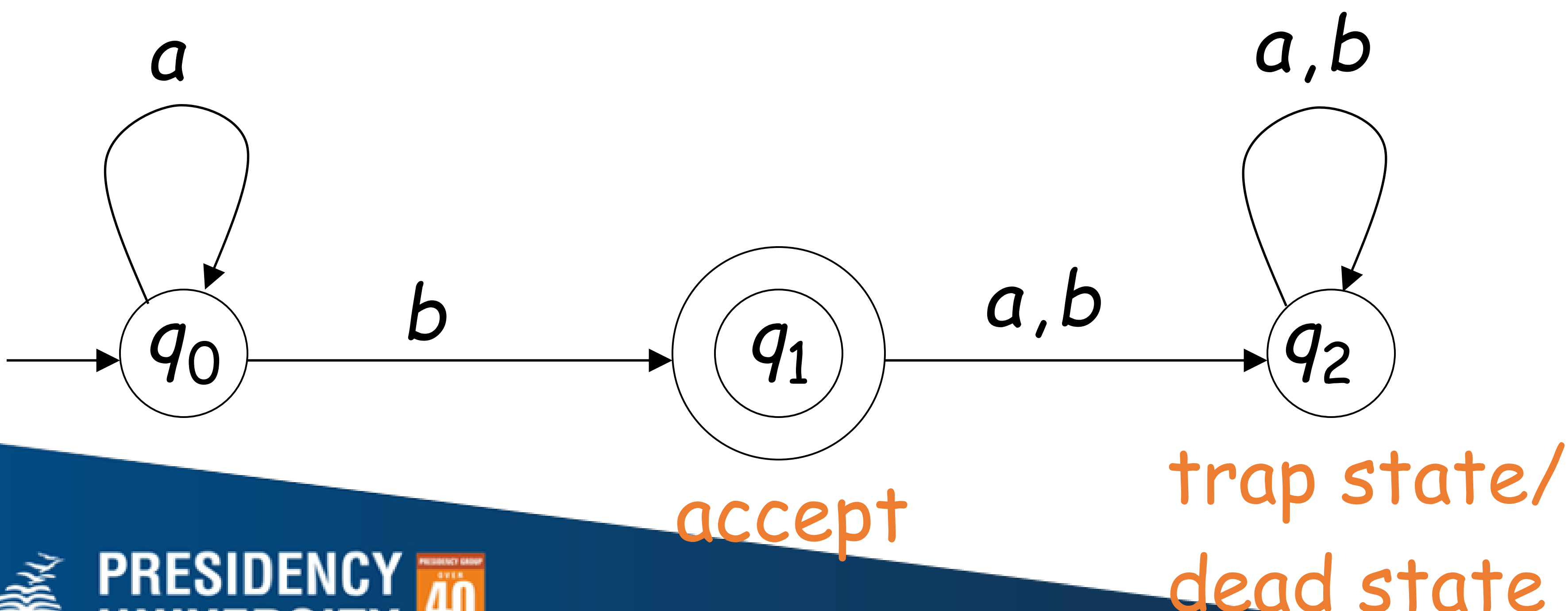
$$L(M) = \{\lambda, ab, abba\}$$

M



Example

$$L(M) = \{a^n b : n \geq 0\}$$



Formal Definition

- Finite Automaton (FA)

$$M = (Q, \Sigma, \delta, q_0, F)$$

Q : set of states

Σ : input alphabet

δ : transition function

q_0 : initial state

F : set of accepting states

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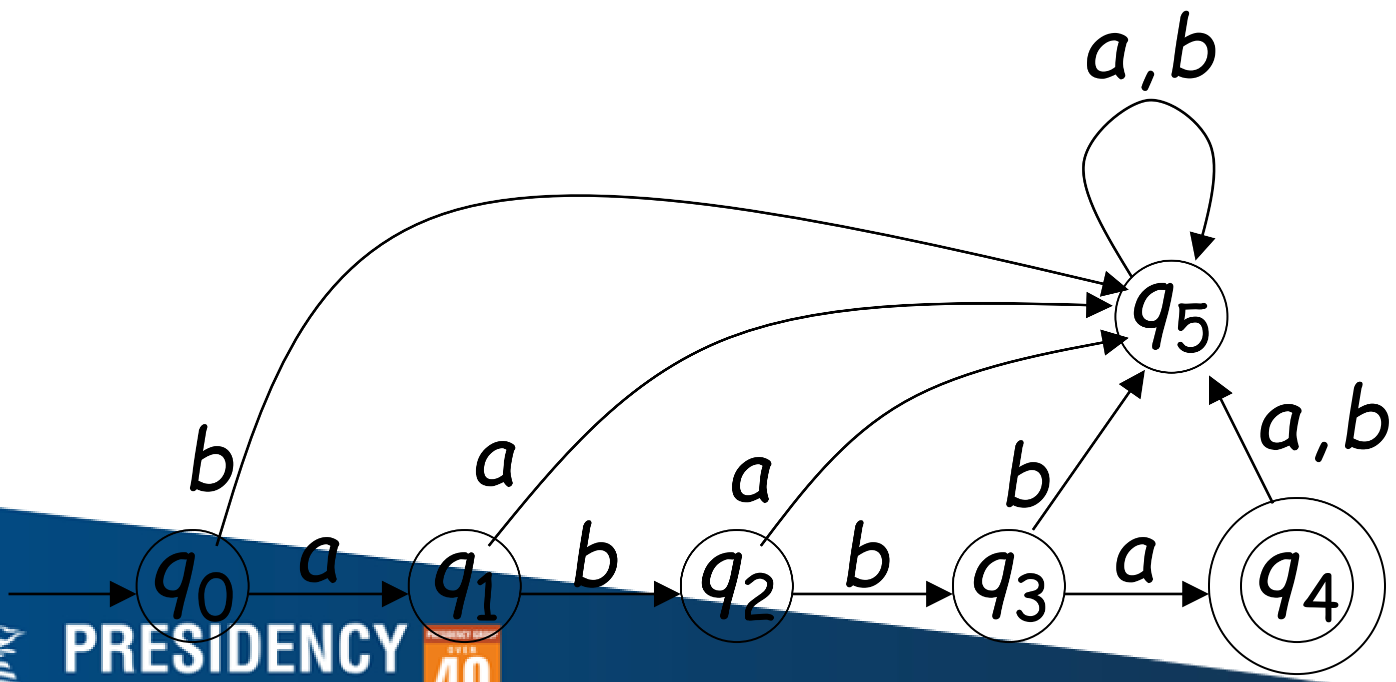
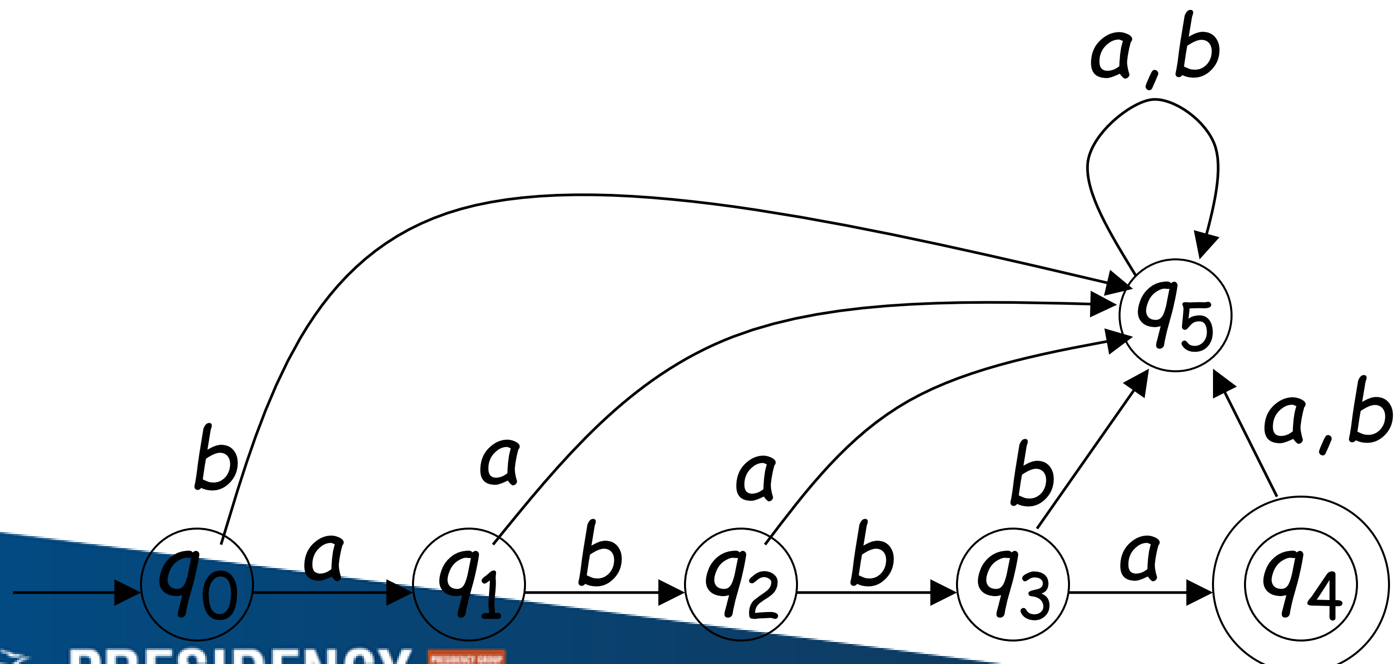
Input Alphabet Σ

- $\Sigma = \{a, b\}$



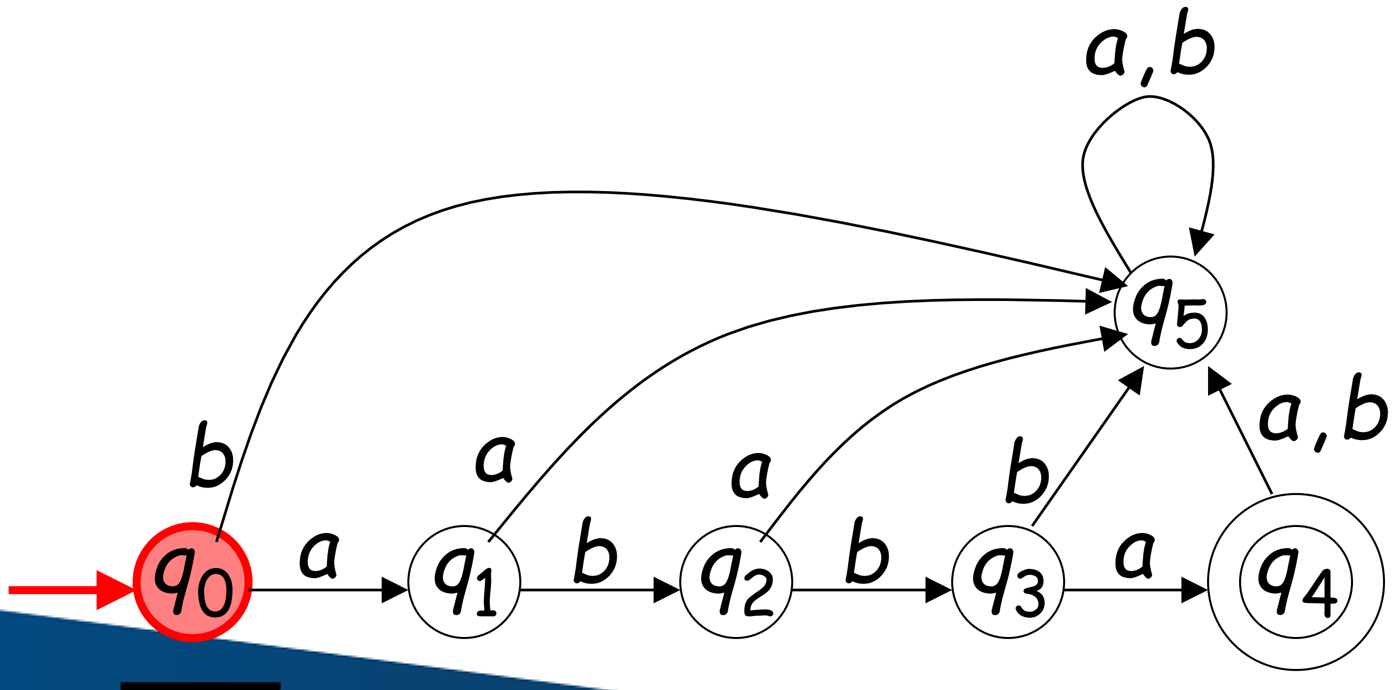
Set of States Q

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$$



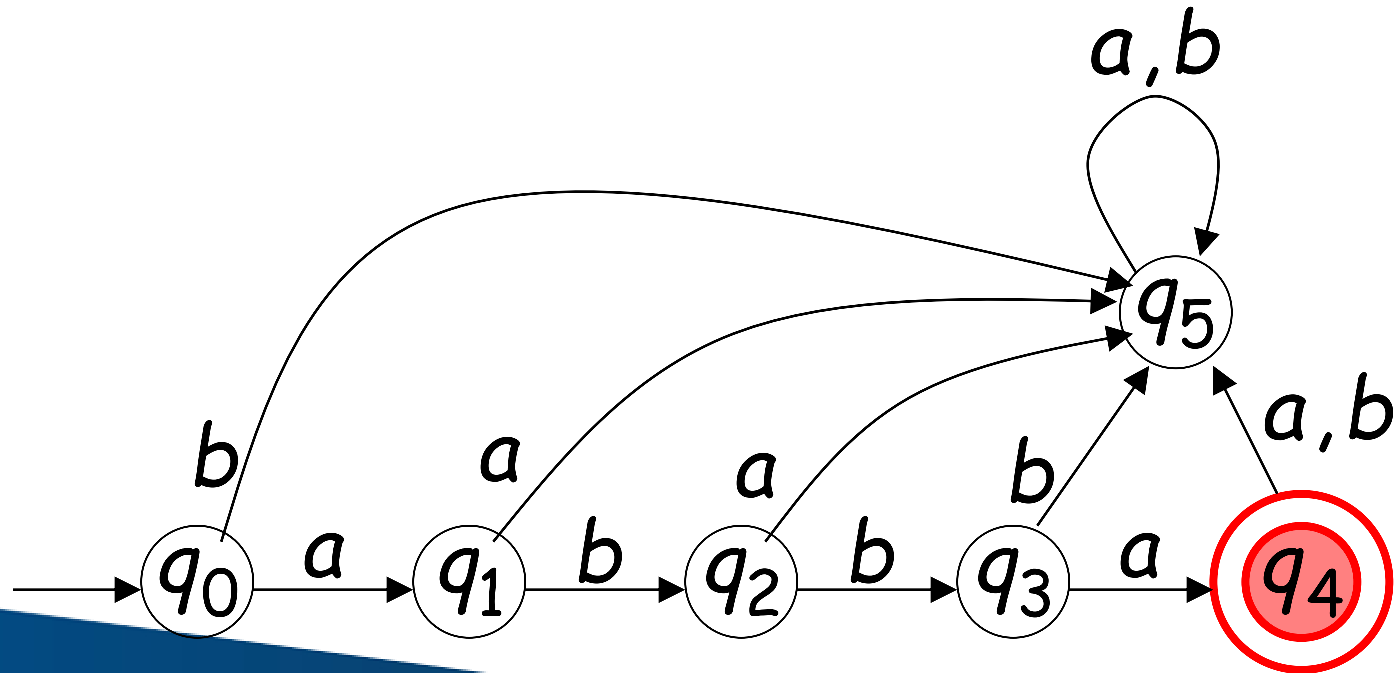
Initial State q_0

•



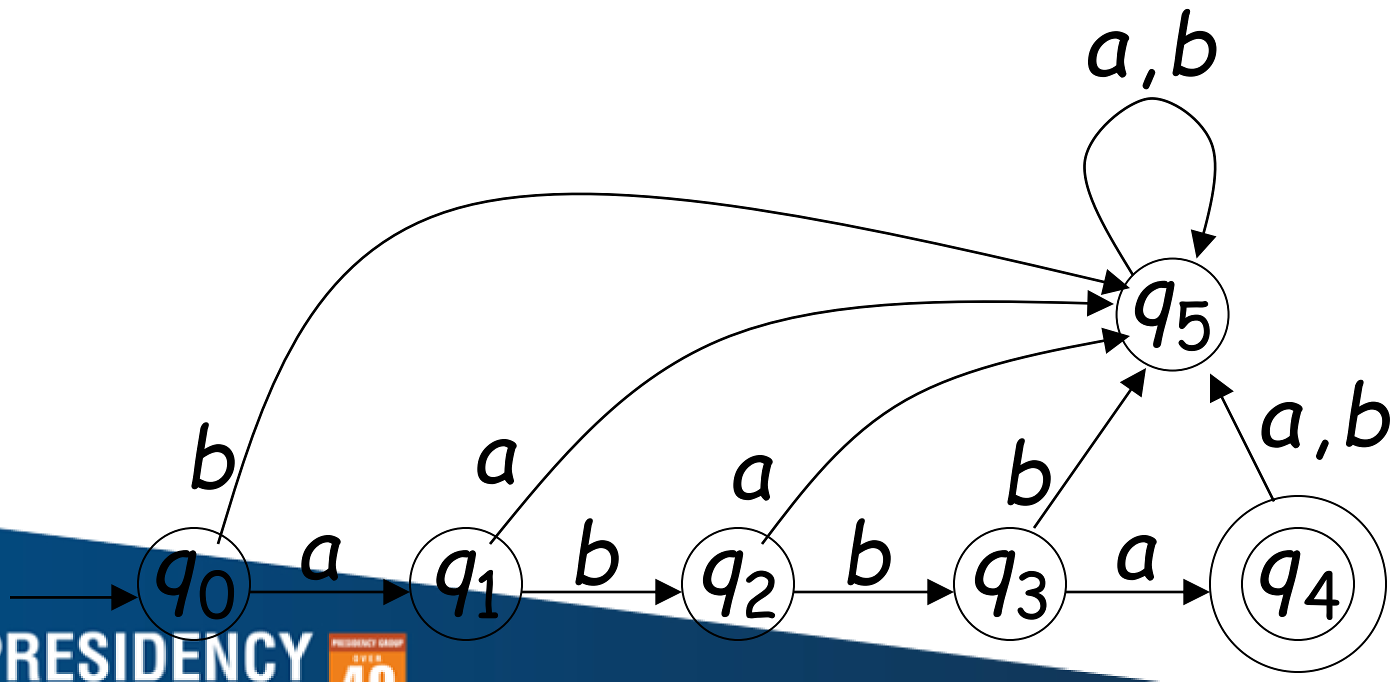
Set of Accepting States F

- $F = \{q_4\}$

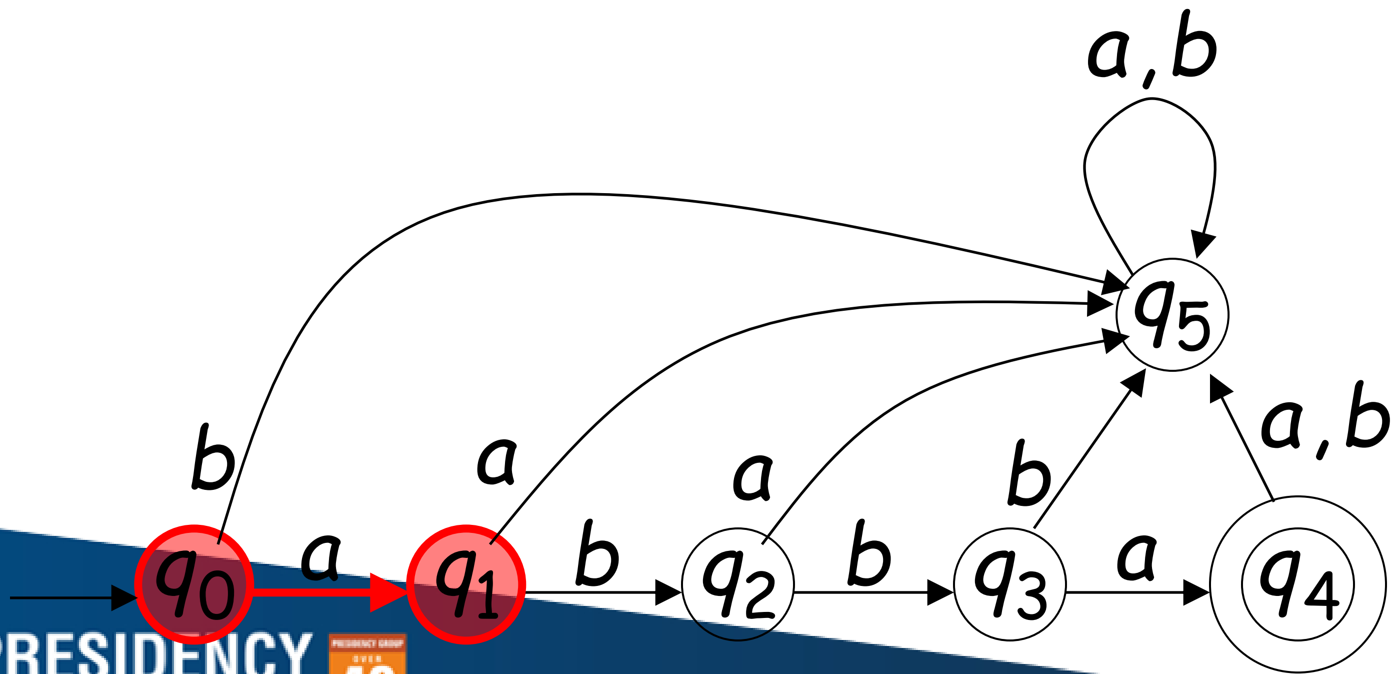


Transition Function δ

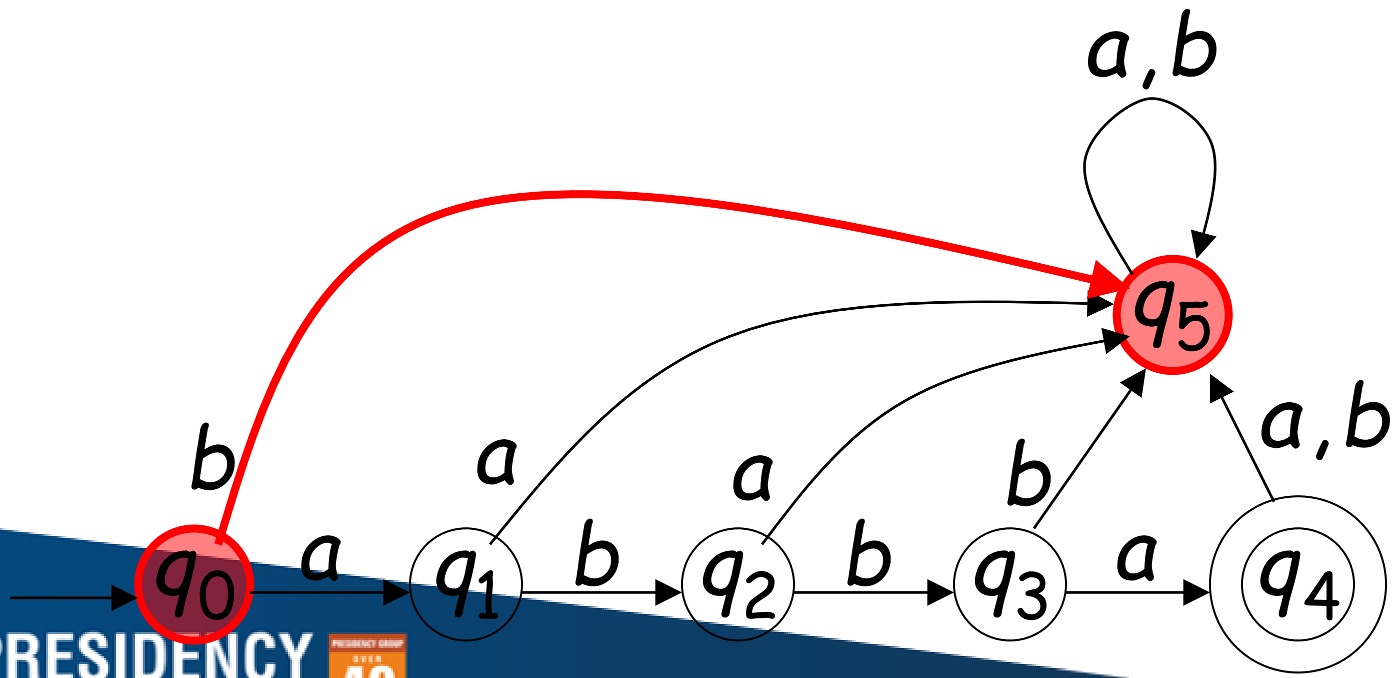
- $$\delta : Q \times \Sigma \rightarrow Q$$



$$\delta(q_0, a) = q_1$$



$$\delta(q_0, b) = q_5$$



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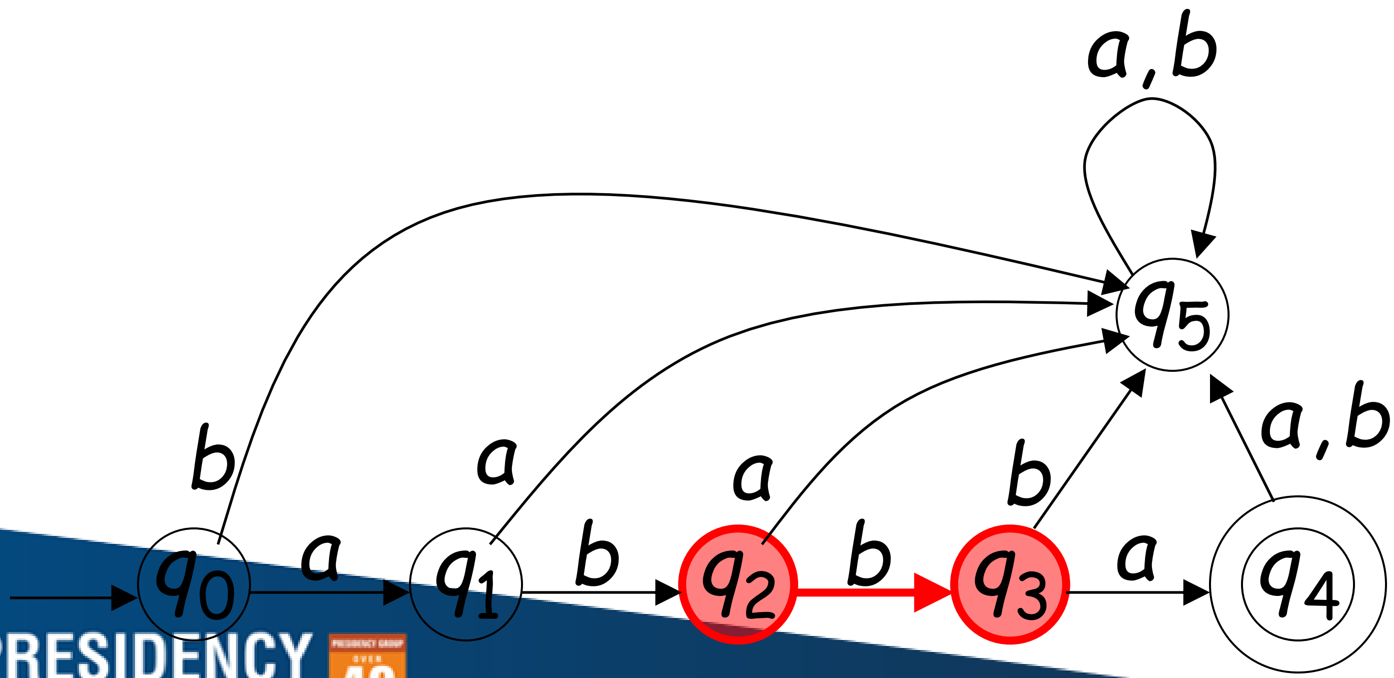
$$\delta(q_2, b) = q_3$$



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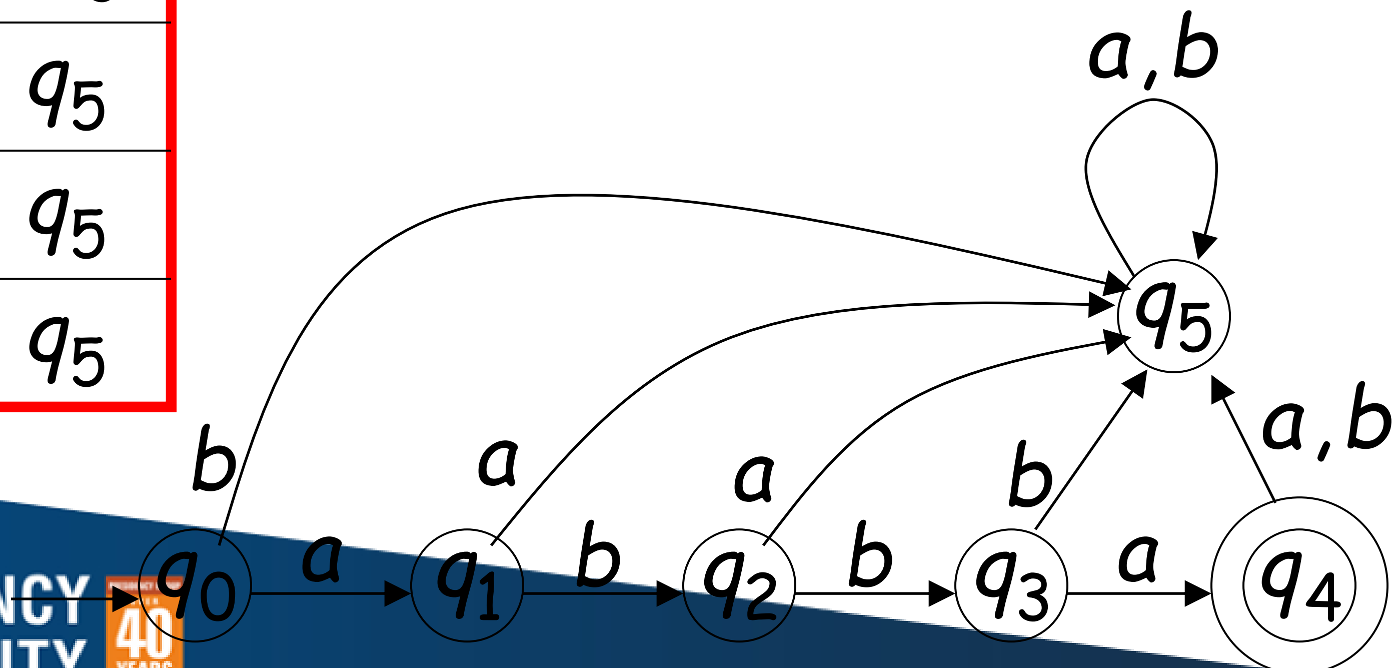
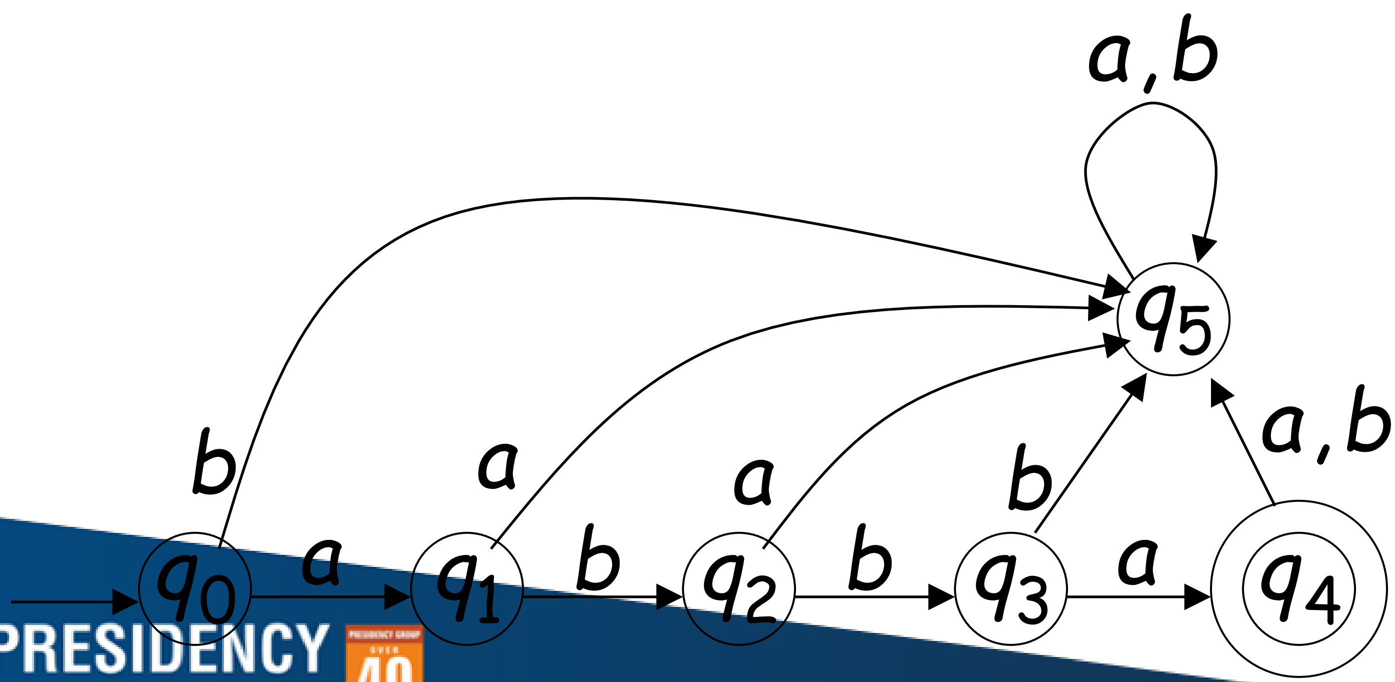
Transition Table δ

δ	a	b
$\rightarrow q_0$	q_1	q_5
q_1	q_5	q_2
q_2	q_5	q_3
q_3	q_4	q_5
q_4	q_5	q_5
q_5 *	q_5	q_5

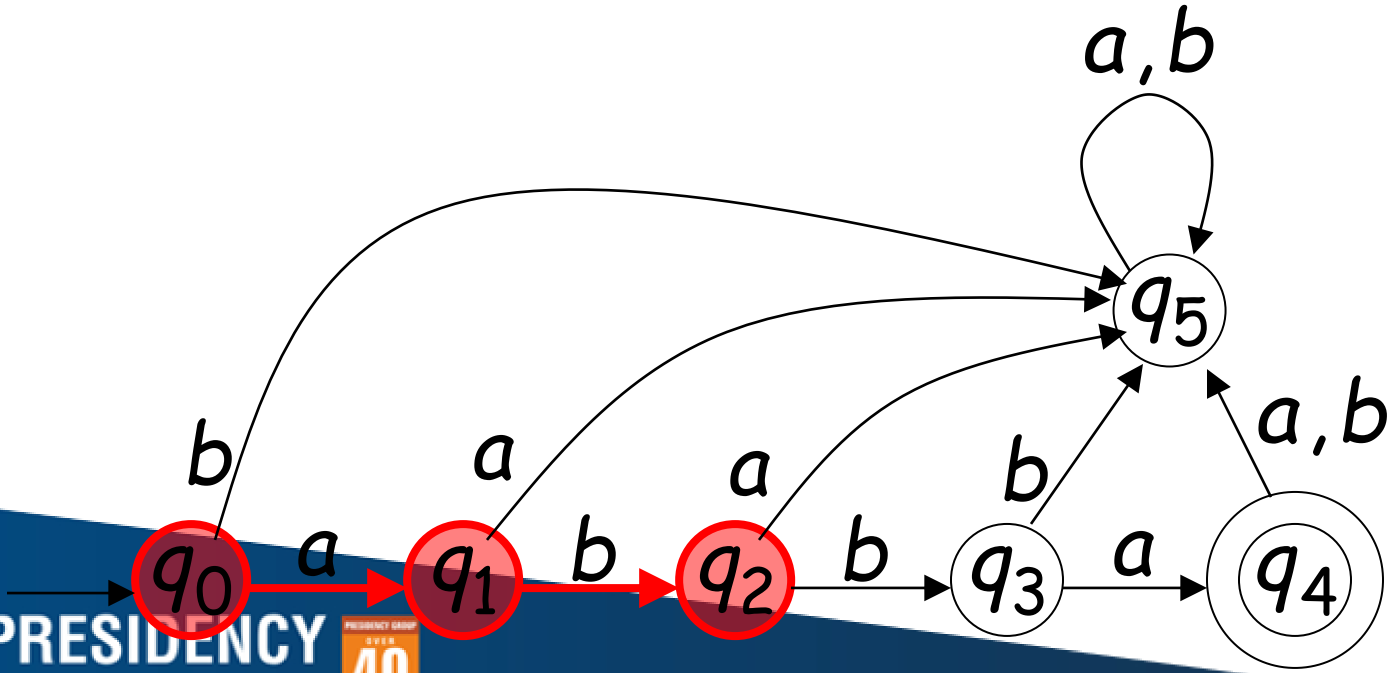


Extended Transition Function δ^*

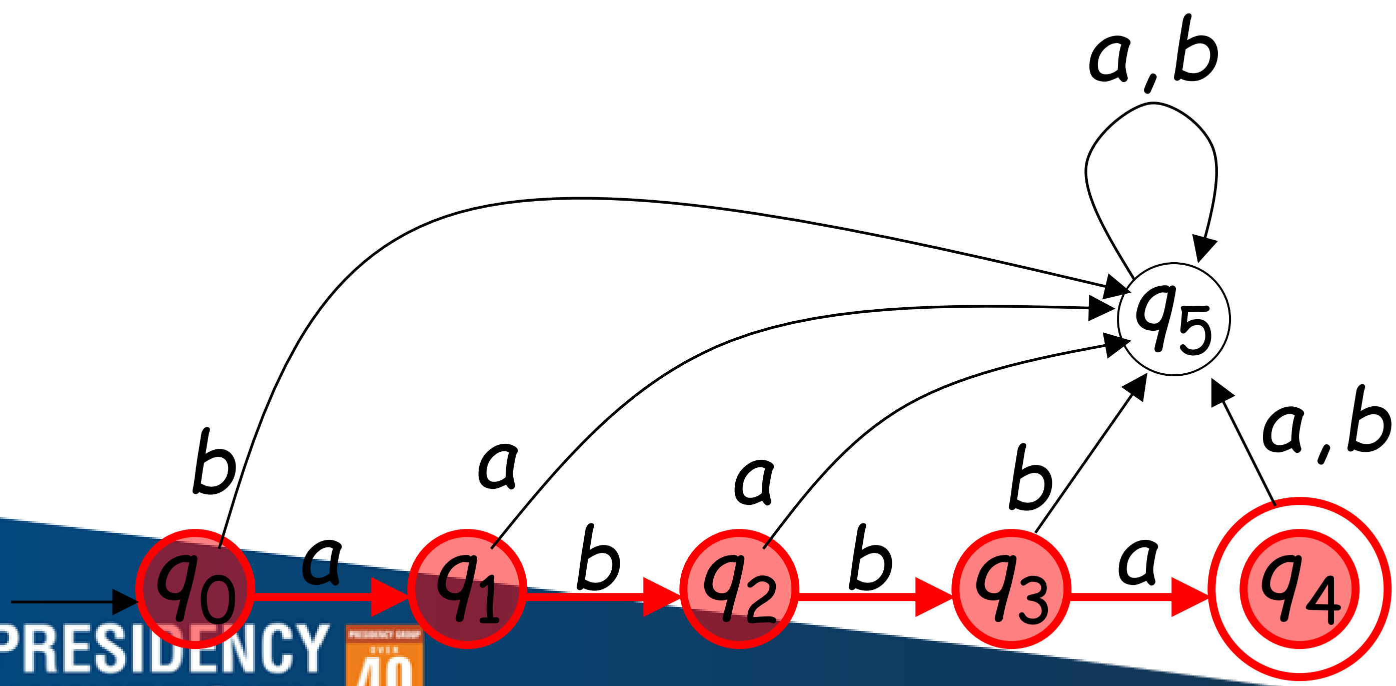
- $$\delta^* : Q \times \Sigma^* \rightarrow Q$$



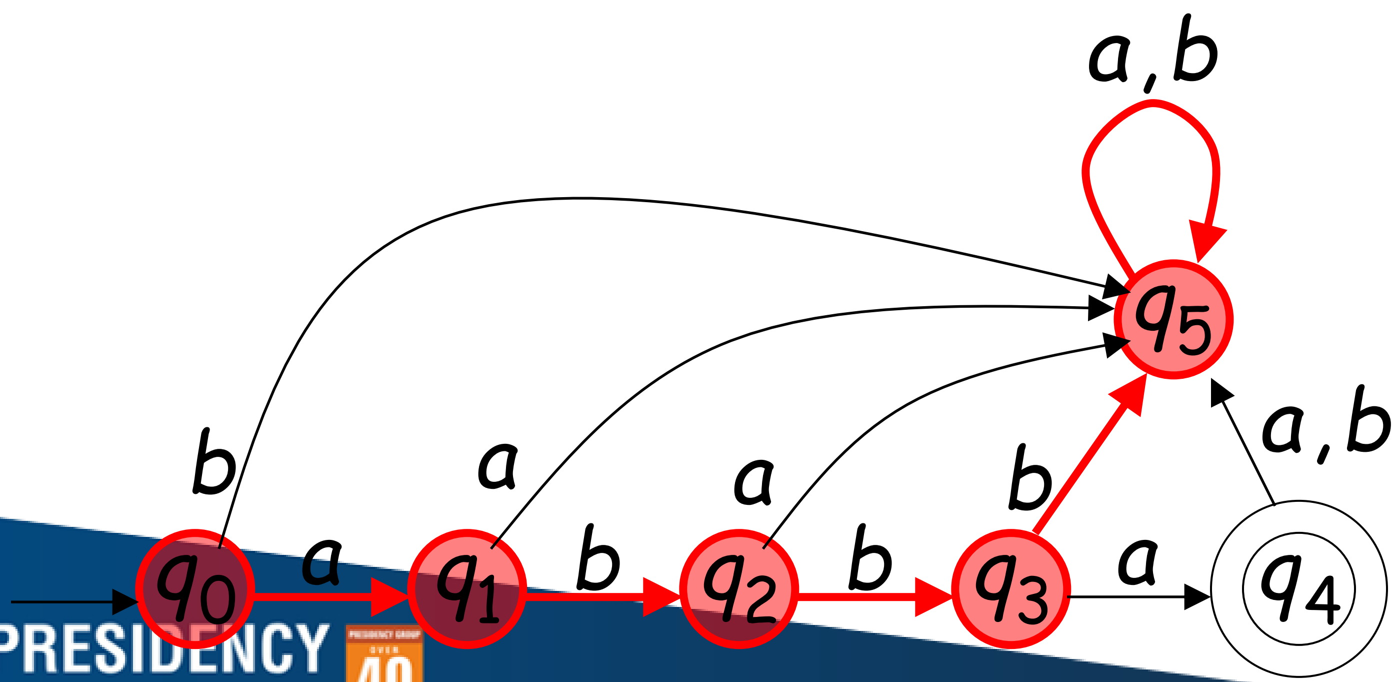
$$\delta^*(q_0, ab) = q_2$$



$$\delta^*(q_0, abba) = q_4$$

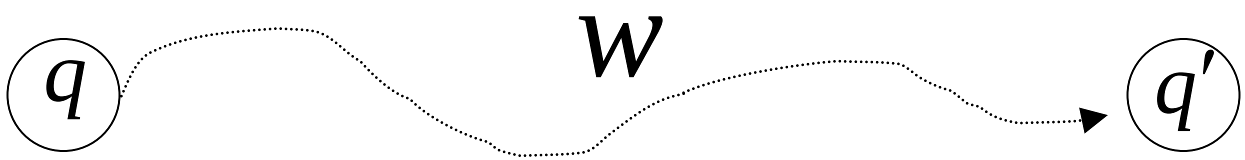


$$\delta^*(q_0, abbbaa) = q_5$$

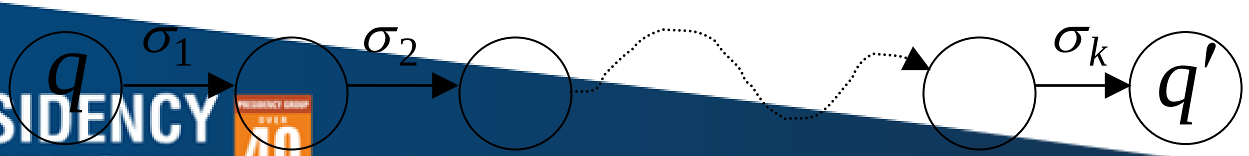


Observation: if there is a walk from q to q' with label w then

$$\delta^*(q,w) = q'$$



$$w = \sigma_1 \sigma_2 \cdots \sigma_k$$



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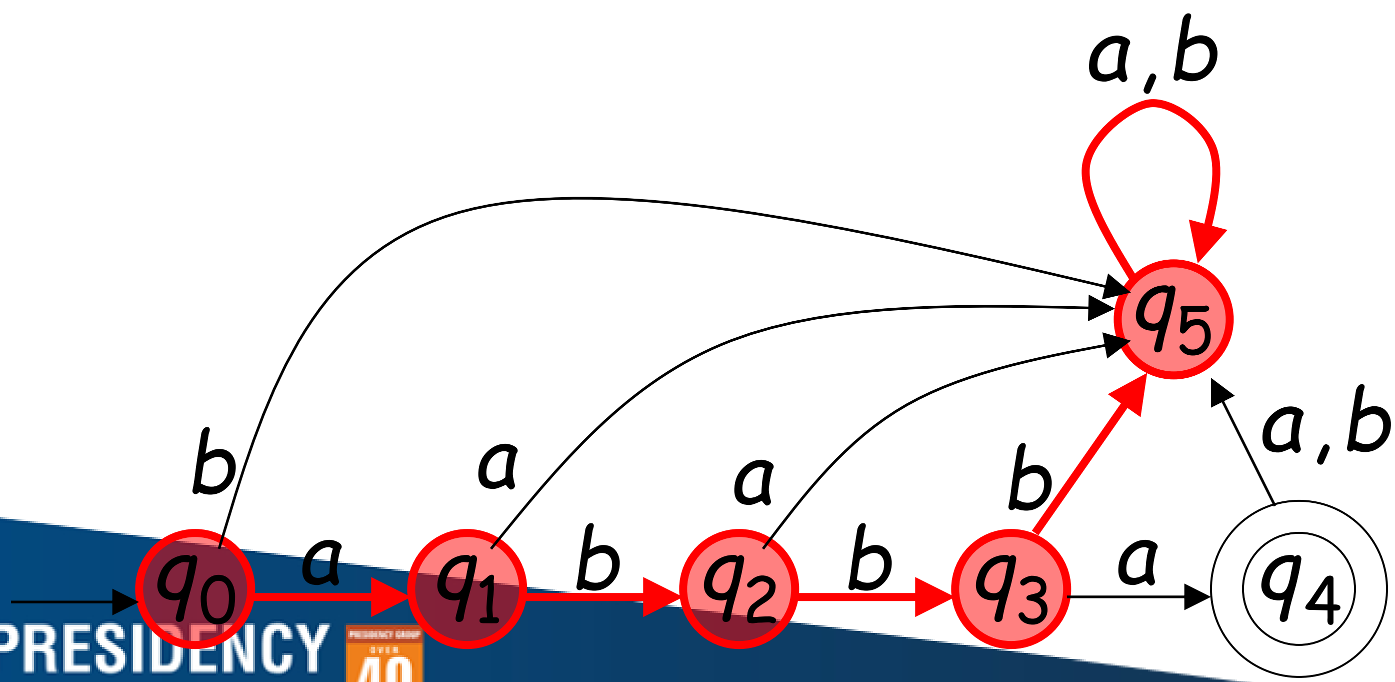
Example: There is a walk from q_0 to q_5 with label $abbbaa$

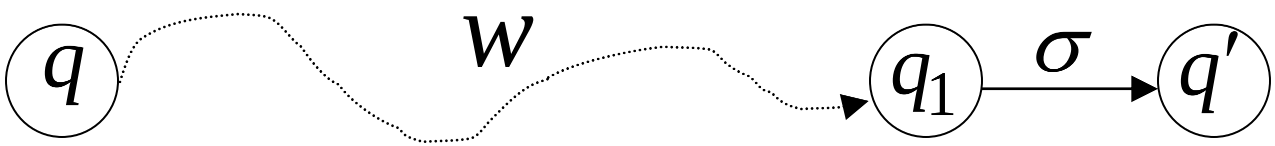
$$\delta^*(q_0, abbbaa) = q_5$$



Recursive Definition

$$\delta^*(q,\lambda) = q$$

$$\delta^*(q,w\sigma) = \delta(\delta^*(q,w),\sigma)$$



$$\left.\begin{array}{l}\delta^*(q,w\sigma) = q' \\ \\ \delta(q_1,\sigma) = q'\end{array}\right\} \Rightarrow \delta^*(q,w\sigma) = \delta(q_1,\sigma)$$

$$\left.\begin{array}{l}\delta^*(q,w\sigma) = \delta(q_1,\sigma) \\ \\ \delta^*(q,w) = q_1\end{array}\right\} \Rightarrow \delta^*(q,w\sigma) = \delta(\delta^*(q,w),\sigma)$$

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$$\delta^*(q_0, ab) =$$

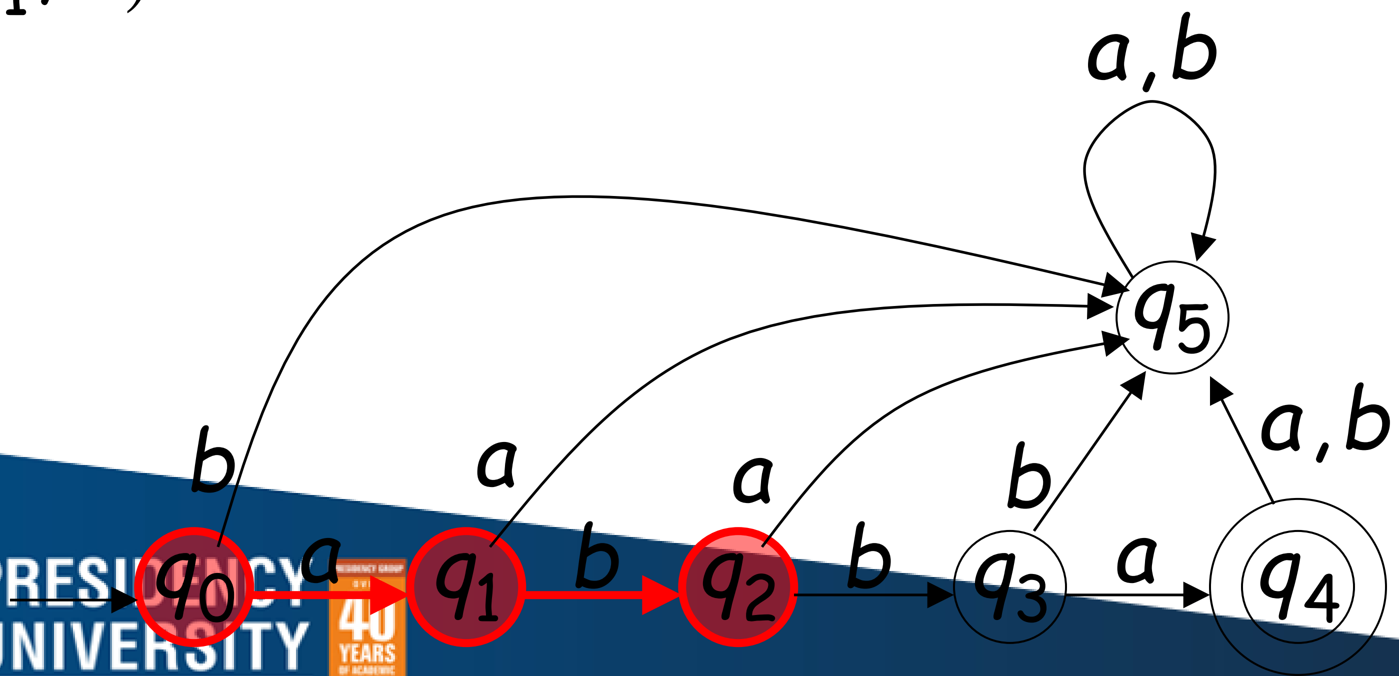
$$\delta(\delta^*(q_0, a), b) =$$

$$\delta(\delta(\delta^*(q_0, \lambda), a), b) =$$

$$\delta(\delta(q_0, a), b) =$$

$$\delta(q_1, b) =$$

$$q_2$$



Check the string acceptance of abba

- $\delta^*(qo,abba)= \delta(\delta^*(q0,abb) , a)$
- $= \delta(\delta (\delta^*(q0,ab),b) , a)$
- $= \delta(\delta (\delta (\delta^*(q0,a),b),b) , a)$
- $= \delta(\delta (\delta (\delta (\delta^*(q0,\lambda),a),b),b) , a)$
- $= \delta(\delta (\delta (\delta (q0,a),b),b) , a)$
- $=\delta(\delta (\delta (q1,b),b) , a)$
- $=\delta(\delta (q2,b) , a)$
- $=\delta(q3, a)$
- $=q4 \in F$
- String abba is accepted as q4 is a final state

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Language Accepted by FAs

- For a FA $M=(Q,\Sigma,\delta,q_0,F)$

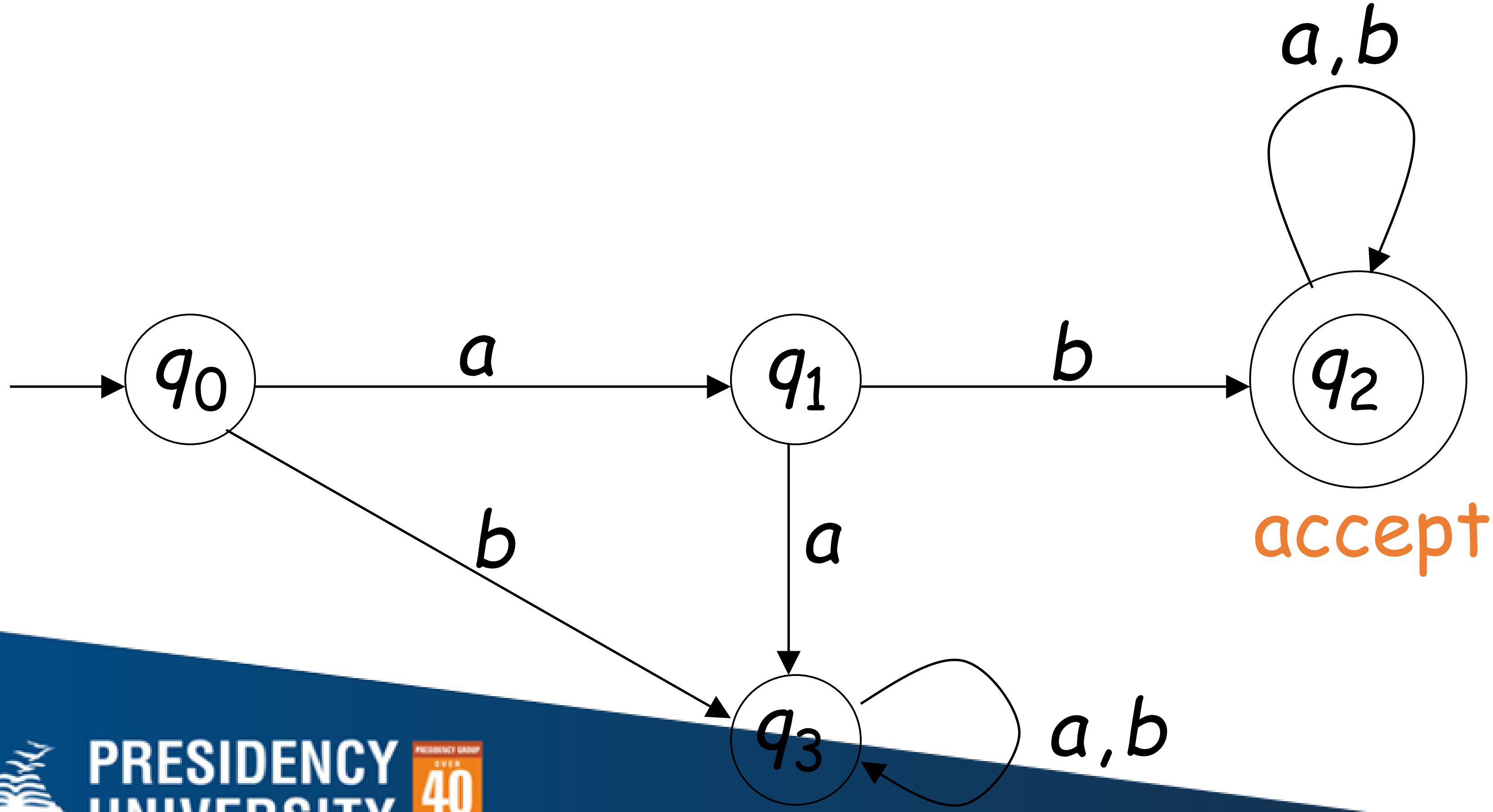
- Language accepted by M :

- $$L(M)=\{w\in\Sigma^*:\delta^*(q_0,w)\in F\}$$

$q'\in F$

Example

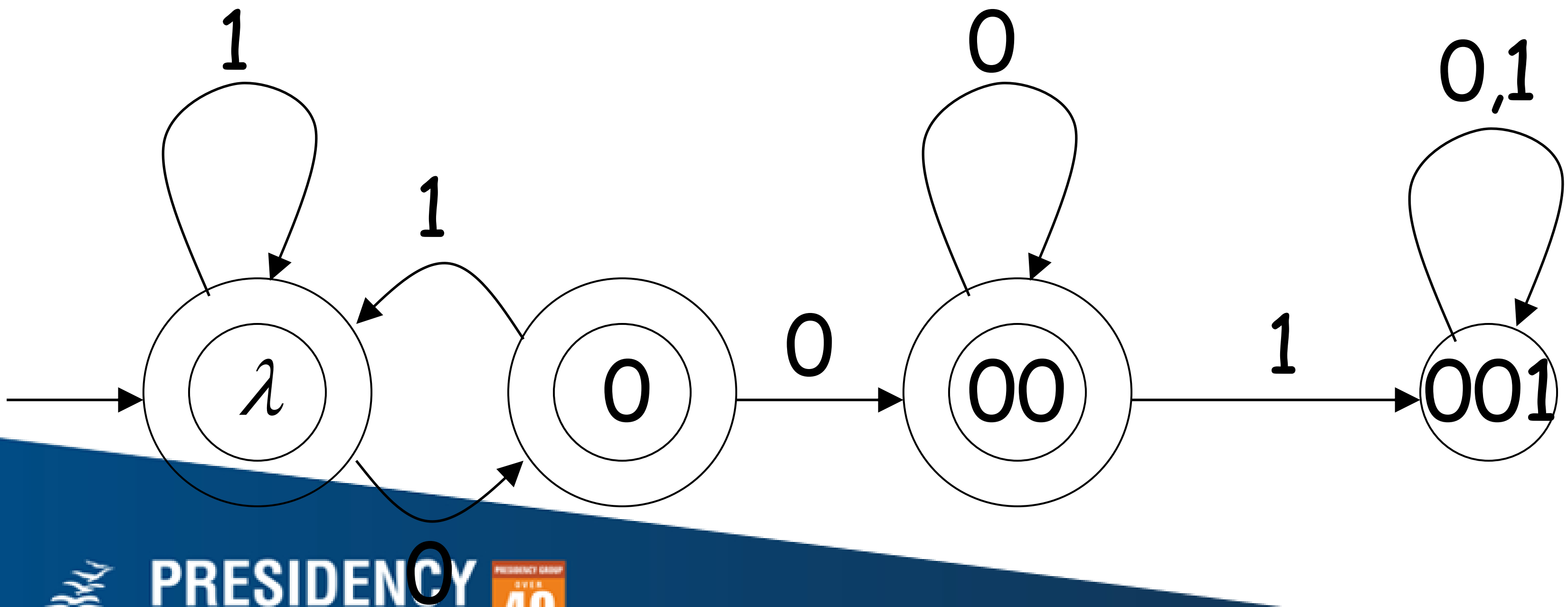
- $L(M)$ = { all strings with prefix ab }



L=strings with substring ‘101’ over {0, 1}

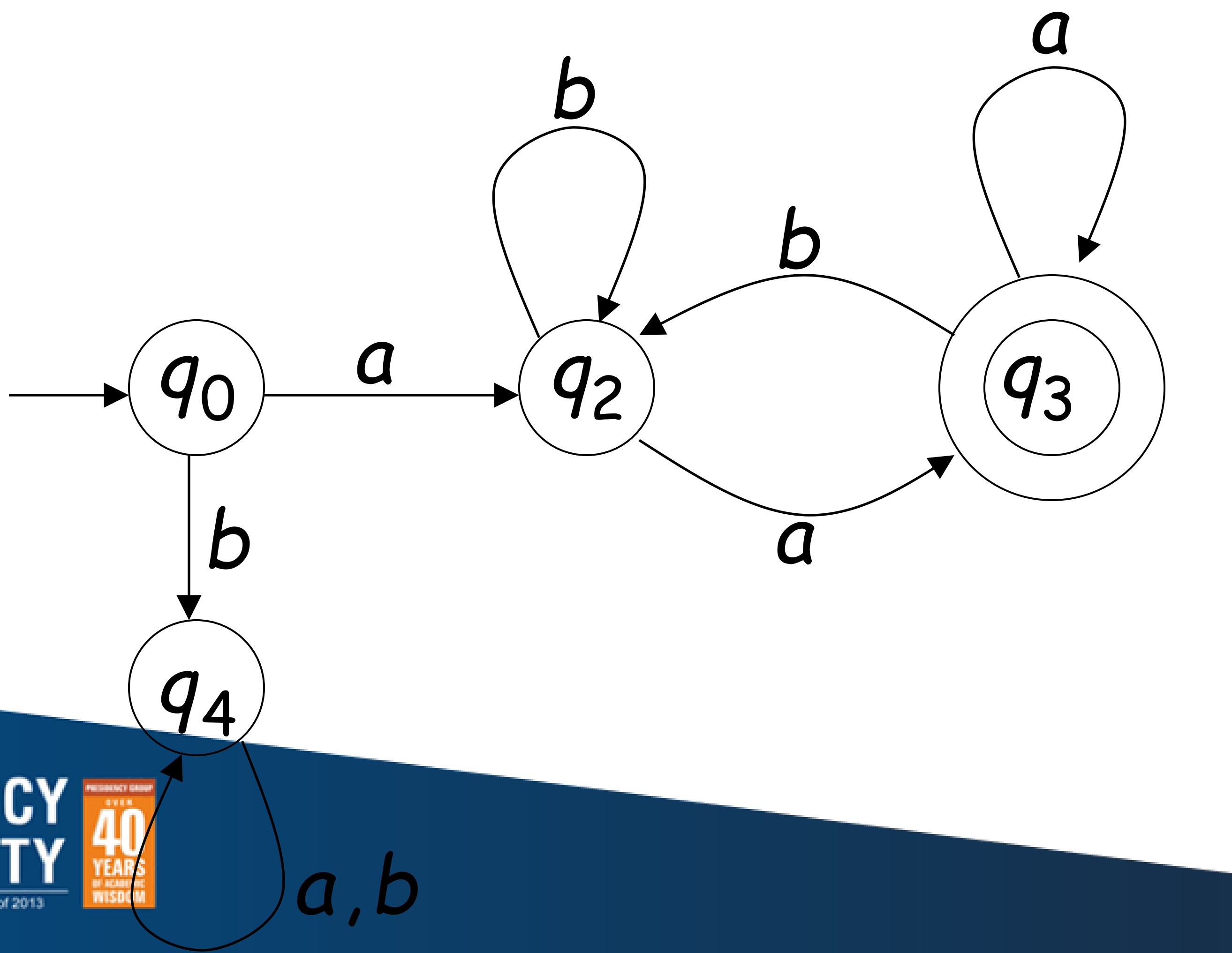
Example

- $L(M) = \{$ all strings without substring 001 $\}$



Example

$$L(M) = \{awa : w \in \{a,b\}^*\}$$

- 

Regular Languages

- Definition:
- A language L is regular if there is
- FA M such that $L = L(M)$

- Observation:
- All languages accepted by FAs
- form the family of regular languages
  -

Examples of regular languages:

$\{abba\}$ $\{\lambda, ab, abba\}$

$\{awa : w \in \{a,b\}^*\}$ $\{a^n b : n \geq 0\}$

{ all strings with prefix ab }

{ all strings without substring 001 }

There exist automata that accept these Languages

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There exist languages which are <u>not</u> Regular:

Example: $L = \{a^n b^n : n \geq 0\}$

There is no FA that accepts such a language

(we will prove this later in the class)

Deterministic Finite Automata

- Every State should have transition over every input symbol
- There should be only One next state for each transition
- DFA is defined as

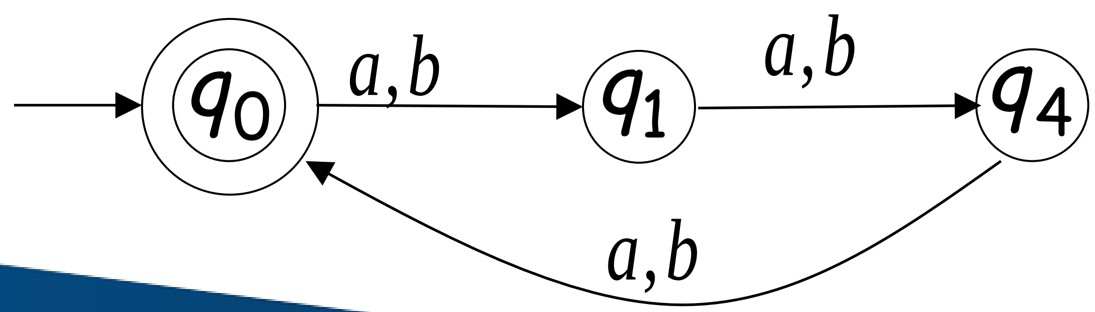
$$M = (Q, \Sigma, \delta, q_0, F)$$

DFA Example

Design DFA that accepts language L= { w : |w| mod 3 = 0 } over ∑ = {a, b}

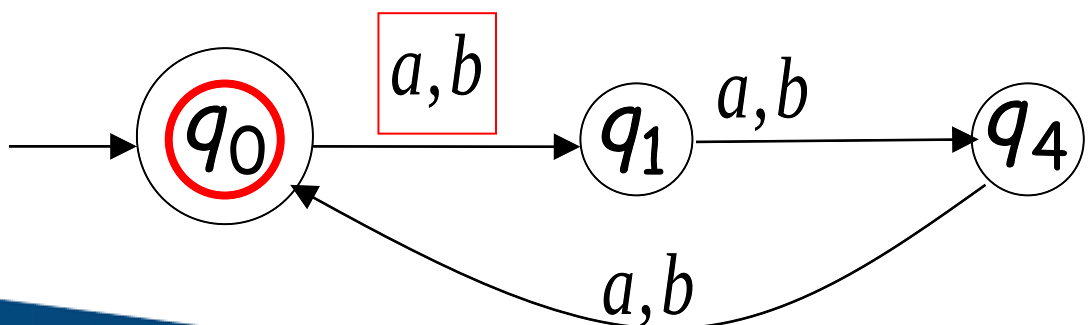
Solution

- **Strings accepted= {ϵ, aaa, bbb, aba, aab, bab, aaabbb, ababab, ...}**
- **Strings rejected= {a, b, ab, ba, abab, baba, bbaa, aaabb, …}**
- **Transition Diagram**



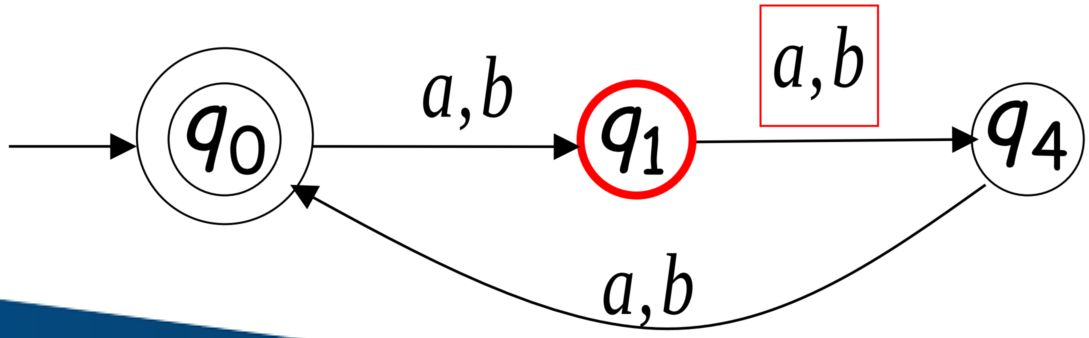
DFA Example

- Consider Sample String : **a a b**



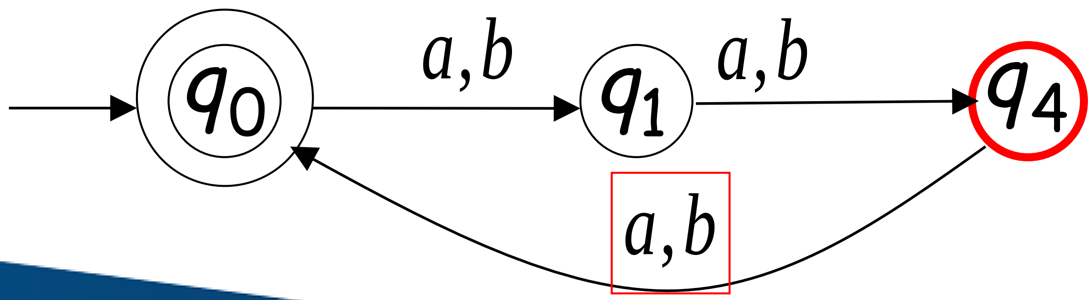
DFA Example

- Consider Sample String : **a a b**



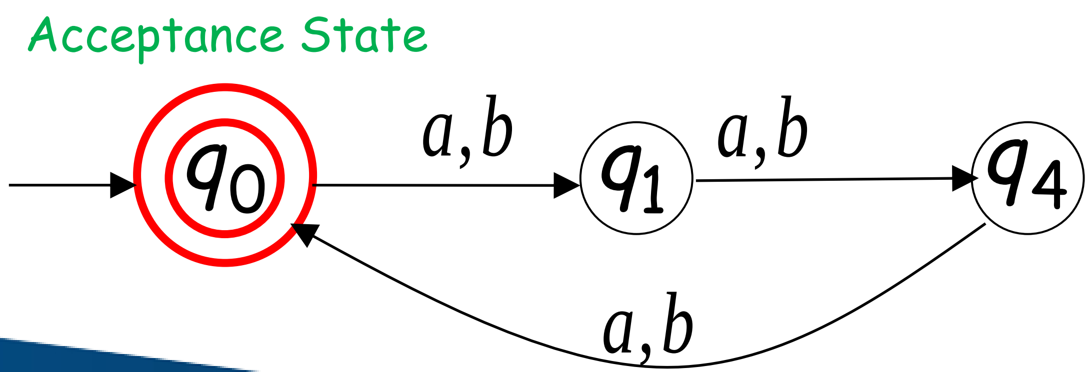
DFA Example

- Consider Sample String : **a a b**



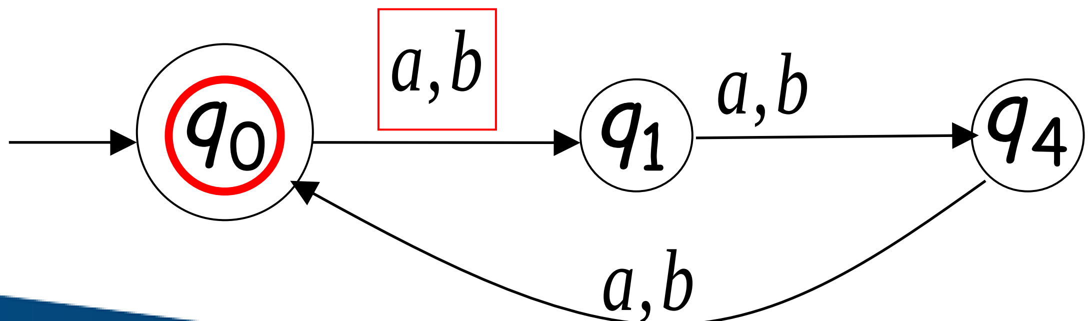
DFA Example

- ➢Consider Sample String : **a a b**
- ➢ String is accepted



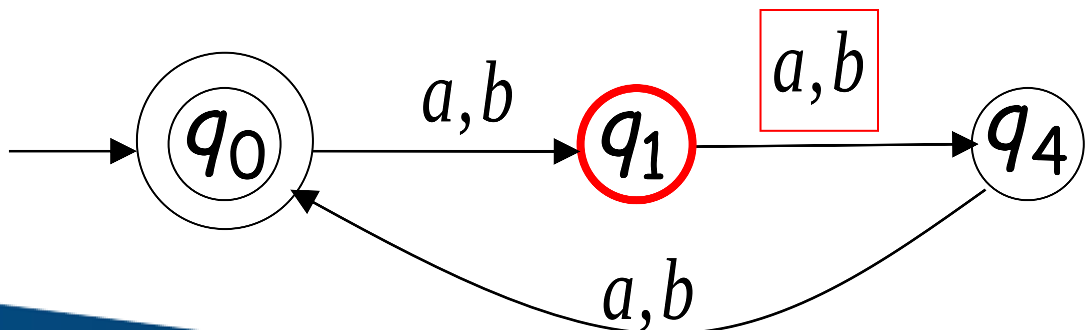
DFA Example

- Consider Another Sample String : **b a b a**



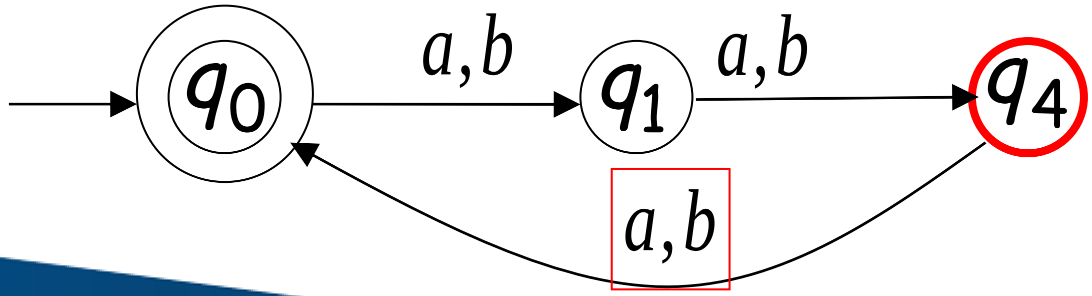
DFA Example

- Consider Another Sample String : **b a b a**



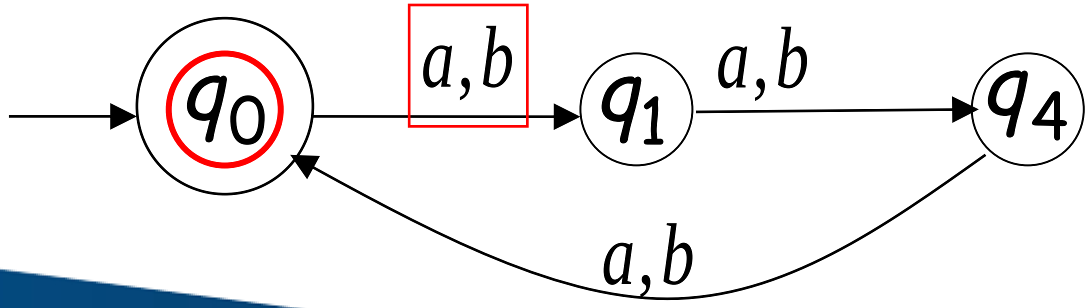
DFA Example

- Consider Another Sample String : **b a b a**



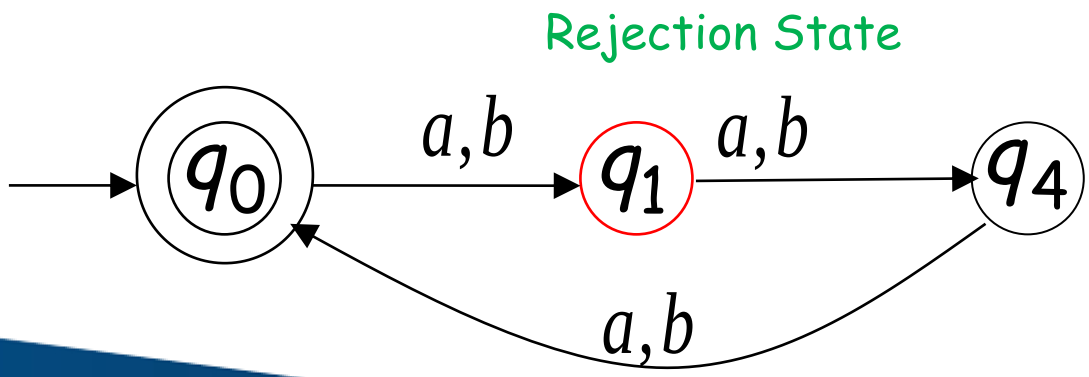
DFA Example

- Consider Another Sample String : **b a b a**



DFA Example

- ➢ **Consider Another Sample String :** **b a b a**
- ➢ **String is rejected**



DFA Example

- DFA Tuples $M=(Q, \Sigma, \delta, q0, F)$
- $Q= \{q0, q1, q4\}$
- $\Sigma= \{a, b\}$
- δ = Transition function represented by Transition table
- q0= Initial State
- F= {q0} → Acceptance State

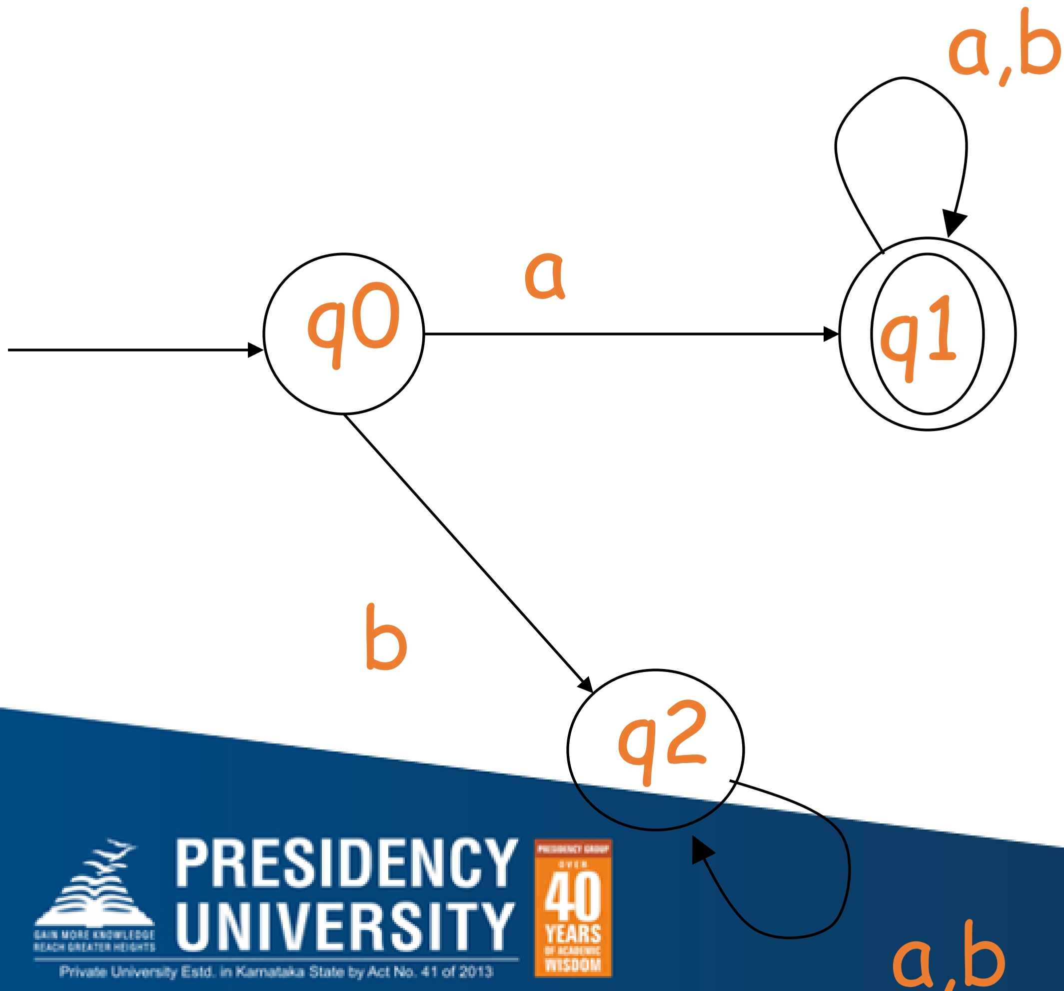
DFA Example

➢Transition Table

Σ Q	a	b
→*q0	q1	q1
q1	q4	q4
q4	q0*	q0*

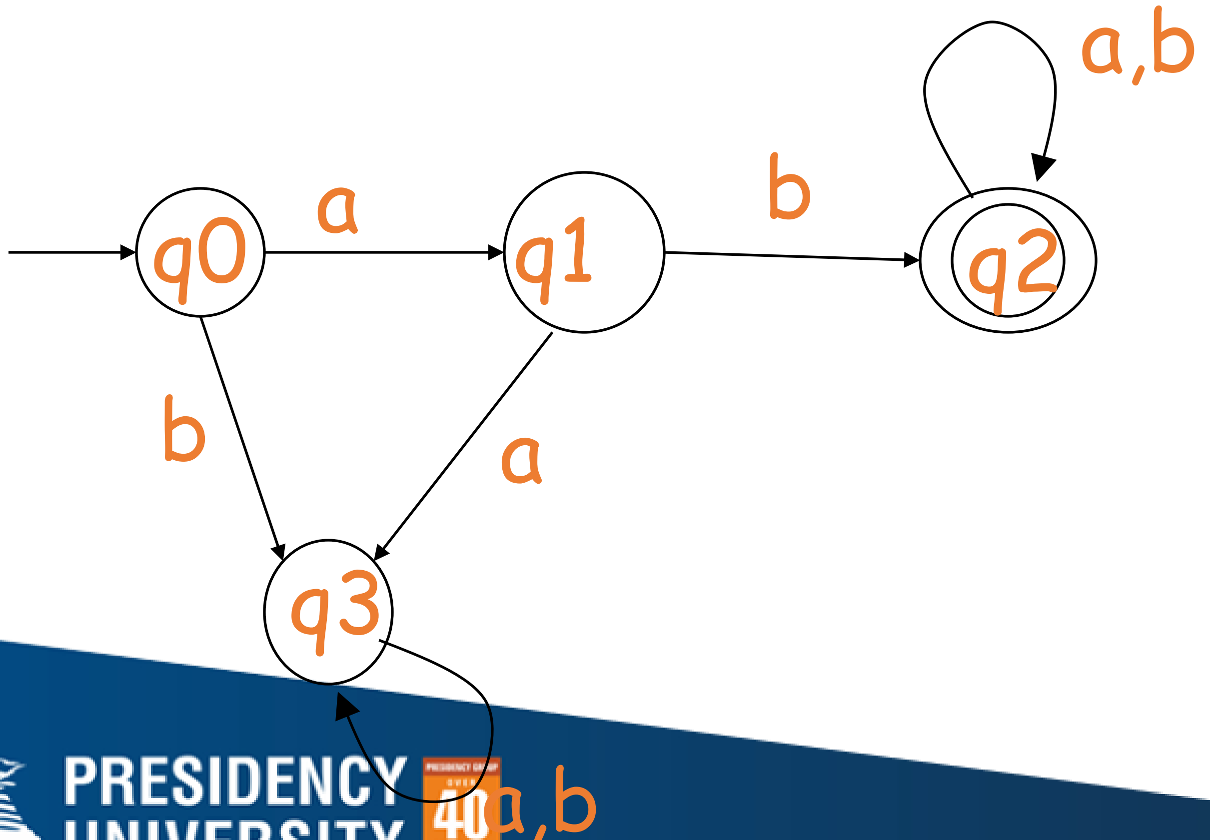
DFA - Starts with a , $\sum$={a,b}

- L= { a,aa,ab,aaa,aab,aba,….}



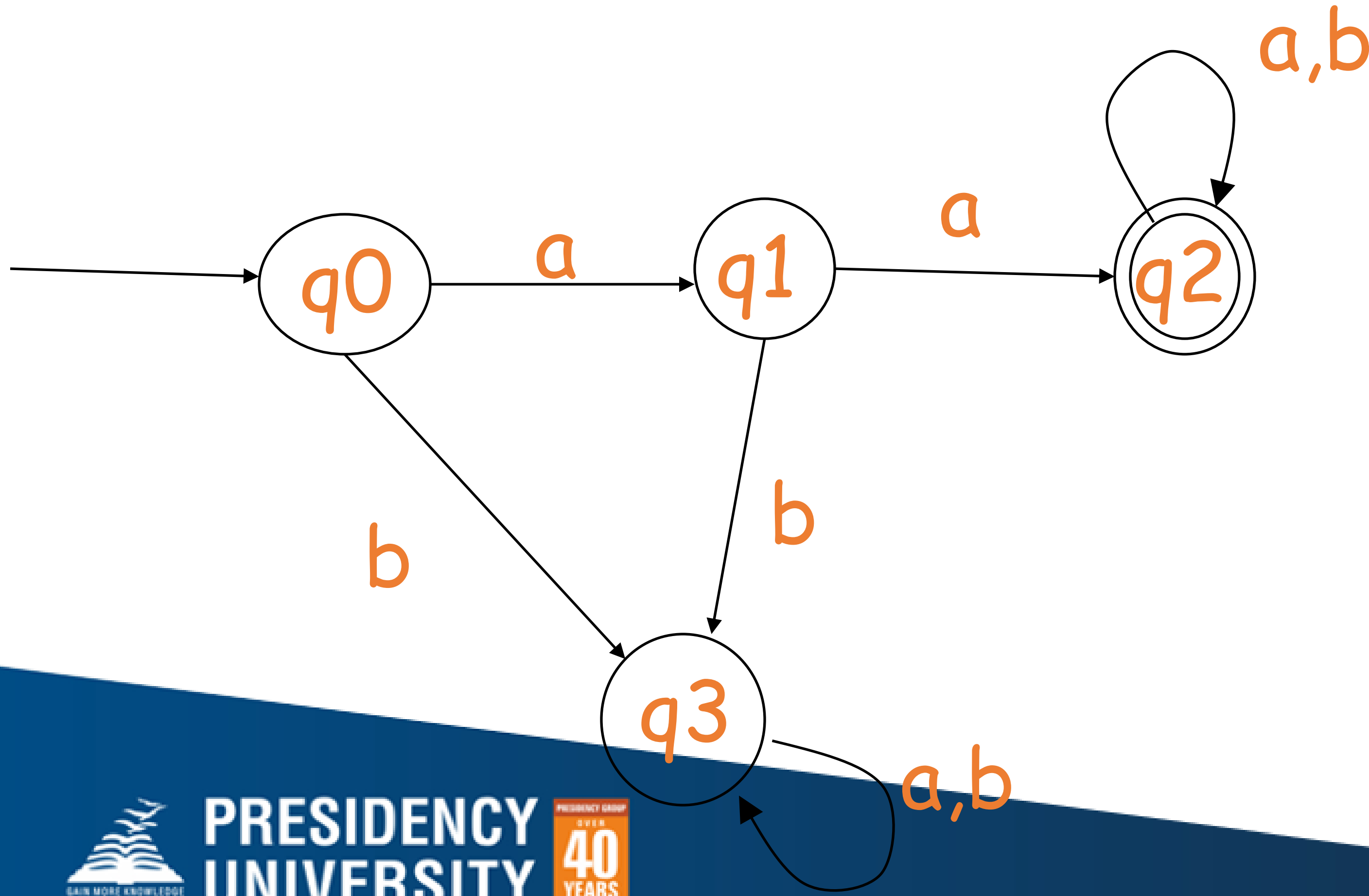
DFA - Starts with ab , ∑={a,b}

- L= { ab,aba,abb,….}



DFA - Starts with aa , $\Sigma=\{a,b\}$

- L = { aa,aab,aaba,....}



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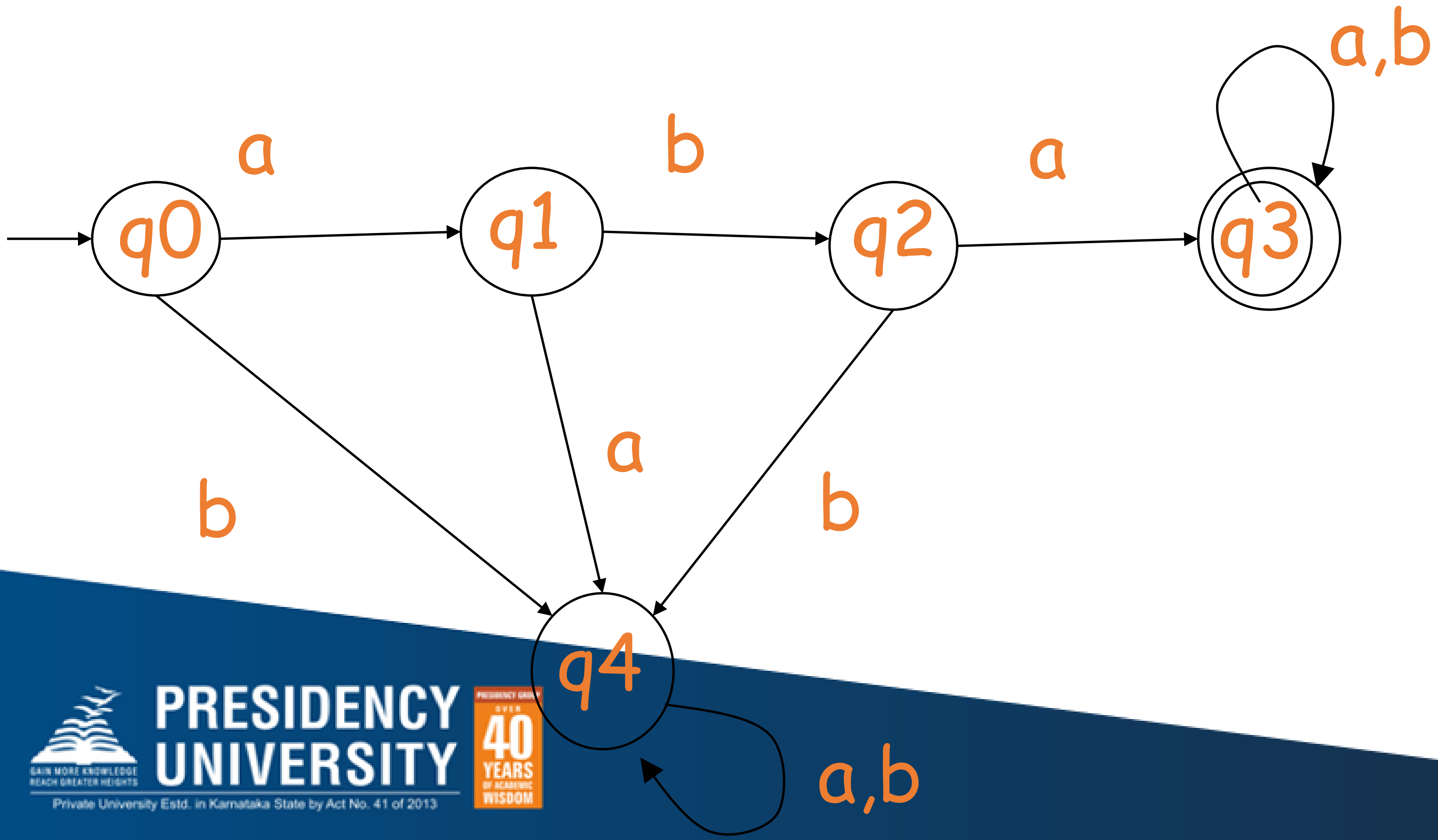
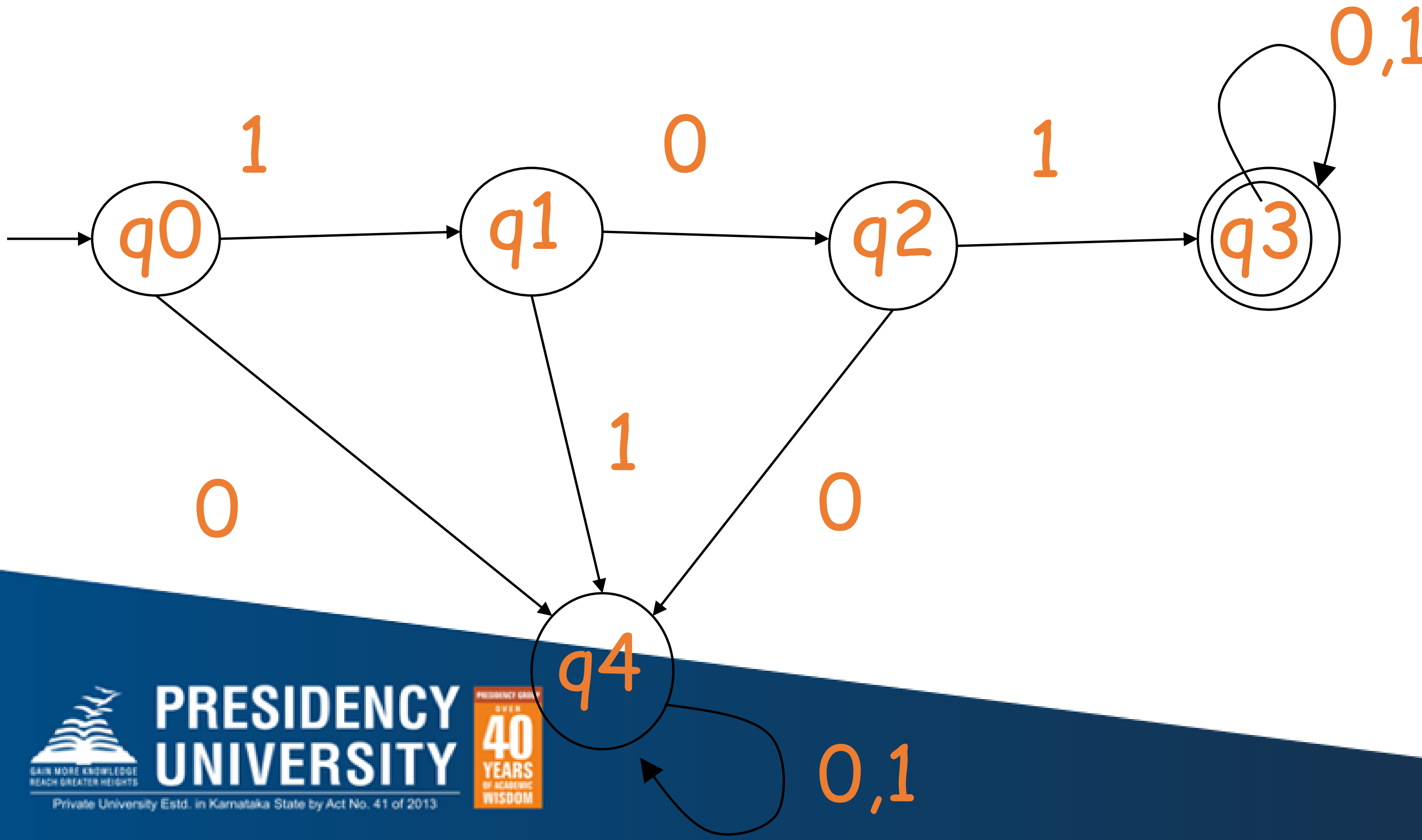
DFA - Starts with aba , $\sum$={a,b}

- L={ aba,abaa,abab,abaaa,…}



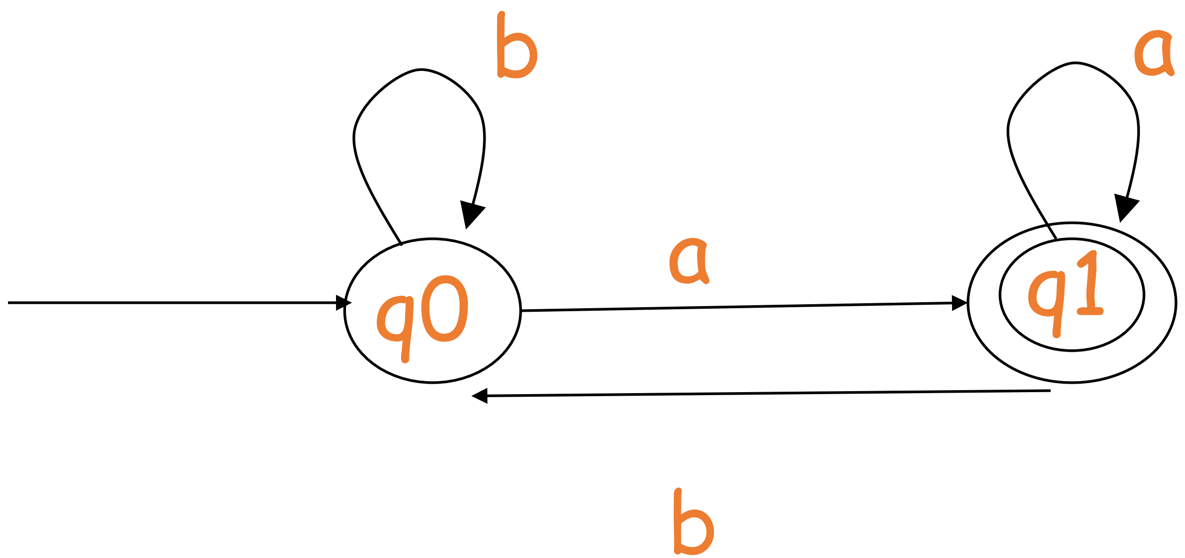
DFA - Starts with 101 , ∑={0,1}

- L={ 101,1010,1011,101101,...}



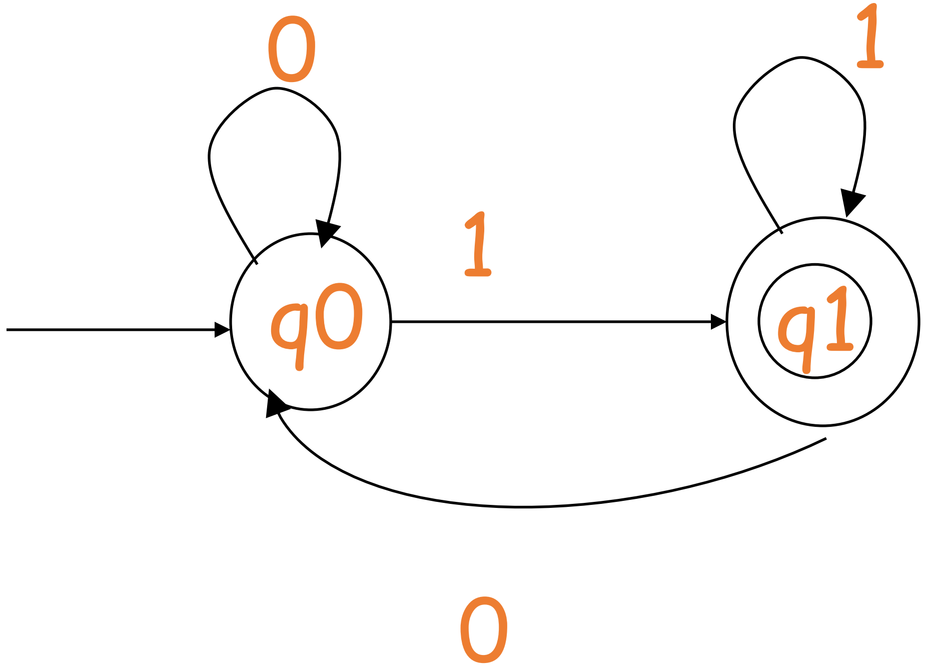
Ends with a

- L={a,aa,ba,aaa,aba,...}



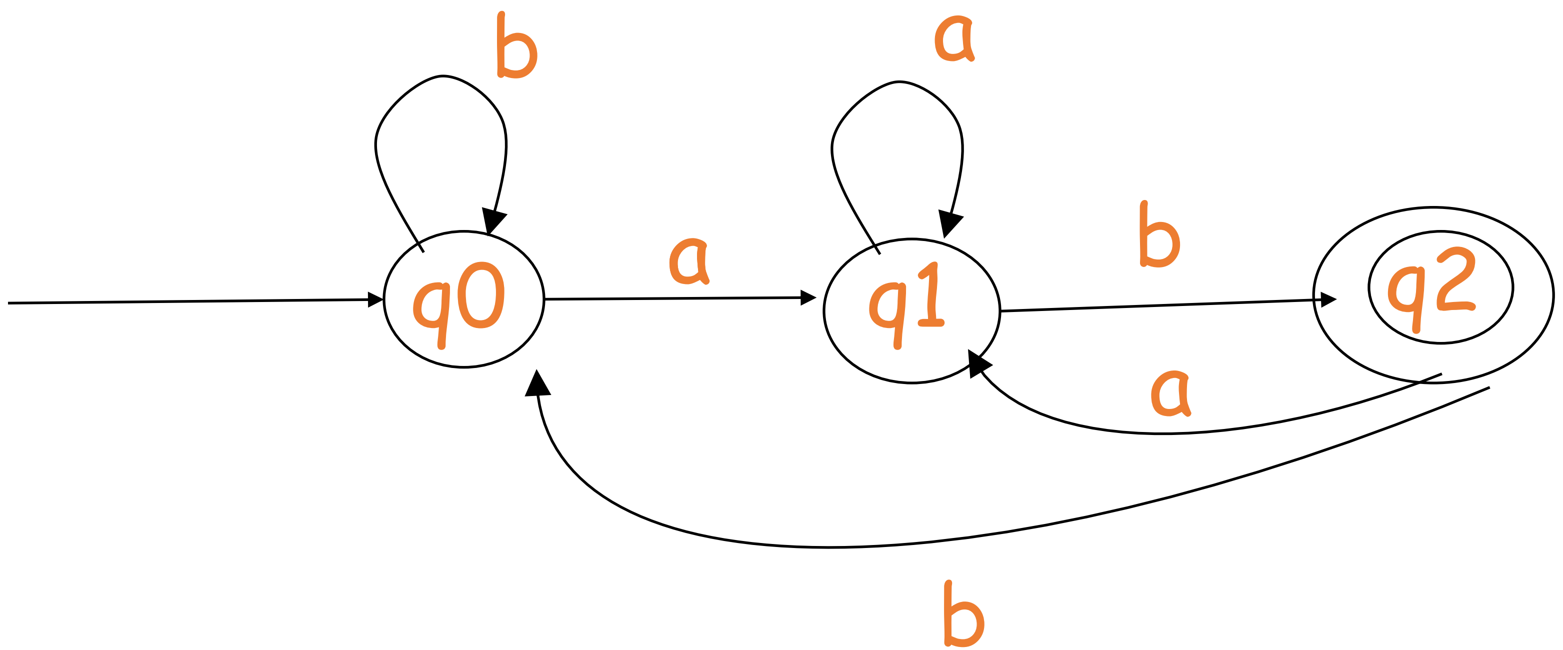
Ends with 1 , ∑={ 0,1)

- L={ 1,01,11,...}

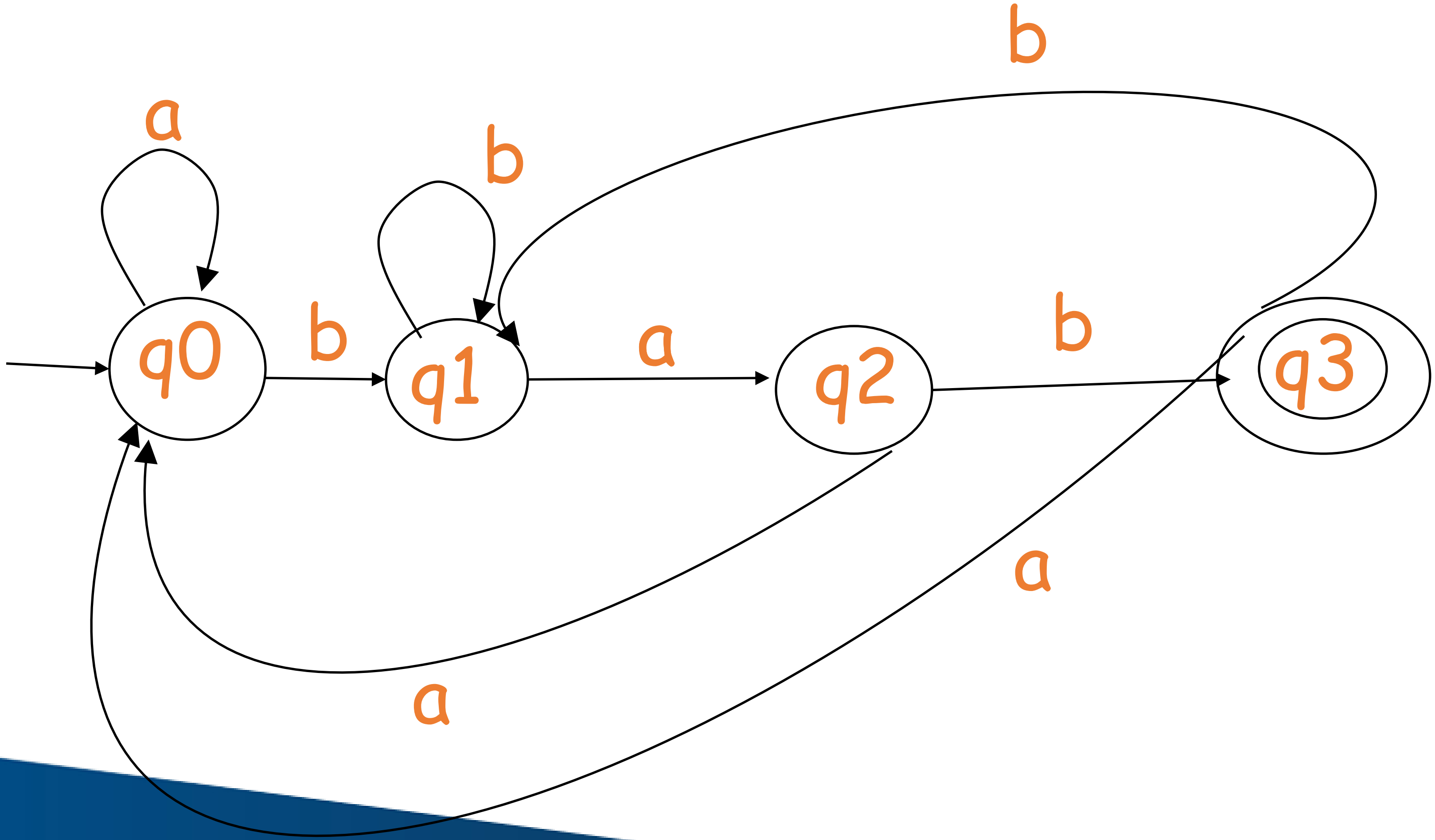


Ends with ab

- L={ab,aab,bab,aaab,bbbbab,abab,…}



Ending with bab



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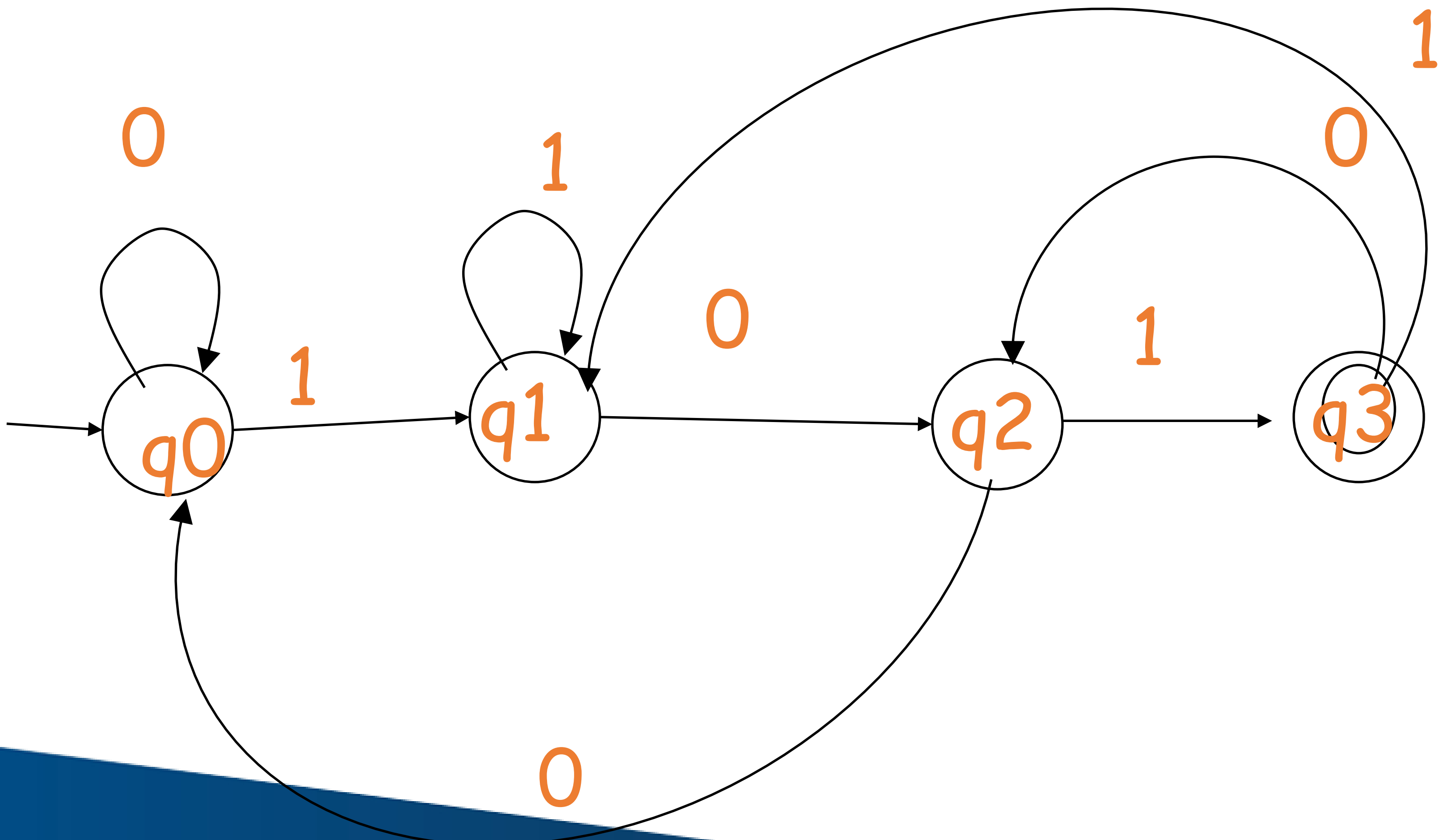
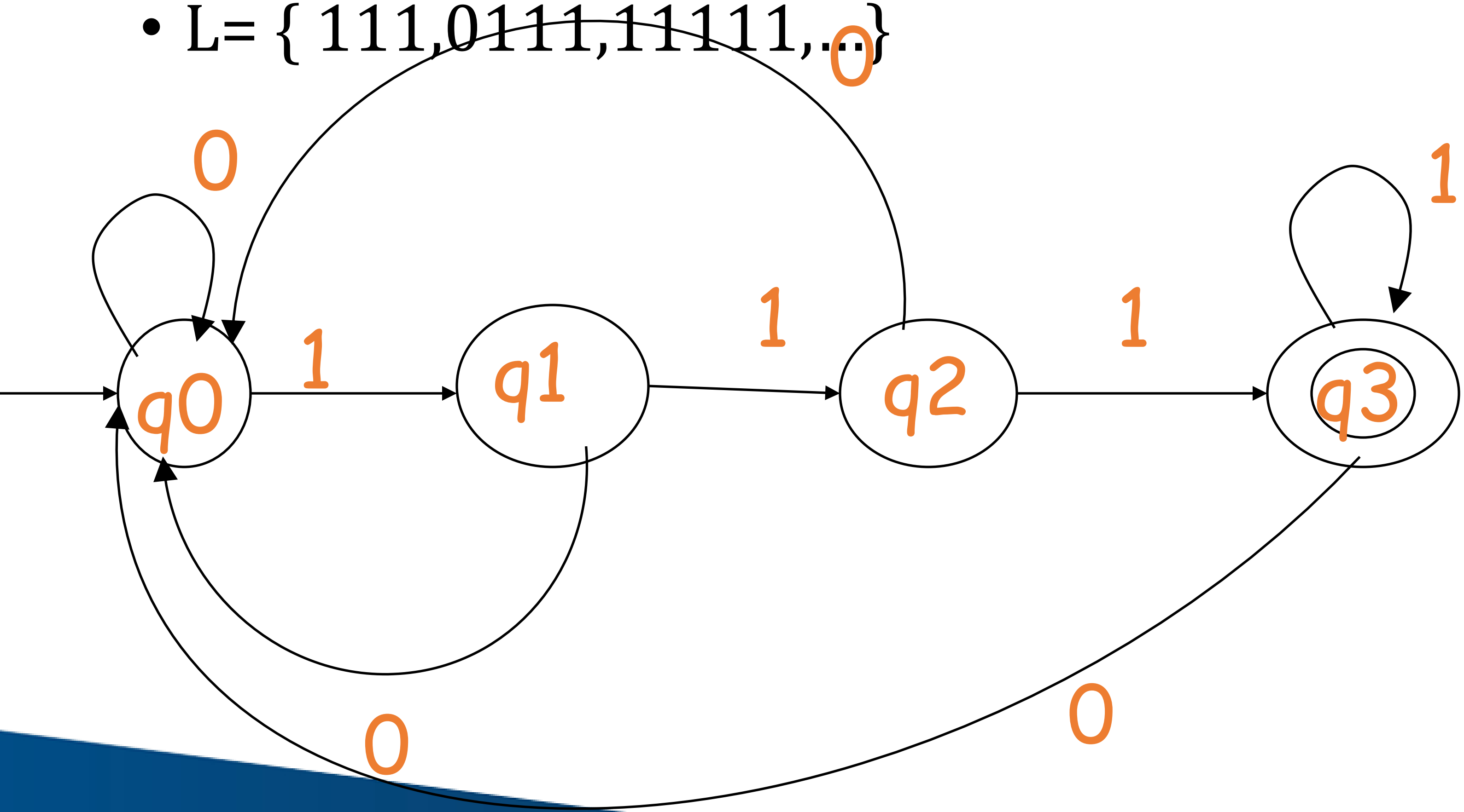
DFA – Ends with 101, Σ={ 0,1}

- L={ 101,0101,1101,000101,111101,…}



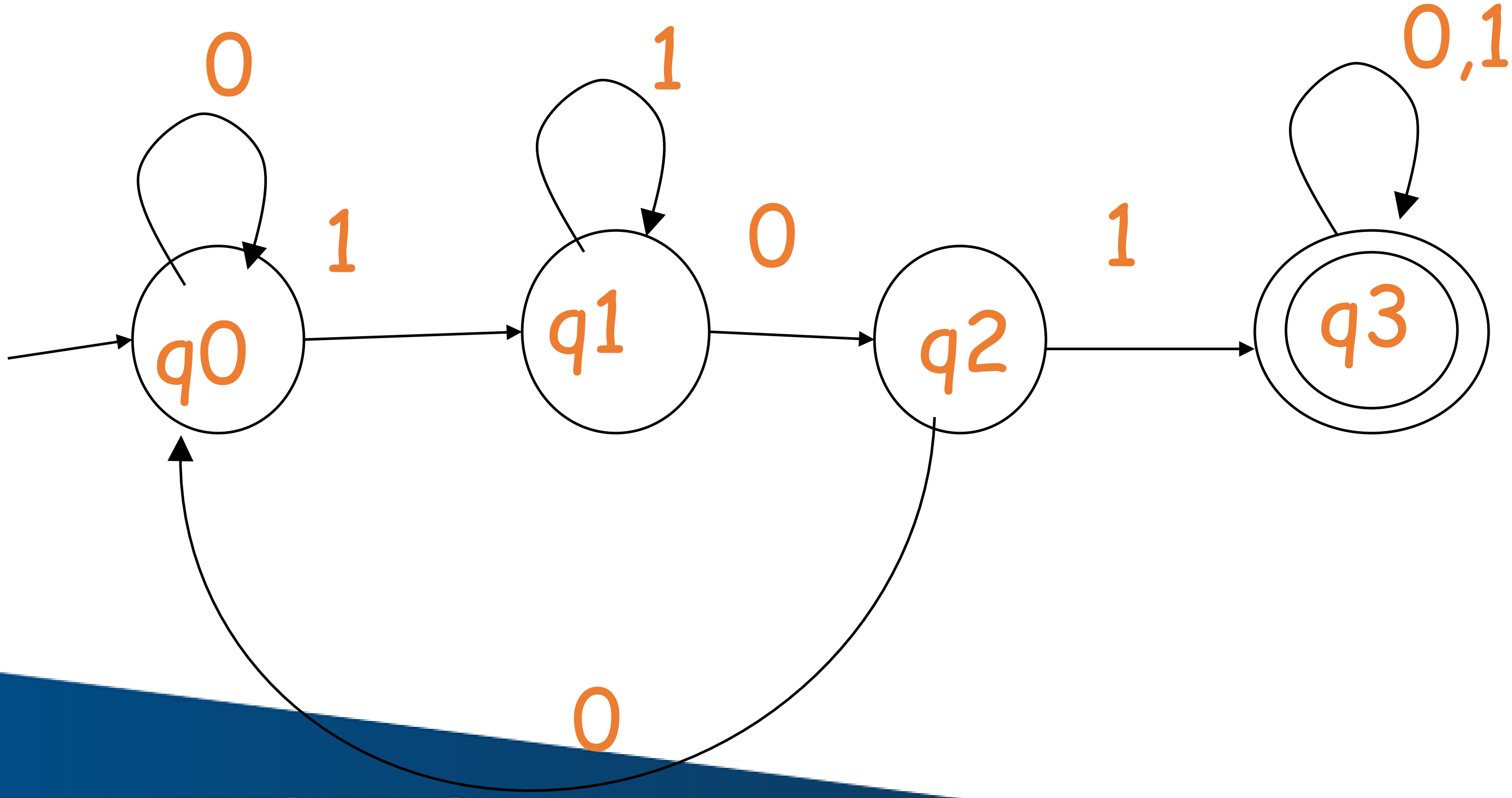
DFA – Ends with 111 , ∑={ 0,1}

- L= { 111,0111,11111,...}



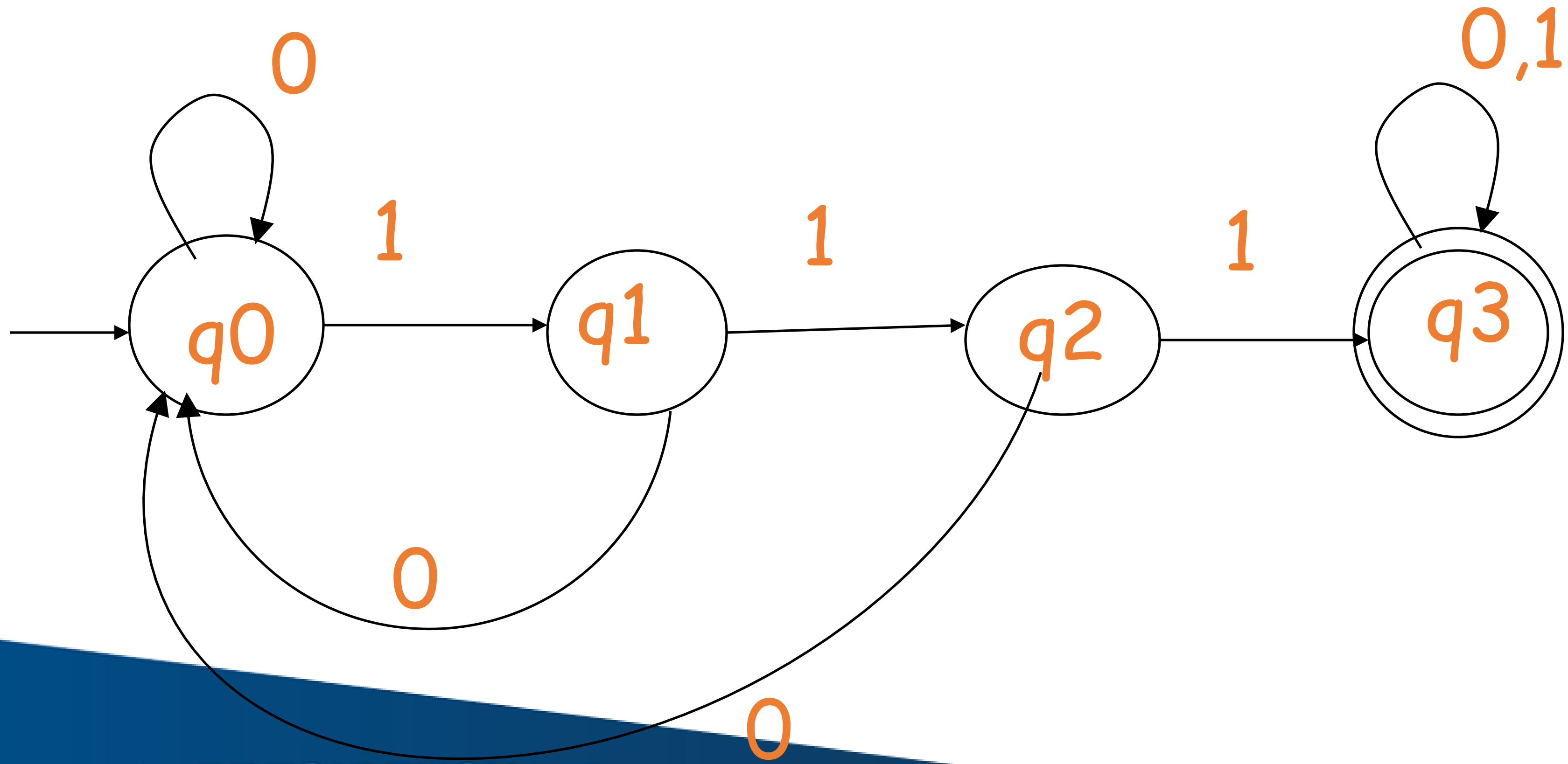
DfA – Substring 101

- L={ 101,0101,1101,1010,1011,00010111,...}



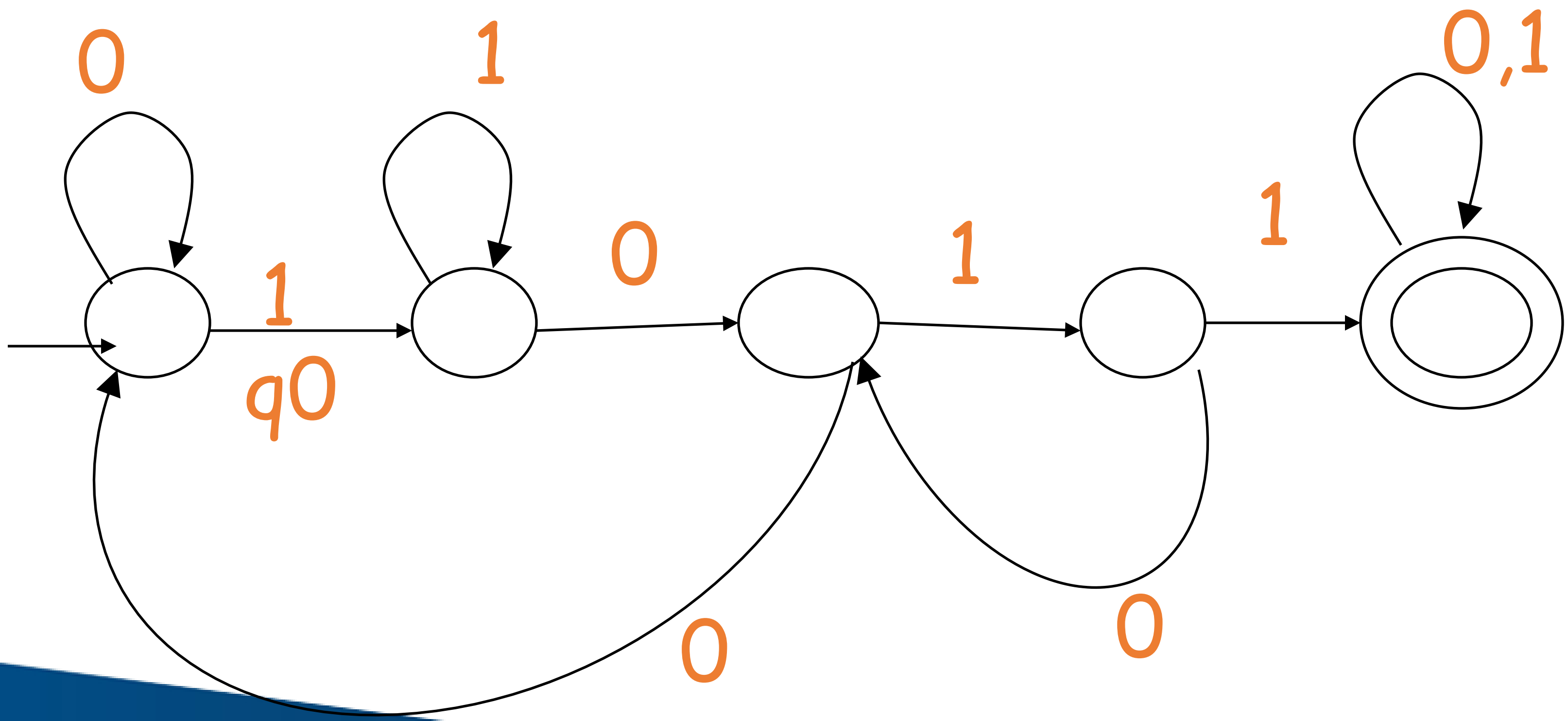
DFA- Contains 111

- L={ 111,0111,1111,000111000,11111000,..}



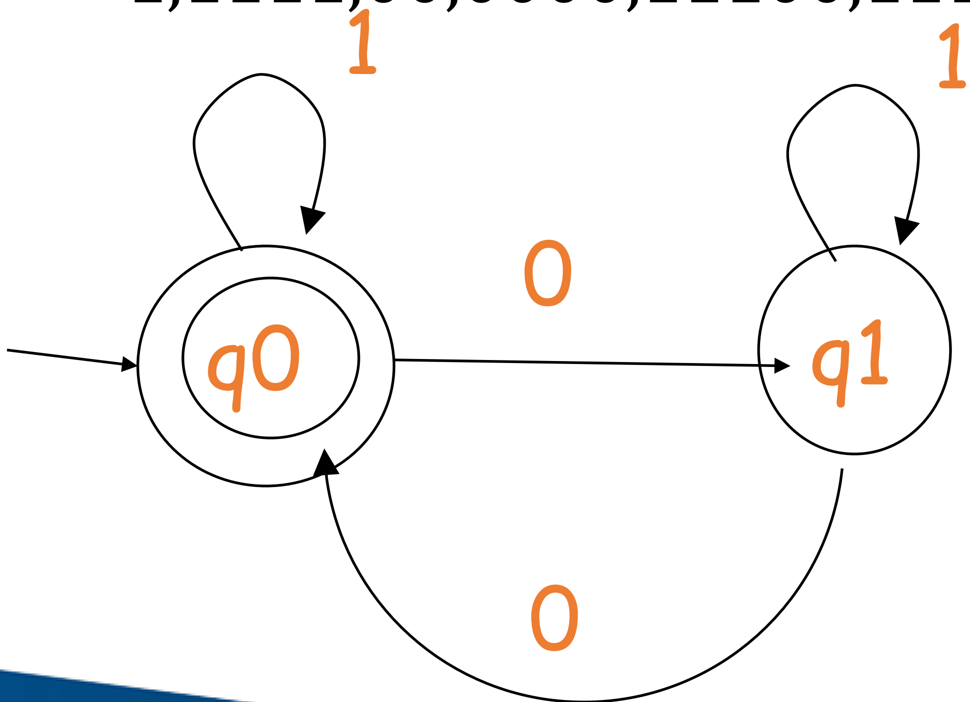
DFA – Contains 1011

- L= { 1011,01011,11011,00010111,..}



DFA- Even no of 0s, ∑={ 0,1} or n0(w)mod2=0

- L={ λ, 1,1111,00,0000,11100,1110000,0000111,..}

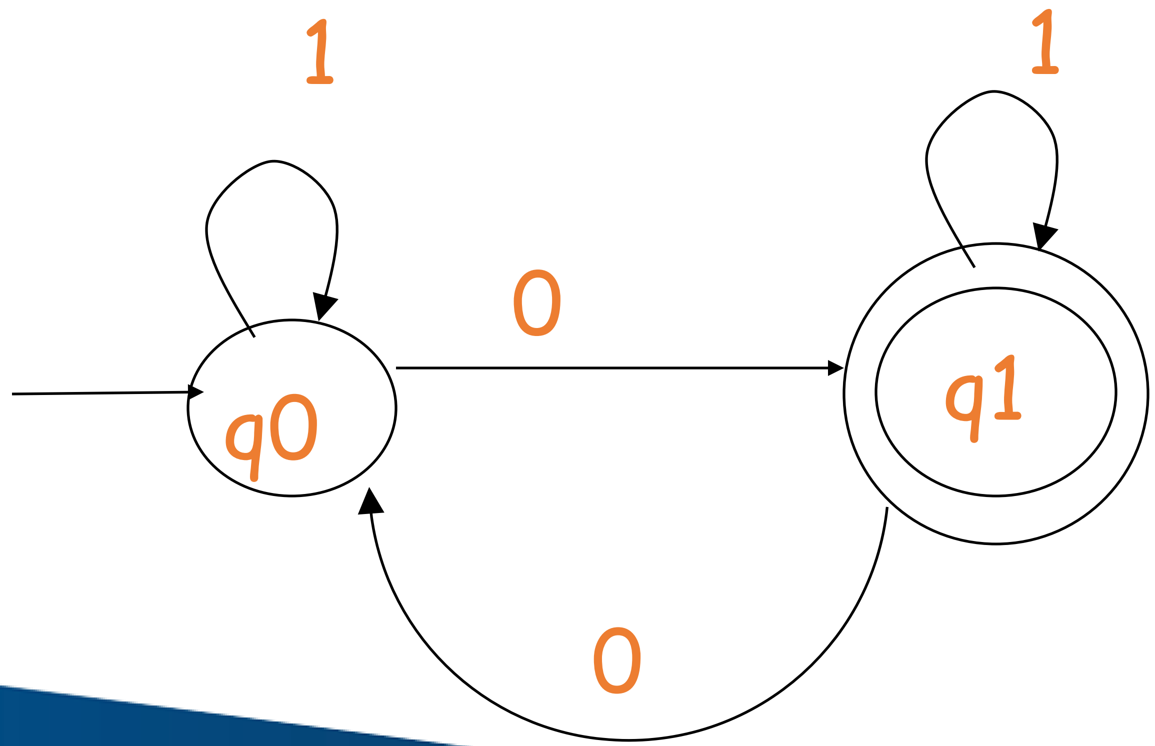


Transition	o	1
*->q0	q1	q0
q1	qo	q1

DFA- strings with even number of 0’s over {0, 1}

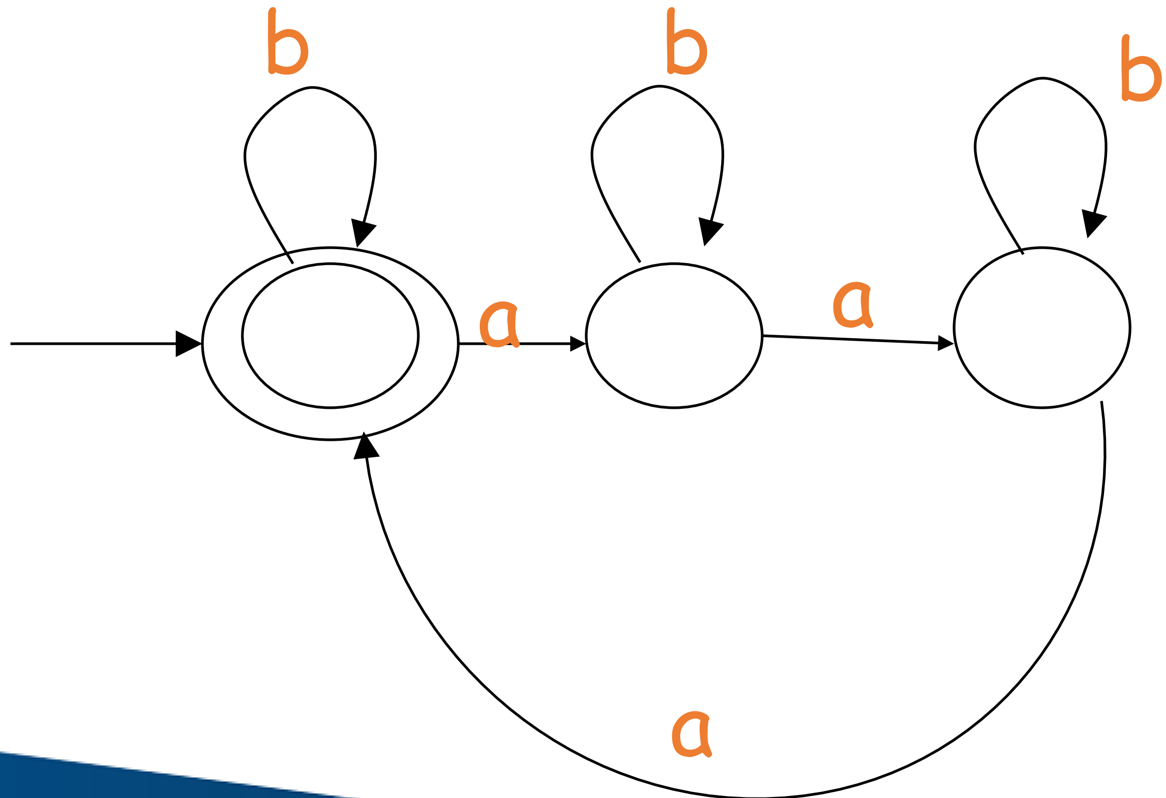
DFA – Odd no of 0's , ∑={ 0,1} or n0(w)mod2=1

- L={ 0,01,1110,000,000001111,...}

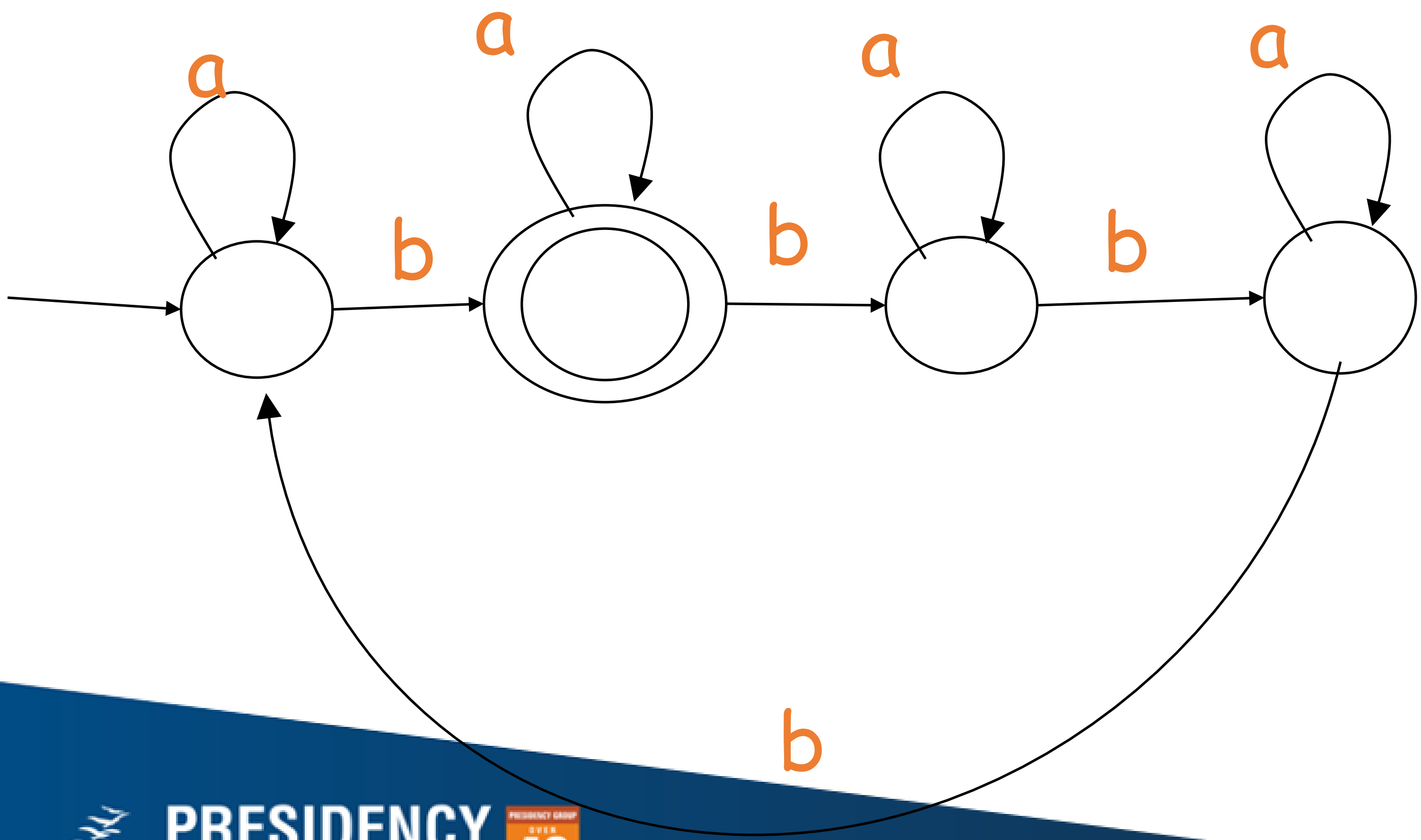


DFA - na(w)mod3=0,∑={ a,b}

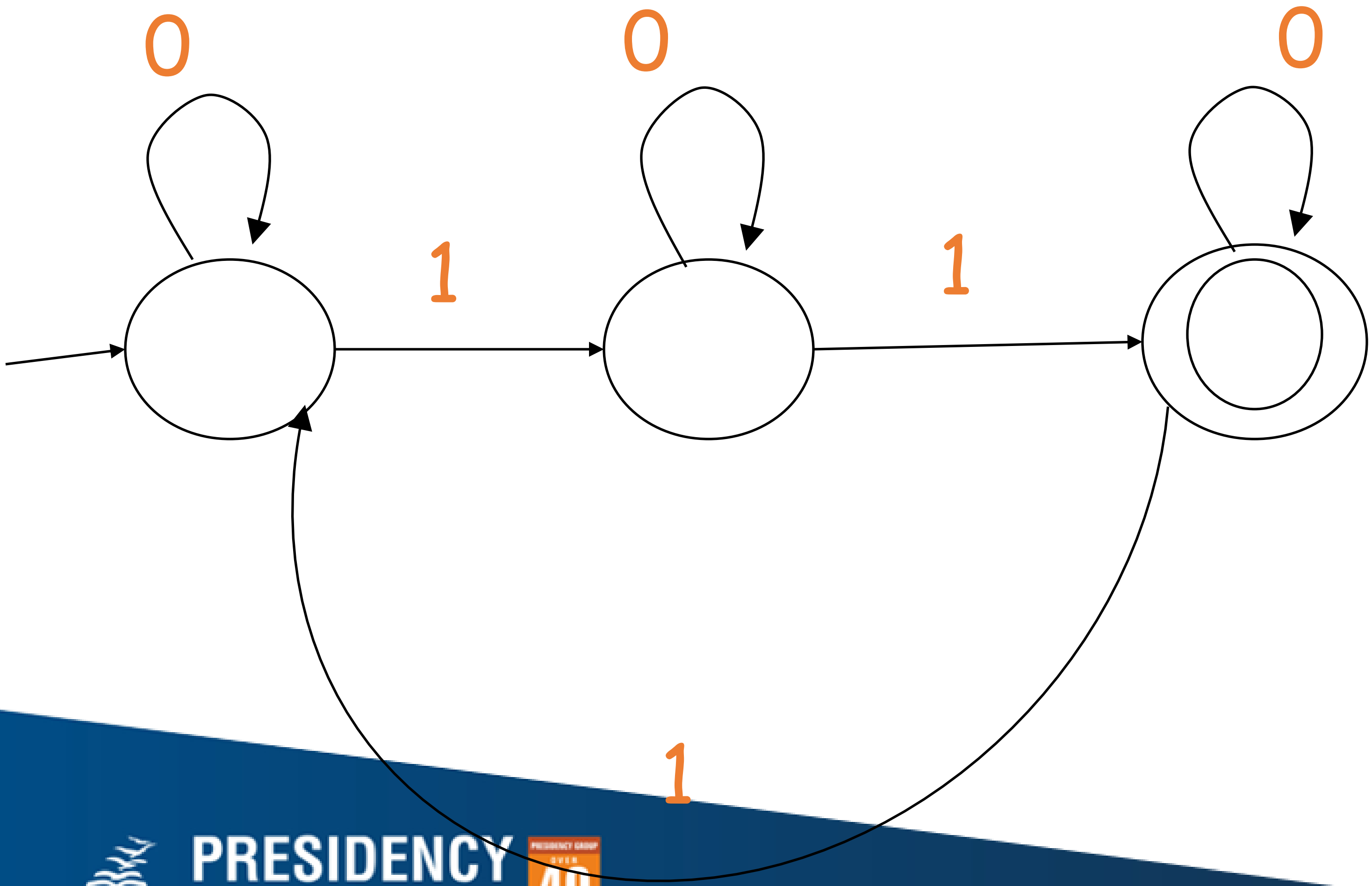
L={ λ,bbbb,aaabbb,aaaaaabbb,....}



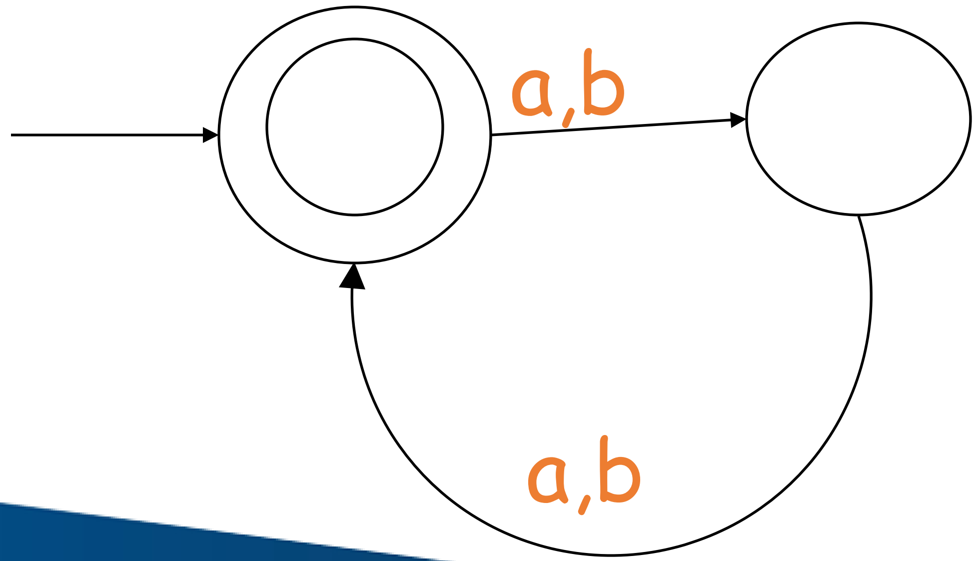
DFA- nb(w)mod4=1, ∑={ a,b}



DfA – n1(w)mod3=2, ∑={ 0,1}

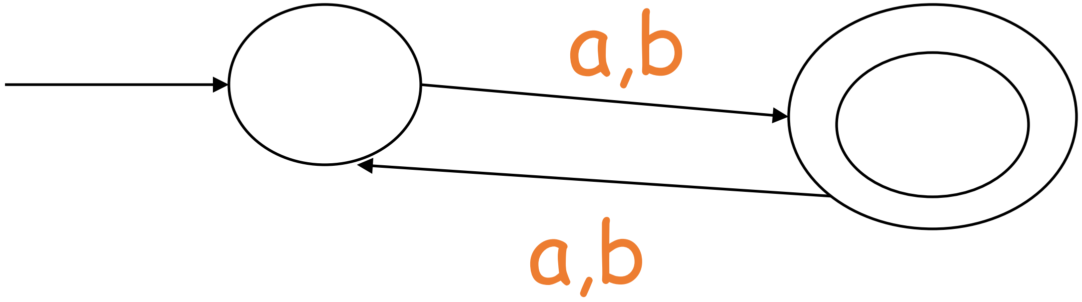


Dfa –lwlmod2=0,

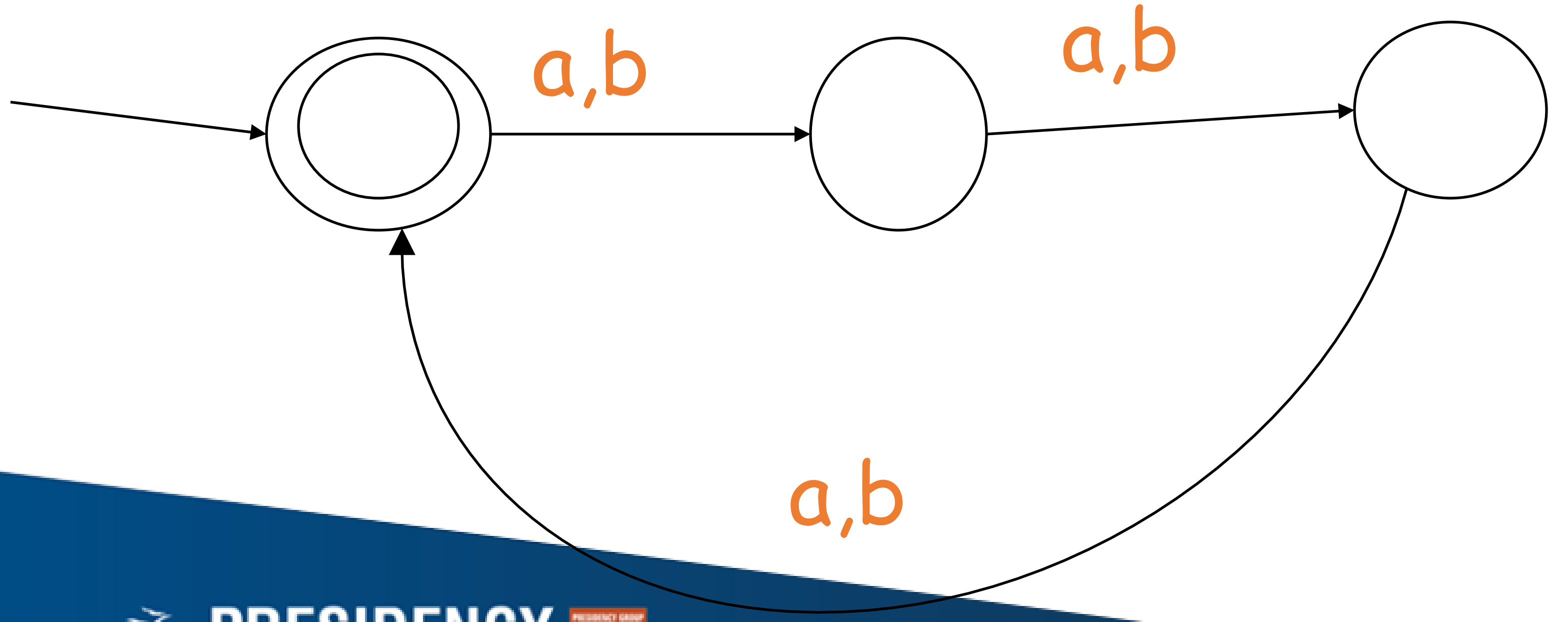


DfA – lwlmod2=1

L={a,b,aaa,aba,aab,baa,bbb,bab a,bba,....

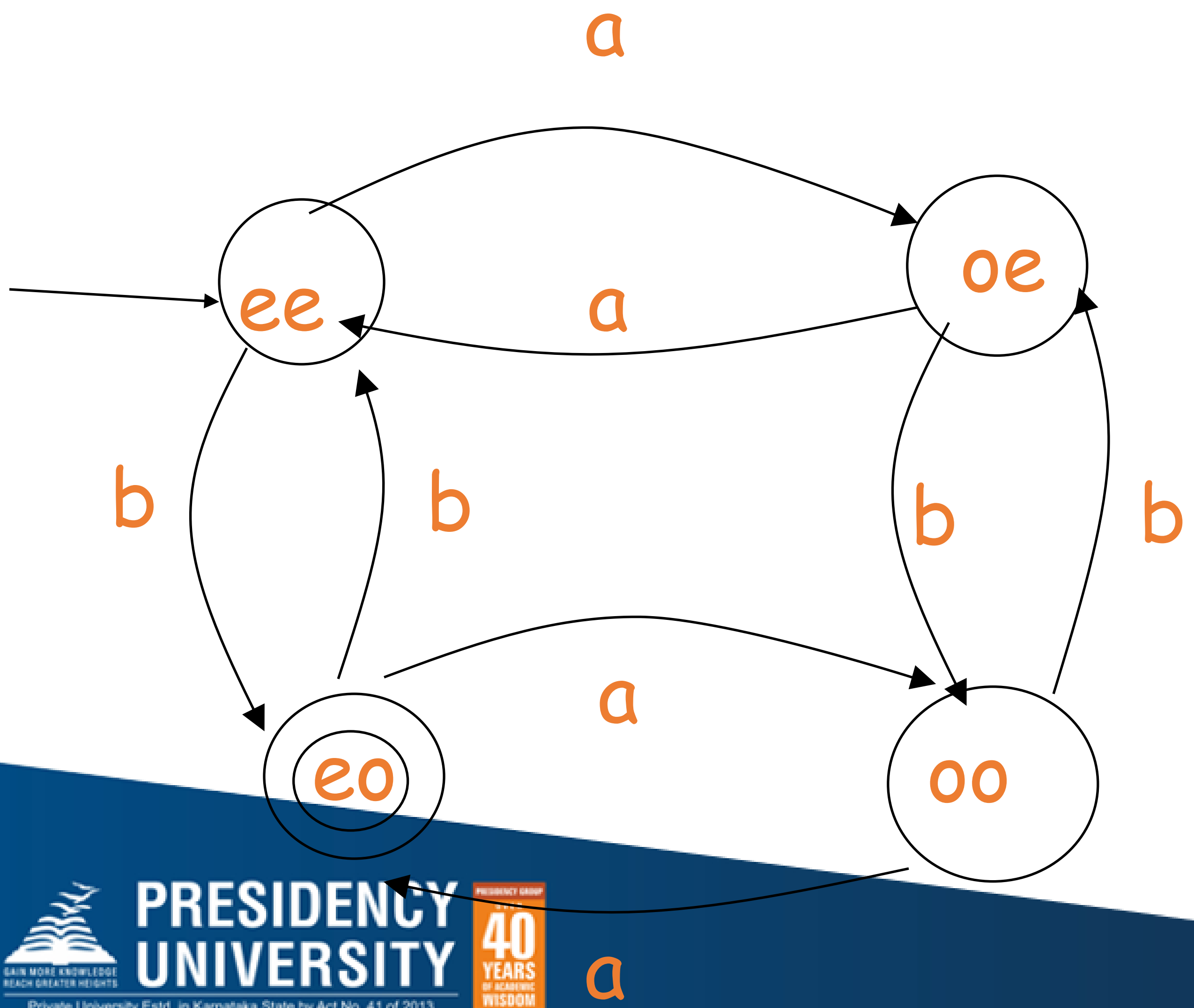


DFA –lwlmod3=0, ∑={ 0,1}

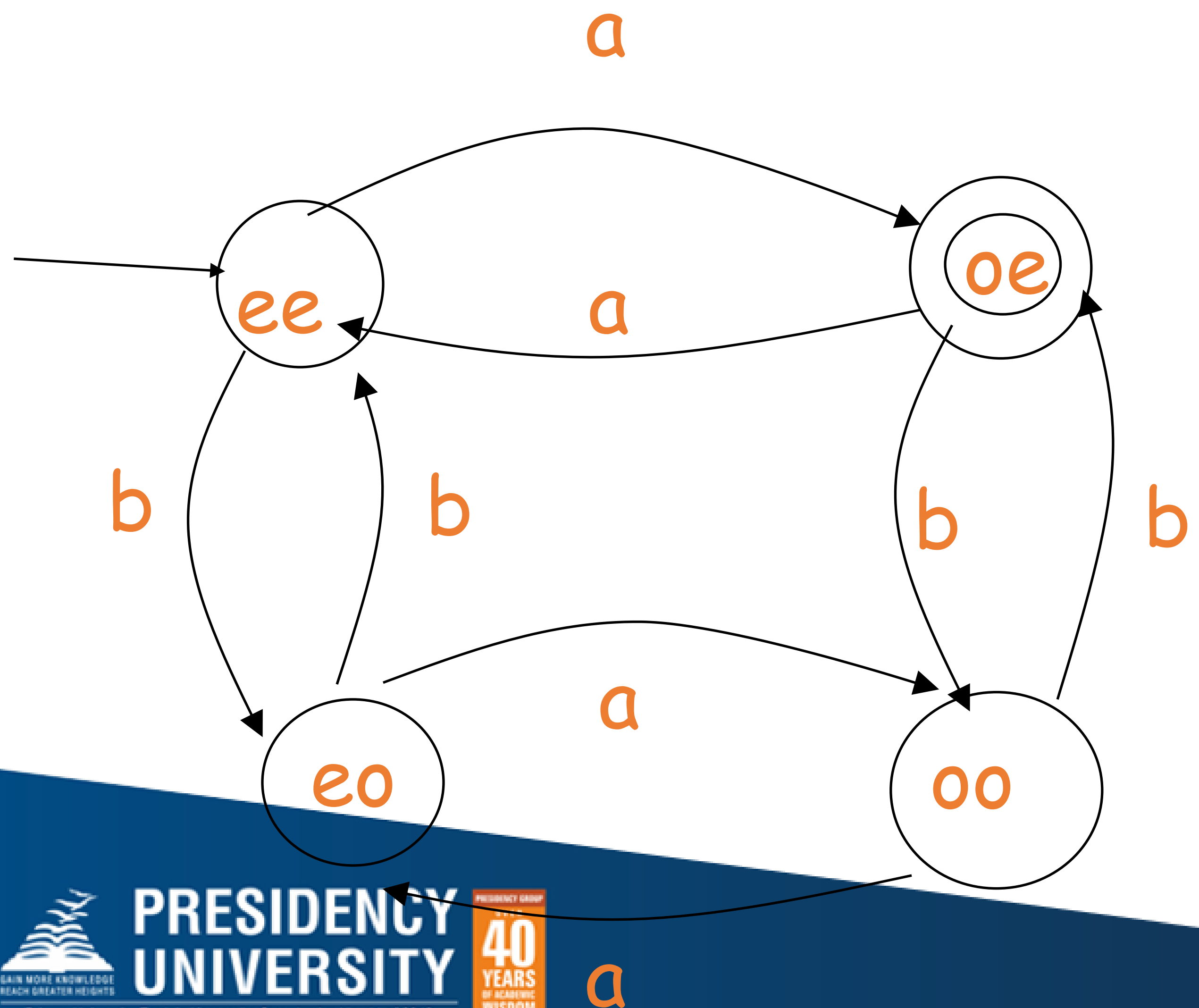


DFA : string with even number of 0’s & odd number of 1’s over {0, 1}

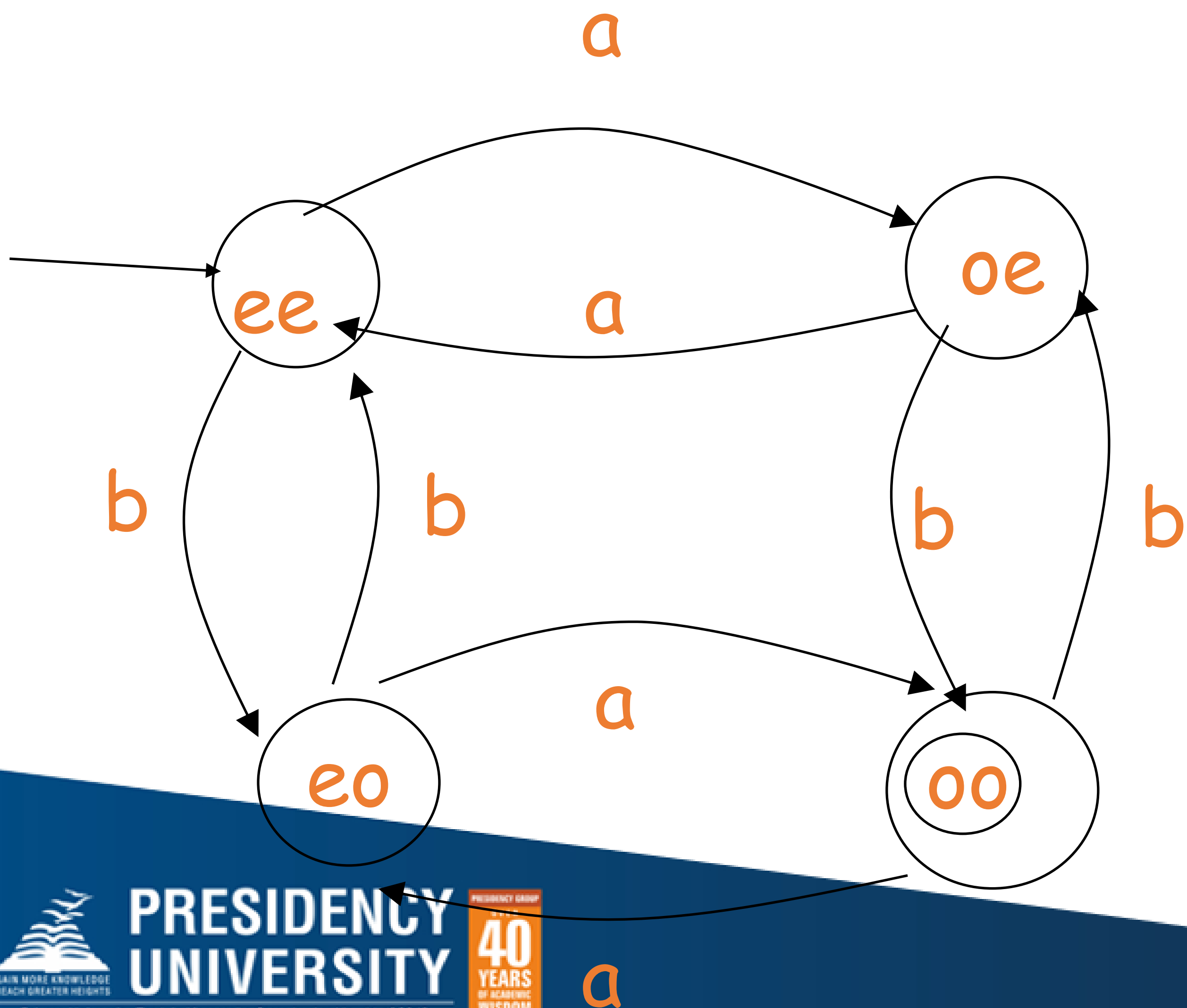
No of a's is even and no of b's odd-eo



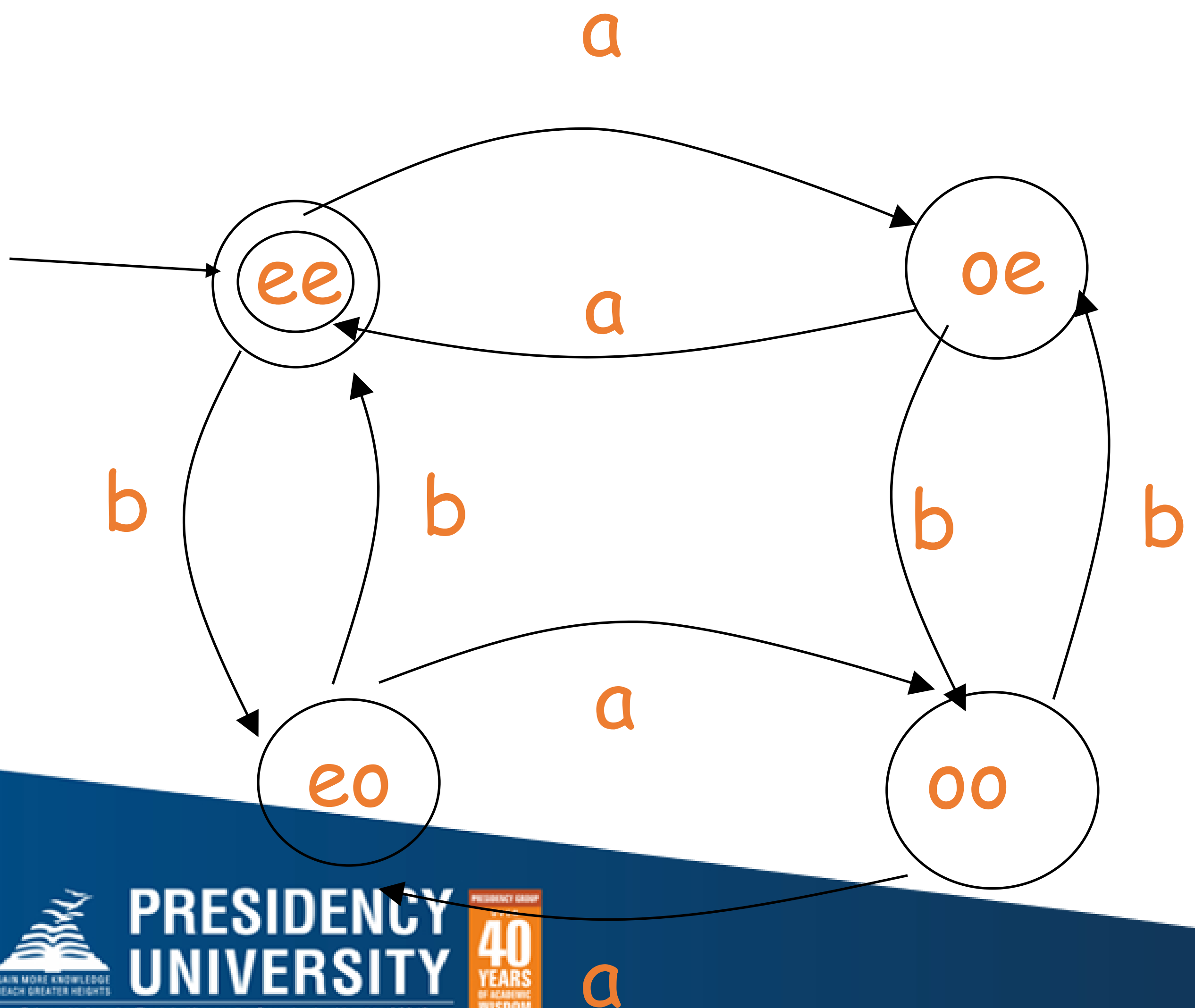
No of a's is odd and no of b's even=oe



No of a’s is odd and no of b’s odd-oo



No of a's is even and no of b's even-ee



NFA Definition

NFA is defined as $M=(Q, \sum, \delta, q0, F)$

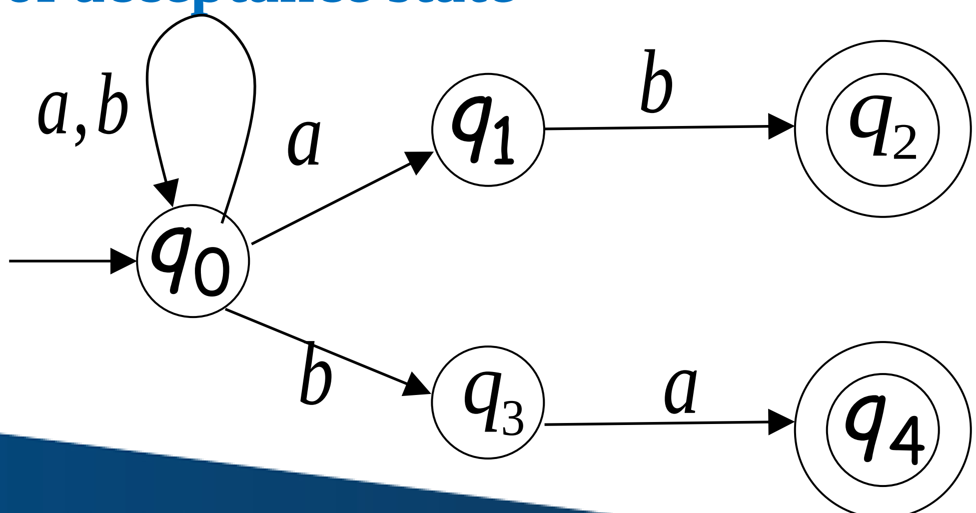
Where, Q= set of states

Σ = input alphabet

δ = transition function $\delta: Q \times (\Sigma \cup \{\epsilon\}) \rightarrow 2^Q$

q0→ Start / Initial State

F → set of acceptance state

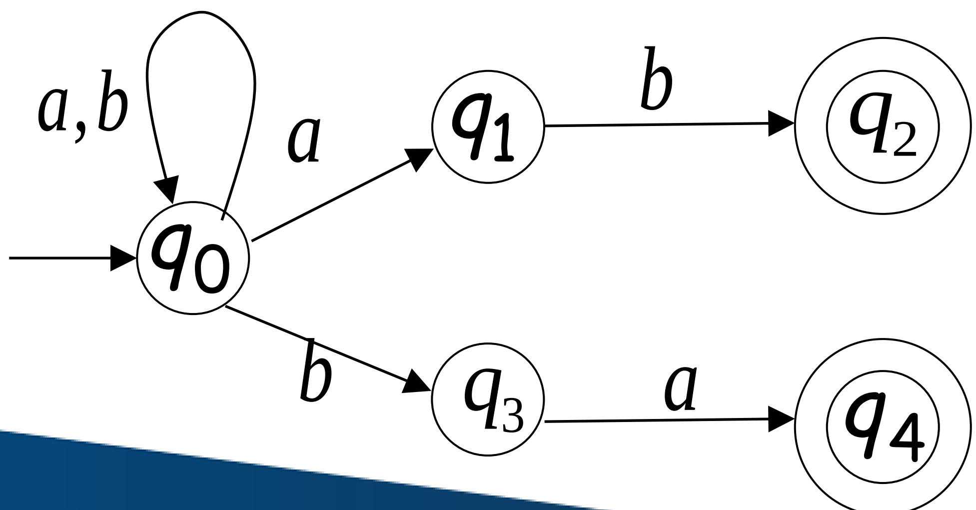


NFA Example

Design NFA that accepts language of strings ending with ab/ba over ∑ = {a, b}

Solution

- **Strings accepted= {ab, ba, abab, baba, bbab, aaaba, …}**
- **Strings rejected= {ϵ, a, b, aa, bb, aaa, abb, aaabbb, ...}**
- **Transition Diagram**

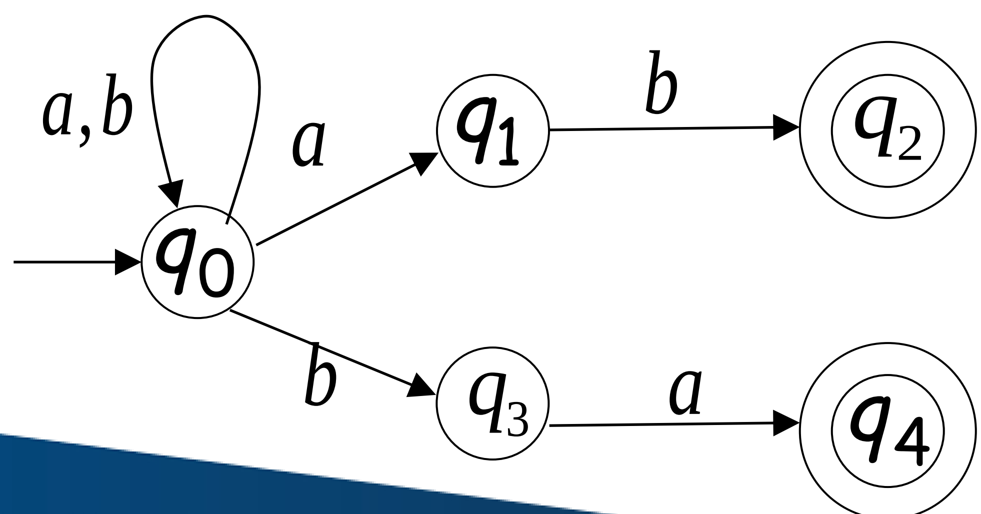


NFA to DFA Conversion Example

Convert NFA to DFA that accepts language of strings ending with ab/ba over ∑ = {a, b}

Solution

- **Draw NFA for given language**
- **Convert that NFA into DFA using subset construction method**



NFA to DFA Conversion Example

- Using Subset Construction Method
- Consider {q0} as Start State
- δ(q0, a)={q0 q1} newly formed state
- δ({q0 q1}, a)= δ(q0, a)u δ({q1, a})
  ={q0, q1} u {Ø}
  ={q0 q1}
- δ({q0 q1}, b)= δ(q0, b)u δ({q1, b})
  ={q0, q3} u {q2}
  ={q0 q2 q3}

NFA	a	b
q0	{q0, q1}	{q0, q3}
q1	-	{q2}
q2	-	-
q3	{q4}	-
q4	-	-

DFA	a	b
q0	{q0 q1}	{q0 q3}
{q0 q1}	{q0 q1}	{q0 q2 q3}

NFA to DFA Conversion Example

- Using Subset Construction Method
- Consider {q0} as Start State
- δ(q0, a)={q0 q1} newly formed state
- δ({q0 q1}, a)= δ(q0, a)u δ({q1, a})
  ={q0, q1} u {Ø}
  ={q0 q1}
- δ({q0 q1}, b)= δ(q0, b)u δ({q1, b})
  ={q0, q3} u {q2}
  ={q0 q2 q3}
- {q0 q2 q3} is newly formed state
- δ({q0 q2 q3}, a)= δ(q0, a) u δ({q2, a}) u δ({q3, a})
  ={q0, q1} u {Ø} u {q4}
  ={q0 q1 q4}
- δ({q0 q2 q3}, b)= δ(q0, b)u δ({q2, b}) u δ({q3, b})
  ={q0, q3} u {Ø} u {Ø} ={q0 q3}

NFA	a	b
q0	{q0, q1}	{q0, q3}
q1	-	{q2}
q2	-	-
q3	{q4}	-
q4	-	-

DFA	a	b
q0	{q0 q1}	{q0 q3}
{q0 q1}	{q0 q1}	{q0 q2 q3}
{q0 q2 q3)	{q0 q1 q4}	{q0 q3}

NFA to DFA Conversion Example

- {q0 q3} is newly formed state
- δ({q0 q3}, a)= δ(q0, a)∪ δ({q3, a})
  ={q0, q1} ∪ {q4}
  ={q0 q1 q4}
- δ({q0 q3}, b)= δ(q0, b)∪ δ({q3, b})
  ={q0, q3} ∪ {Ø}
  ={q0 q3}

NFA	a	b
q0	{q0, q1}	{q0, q3}
q1	-	{q2}
q2	-	-
q3	{q4}	-
q4	-	-

DFA	a	b
q0	{q0 q1}	{q0 q3}
{q0 q1}	{q0 q1}	{q0 q2 q3}
{q0 q2 q3)	{q0 q1 q4}	{q0 q3}
{q0 q3}	{q0 q1 q4}	{q0 q3}

NFA to DFA Conversion Example

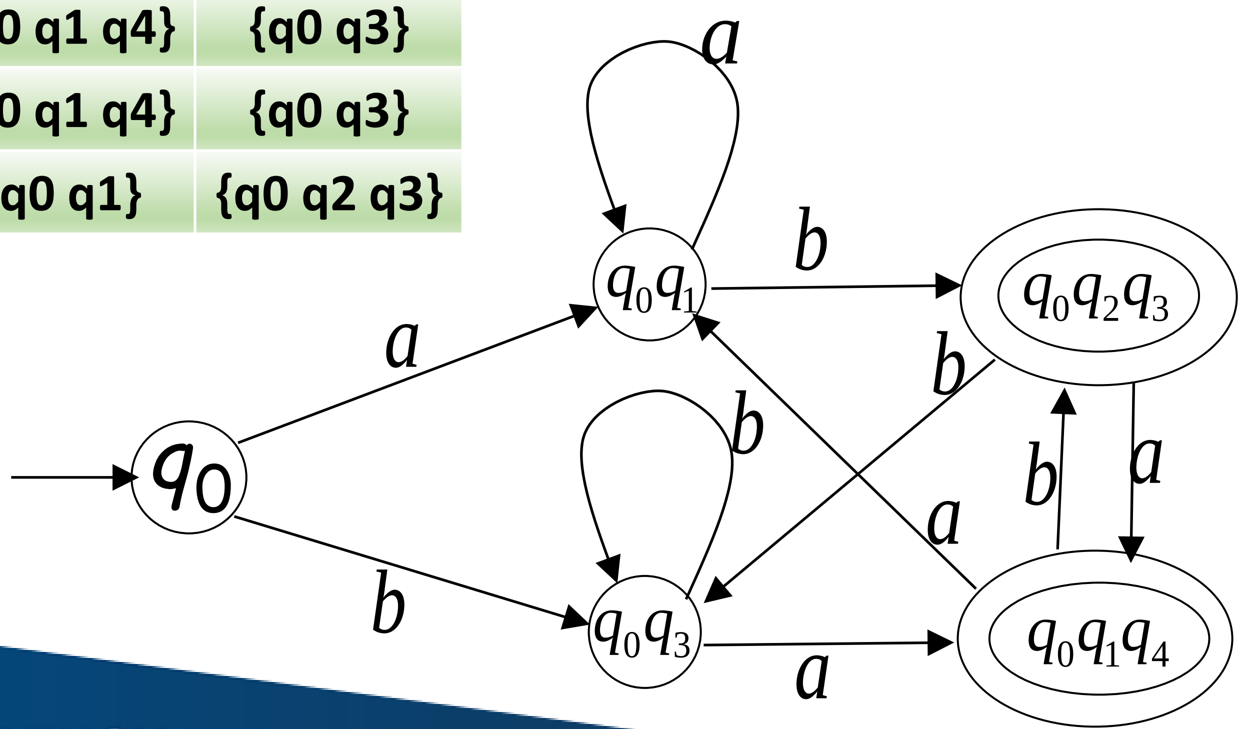
- {q0 q3} is newly formed state
- δ({q0 q3}, a)= δ(q0, a)∪ δ({q3, a})
  ={q0, q1} ∪ {q4}
  ={q0 q1 q4}
- δ({q0 q3}, b)= δ(q0, b)∪ δ({q3, b})
  ={q0, q3} ∪ {Ø}
  ={q0 q3}
- {q0 q1 q4} is newly formed state
- δ({q0 q1 q4}, a)= δ(q0, a) ∪ δ({q1, a}) ∪ δ({q4, a})
  ={q0, q1} ∪ {Ø} ∪ {Ø}
  ={q0 q1}
- δ({q0 q1 q4}, b)= δ(q0, b)∪ δ({q1, b}) ∪ δ({q4, b})
  ={q0, q3} ∪ {q2} ∪ {Ø}
  ={q0 q2 q3}

NFA	a	b
q0	{q0, q1}	{q0, q3}
q1	-	{q2}
q2	-	-
q3	{q4}	-
q4	-	-

DFA	a	b
q0	{q0 q1}	{q0 q3}
{q0 q1}	{q0 q1}	{q0 q2 q3}
{q0 q2 q3)	{q0 q1 q4}	{q0 q3}
{q0 q3}	{q0 q1 q4}	{q0 q3}
{q0 q1 q4}	{q0 q1}	{q0 q2 q3}

NFA to DFA Conversion Example

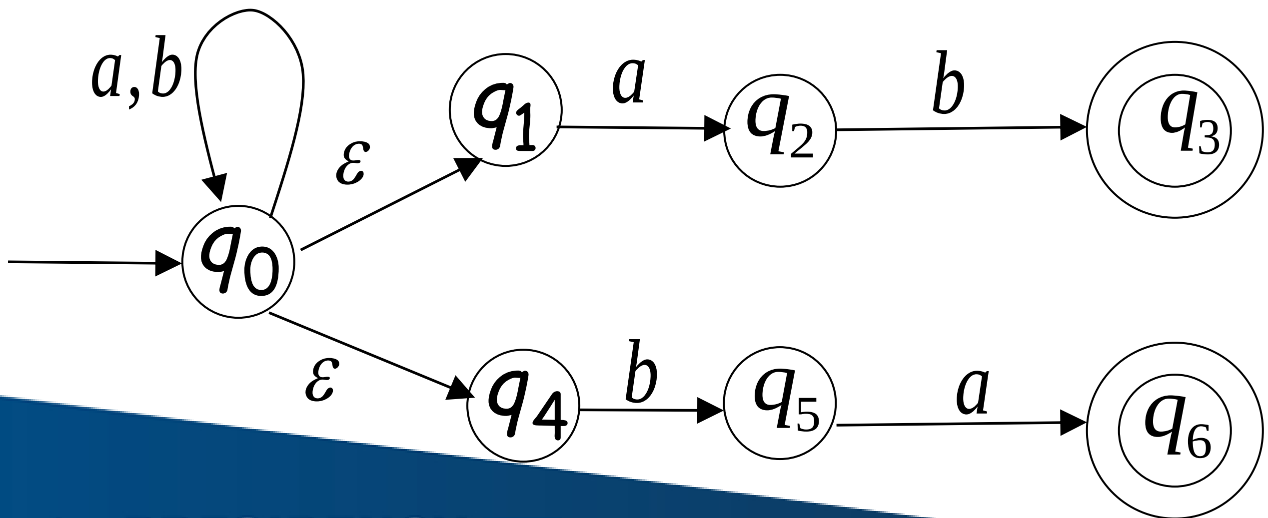
DFA	a	b
q0	{q0 q1}	{q0 q3}
{q0 q1}	{q0 q1}	{q0 q2 q3}
{q0 q2 q3)	{q0 q1 q4}	{q0 q3}
{q0 q3}	{q0 q1 q4}	{q0 q3}
{q0 q1 q4}	{q0 q1}	{q0 q2 q3}



ϵ-NFA Example

ϵ- NFA allows transition on null string or no inputs

It means machine can make transition without input

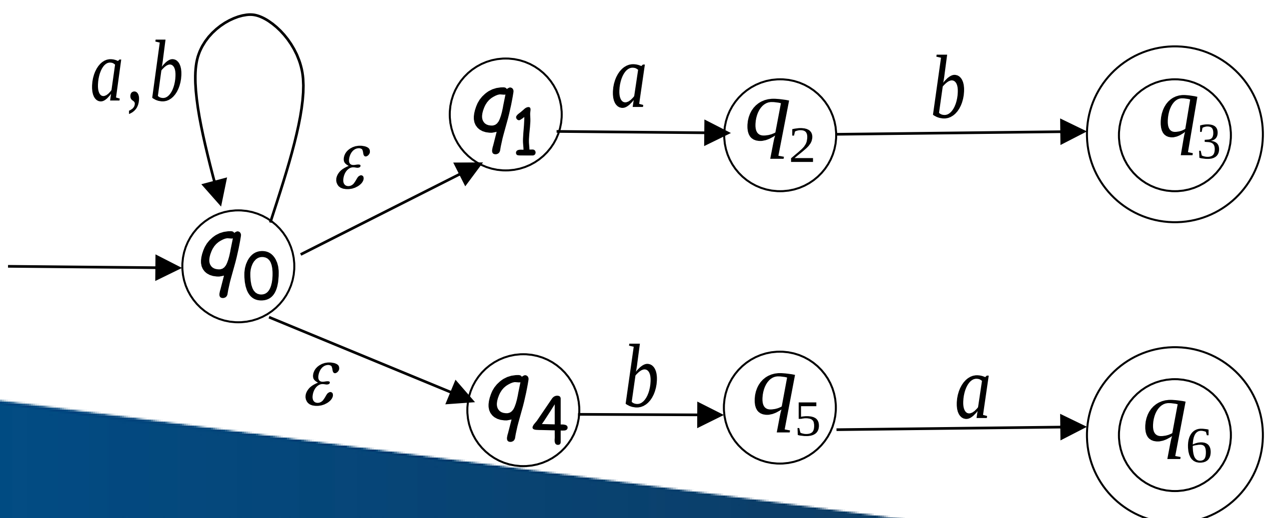


ϵ-NFA Example

Design ϵ-NFA that accepts language of strings ending with ab/ba over ∑ = {a, b}

Solution

- **Strings accepted= {ab, ba, abab, baba, bbab, aaaba, …}**
- **Strings rejected= {ϵ, a, b, aa, bb, aaa, abb, aaabbb, ...}**
- **Transition Diagram**

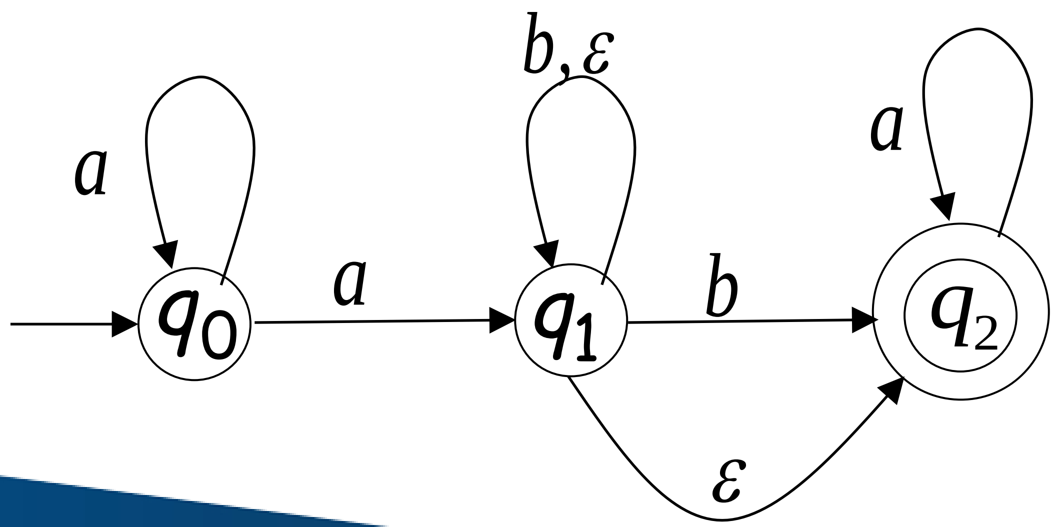


ϵ-NFA to DFA Conversion Example

Convert the following ϵ-NFA to DFA

Solution

- **Find ϵ-closure of each state**
- **Convert that ϵ-NFA into DFA using subset construction method**



ϵ-NFA to DFA Conversion Example

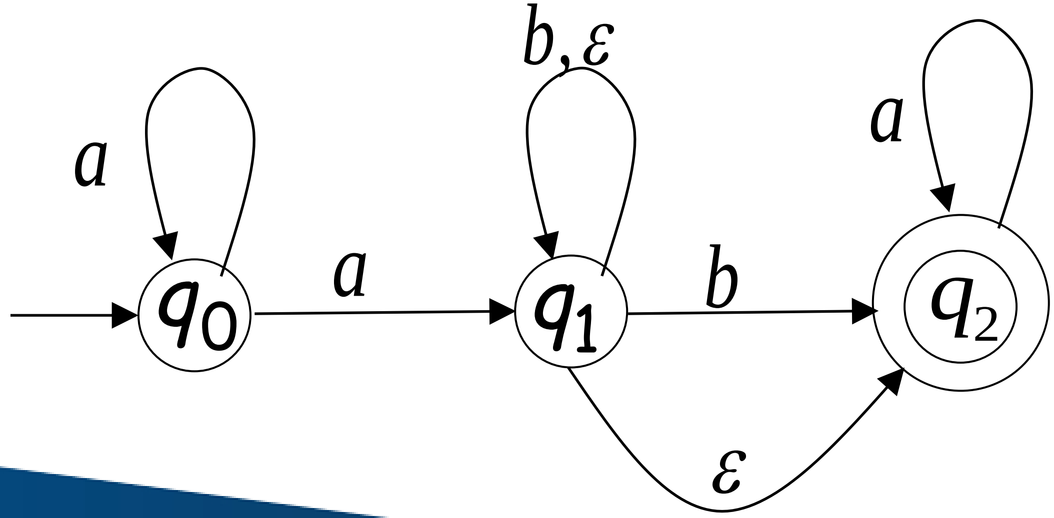
ϵ- Closure : of a state qi is the set of states including qi, also states having ϵ-transitions from state qi and so on…

ϵ- Closure of state qi includes:

-qi

-set of states reachable from qi over ϵ- move

-set of states reachable from states of state qi over ϵ- move and so on…



ϵ-NFA to DFA Conversion Example

- ϵ-closure of state is define as a set of states that includes
- that state itself
- set of states reachable from state Q over ϵ-transaction
- set of states reachable from existing states in ϵ-closure over ϵ-transaction and so on..

States	ϵ-closure
q0	{q0}
q1	{q1, q2}
q2	{q2}

ϵ-NFA to DFA Conversion Example

- Using Subset Construction Method
- Consider ϵ-closure of {q0}= {q0} as Start State
- a-successor of {q0}
  = ϵ-closure (δ(q0, a))
  = ϵ-closure ({q0,q1})
  = ϵ-closure (q0) ∪ ϵ-closure (q1)
  = {q0} ∪ {q1, q2}
  ={q0,q1,q2}
- b-successor of {q0}
  = ϵ-closure (δ(q0, b))={Ø}

DFA	a	b
{q0}	{q0 q1 q2}	{Ø}

ϵ-NFA to DFA Conversion Example

- Using Subset Construction Method
- Consider {q0 q1 q2} as a new state
- a-successor of {q0 q1 q2}

  = ϵ-closure (δ(q0, a)∪δ(q1, a)∪δ(q2, a))

  = ϵ-closure ({q0 q1}∪{Ø}∪{q2})

  = ϵ-closure ({q0 q1 q2})

  = ϵ-closure (q0) ∪ ϵ-closure (q1) ∪ ϵ-closure (q2)

  = {q0} ∪ {q0, q1} ∪ {q2}={q0,q1,q2}
- b-successor of {q0 q1 q2}

  ϵ-closure (δ(q0, b)∪δ(q1, b)∪δ(q2, b))

  = ϵ-closure ({Ø}∪{q1 q2}∪{Ø})

  = ϵ-closure ({q1 q2})

  = ϵ-closure (q1) ∪ ϵ-closure (q2) = {q1 q2} ∪ {q2}={q1 q2}

DFA	a	b
{q0}	{q0 q1 q2}	{Ø}
{q0 q1 q2}	{q0 q1 q2}	{q1 q2}

ϵ-NFA to DFA Conversion Example

- **Using Subset Construction Method**
- **Consider {q1 q2} as a new state**
- **a-successor of {q1 q2}**

  = ϵ-closure (δ(q1, a)∪δ(q2, a))

  = ϵ-closure ({Ø}∪{q2})

  = ϵ-closure ({q2})

  = {q2}
- **b-successor of {q1 q2}**

  ϵ-closure (δ(q1, b)∪δ(q2, b))

  = ϵ-closure ({q1 q2}∪{Ø})

  = ϵ-closure ({q1 q2})

  = ϵ-closure (q1) ∪ ϵ-closure (q2) = {q1 q2} ∪ {q2}={q1 q2}

DFA	a	b
{q0}	{q0 q1 q2}	{Ø}
{q0 q1 q2}	{q0 q1 q2}	{q1 q2}
{q1 q2)	{q2}	{q1 q2}

ϵ-NFA to DFA Conversion Example

- Using Subset Construction Method
- Consider {q2} as a new state
- a-successor of {q2}

  = ϵ-closure (δ(q2, a))

  = ϵ-closure ({q2})

  = {q2}
- b-successor of {q2}

  =ϵ-closure (δ(q2, b))

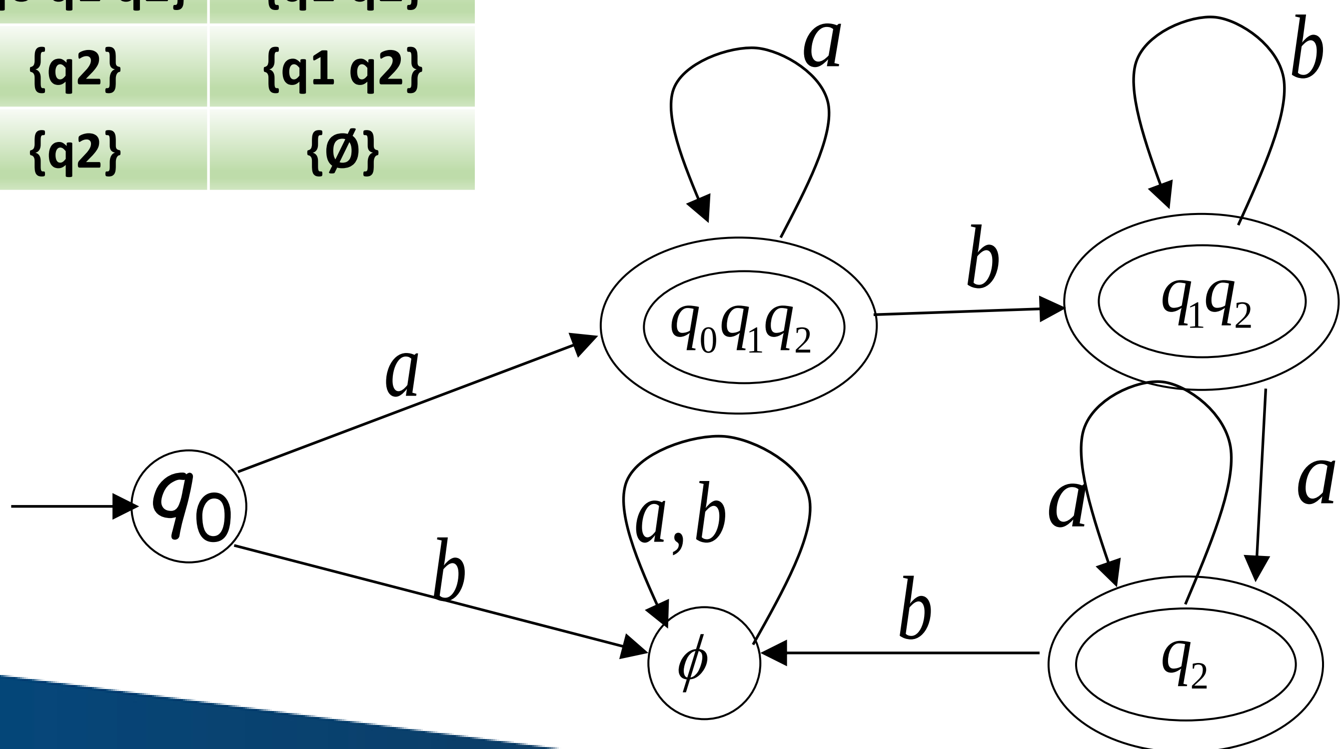
  = ϵ-closure ({Ø})

  ={Ø}

DFA	a	b
{q0}	{q0 q1 q2}	{Ø}
{q0 q1 q2}	{q0 q1 q2}	{q1 q2}
{q1 q2)	{q2}	{q1 q2}
{q2}	{q2}	{Ø}

ϵ-NFA to DFA Conversion Example

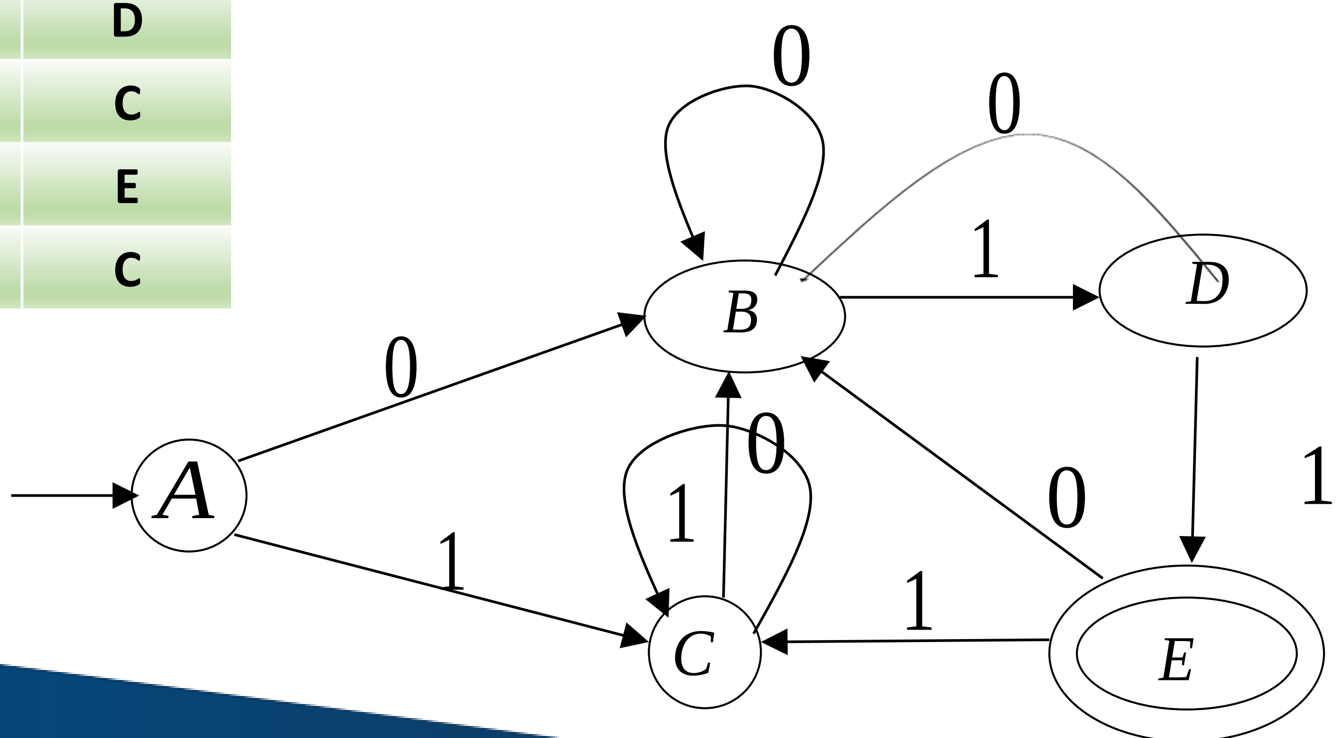
DFA	a	b
{q0}	{q0 q1 q2}	{Ø}
{q0 q1 q2}	{q0 q1 q2}	{q1 q2}
{q1 q2)	{q2}	{q1 q2}
{q2}	{q2}	{Ø}



Minimization of DFA Example

Minimize the following DFA

DFA	0	1
→A	B	C
B	B	D
C	B	C
D	B	E
E*	B	C



Minimization of DFA Example

Minimize the following DFA

- Use State Equivalence Method
- Write 0'Equivalence as

  {A, B, C, D}   {E}
- Write 1'Equivalence

  {A, B, C} {D} {E}
- Write 2'Equivalence

  {A, C} {B} {D} {E}
- Write 3'Equivalence

  {A, C} {B} {D} {E}

DFA	0	1
→A	B	C
B	B	D
C	B	C
D	B	E
E*	B	C

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Minimization of DFA Example

Minimize the following DFA

- **Use State Equivalence Method**
- **Write 0'Equivalence as**

  {A, B, C, D} {E}
- **Write 1'Equivalence**

  {A, B, C} {D} {E}
- **Write 2'Equivalence**

  {A, C} {B} {D} {E}
- **Write 3'Equivalence**

  {A, C} {B} {D} {E}

DFA	0	1
→A,C	B	C
B	B	D
D	B	E
E*	B	C