Practical 6

Show that the image of the right half

plane A = {z : Re z ≥ 1/2} under the mapping

$$w = f(z) = 1/z$$

is the closed disk B = {w : |w – 1| ≤ 1} in the

w - plane.

1

→ kill(all);

(%o0) *done*

→ f(z):=block(
    [x, y],
    x:realpart(z),
    y:imagpart(z),
    w:rectform(1/(x+y·%i))
);

(%o1) f*(z):=* block

$$\left(\textbf{[x,y]}, x\textbf{:}\text{realpart}\textbf{(z)}, y\textbf{:}\text{imagpart}\textbf{(z)}, w\textbf{:}\text{rectform}\left(\frac{1}{x+y \text{ \%i}} \right) \right)$$

→ f(1);

(%o2) 1

→ f(%i);

(%o3) − %i

→ f(1+%i);

(%o4) $\dfrac{1}{2} - \dfrac{\%i}{2}$

→ r(t, s):=(s+%i·t);
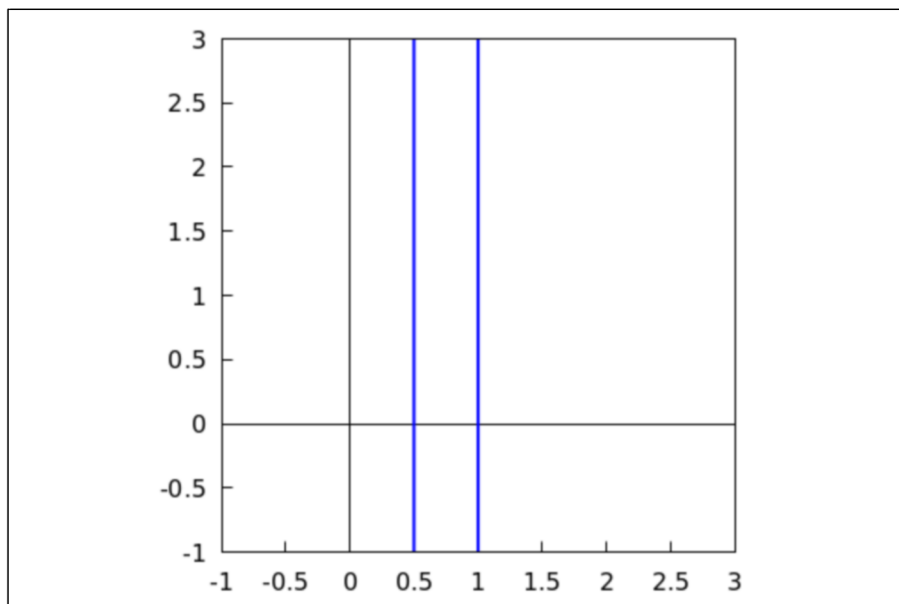
(%o5) r*(t,s):=* s + %i t

→ zdomain:makelist(parametric(realpart(r(t, s)), imagpart(r(t, s)), t, −1, 3), s,

(zdomain) *[* parametric$\left(\dfrac{1}{2}, t, t, -1, 3 \right)$, parametric*(1,t,t,−1,3)]*

```
→    wxdraw2d(
        xaxis = true, xaxis_type = solid, xrange = [−1,  3],
        yaxis = true, yaxis_type = solid, yrange = [−1,  3],
        proportional_axes = xy,
        line_width = 2,
        nticks = 600,
        zdomain
     );
```
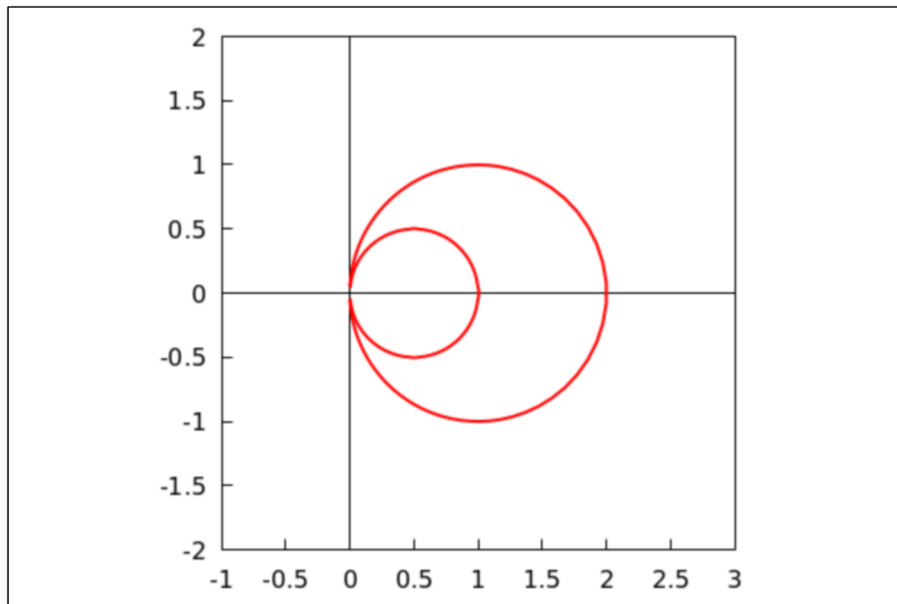
(%t7)

(%o7)

```
→    w(t,  s):=f(r(t,  s));
```

(%o8) $\mathrm{w}(t,s):=\mathrm{f}(\mathrm{r}(t,s))$

```
→    wdomain:makelist(parametric(realpart(w(t,  s)), imagpart(w(t, s)),  t,  −20, 20
```

(wdomain) $\left[\mathrm{parametric}\left(\dfrac{1}{2\left(t^2+\dfrac{1}{4}\right)}, -\dfrac{t}{t^2+\dfrac{1}{4}}, t, -20, 20\right),\right.$

$\left.\mathrm{parametric}\left(\dfrac{1}{t^2+1}, -\dfrac{t}{t^2+1}, t, -20, 20\right)\right]$

```
→    wxdraw2d(
        xaxis = true, xaxis_type = solid, xrange = [−1,  3],
        yaxis = true, yaxis_type = solid, yrange = [−2,  2],
        proportional_axes = xy,
        nticks = 1000,
        line_width = 2,
        color = red,
        wdomain
     );
```

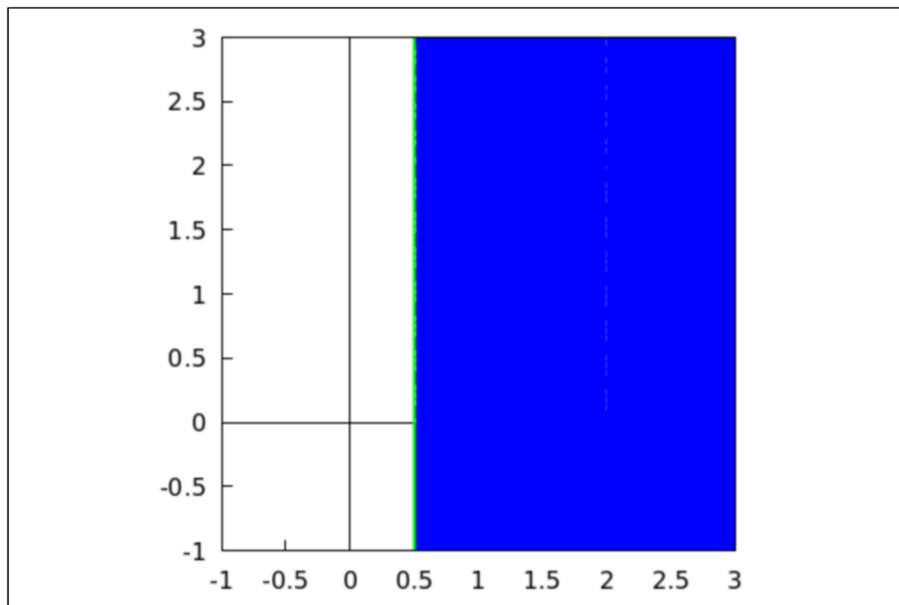(%t10)

(%o10)

2

➜   wxdraw2d(
        xaxis = true, xaxis_type = solid, xrange = [−1, 3],
        yaxis = true, yaxis_type = solid, yrange = [−1, 3],
        proportional_axes = xy,

        fill_color = blue,
        region(x>0.5, x, −10, 10, y, −10, 10),
        line_width = 2,
        color = green,
        parametric(0.5, t, t, −10, 10)

     );

(%t11)

(%o11)

3

➜   W:u+%i·v;

(W)   %i $v + u$

➜   sol:solve(W=f(z), z);

(sol) $[z = \dfrac{1}{\%i\ v + u}]$

➜   sol[1];

(%o14) $z = \dfrac{1}{\%i\ v + u}$

➜   q:rhs(sol[1]);

(q) $\dfrac{1}{\%i\ v + u}$

→   realpart(q)>1;

(%o16) $\dfrac{u}{v^2 + u^2} > 1$

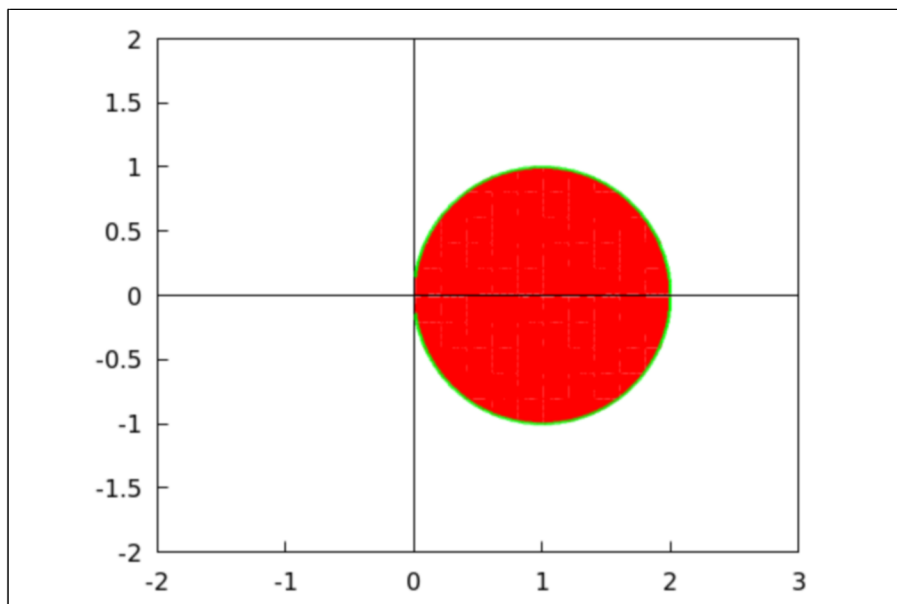→   eq:realpart(q)=1/2;

(eq) $\dfrac{u}{v^2 + u^2} = \dfrac{1}{2}$

→   wxdraw2d(
       xaxis = true, xaxis_type = solid, xrange = [−2, 3],
       yaxis = true, yaxis_type = solid, yrange = [−2, 2],
       proportional_axes = xy,

       region(realpart(q)>1/2, u, 1/100, 2, v, 1/100, 2),
       region(realpart(q)>1/2, u, 1/100, 2, v, −2, −1/100),
       line_width = 2,
       color = green,
       implicit(eq, u, 1/100, 2, v, 1/100, 2),
       implicit(eq, u, 1/100, 2, v, −2, −1/100)

     );

(%t18)

(%o18)

Exercise

1. Show that the image of
the half plane
                            x < -1/2
under the transformation w = 1/z is the interior
of a circle.

2. Show that the image of the half plane
                            y > 1/2
under the transformation w = 1/z is the
interior of a circle.