Practical 1

Make a geometric plot to show that the n th roots of unity are equally spaced points that lie on the unit circle C(0, 1) = {z | |z| = 1}
 and
form the vertices of a regular polygon with n sides, for n = 4, 5, 6, 7, 8.

1

```
kill(all);
```

(%o0) *done*

```
for n:1 thru 5 do display(solve(1 = x^n, x));
```

$$\mathrm{solve}(1=x,x)=[x=1]$$
$$\mathrm{solve}(1=x^2,x)=[x=-1,x=1]$$
$$\mathrm{solve}(1=x^3,x)=[x=\frac{\sqrt{3}\,\%i-1}{2},x=-\frac{\sqrt{3}\,\%i+1}{2},x=1]$$
$$\mathrm{solve}(1=x^4,x)=[x=\%i,x=-1,x=-\%i,x=1]$$
$$\mathrm{solve}(1=x^5,x)=[x=\%e^{\frac{2\,\%i\,\pi}{5}},x=\%e^{\frac{4\,\%i\,\pi}{5}},x=\%e^{-\frac{4\,\%i\,\pi}{5}},x=\%e^{-\frac{2\,\%i\,\pi}{5}},x=1]$$

(%o1) *done*

2

```
kill(all);
```

(%o0) *done*

```
root:solve(z^3=1,    z);
```

(root) $$[z=\frac{\sqrt{3}\,\%i-1}{2},z=-\frac{\sqrt{3}\,\%i+1}{2},z=1]$$

```
sol:map(rhs,   root);
```

(sol) $$[\frac{\sqrt{3}\,\%i-1}{2},-\frac{\sqrt{3}\,\%i+1}{2},1]$$

```
rsol:map(realpart, sol);
isol:map(imagpart, sol);
```

(rsol) $[-\frac{1}{2},-\frac{1}{2},1]$

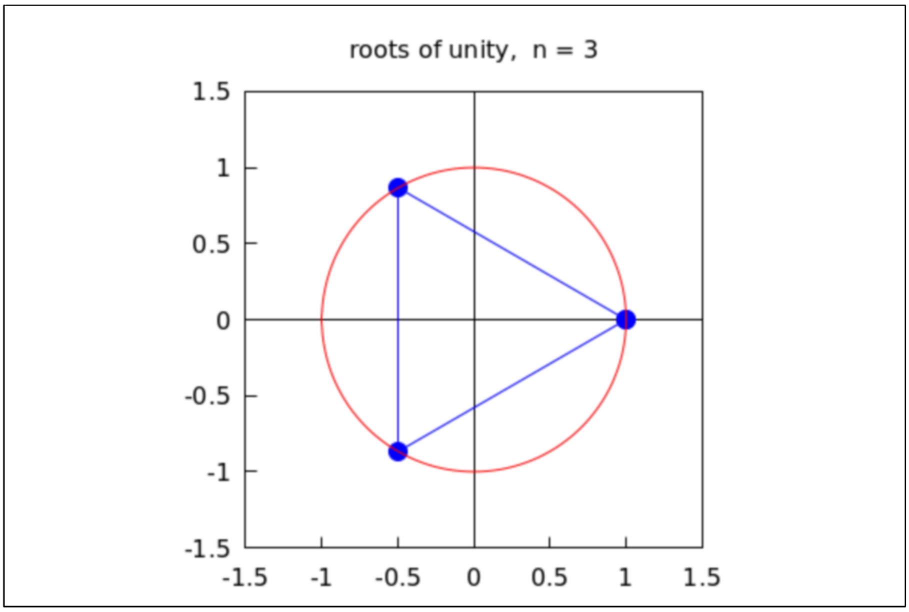
(isol) $[\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2},0]$

```
rsol1:cons(1, rsol);
isol1:cons(0, isol);
```

(rsol1) $[1,-\frac{1}{2},-\frac{1}{2},1]$

(isol1) $[0,\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2},0]$

```
wxdraw2d(
    title = concat("roots of unity, n = ", 3),
    xaxis = true, xaxis_type = solid, xrange = [−1.5, 1.5],
    yaxis = true, yaxis_type = solid, yrange = [−1.5, 1.5],
    proportional_axes = xy,
    point_size = 2,
    point_type = 7,
    points_joined = true,
    points(rsol1, isol1),
    color = red,
    nticks = 200,
    parametric(cos(t), sin(t), t, 0, 2·%pi)
);
```

(%t7) 

(%o7)

3

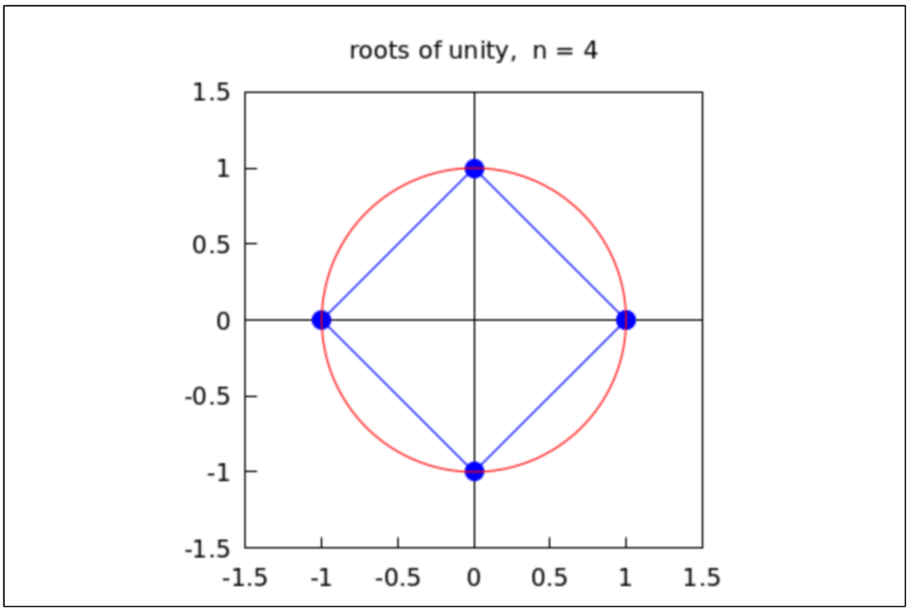
```
→ kill(all);
(%o0) done

→ plotRoots(n):=block(
    root:solve(z^n=1,   z),
    sol:map(rhs,   root),
    rsol:map(realpart,   sol),
    isol:map(imagpart,   sol),
    rsol1:cons(1,  rsol),
    isol1:cons(0,   isol),
    wxdraw2d(
    title = concat("roots of unity,  n = ",  n),
    xaxis = true,  xaxis_type = solid,  xrange = [−1.5,  1.5],
    yaxis = true,  yaxis_type = solid,  yrange = [−1.5,  1.5],
    proportional_axes = xy,
    point_size = 2,
    point_type = 7,
    points_joined = true,
    points(rsol1,   isol1),
    color = red,
    nticks = 200,
    parametric(cos(t),   sin(t),    t,   0,    2·%pi)
  )
  );
```

(%o8) plotRoots(n):=block$(root$:solve$(z^n=1,z),sol$: map$(rhs,root),rsol$:map$(realpart,sol),isol$:map$(imagpart,sol),$ $rsol1$:cons$(1,rsol),isol1$:cons$(0,isol)$,wxdraw2d$(title=$ concat$($*roots of unity, n = *$,n),xaxis=$**true**$,xaxis_type=solid,$ $xrange=[-1.5,1.5],yaxis=$**true**$,yaxis\_type=solid,yrange=[-1.5,1.5],proportional\_axes=xy,point\_size=2,point\_type=7,$ $points\_joined=$**true**$,$points$(rsol1,isol1),color=red,nticks=200,$ parametric$($cos$(t),$sin$(t),t,0,2\pi))$)

➔ plotRoots(4);

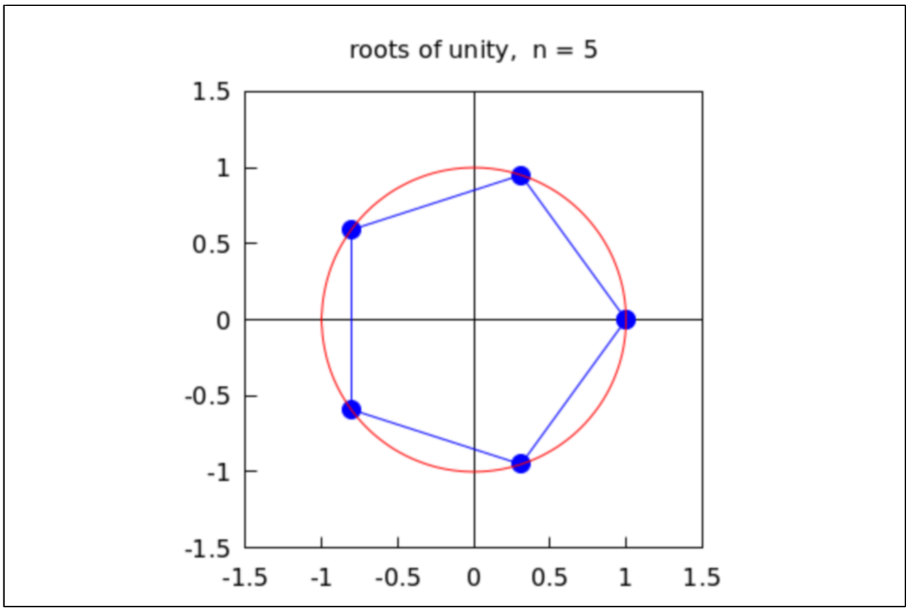
(%t9)



(%o9)

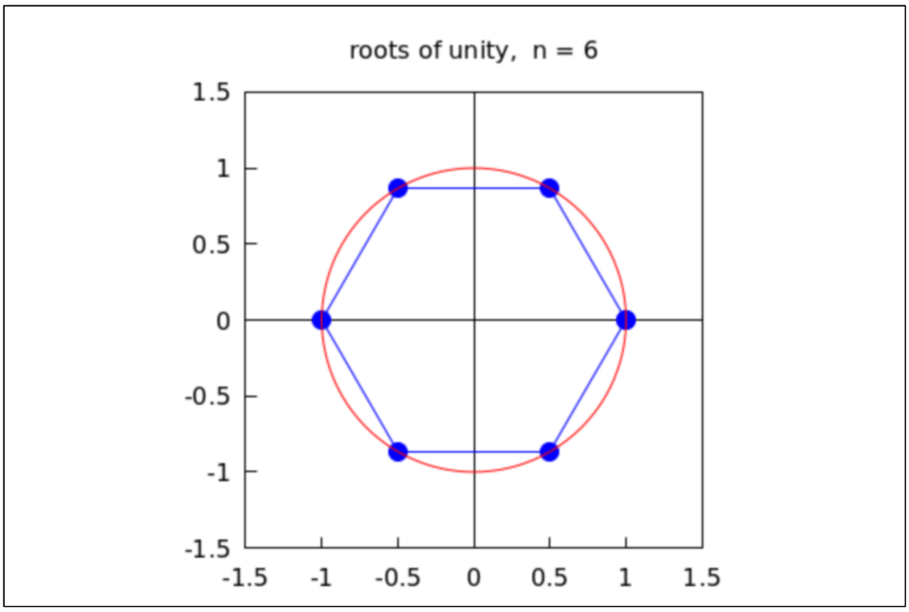
➔ plotRoots(5);

(%t10)



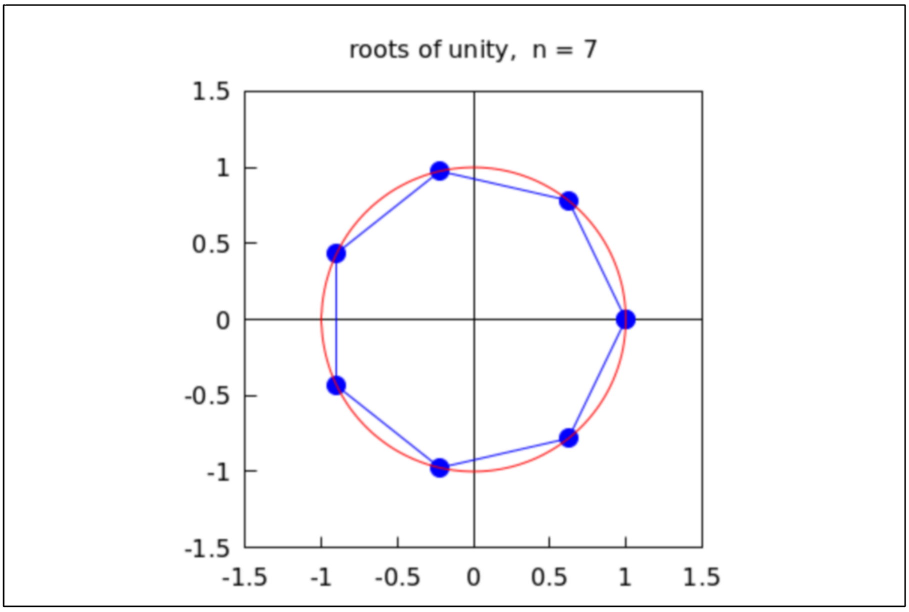
(%o10)

➔ plotRoots(6);

(%t11) 

(%o11)

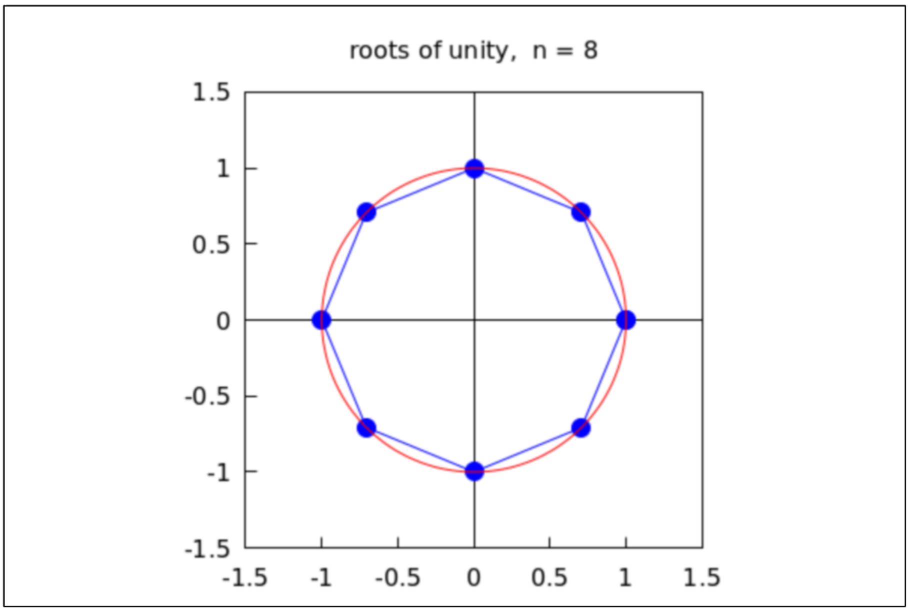
➔ plotRoots(7);

(%t12) 

(%o12)

```
plotRoots(8);
```

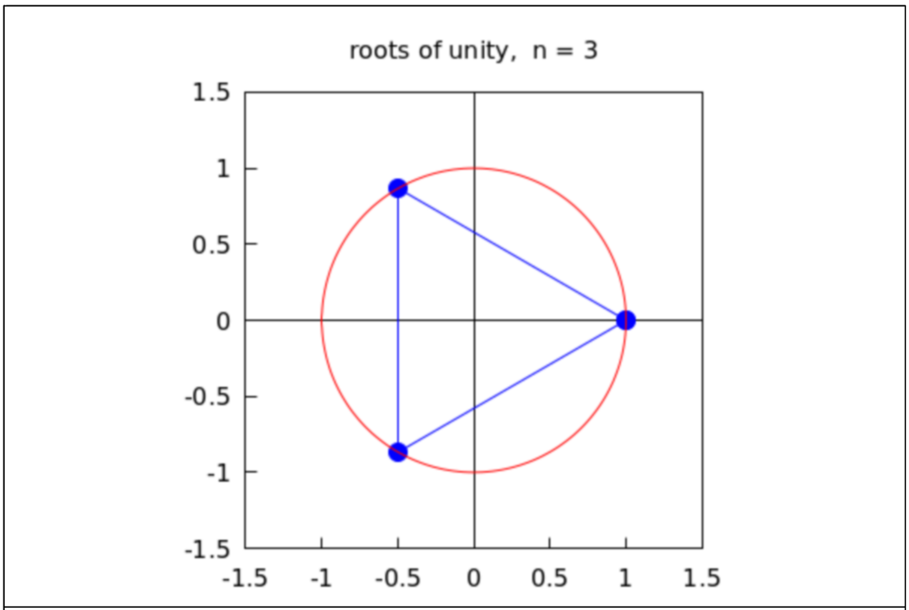
(%t13)



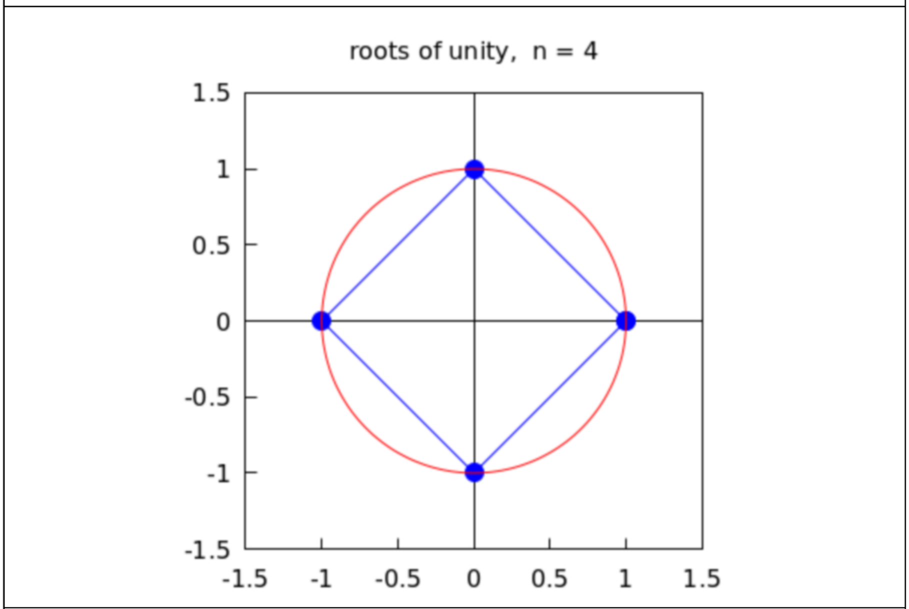
(%o13)

```
for k:3 thru 8 do plotRoots(k);
```

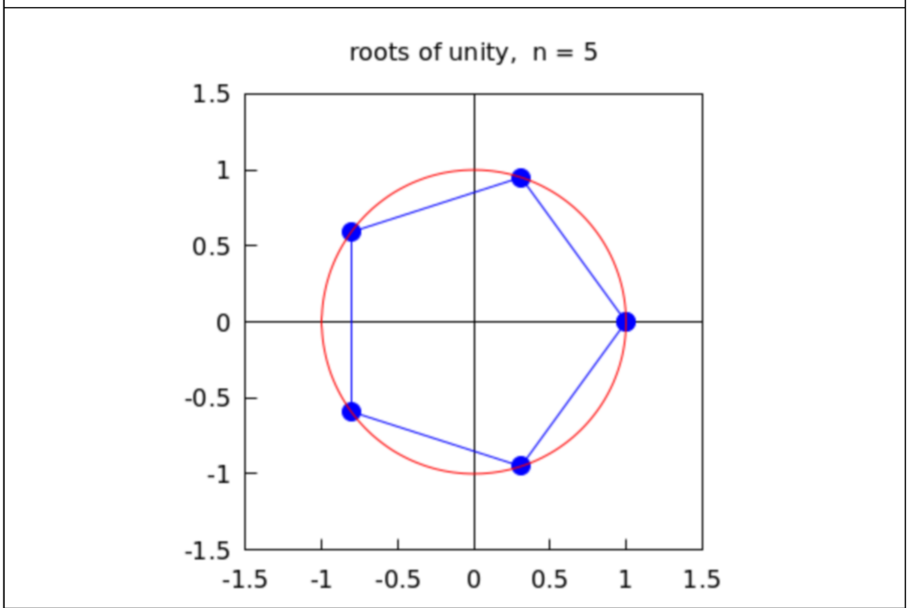
(%t14)

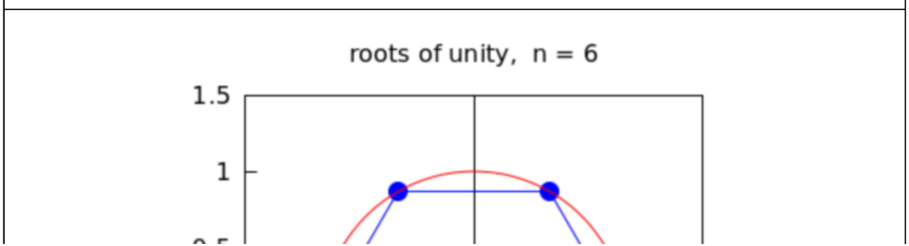


(%t15)



(%t16)





4

Exercise

1. Find and plot the cube roots of 2.

2. Find and plot the cube roots of i.

3. Find and plot the fourth roots of i.

4. Find and plot the fourth roots of -16i.

5. Find and plot the fourth roots of 8i.

6. Find and plot the cube roots of 8i.

7. Find the roots of the equation
   z^2+2z+(1-i) = 0