Practical 3

Write parametric equations and make a parametric plot for an ellipse centered at the origin with horizontal major axis of 4 units and vertical minor axis of 2 units.

Show the effect of rotation of this ellipse by an angle of pi/6 radians and shifting of the centre from (0,0) to (2,1), by making a parametric plot.

1

The transformation w = R(z) that rotates the point z about the origin thru an angle α(fixed real no) to the new position w is given by

Figure 1:

$$w = R(z) = ze^{i\alpha} = re^{i\theta}e^{i\alpha} = re^{i(\theta+\alpha)}$$

2

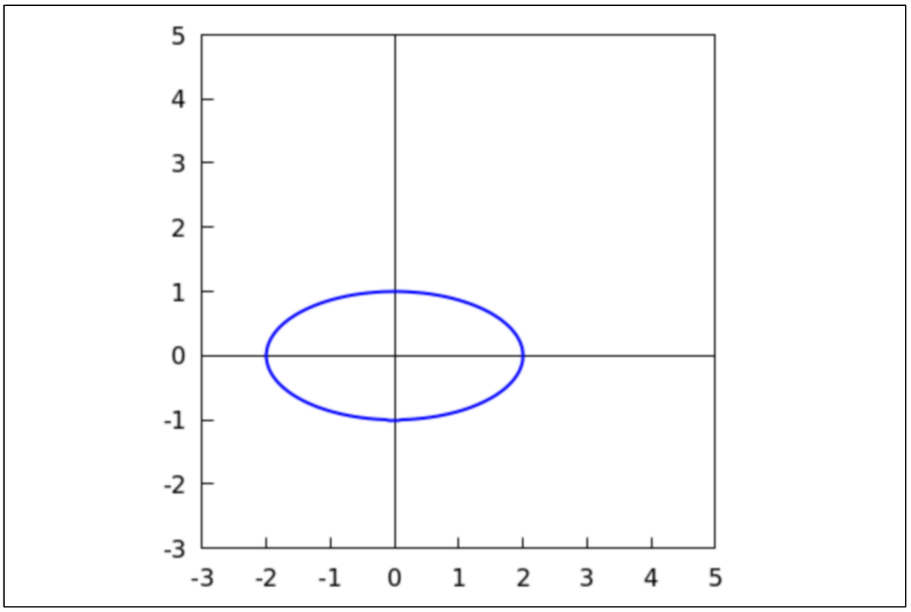
plot of the ellipse (2cos(t), sin(t)), t in [0, 2pi]

```
(%i1) kill(all);
(%o0) done

(%i4) s(t):=2·cos(t) +%i·sin(t);
(%o4) s(t):=2 cos(t)+%i sin(t)
```

```
wxdraw2d(
  xaxis = true, xaxis_type = solid, xrange = [−3,  5],
  yaxis = true, yaxis_type = solid, yrange = [−3,  5],
  proportional_axes = xy,
  nticks = 200,
  line_width = 2,
  parametric(realpart(s(t)),  imagpart(s(t)),  t,  0,  2·%pi)
);
```

(%t7)



(%o7)

rotate the ellipse about the origin by
an angle of pi/6

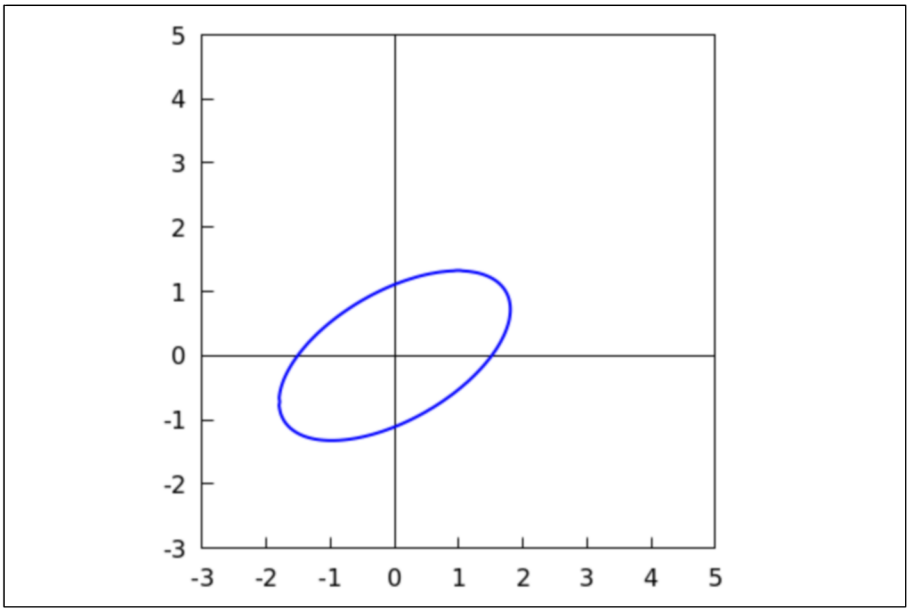
```
theta:%pi/6;
r(t):=s(t)·exp(%i·theta);
```

(theta) $\frac{\pi}{6}$

(%o9) $r(t):=s(t)\,\exp(\%i\,\theta)$

```
wxdraw2d(
  xaxis = true, xaxis_type = solid, xrange = [−3,  5],
  yaxis = true, yaxis_type = solid, yrange = [−3,  5],
  proportional_axes = xy,
  nticks = 200,
  line_width = 2,
  parametric(realpart(r(t)),  imagpart(r(t)),  t,  0,  2·%pi)
);
```

(%t10)



(%o10)

rotate the ellipse about the origin by
an angle of -pi/6

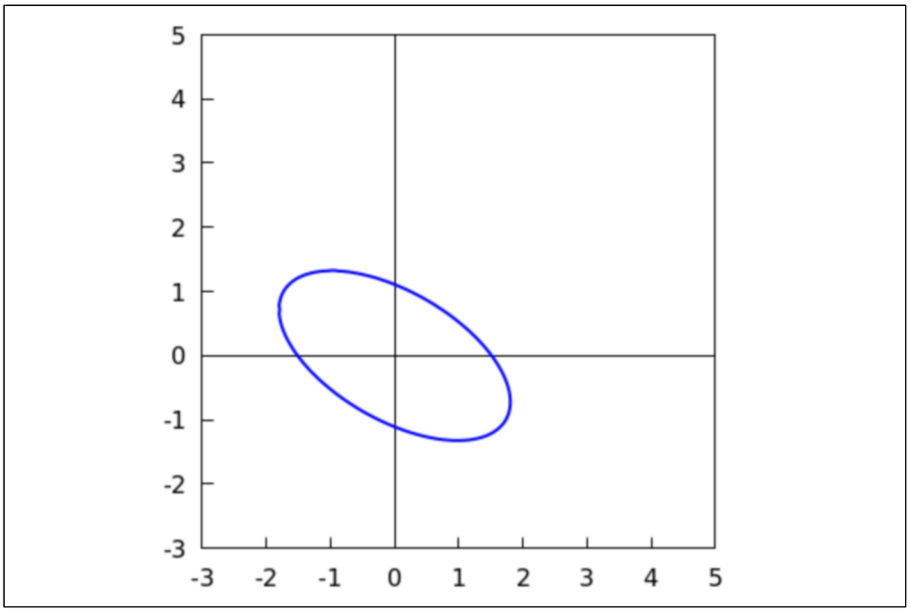
```
theta:−%pi/6;
r(t):=s(t)·exp(%i·theta);
```

(theta) $-\frac{\pi}{6}$

(%o12) $r(t):=s(t)\,\exp(\%i\,\theta)$

```
wxdraw2d(
  xaxis = true, xaxis_type = solid, xrange = [−3,  5],
  yaxis = true, yaxis_type = solid, yrange = [−3,  5],
  proportional_axes = xy,
  nticks = 200,
  line_width = 2,
  parametric(realpart(r(t)),  imagpart(r(t)),  t,  0,  2·%pi)
);
```

(%t13)



(%o13)

3

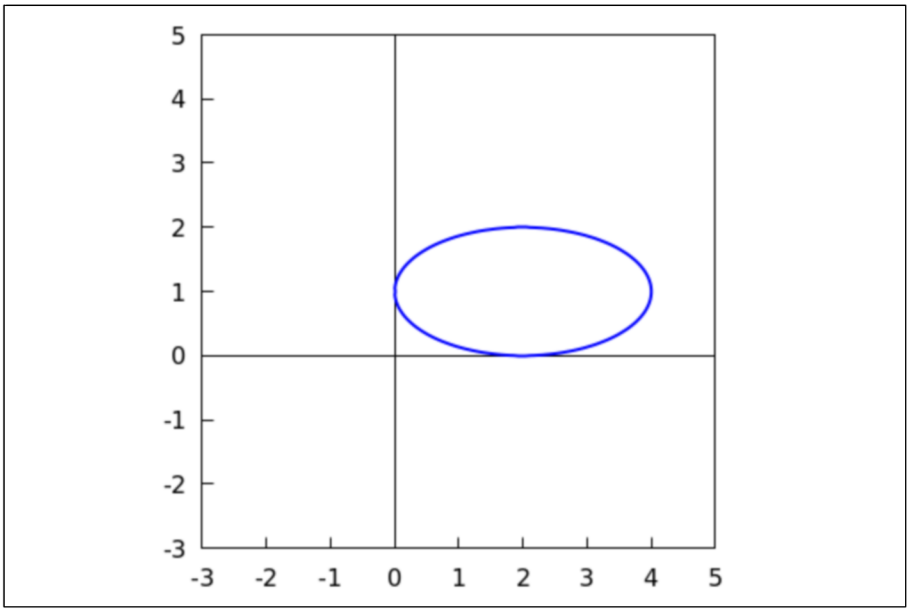
shift the center of the ellipse
(2cos(t), sin(t)), t in [0, 2pi]
to (2, 1)

```
r(t):=s(t)+(2+%i);
```

(%o14) $r(t):=s(t)+(2+\%i)$

```
wxdraw2d(
  xaxis = true, xaxis_type = solid, xrange = [−3,  5],
  yaxis = true, yaxis_type = solid, yrange = [−3,  5],
  proportional_axes = xy,
  nticks = 200,
  line_width = 2,
  parametric(realpart(r(t)),  imagpart(r(t)),  t,  0,  2·%pi)
);
```

(%t15)



(%o15)

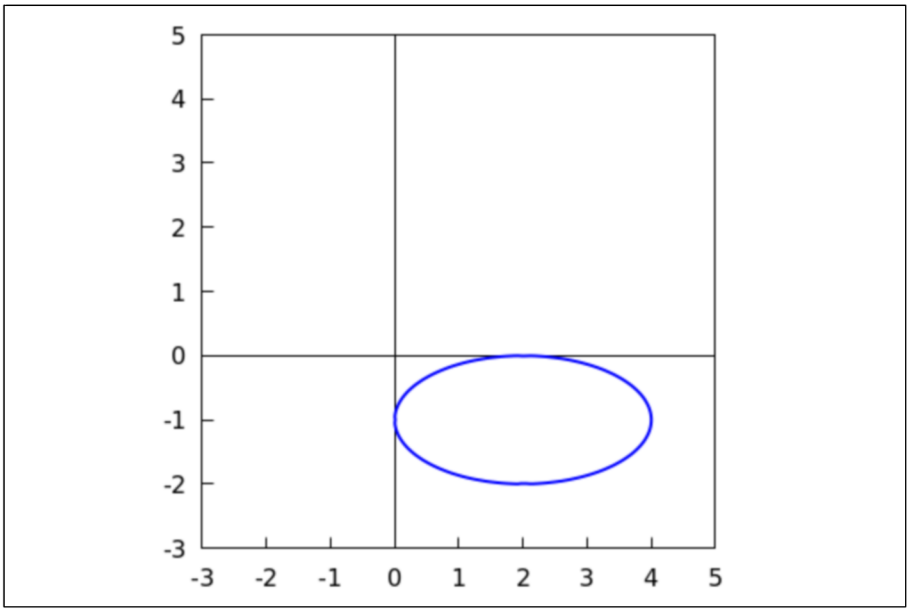
shift the center of the ellipse to (2, -1)

```
r(t):=s(t)+(2−%i);
```

(%o16) $r(t):=s(t)+(2-\%i)$

```
wxdraw2d(
  xaxis = true, xaxis_type = solid, xrange = [−3,  5],
  yaxis = true, yaxis_type = solid, yrange = [−3,  5],
  proportional_axes = xy,
  nticks = 200,
  line_width = 2,
  parametric(realpart(r(t)),  imagpart(r(t)),  t,  0,  2·%pi)
);
```

(%t17)



(%o17)

4

rotate the ellipse by pi/ 6 and shift the center
of the ellipse to (2, 1)

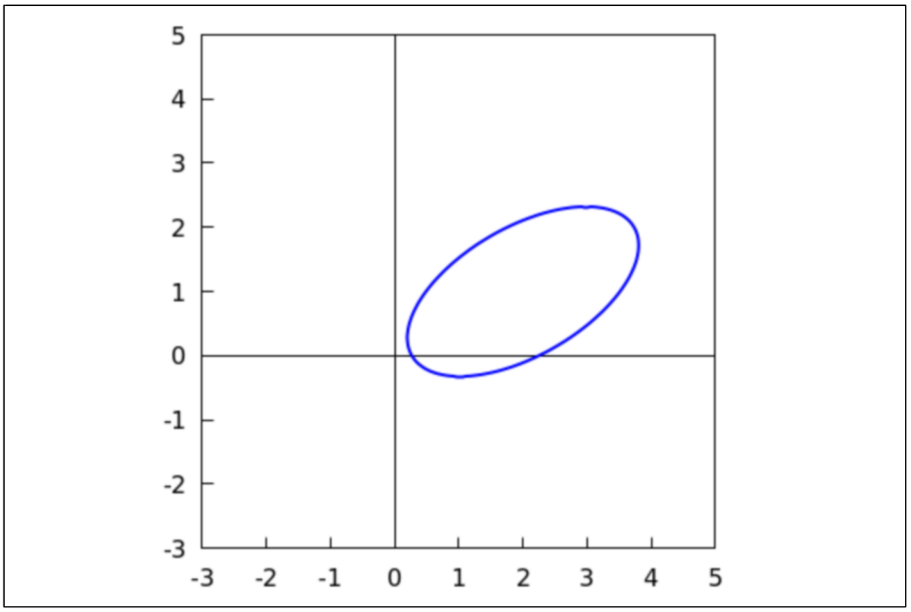
```
theta:%pi/6;
r(t):=s(t)·exp(%i·theta)+(2+%i);
```

(theta) $\frac{\pi}{6}$

(%o20) $r(t):=s(t)\exp(\%i\,\theta)+(2+\%i)$

```
wxdraw2d(
  xaxis = true, xaxis_type = solid, xrange = [−3,  5],
  yaxis = true, yaxis_type = solid, yrange = [−3,  5],
  proportional_axes = xy,
  nticks = 200,
  line_width = 2,
  parametric(realpart(r(t)),  imagpart(r(t)),  t,  0,  2·%pi)
);
```

(%t21)



(%o21)

5

Exercise

1. Rotate the parabola (t, t^2),
t in [-2, 2], by an angle pi/6 and
shift the vertex to (-1, 1).

2. Rotate the curve (t, t^3),
t in [-2, 2], by an angle pi/6.

3. Rotate the hyperbola y^2-x^2=1,
by an angle pi/6.

4. Rotate the hyperbola xy=1
by an angle pi/3.

5. Rotate the triangle joining the points
(0, 0), (2, 0), (1, sqrt(3))
by an angle pi/12.

6. Find the image of the rectangle
0, 1, 1+2i, 2i
under the mapping
w = (1+i)z+2.