

Practical 5

Show that the image of the right half plane $\operatorname{Re} z = x > 1$ under the linear transformation $w = (-1 + i)z - 2 + 3i$ is the half plane $v > u + 7$, where $u = \operatorname{Re}(w)$, etc. Plot the map.

1

```
→ kill(all);
(%o0) done

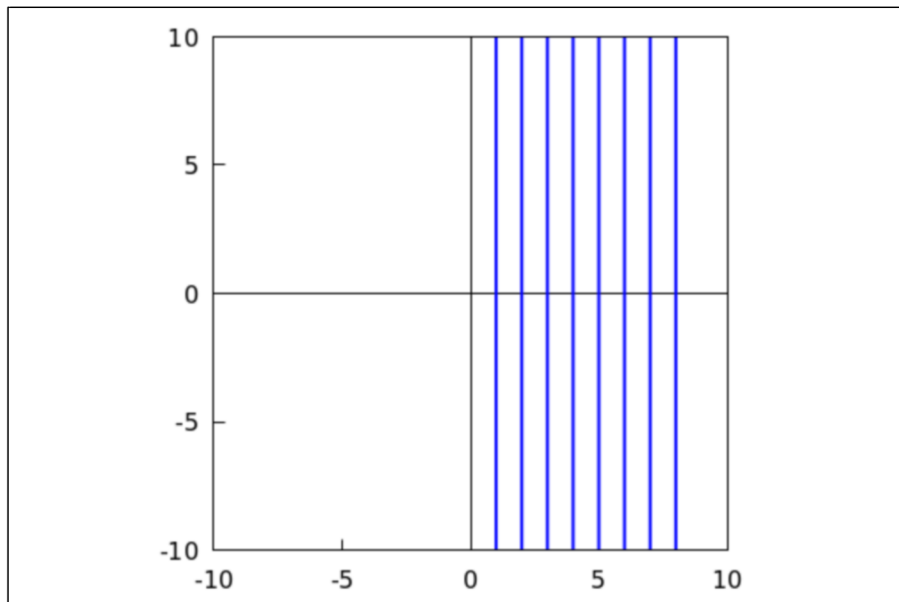
→ f(z):=block(
    [x, y],
    x:realpart(z),
    y:imagpart(z),
    w:rectform((-1+%i)·(x+y·%i)+(-2+3·%i))
);
(%o1) f(z):=block([x,y],x:realpart(z),y:imagpart(z),w:
    rectform((-1+%i) (x+y %i) + (-2+3 %i)))

→ r(t, s):=(s+%i·t);
(%o2) r(t,s):=s+%i t

→ zdomain:makelist(parametric(realpart(r(t, s)), imagpart(r(t, s)), t, -10, 10),
(zdomain) [parametric(1,t,t,-10,10),parametric(2,t,t,-10,10),
    parametric(3,t,t,-10,10),parametric(4,t,t,-10,10),
    parametric(5,t,t,-10,10),parametric(6,t,t,-10,10),
    parametric(7,t,t,-10,10),parametric(8,t,t,-10,10)]
```

```
→ wxdraw2d(
    xaxis = true, xaxis_type = solid, xrange = [-10, 10],
    yaxis = true, yaxis_type = solid, yrange = [-10, 10],
    proportional_axes = xy,
    line_width = 2,
    nticks = 600,
    zdomain
);
```

(%t4)



(%o4)

```
→ w(t, s):=f(r(t, s));
```

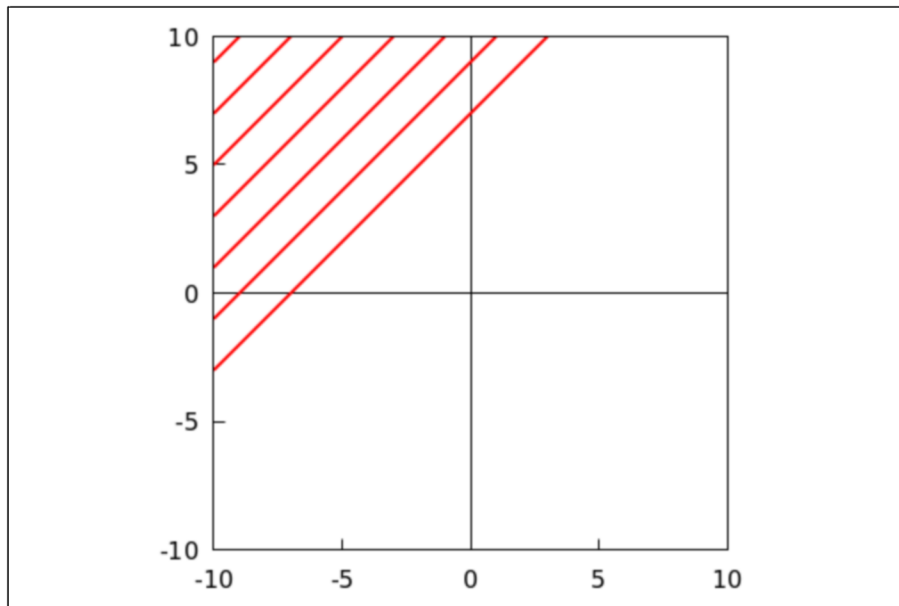
(%o5) w(t,s):=f(r(t,s))

```
→ wdomain:makelist(parametric(realpart(w(t, s)), imagpart(w(t, s)), t, -10, 10
```

(wdomain) [parametric(-t-3, 4-t, t, -10, 10),
parametric(-t-4, 5-t, t, -10, 10),
parametric(-t-5, 6-t, t, -10, 10),
parametric(-t-6, 7-t, t, -10, 10),
parametric(-t-7, 8-t, t, -10, 10),
parametric(-t-8, 9-t, t, -10, 10),
parametric(-t-9, 10-t, t, -10, 10),
parametric(-t-10, 11-t, t, -10, 10)]

```
→ wxdraw2d(  
    xaxis = true, xaxis_type = solid, xrange = [-10, 10],  
    yaxis = true, yaxis_type = solid, yrange = [-10, 10],  
    proportional_axes = xy,  
    nticks = 600,  
    line_width = 2,  
    color = red,  
    wdomain  
);
```

(%t7)



(%o7)

2

```

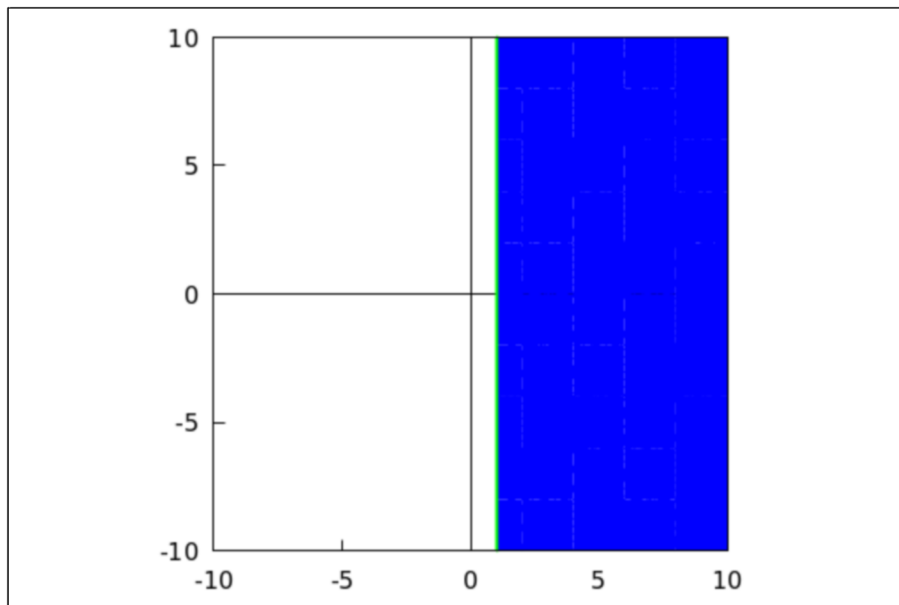
→ wxdraw2d(
    xaxis = true, xaxis_type = solid, xrange = [-10, 10],
    yaxis = true, yaxis_type = solid, yrange = [-10, 10],
    proportional_axes = xy,

    fill_color = blue,
    region(x>1, x, -10, 10, y, -10, 10),
    line_width = 2,
    color = green,
    parametric(1, t, t, -10, 10)

);

```

(%t16)



(%o16)

3

```
→ W:u+%i·v;
```

```
(W) %i v + u
```

```
→ sol:solve(W=f(z), z);
```

```
(sol) [z =  $\frac{\%i v + u - 3 \%i + 2}{\%i - 1}$ ]
```

```
→ sol[1];
```

```
(%o11) z =  $\frac{\%i v + u - 3 \%i + 2}{\%i - 1}$ 
```

```
→ q:rhs(sol[1]);
```

```
(q)  $\frac{\%i v + u - 3 \%i + 2}{\%i - 1}$ 
```

→ `realpart(q)>1;`

(%o13) $\frac{v-3}{2} - \frac{u+2}{2} > 1$

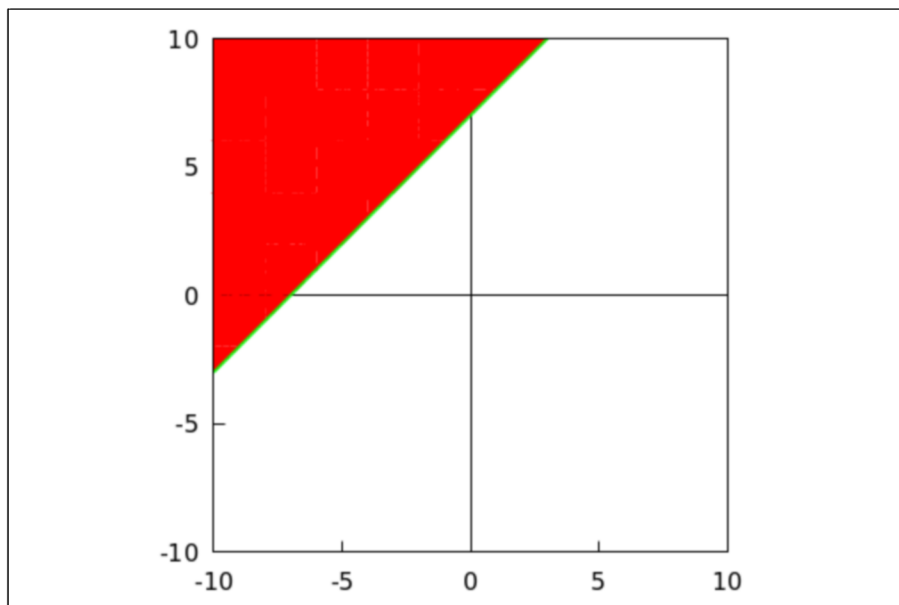
→ `eq:realpart(q)=1;`

(eq) $\frac{v-3}{2} - \frac{u+2}{2} = 1$

→ `wxdraw2d(
 xaxis = true, xaxis_type = solid, xrange = [-10, 10],
 yaxis = true, yaxis_type = solid, yrange = [-10, 10],
 proportional_axes = xy,
 region(realpart(q)>1, u, -10, 10, v, -10, 10),
 line_width = 2,
 color = green,
 implicit(eq, u, -10, 10, v, -10, 10)`

`);`

(%t17)



(%o17)

Exercise

Show that the linear transformation

$$w = iz + i$$

maps the right half-plane

$$\operatorname{Re}(z) > 1$$

onto the upper half-plane

$$\operatorname{Im}(w) > 2.$$