1. Please read the article "The Concept of 'Cat Face'" available on blackboard, and answer the following question:

What was the difference that philosopher Hubert Dreyfus identified between human intelligence and digital computation? (One sentence please.)

Ans:

Philosopher Hubert Dreyfus said that the human intelligence interprets information in context that are not always explicitly mentioned (that is, humans can be intuitive) whereas digital computation doesn't know the context and can't include those information in computation unless specified as rules and included by programmer.

2.5 What is the distance between two parallel hyperplanes $\{x \in \mathbf{R}^n \mid a^T x = b_1\}$ and $\{x \in \mathbf{R}^n \mid a^T x = b_2\}$?

2.5 Hyperplane $(H_1) = \{x \mid a^T x = b_1\}$ For all $x \in R$

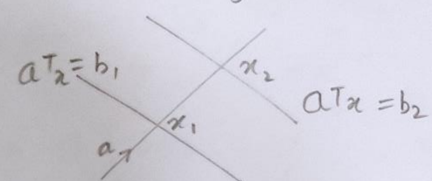
$= \{x \mid a^T x - b_1 = 0\}$

Hyperplane $(H_2) = \{x \mid a^T x = b_2\}$ For all $x \in R$

$= \{x \mid a^T x - b_2 = 0\}$

We saw in class that for any given point in the hyperplane the distance is

$$d(z, H) = \frac{a^T z - b}{\|a\|_2} \rightarrow (1)$$



Using eqn (1) we can say:-

$$d(z, H_1) = \frac{a^T z - b_1}{\|a\|_2} \quad \text{and} \quad d(z, H_2) = \frac{a^T z - b_2}{\|a\|_2} \rightarrow (2)$$

Therefore the distance for Hyperplanes H_1 & H_2 from origin is:-

$$\Rightarrow d(0, H_1) = \frac{-b_1}{\|a\|_2} \quad \text{and} \quad d(0, H_2) = \frac{-b_2}{\|a\|_2} \rightarrow (3)$$

Thus, the distance between 2 parallel hyperplanes is

$$d(H_1, H_2) = d(0, H_1) - d(0, H_2) \quad \{\text{using } (3)\}$$

$$= \frac{-b_1}{\|a\|_2} - \left(\frac{-b_2}{\|a\|_2}\right)$$

$$= \frac{b_2 - b_1}{\|a\|_2}$$ → This is a signed distance

But, we are not completely certain of H_1 and H_2 and their positioning from origin so we will use absolute value.

$$\Rightarrow d(H_1, H_2) = \frac{|b_2 - b_1|}{\|a\|_2}$$

2.7 *Voronoi description of halfspace.* Let a and b be distinct points in $\mathbf{R}^n$. Show that the set of all points that are closer (in Euclidean norm) to a than b, *i.e.*, $\{x \mid \|x-a\|_2 \le \|x-b\|_2\}$, is a halfspace. Describe it explicitly as an inequality of the form $c^T x \le d$. Draw a picture.

2.7 Since a norm is always non-negative and we have $\|x-a\|_2 \le \|x-b\|_2$

$$\Rightarrow \|x-a\|_2^2 \le \|x-b\|_2^2$$

on expanding on both L.H.S and R.H.S. we have,

$$\Rightarrow (x-a)^T(x-a) \le (x-b)^T(x-b)$$

$$\Rightarrow \cancel{x^T x} - 2a^T x + a^T a \le \cancel{x^T x} - 2b^T x + b^T b$$

$$\Rightarrow -2a^T x + 2b^T x \le b^T b - a^T a$$

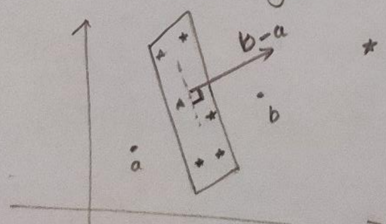
$$\Rightarrow 2(b-a)^T x \le b^T b - a^T a. \rightarrow (1)$$

If we define equ^n (1) in the form of $C^T x \le d$ that gives us :-

$$C = 2(b-a) \text{ and } d = b^T b - a^T a$$

This proves that the set $\{x \mid \|x-a\|_2 \le \|x-b\|_2\}$ is surely a half space and is in the form of $\{x \mid C^T x \le d\}$ for all $x \in R$.

Therefore, points equidistant from a & b can be represented by a hyperplane normal to b−a.



$*$ → points on the hyperplane equidistant from a & b.

↳ and it is represented by a hyperplane normal to (b−a) and ($d = b^T b - a^T a$) is the offset.

Thus, $\{x \mid C^T x \le d\}$ is a halfspace and also a convex.