

**STA 9705: HW4**

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**# 10.14 (a)**

**HW4 Q-10.14**

The REG Procedure  
Model: MODEL1  
Dependent Variable: Y1

|                             |    |
|-----------------------------|----|
| Number of Observations Read | 46 |
| Number of Observations Used | 46 |

| Analysis of Variance |    |                |             |         |        |
|----------------------|----|----------------|-------------|---------|--------|
| Source               | DF | Sum of Squares | Mean Square | F Value | Pr > F |
| Model                | 3  | 0.18812        | 0.06271     | 4.88    | 0.0053 |
| Error                | 42 | 0.54006        | 0.01286     |         |        |
| Corrected Total      | 45 | 0.72818        |             |         |        |

|                |          |          |        |
|----------------|----------|----------|--------|
| Root MSE       | 0.11340  | R-Square | 0.2583 |
| Dependent Mean | 0.91783  | Adj R-Sq | 0.2054 |
| Coeff Var      | 12.35484 |          |        |

**HW4 Q-10.14**

The REG Procedure  
Model: MODEL1  
Dependent Variable: Y2

|                             |    |
|-----------------------------|----|
| Number of Observations Read | 46 |
| Number of Observations Used | 46 |

| Analysis of Variance |    |                |             |         |        |
|----------------------|----|----------------|-------------|---------|--------|
| Source               | DF | Sum of Squares | Mean Square | F Value | Pr > F |
| Model                | 3  | 51.72482       | 17.24161    | 0.23    | 0.8737 |
| Error                | 42 | 3123.42735     | 74.36732    |         |        |
| Corrected Total      | 45 | 3175.15217     |             |         |        |

|                |          |          |         |
|----------------|----------|----------|---------|
| Root MSE       | 8.62365  | R-Square | 0.0163  |
| Dependent Mean | 90.41304 | Adj R-Sq | -0.0540 |
| Coeff Var      | 9.53806  |          |         |

| Parameter Estimates |    |                    |                |         |         |
|---------------------|----|--------------------|----------------|---------|---------|
| Variable            | DF | Parameter Estimate | Standard Error | t Value | Pr >  t |
| Intercept           | 1  | 0.62639            | 0.17573        | 3.56    | 0.0009  |
| X1                  | 1  | 0.00090822         | 0.00052516     | 1.73    | 0.0911  |
| X2                  | 1  | -0.00095571        | 0.00041408     | -2.31   | 0.0260  |
| X3                  | 1  | 0.00149            | 0.00042503     | 3.51    | 0.0011  |

| Parameter Estimates |    |                    |                |         |         |
|---------------------|----|--------------------|----------------|---------|---------|
| Variable            | DF | Parameter Estimate | Standard Error | t Value | Pr >  t |
| Intercept           | 1  | 83.24254           | 13.36386       | 6.23    | <.0001  |
| X1                  | 1  | 0.02870            | 0.03994        | 0.72    | 0.4763  |
| X2                  | 1  | -0.01272           | 0.03149        | -0.40   | 0.6882  |
| X3                  | 1  | -0.00441           | 0.03232        | -0.14   | 0.8922  |

10.14 (a) We use the parameter estimates for  $Y_1$  and  $Y_2$  from our SAS output to get  $\hat{\beta}$ .

$$\hat{\beta} = \begin{pmatrix} 0.6264 & 83.2425 \\ 0.0009 & 0.0287 \\ -0.0010 & -0.0127 \\ 0.0015 & -0.0044 \end{pmatrix}$$

**10.14 (b)**

| HW4 Q-10.14                                                                            |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
|----------------------------------------------------------------------------------------|-----------------------|--------------------------------|----------------------------|-------------------------------|------------|------------|------------|------------|------------------|---------------------|--------|--------|--------|
| The REG Procedure<br>Model: MODEL1<br>Multivariate Test: OVERALL                       |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| Error Matrix (E)                                                                       |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| 0.5400626855 9.0162268852<br>9.0162268852 3123.4273529                                 |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| Hypothesis Matrix (H)                                                                  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| 0.1881199232 0.7050774626<br>0.7050774626 51.724820999                                 |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| Eigenvalues of Inv(E) <sup>H</sup><br>= CanRsq/(1-CanRsq)                              |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| Test of H0: The canonical correlations in the current row and all that follow are zero |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| 1                                                                                      | Canonical Correlation | Adjusted Canonical Correlation | Approximate Standard Error | Squared Canonical Correlation | Eigenvalue | Difference | Proportion | Cumulative | Likelihood Ratio | Approximate F Value | Num DF | Den DF | Pr > F |
| 1                                                                                      | 0.514205              | 0.469242                       | 0.109656                   | 0.264407                      | 0.3594     | 0.3434     | 0.9574     | 0.9574     | 0.72400963       | 2.39                | 6      | 82     | 0.0350 |
| 2                                                                                      | 0.125488              | 0.047831                       | 0.146724                   | 0.015747                      | 0.0160     |            | 0.0426     | 1.0000     | 0.98425273       | 0.34                | 2      | 42     | 0.7165 |
| Multivariate Statistics                                                                |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| S=2 M=0 N=19.5                                                                         |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| Statistic                                                                              |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| Wilks' Lambda                                                                          |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| 0.72400963                                                                             |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| Pillai's Trace                                                                         |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| 0.28015406                                                                             |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| Hotelling-Lawley Trace                                                                 |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| 0.37544623                                                                             |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| Roy's Greatest Root                                                                    |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| 0.35944702                                                                             |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |

(b) From the data - T34-DIABETES.dat we have.

$$P=2, q=3, n=46, V_H=q=3,$$

$$V_E = n-q-1 = 46-3-1 = 42,$$

$$S = \min(P, q) = \min(2, 3) = 2$$

$$M = \frac{1}{2} (|q-p|-1) = \frac{1}{2} (|3-2|-1) = 0$$

$$N = \frac{1}{2} (n-q-p-2) = 0.5(46-3-2-2) = 19.5$$

stating the hypothesis:-

$$H_0: B_1 = 0 \text{ and } H_a: B_1 \neq 0.$$

For model 1  $\rightarrow$  overall the eigen values from SAS is:-

$$\lambda_1 = 0.3594 \text{ and } \lambda_2 = 0.0160$$

Comparing all 4 test to check the significance:-

i) Wilks' test :-

$$\Lambda = \frac{|E|}{|E| + |H|} = \prod_{i=1}^S \frac{1}{1+\lambda_i} = \prod_{i=1}^2 \frac{1}{1+\lambda_i} = \frac{1}{(1+0.3594)} \times \frac{1}{(1+0.0160)} = 0.724$$

From table A.9 we look for  $\Lambda_{0.05}(2, 3, 42)$

We see  $\Lambda_{0.05}(2, 3, 42) \approx \Lambda_{0.05}(2, 3, 40)$   
 $\approx 0.730$ .

We check for critical value

$$\Lambda_{0.05}(2, 3, 42) > \Lambda_{0.05}(2, 3, 40) > \Lambda$$

$$\text{i.e. } 0.730 > 0.724$$

$\Rightarrow$  We reject  $H_0$  in Wilk's test.

ii) Ray's test :-

$\Theta = \frac{\lambda_1}{1+\lambda_1}$  where  $\lambda_1$  is the largest eigen value of  $E^{-1}H$ .

$$\Rightarrow \Theta = \frac{0.3594}{1+0.3594} = 0.264$$

From table A.10 we look for  $\Theta_{0.05}(2, 0, 19.5)$

We see  $\Theta_{0.05}(2, 0, 19.5) \approx \Theta_{0.05}(2, 0, 20)$   
 $\approx 0.221$

We check for critical value

$$\Theta > \Theta_{(s, m, N)} \Rightarrow 0.264 > 0.221$$

$\Rightarrow$  We reject  $H_0$  in Ray's test.

iii) Pillai's test :-

$$V^{(s)} = \text{tr} [H (E + H)^{-1}] = \sum_{i=1}^s \frac{\lambda_i}{1+\lambda_i}$$

$$V^{(2)} = \frac{0.3594}{1+0.3594} + \frac{0.0160}{1+0.0160} = 0.280$$

From table A.11 we look for  $V_{0.05}^{(2)}(2, 0, 19.5)$

We see  $V_{0.05}^{(2)}(2, 0, 19.5) \approx V^{(2)}(2, 0, 20)$   
 $\approx 0.263$ .

We check for critical value

$$V^{(2)} > V_{0.05}^{(2)}(s, m, N)$$

$\Rightarrow$  We reject  $H_0$  in Pillai's test.

iv) Lawley - Hotelling test:-

$$U^{(s)} = \text{tr}(E^{-1}H) = \sum_{i=1}^s d_i$$

$$U^{(2)} = 0.3594 + 0.0160 = 0.375$$

$$\therefore \frac{V_E}{V_H} (U^{(2)}) = \frac{42}{3} (0.375) = 5.25$$

From table A.12 we look for  $\tilde{U}_{0.05}^{(2)}(2, 3, 42) \quad \{ p < V_H \}$

We see  $\tilde{U}_{0.05}^{(2)}(2, 3, 42) \approx \tilde{U}_{0.05}^{(2)}(2, 3, 40)$   
 $\approx 4.7424$ .

We check for critical value

$$\frac{V_E}{V_H} (U^{(2)}) > \tilde{U}_{0.05}^{(2)}(p=2, V_H=3, V_E=42) \Rightarrow 5.25 > 4.7424$$

$\Rightarrow$  We reject  $H_0$  in Lawley - Hotelling's test.

For  $\alpha = 0.05$  all four test appear significant.

$\therefore$  There is a linear relationship between dependent variables  $y_1, y_2$  and independent variables  $x_1, x_2, x_3$ .

### 10.14 (c)

(c)  $\lambda_1 = 0.3594 \gg \lambda_2 = 0.0160.$

$\lambda_1$  is 95.74% of the total share and is thus the only dominant eigen value.

$\Rightarrow$  The essential rank of  $\hat{B}$ , is 1.

The power ranking is  $\Theta > V^{(s)} > \Lambda > V^{(s)}$ .

### 10.14 (d)

| HW4 Q-10.14                                                                            |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
|----------------------------------------------------------------------------------------|-----------------------|--------------------------------|----------------------------|-------------------------------|------------|------------|------------|------------|------------------|---------------------|--------|--------|--------|
| The REG Procedure<br>Model: MODEL1<br>Multivariate Test: PARTIAL_X1                    |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| Error Matrix (E)                                                                       |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| 0.5400626855 9.0162268852<br>9.0162268852 3123.4273529                                 |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| Hypothesis Matrix (H)                                                                  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| 0.0384591746 1.2153432557<br>1.2153432557 38.405900428                                 |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| Eigenvalues of Inv(E)'H<br>= CanRsq/(1-CanRsq)                                         |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| Test of H0: The canonical correlations in the current row and all that follow are zero |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |
| 1                                                                                      | Canonical Correlation | Adjusted Canonical Correlation | Approximate Standard Error | Squared Canonical Correlation | Eigenvalue | Difference | Proportion | Cumulative | Likelihood Ratio | Approximate F Value | Num DF | Den DF | Pr > F |
| 1                                                                                      | 0.262634              | 0.224257                       | 0.141980                   | 0.068977                      | 0.0741     |            | 1.0000     | 1.0000     | 0.93102343       | 1.52                | 2      | 41     | 0.2310 |

Note: The F statistic is exact.

| Multivariate Statistics |            |         |  |
|-------------------------|------------|---------|--|
| S=1 M=0 N=19.5          |            |         |  |
| Statistic               | Value      | P-Value |  |
| Wilks' Lambda           | 0.93102343 | 0.2310  |  |
| Pillai's Trace          | 0.06897657 | 0.2310  |  |
| Hotelling-Lawley Trace  | 0.07408682 | 0.2310  |  |
| Roy's Greatest Root     | 0.07408682 | 0.2310  |  |

#### HW4 Q-10.14

The REG Procedure  
Model: MODEL1  
Multivariate Test: PARTIAL\_X2

| Error Matrix (E) |              |
|------------------|--------------|
| 0.5400626855     | 9.0162268852 |
| 9.0162268852     | 3123.4273529 |

| Hypothesis Matrix (H) |              |
|-----------------------|--------------|
| 0.068497593           | 0.9119819959 |
| 0.9119819959          | 12.142195433 |

|   | Canonical Correlation | Adjusted Canonical Correlation | Approximate Standard Error | Squared Canonical Correlation | Eigenvalues of Inv(E)'H<br>= CanRsq/(1-CanRsq) |            |            |            | Test of H0: The canonical correlations in the current row and all that follow are zero |                     |        |        |        |
|---|-----------------------|--------------------------------|----------------------------|-------------------------------|------------------------------------------------|------------|------------|------------|----------------------------------------------------------------------------------------|---------------------|--------|--------|--------|
|   |                       |                                |                            |                               | Eigenvalue                                     | Difference | Proportion | Cumulative | Likelihood Ratio                                                                       | Approximate F Value | Num DF | Den DF | Pr > F |
| 1 | 0.335804              | 0.308546                       | 0.135302                   | 0.112764                      | 0.1271                                         |            | 1.0000     | 1.0000     | 0.88723586                                                                             | 2.61                | 2      | 41     | 0.0861 |

Note: The F statistic is exact.

| Multivariate Statistics |            |         |
|-------------------------|------------|---------|
| S=1 M=0 N=19.5          |            |         |
| Statistic               | Value      | P-Value |
| Wilks' Lambda           | 0.88723586 | 0.0861  |
| Pillai's Trace          | 0.11276414 | 0.0861  |
| Hotelling-Lawley Trace  | 0.12709601 | 0.0861  |
| Roy's Greatest Root     | 0.12709601 | 0.0861  |

#### HW4 Q-10.14

The REG Procedure  
Model: MODEL1  
Multivariate Test: PARTIAL\_X3

| Error Matrix (E) |              |
|------------------|--------------|
| 0.5400626855     | 9.0162268852 |
| 9.0162268852     | 3123.4273529 |

| Hypothesis Matrix (H) |              |
|-----------------------|--------------|
| 0.1579729491          | -0.467207454 |
| -0.467207454          | 1.3817733104 |

|   | Canonical Correlation | Adjusted Canonical Correlation | Approximate Standard Error | Squared Canonical Correlation | Eigenvalues of Inv(E)'H<br>= CanRsq/(1-CanRsq) |            |            |            | Test of H0: The canonical correlations in the current row and all that follow are zero |                     |        |        |        |
|---|-----------------------|--------------------------------|----------------------------|-------------------------------|------------------------------------------------|------------|------------|------------|----------------------------------------------------------------------------------------|---------------------|--------|--------|--------|
|   |                       |                                |                            |                               | Eigenvalue                                     | Difference | Proportion | Cumulative | Likelihood Ratio                                                                       | Approximate F Value | Num DF | Den DF | Pr > F |
| 1 | 0.488266              | 0.474452                       | 0.116142                   | 0.238403                      | 0.3130                                         |            | 1.0000     | 1.0000     | 0.76159667                                                                             | 6.42                | 2      | 41     | 0.0038 |

Note: The F statistic is exact.

| Multivariate Statistics |            |         |
|-------------------------|------------|---------|
| S=1 M=0 N=19.5          |            |         |
| Statistic               | Value      | P-Value |
| Wilks' Lambda           | 0.76159667 | 0.0038  |
| Pillai's Trace          | 0.23840333 | 0.0038  |
| Hotelling-Lawley Trace  | 0.31303096 | 0.0038  |
| Roy's Greatest Root     | 0.31303096 | 0.0038  |

(d) When we run partial test the value of  $V_H$  is no more equal to  $q$  but  $h = 1$ .

In this case where  $V_H = 1$ , we can simply test any one of 4 MANOVA tests, all of which will give same results for  $\chi^2$ -value and equal F-transformed.

For all three  $x$ 's we have the following info

|                  | $\Lambda$ | F    | p-value |
|------------------|-----------|------|---------|
| $x_1   x_2, x_3$ | 0.9310    | 1.52 | 0.2310  |
| $x_2   x_1, x_3$ | 0.8872    | 2.61 | 0.0861  |
| $x_3   x_1, x_2$ | 0.7616    | 6.42 | 0.0038  |

From the table above we can observe that  $x_3$  adjusted for other two  $x$ 's shows significant result for  $\alpha = 0.05$ .

However for  $x_1$  and  $x_2$  when adjusted for other  $x$ 's result insignificant results and thus we need reduced model.

# 10.17 (a)

| HW4 Q-10.17                                                  |    |                    |                             |         |         | HW4 Q-10.17                                                  |    |                    |                             |         |         |
|--------------------------------------------------------------|----|--------------------|-----------------------------|---------|---------|--------------------------------------------------------------|----|--------------------|-----------------------------|---------|---------|
| The REG Procedure<br>Model: MODEL1<br>Dependent Variable: Y1 |    |                    |                             |         |         | The REG Procedure<br>Model: MODEL1<br>Dependent Variable: Y2 |    |                    |                             |         |         |
| Number of Observations Read                                  |    | 30                 | Number of Observations Used |         | 30      | Number of Observations Read                                  |    | 30                 | Number of Observations Used |         | 30      |
| Analysis of Variance                                         |    |                    |                             |         |         |                                                              |    |                    |                             |         |         |
| Source                                                       | DF | Sum of Squares     | Mean Square                 | F Value | Pr > F  | Source                                                       | DF | Sum of Squares     | Mean Square                 | F Value | Pr > F  |
| Model                                                        | 8  | 1.41711            | 0.17714                     | 1.17    | 0.3603  | Model                                                        | 8  | 1.40951            | 0.17619                     | 0.89    | 0.5440  |
| Error                                                        | 21 | 3.17256            | 0.15107                     |         |         | Error                                                        | 21 | 4.17349            | 0.19874                     |         |         |
| Corrected Total                                              | 29 | 4.58967            |                             |         |         | Corrected Total                                              | 29 | 5.58300            |                             |         |         |
| Root MSE                                                     |    | 0.38868            | R-Square                    | 0.3088  |         | Root MSE                                                     |    | 0.44580            | R-Square                    | 0.2525  |         |
| Dependent Mean                                               |    | 0.26333            | Adj R-Sq                    | 0.0454  |         | Dependent Mean                                               |    | 0.07000            | Adj R-Sq                    | -0.0323 |         |
| Coeff Var                                                    |    | 147.60102          |                             |         |         | Coeff Var                                                    |    | 636.85715          |                             |         |         |
| Parameter Estimates                                          |    |                    |                             |         |         |                                                              |    |                    |                             |         |         |
| Variable                                                     | DF | Parameter Estimate | Standard Error              | t Value | Pr >  t | Variable                                                     | DF | Parameter Estimate | Standard Error              | t Value | Pr >  t |
| Intercept                                                    | 1  | -4.14039           | 4.78081                     | -0.87   | 0.3963  | Intercept                                                    | 1  | 4.93526            | 5.48336                     | 0.90    | 0.3783  |
| X1                                                           | 1  | 1.10259            | 0.78372                     | 1.41    | 0.1741  | X1                                                           | 1  | -0.95500           | 0.89888                     | -1.06   | 0.3001  |
| X2                                                           | 1  | 0.23074            | 0.59447                     | 0.39    | 0.7018  | X2                                                           | 1  | -0.22174           | 0.68183                     | -0.33   | 0.7482  |
| X3                                                           | 1  | 1.17138            | 0.72314                     | 1.62    | 0.1202  | X3                                                           | 1  | 1.77275            | 0.82941                     | 2.14    | 0.0445  |
| X4                                                           | 1  | 0.11145            | 0.06278                     | 1.78    | 0.0904  | X4                                                           | 1  | 0.04793            | 0.07201                     | 0.67    | 0.5129  |
| X5                                                           | 1  | 0.61679            | 0.38692                     | 1.59    | 0.1259  | X5                                                           | 1  | -0.05787           | 0.44378                     | -0.13   | 0.8975  |
| X6                                                           | 1  | 0.26683            | 0.38342                     | 0.70    | 0.4941  | X6                                                           | 1  | 0.48537            | 0.43976                     | 1.10    | 0.2822  |
| X7                                                           | 1  | -0.26258           | 0.20081                     | -1.31   | 0.2052  | X7                                                           | 1  | -0.20928           | 0.23032                     | -0.91   | 0.3739  |
| X8                                                           | 1  | -0.00432           | 0.00398                     | -1.09   | 0.2901  | X8                                                           | 1  | -0.00407           | 0.00456                     | -0.89   | 0.3824  |

10.17 (a) We use the parameter estimates for  $Y_1$  and  $Y_2$  from our SAS output to get  $\hat{B}$

$$\hat{B} = \begin{pmatrix} -4.1404 & 4.9353 \\ 1.1026 & -0.9550 \\ 0.2370 & -0.2217 \\ 1.1714 & 1.7728 \\ 0.1115 & 0.0479 \\ 0.6168 & -0.0579 \\ 0.2668 & 0.4854 \\ -0.2626 & -0.2093 \\ -0.0043 & -0.0041 \end{pmatrix}$$

| HW4 Q-10.17                                                      |          |                                |          |                            |        |                               |                          |            |            |            |            |                  |                     |               |        |        |  |
|------------------------------------------------------------------|----------|--------------------------------|----------|----------------------------|--------|-------------------------------|--------------------------|------------|------------|------------|------------|------------------|---------------------|---------------|--------|--------|--|
| The REG Procedure<br>Model: MODEL1<br>Multivariate Test: OVERALL |          |                                |          |                            |        |                               |                          |            |            |            |            |                  |                     |               |        |        |  |
| Error Matrix (E)                                                 |          |                                |          |                            |        |                               |                          |            |            |            |            |                  |                     |               |        |        |  |
| 3.1725587246 2.2483661643                                        |          |                                |          |                            |        |                               | 2.2483661643 4.173490596 |            |            |            |            |                  |                     |               |        |        |  |
| Hypothesis Matrix (H)                                            |          |                                |          |                            |        |                               | 1.417107942 0.5786338357 |            |            |            |            |                  |                     |               |        |        |  |
| 0.5786338357 1.409509404                                         |          |                                |          |                            |        |                               |                          |            |            |            |            |                  |                     |               |        |        |  |
| Eigenvalues of Inv(E) <sup>T</sup> H<br>= CanRsq/(1-CanRsq)      |          |                                |          |                            |        |                               |                          |            |            |            |            |                  |                     |               |        |        |  |
| Canonical Correlation                                            |          | Adjusted Canonical Correlation |          | Approximate Standard Error |        | Squared Canonical Correlation |                          | Eigenvalue | Difference | Proportion | Cumulative | Likelihood Ratio | Approximate F Value | Num DF        | Den DF | Pr > F |  |
| 1                                                                | 0.620790 | 0.454318                       | 0.114132 | 0.385380                   | 0.6270 | 0.3031                        | 0.6594                   | 0.6594     | 0.46423605 | 1.17       | 16         | 40               | 0.3321              |               |        |        |  |
| 2                                                                | 0.494650 | 0.399406                       | 0.140260 | 0.244678                   | 0.3239 |                               | 0.3406                   | 1.0000     | 0.75532164 | 0.97       | 7          | 21               | 0.4766              |               |        |        |  |
| Multivariate Statistics                                          |          |                                |          |                            |        |                               |                          |            |            |            |            |                  |                     | S=2 M=2.5 N=9 |        |        |  |
| Statistic                                                        |          | Value                          |          | P-Value                    |        |                               |                          |            |            |            |            |                  |                     |               |        |        |  |
| Wilks' Lambda                                                    |          | 0.46423605                     |          | 0.3321                     |        |                               |                          |            |            |            |            |                  |                     |               |        |        |  |
| Pillai's Trace                                                   |          | 0.63005801                     |          | 0.2973                     |        |                               |                          |            |            |            |            |                  |                     |               |        |        |  |
| Hotelling-Lawley Trace                                           |          | 0.95095995                     |          | 0.3598                     |        |                               |                          |            |            |            |            |                  |                     |               |        |        |  |

From the data  $\rightarrow$  T7-1-SEISHU.DAT we have,

$P=2$ ,  $q_1=8$ ,  $n=30$ ,  $V_n=q_1=8$ ,

$V_E = n - q_1 - 1 = 30 - 8 - 1 = 21$ ,

$S = \min(P, q_1) = \min(2, 8) = 2$

$m = \frac{1}{2} (|q_1 - P| - 1) = \frac{1}{2} (|8 - 2| - 1) = 2.5$

$N = 9$

Stating the hypothesis:-

$H_0: B_1 = 0$  and  $H_a: B_1 \neq 0$

Now, compare all 4 test to check for significance.-

Wilks' test,  $\Lambda = \frac{1}{(1+0.6270)} \times \frac{1}{(1+0.3239)} = 0.4642$

and  $\Lambda_{0.05(2, 8, 21)} = 0.322$  for table A-9

$$\Rightarrow \Lambda_{0.5}(2,8,21) < \Lambda \Rightarrow 0.322 < 0.464$$

$\Rightarrow$  we cannot reject  $H_0$  in Wilk's test

(ii) Roy's test :-

$$\Theta = \frac{\lambda_1}{1+\lambda_1} \text{ where } \lambda_1 \text{ is the largest eigen value of } E^{-1}H$$

$$\Rightarrow \Theta = \frac{0.6270}{1+0.6270} = 0.3854$$

From table A.10 we look for  $\Theta_{0.05}(2,2.5,9)$

$$\text{we see } \Theta_{0.05}(2,2.5,9) \approx \Theta_{0.05}(2,2,10) \\ = 0.514.$$

we check for critical value and

$$\Theta < \Theta_{(S,M,N)} \Rightarrow 0.3854 < 0.514.$$

$\Rightarrow$  we fail to reject  $H_0$  in Roy's test.

(iii) Pillai's Test :-

$$V^{(2)} = \text{tr}[H(E+H)^{-1}] = \sum_{i=1}^s \frac{\lambda_i}{1+\lambda_i}$$

$$V^{(2)} = \frac{0.6270}{1+0.6270} + \frac{0.3239}{1+0.3239} = 0.6301$$

From table A.11 we look for  $V_{0.05}^{(2)}(2,2.5,9)$

$$\text{we see } V_{0.05}^{(2)}(2,2.5,9) \approx V_{0.05}^{(2)}(2,2,9) \\ = 0.772$$

We check for critical values:-

$$V_{0.05}^{(2)}(2, 1.5, 9) > V_{0.05}^{(2)}(2, 2, 9) > V^{(2)}$$

$$\Rightarrow 0.772 > 0.6301$$

$\Rightarrow$  we fail to reject  $H_0$  in Pillai's test.

(iv) Lawley-Hotelling test:-

$$U^{(s)} = \text{tr}(E^{-1}H) = \sum_{i=1}^s \lambda_i$$

$$U^{(s)} = 0.6270 + 0.3239 = 0.9509$$

$$\therefore \frac{U_E}{U_H} U^{(s)} = \frac{21}{8} (0.9509) = 2.496.$$

From table A.12 we look for  $\tilde{U}_{0.05}^{(2)}(2, 8, 21)$

$$\text{we see } \tilde{U}_{0.05}^{(2)}(2, 8, 21) > \tilde{U}_{0.05}^{(2)}(2, 8, 25) \\ = 4.147.$$

We check for critical value

$$\frac{U_E}{U_H} U^{(s)} < \tilde{U}_{0.05}^{(2)}(2, 8, 25) \Rightarrow 2.496 < 4.147$$

$\Rightarrow$  We fail to reject  $H_0$  in Lawley-Hotelling's test.

$\therefore$  For  $\alpha = 0.05$  all four tests are insignificant  
and there is no linear relationship between  $(y_1, y_2)$   
and all other  $x$ 's.

# 10.17 (b)

| HW4 Q-10.17                                                                            |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |
|----------------------------------------------------------------------------------------|-----------------------|--------------------------------|----------------------------|-------------------------------|------------|------------|------------|------------|------------------|---------------------|--------|--------|--------|--|
| The REG Procedure<br>Model: MODEL1<br>Multivariate Test: PARTIAL_X7_X8                 |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |
| Error Matrix (E)                                                                       |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |
| 3.1725587246 2.2483661643<br>2.2483661643 4.173490596                                  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |
| Hypothesis Matrix (H)                                                                  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |
| 0.5165001532 0.4433479905<br>0.4433479905 0.3825011352                                 |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |
| Eigenvalues of Inv(E) <sup>T</sup> H<br>= CanRsq/(t-CanRsq)                            |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |
| Test of H0: The canonical correlations in the current row and all that follow are zero |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |
| 1                                                                                      | Canonical Correlation | Adjusted Canonical Correlation | Approximate Standard Error | Squared Canonical Correlation | Eigenvalue | Difference | Proportion | Cumulative | Likelihood Ratio | Approximate F Value | Num DF | Den DF | Pr > F |  |
| 1                                                                                      | 0.378587              | 0.286820                       | 0.178629                   | 0.143328                      | 0.1673     | 0.1666     | 0.9956     | 0.9956     | 0.85604439       | 0.81                | 4      | 40     | 0.5274 |  |
| 2                                                                                      | 0.027072              | .                              | 0.208362                   | 0.000733                      | 0.0007     |            | 0.0044     | 1.0000     | 0.99926709       | 0.02                | 1      | 21     | 0.9024 |  |
| Multivariate Statistics                                                                |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |
| S=2 M=0.5 N=9                                                                          |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |
| Statistic                                                                              |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |
| Wilks' Lambda                                                                          |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |
| 0.85604439 0.5274                                                                      |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |
| Pillai's Trace                                                                         |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |
| 0.14406066 0.5316                                                                      |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |
| Hotelling-Lawley Trace                                                                 |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |
| 0.16804101 0.5213                                                                      |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |
| Roy's Greatest Root                                                                    |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |
| 0.16730756 0.4651                                                                      |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |

(b) Stating the hypothesis:-

$$H_0: B_d = 0 \text{ and } H_a: B_d \neq 0.$$

where  $B_d$  will have slopes relative to  $x_7$  &  $x_8$ .  
From the SAS output we have.

$$h=2, p=2, V_H=h=2,$$

$$V_E = n-q-1 = 30-8-1 = 21$$

$$S = \min(p, h) = \min(2, 2) = 2.$$

$$m = \frac{1}{2} (|h-p|-1) = \frac{1}{2} (|2-2|-1) = -0.5.$$

$$N = 9.$$

From class chapt 6, we know that all four tests shows exact tests for  $H_0$  and rejects it with same probability.

In practice,  $\Lambda$  and  $V^{(s)}$  are often preferred but  $\Lambda$  is flexible and has historical precedence.

Therefore we test our hypothesis through Wilk's test.

Wilk's test :-

$$\Lambda = \frac{|E|}{|E| + |H|} = \prod_{i=1}^s \frac{1}{1+d_i}$$

Here,  $\Lambda = 0.8560$  and  $\Lambda_{0.05}(2, 2, 21) = 0.629$ .

since  $\Lambda_{0.05}(2, 2, 21) < \Lambda \Rightarrow 0.629 < 0.8560$   
 $\Rightarrow$  We fail to reject  $H_0$ .

For  $\alpha = 0.05$  we see all the four test have results that are not significant with p-value being greater than  $\alpha$ .

$\Rightarrow$  Even with the reduced model  $x_7$  &  $x_8$  are not significant predictors of other  $x_i$ 's.

# 10.17 (c)

| HW4 Q-10.17                                                               |          |          |          |          |        |                                                  |              |            |            |                                                                                        |                     |        |        |        |
|---------------------------------------------------------------------------|----------|----------|----------|----------|--------|--------------------------------------------------|--------------|------------|------------|----------------------------------------------------------------------------------------|---------------------|--------|--------|--------|
| The REG Procedure<br>Model: MODEL1<br>Multivariate Test: PARTIAL_X4_X5_X6 |          |          |          |          |        |                                                  |              |            |            |                                                                                        |                     |        |        |        |
|                                                                           |          |          |          |          |        | Error Matrix (E)                                 |              |            |            |                                                                                        |                     |        |        |        |
|                                                                           |          |          |          |          |        | 3.1725587246                                     | 2.2483661643 |            |            |                                                                                        |                     |        |        |        |
|                                                                           |          |          |          |          |        | 2.2483661643                                     | 4.173490596  |            |            |                                                                                        |                     |        |        |        |
|                                                                           |          |          |          |          |        | Hypothesis Matrix (H)                            |              |            |            |                                                                                        |                     |        |        |        |
|                                                                           |          |          |          |          |        | 0.9774657385                                     | 0.3758022424 |            |            |                                                                                        |                     |        |        |        |
|                                                                           |          |          |          |          |        | 0.3758022424                                     | 0.4143043703 |            |            |                                                                                        |                     |        |        |        |
|                                                                           |          |          |          |          |        | Eigenvalues of Inv(E)'H<br>= CanResq/(1-CanResq) |              |            |            | Test of H0: The canonical correlations in the current row and all that follow are zero |                     |        |        |        |
|                                                                           |          |          |          |          |        | Eigenvalue                                       | Difference   | Proportion | Cumulative | Likelihood Ratio                                                                       | Approximate F Value | Num DF | Den DF | Pr > F |
| 1                                                                         | 0.516577 | 0.416761 | 0.149653 | 0.266852 | 0.3640 | 0.2755                                           | 0.8044       | 0.8044     | 0.67352570 | 1.46                                                                                   | 6                   | 40     | 0.2178 |        |
| 2                                                                         | 0.285173 | 0.266653 | 0.187524 | 0.081324 | 0.0885 |                                                  | 0.1956       | 1.0000     | 0.91867645 | 0.93                                                                                   | 2                   | 21     | 0.4104 |        |
| Multivariate Statistics                                                   |          |          |          |          |        |                                                  |              |            |            |                                                                                        |                     |        |        |        |
| S=2 M=0 N=9                                                               |          |          |          |          |        |                                                  |              |            |            |                                                                                        |                     |        |        |        |
|                                                                           |          |          |          |          |        | Statistic                                        | Value        | P-Value    |            |                                                                                        |                     |        |        |        |
|                                                                           |          |          |          |          |        | Wilks' Lambda                                    | 0.67352570   | 0.2178     |            |                                                                                        |                     |        |        |        |
|                                                                           |          |          |          |          |        | Pillai's Trace                                   | 0.34817566   | 0.2034     |            |                                                                                        |                     |        |        |        |
|                                                                           |          |          |          |          |        | Hotelling-Lawley Trace                           | 0.45250380   | 0.2264     |            |                                                                                        |                     |        |        |        |
|                                                                           |          |          |          |          |        | Roy's Greatest Root                              | 0.36398128   | 0.2529     |            |                                                                                        |                     |        |        |        |

(c) Stating the hypothesis:-  
 $H_0: B_d = 0$  and  $B_d \neq 0$ .

where  $B_d$  will have slopes relative to  $x_4, x_5$  &  $x_6$ .

From the SAS output we have :-

$$h=3, p=2, V_n=h=3$$

$$V_E = m-p-1 = 21, S = \min(p, h) = \min(2, 3) = 2.$$

$$m = \frac{1}{2} (|h-p|-1) = \frac{1}{2} (|3-2|-1) = 0.$$

$$N = 9.$$

From class chap 6, we know that all four tests shows exact tests for  $H_0$  and rejects it with same probability.

In practice,  $\Lambda$  and  $V^{(s)}$  are often preferred but  $\Lambda$  is flexible and has historical precedence.

Therefore we test our hypothesis through Wilk's test.

Wilk's test :-

$$\Lambda = \frac{|E|}{|E| + |H|} = \prod_{i=1}^s \frac{1}{1+d_i}$$

Here,  $\Lambda = 0.6735$  and  $\Lambda_{0.05}(2, 3, 21) = 0.548$

since  $\Lambda_{0.5}(2, 3, 21) < \Lambda \Rightarrow 0.548 < 0.6735$

$\Rightarrow$  We fail to reject  $H_0$ .

For  $\alpha = 0.05$  we see all the four test have results that are not significant with p-value greater than  $\alpha$ .

$\Rightarrow$  Even with the reduced model  $x_4, x_5$  &  $x_6$  are not significant predictor of other  $x$ 's.

# 10.17 (d)

| HW4 Q-10.17                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                                |                            |                               |            |            |            |                                                        |                  |                     |        |        |        |        |                                                                           |  |  |  |  |  |  |  |  |  |  |  |  |  |                                                                                        |  |  |  |  |  |  |  |  |  |  |  |  |  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |               |            |          |          |          |        |        |        |        |            |      |   |    |        |                |            |  |          |          |        |  |        |        |            |      |   |    |        |                        |            |  |  |        |  |  |  |  |  |  |  |  |  |                     |            |  |  |        |  |  |  |  |  |  |  |  |  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------|----------------------------|-------------------------------|------------|------------|------------|--------------------------------------------------------|------------------|---------------------|--------|--------|--------|--------|---------------------------------------------------------------------------|--|--|--|--|--|--|--|--|--|--|--|--|--|----------------------------------------------------------------------------------------|--|--|--|--|--|--|--|--|--|--|--|--|--|-----------------------|--------------------------------|----------------------------|-------------------------------|------------|------------|------------|------------|------------------|---------------------|--------|--------|--------|--|---------------|------------|----------|----------|----------|--------|--------|--------|--------|------------|------|---|----|--------|----------------|------------|--|----------|----------|--------|--|--------|--------|------------|------|---|----|--------|------------------------|------------|--|--|--------|--|--|--|--|--|--|--|--|--|---------------------|------------|--|--|--------|--|--|--|--|--|--|--|--|--|
| The REG Procedure<br>Model: MODEL1<br>Multivariate Test: PARTIAL_X1_X2_X3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                |                            |                               |            |            |            |                                                        |                  |                     |        |        |        |        |                                                                           |  |  |  |  |  |  |  |  |  |  |  |  |  |                                                                                        |  |  |  |  |  |  |  |  |  |  |  |  |  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |               |            |          |          |          |        |        |        |        |            |      |   |    |        |                |            |  |          |          |        |  |        |        |            |      |   |    |        |                        |            |  |  |        |  |  |  |  |  |  |  |  |  |                     |            |  |  |        |  |  |  |  |  |  |  |  |  |
| Error Matrix (E)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                |                            |                               |            |            |            |                                                        |                  |                     |        |        |        |        |                                                                           |  |  |  |  |  |  |  |  |  |  |  |  |  |                                                                                        |  |  |  |  |  |  |  |  |  |  |  |  |  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |               |            |          |          |          |        |        |        |        |            |      |   |    |        |                |            |  |          |          |        |  |        |        |            |      |   |    |        |                        |            |  |  |        |  |  |  |  |  |  |  |  |  |                     |            |  |  |        |  |  |  |  |  |  |  |  |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                |                            |                               |            |            |            | 3.1725587246 2.2483661643<br>2.2483661643 4.173490596  |                  |                     |        |        |        |        |                                                                           |  |  |  |  |  |  |  |  |  |  |  |  |  |                                                                                        |  |  |  |  |  |  |  |  |  |  |  |  |  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |               |            |          |          |          |        |        |        |        |            |      |   |    |        |                |            |  |          |          |        |  |        |        |            |      |   |    |        |                        |            |  |  |        |  |  |  |  |  |  |  |  |  |                     |            |  |  |        |  |  |  |  |  |  |  |  |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                |                            |                               |            |            |            | Hypothesis Matrix (H)                                  |                  |                     |        |        |        |        |                                                                           |  |  |  |  |  |  |  |  |  |  |  |  |  |                                                                                        |  |  |  |  |  |  |  |  |  |  |  |  |  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |               |            |          |          |          |        |        |        |        |            |      |   |    |        |                |            |  |          |          |        |  |        |        |            |      |   |    |        |                        |            |  |  |        |  |  |  |  |  |  |  |  |  |                     |            |  |  |        |  |  |  |  |  |  |  |  |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                |                            |                               |            |            |            | 1.0505812926 0.5054338805<br>0.5054338805 1.0273346775 |                  |                     |        |        |        |        |                                                                           |  |  |  |  |  |  |  |  |  |  |  |  |  |                                                                                        |  |  |  |  |  |  |  |  |  |  |  |  |  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |               |            |          |          |          |        |        |        |        |            |      |   |    |        |                |            |  |          |          |        |  |        |        |            |      |   |    |        |                        |            |  |  |        |  |  |  |  |  |  |  |  |  |                     |            |  |  |        |  |  |  |  |  |  |  |  |  |
| <table border="1"> <thead> <tr> <th colspan="14">Eigenvalues of <math>\text{Inv}(E)^T H</math><br/><math>= \text{CanRsq}/(1-\text{CanRsq})</math></th></tr> <tr> <th colspan="14">Test of H0: The canonical correlations in the current row and all that follow are zero</th></tr> <tr> <th>Canonical Correlation</th><th>Adjusted Canonical Correlation</th><th>Approximate Standard Error</th><th>Squared Canonical Correlation</th><th>Eigenvalue</th><th>Difference</th><th>Proportion</th><th>Cumulative</th><th>Likelihood Ratio</th><th>Approximate F Value</th><th>Num DF</th><th>Den DF</th><th>Pr &gt; F</th><th></th></tr> </thead> <tbody> <tr> <td>1</td><td>0.540038</td><td>0.386148</td><td>0.144593</td><td>0.291641</td><td>0.4117</td><td>0.1673</td><td>0.6275</td><td>0.6275</td><td>0.56921187</td><td>2.17</td><td>6</td><td>40</td><td>0.0663</td></tr> <tr> <td>2</td><td>0.443211</td><td></td><td>0.164027</td><td>0.196436</td><td>0.2445</td><td></td><td>0.3725</td><td>1.0000</td><td>0.80356412</td><td>2.57</td><td>2</td><td>21</td><td>0.1006</td></tr> </tbody> </table> |                                |                            |                               |            |            |            |                                                        |                  |                     |        |        |        |        | Eigenvalues of $\text{Inv}(E)^T H$<br>$= \text{CanRsq}/(1-\text{CanRsq})$ |  |  |  |  |  |  |  |  |  |  |  |  |  | Test of H0: The canonical correlations in the current row and all that follow are zero |  |  |  |  |  |  |  |  |  |  |  |  |  | Canonical Correlation | Adjusted Canonical Correlation | Approximate Standard Error | Squared Canonical Correlation | Eigenvalue | Difference | Proportion | Cumulative | Likelihood Ratio | Approximate F Value | Num DF | Den DF | Pr > F |  | 1             | 0.540038   | 0.386148 | 0.144593 | 0.291641 | 0.4117 | 0.1673 | 0.6275 | 0.6275 | 0.56921187 | 2.17 | 6 | 40 | 0.0663 | 2              | 0.443211   |  | 0.164027 | 0.196436 | 0.2445 |  | 0.3725 | 1.0000 | 0.80356412 | 2.57 | 2 | 21 | 0.1006 |                        |            |  |  |        |  |  |  |  |  |  |  |  |  |                     |            |  |  |        |  |  |  |  |  |  |  |  |  |
| Eigenvalues of $\text{Inv}(E)^T H$<br>$= \text{CanRsq}/(1-\text{CanRsq})$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                |                            |                               |            |            |            |                                                        |                  |                     |        |        |        |        |                                                                           |  |  |  |  |  |  |  |  |  |  |  |  |  |                                                                                        |  |  |  |  |  |  |  |  |  |  |  |  |  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |               |            |          |          |          |        |        |        |        |            |      |   |    |        |                |            |  |          |          |        |  |        |        |            |      |   |    |        |                        |            |  |  |        |  |  |  |  |  |  |  |  |  |                     |            |  |  |        |  |  |  |  |  |  |  |  |  |
| Test of H0: The canonical correlations in the current row and all that follow are zero                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                |                            |                               |            |            |            |                                                        |                  |                     |        |        |        |        |                                                                           |  |  |  |  |  |  |  |  |  |  |  |  |  |                                                                                        |  |  |  |  |  |  |  |  |  |  |  |  |  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |               |            |          |          |          |        |        |        |        |            |      |   |    |        |                |            |  |          |          |        |  |        |        |            |      |   |    |        |                        |            |  |  |        |  |  |  |  |  |  |  |  |  |                     |            |  |  |        |  |  |  |  |  |  |  |  |  |
| Canonical Correlation                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | Adjusted Canonical Correlation | Approximate Standard Error | Squared Canonical Correlation | Eigenvalue | Difference | Proportion | Cumulative                                             | Likelihood Ratio | Approximate F Value | Num DF | Den DF | Pr > F |        |                                                                           |  |  |  |  |  |  |  |  |  |  |  |  |  |                                                                                        |  |  |  |  |  |  |  |  |  |  |  |  |  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |               |            |          |          |          |        |        |        |        |            |      |   |    |        |                |            |  |          |          |        |  |        |        |            |      |   |    |        |                        |            |  |  |        |  |  |  |  |  |  |  |  |  |                     |            |  |  |        |  |  |  |  |  |  |  |  |  |
| 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | 0.540038                       | 0.386148                   | 0.144593                      | 0.291641   | 0.4117     | 0.1673     | 0.6275                                                 | 0.6275           | 0.56921187          | 2.17   | 6      | 40     | 0.0663 |                                                                           |  |  |  |  |  |  |  |  |  |  |  |  |  |                                                                                        |  |  |  |  |  |  |  |  |  |  |  |  |  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |               |            |          |          |          |        |        |        |        |            |      |   |    |        |                |            |  |          |          |        |  |        |        |            |      |   |    |        |                        |            |  |  |        |  |  |  |  |  |  |  |  |  |                     |            |  |  |        |  |  |  |  |  |  |  |  |  |
| 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | 0.443211                       |                            | 0.164027                      | 0.196436   | 0.2445     |            | 0.3725                                                 | 1.0000           | 0.80356412          | 2.57   | 2      | 21     | 0.1006 |                                                                           |  |  |  |  |  |  |  |  |  |  |  |  |  |                                                                                        |  |  |  |  |  |  |  |  |  |  |  |  |  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |               |            |          |          |          |        |        |        |        |            |      |   |    |        |                |            |  |          |          |        |  |        |        |            |      |   |    |        |                        |            |  |  |        |  |  |  |  |  |  |  |  |  |                     |            |  |  |        |  |  |  |  |  |  |  |  |  |
| <table border="1"> <thead> <tr> <th colspan="14">Multivariate Statistics</th></tr> <tr> <th colspan="14">S=2 M=0 N=9</th></tr> <tr> <th>Statistic</th><th colspan="3">Value</th><th>P-Value</th><th colspan="9"></th></tr> </thead> <tbody> <tr> <td>Wilks' Lambda</td><td colspan="3">0.56921187</td><td>0.0663</td><td colspan="9"></td></tr> <tr> <td>Pillai's Trace</td><td colspan="3">0.48807689</td><td>0.0495</td><td colspan="9"></td></tr> <tr> <td>Hotelling-Lawley Trace</td><td colspan="3">0.65616933</td><td>0.0836</td><td colspan="9"></td></tr> <tr> <td>Roy's Greatest Root</td><td colspan="3">0.41171357</td><td>0.1955</td><td colspan="9"></td></tr> </tbody> </table>                                                                                                                                                                                                                                                                                                                                                                                                                |                                |                            |                               |            |            |            |                                                        |                  |                     |        |        |        |        | Multivariate Statistics                                                   |  |  |  |  |  |  |  |  |  |  |  |  |  | S=2 M=0 N=9                                                                            |  |  |  |  |  |  |  |  |  |  |  |  |  | Statistic             | Value                          |                            |                               | P-Value    |            |            |            |                  |                     |        |        |        |  | Wilks' Lambda | 0.56921187 |          |          | 0.0663   |        |        |        |        |            |      |   |    |        | Pillai's Trace | 0.48807689 |  |          | 0.0495   |        |  |        |        |            |      |   |    |        | Hotelling-Lawley Trace | 0.65616933 |  |  | 0.0836 |  |  |  |  |  |  |  |  |  | Roy's Greatest Root | 0.41171357 |  |  | 0.1955 |  |  |  |  |  |  |  |  |  |
| Multivariate Statistics                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                |                            |                               |            |            |            |                                                        |                  |                     |        |        |        |        |                                                                           |  |  |  |  |  |  |  |  |  |  |  |  |  |                                                                                        |  |  |  |  |  |  |  |  |  |  |  |  |  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |               |            |          |          |          |        |        |        |        |            |      |   |    |        |                |            |  |          |          |        |  |        |        |            |      |   |    |        |                        |            |  |  |        |  |  |  |  |  |  |  |  |  |                     |            |  |  |        |  |  |  |  |  |  |  |  |  |
| S=2 M=0 N=9                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                                |                            |                               |            |            |            |                                                        |                  |                     |        |        |        |        |                                                                           |  |  |  |  |  |  |  |  |  |  |  |  |  |                                                                                        |  |  |  |  |  |  |  |  |  |  |  |  |  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |               |            |          |          |          |        |        |        |        |            |      |   |    |        |                |            |  |          |          |        |  |        |        |            |      |   |    |        |                        |            |  |  |        |  |  |  |  |  |  |  |  |  |                     |            |  |  |        |  |  |  |  |  |  |  |  |  |
| Statistic                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | Value                          |                            |                               | P-Value    |            |            |                                                        |                  |                     |        |        |        |        |                                                                           |  |  |  |  |  |  |  |  |  |  |  |  |  |                                                                                        |  |  |  |  |  |  |  |  |  |  |  |  |  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |               |            |          |          |          |        |        |        |        |            |      |   |    |        |                |            |  |          |          |        |  |        |        |            |      |   |    |        |                        |            |  |  |        |  |  |  |  |  |  |  |  |  |                     |            |  |  |        |  |  |  |  |  |  |  |  |  |
| Wilks' Lambda                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 0.56921187                     |                            |                               | 0.0663     |            |            |                                                        |                  |                     |        |        |        |        |                                                                           |  |  |  |  |  |  |  |  |  |  |  |  |  |                                                                                        |  |  |  |  |  |  |  |  |  |  |  |  |  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |               |            |          |          |          |        |        |        |        |            |      |   |    |        |                |            |  |          |          |        |  |        |        |            |      |   |    |        |                        |            |  |  |        |  |  |  |  |  |  |  |  |  |                     |            |  |  |        |  |  |  |  |  |  |  |  |  |
| Pillai's Trace                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | 0.48807689                     |                            |                               | 0.0495     |            |            |                                                        |                  |                     |        |        |        |        |                                                                           |  |  |  |  |  |  |  |  |  |  |  |  |  |                                                                                        |  |  |  |  |  |  |  |  |  |  |  |  |  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |               |            |          |          |          |        |        |        |        |            |      |   |    |        |                |            |  |          |          |        |  |        |        |            |      |   |    |        |                        |            |  |  |        |  |  |  |  |  |  |  |  |  |                     |            |  |  |        |  |  |  |  |  |  |  |  |  |
| Hotelling-Lawley Trace                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 0.65616933                     |                            |                               | 0.0836     |            |            |                                                        |                  |                     |        |        |        |        |                                                                           |  |  |  |  |  |  |  |  |  |  |  |  |  |                                                                                        |  |  |  |  |  |  |  |  |  |  |  |  |  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |               |            |          |          |          |        |        |        |        |            |      |   |    |        |                |            |  |          |          |        |  |        |        |            |      |   |    |        |                        |            |  |  |        |  |  |  |  |  |  |  |  |  |                     |            |  |  |        |  |  |  |  |  |  |  |  |  |
| Roy's Greatest Root                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 0.41171357                     |                            |                               | 0.1955     |            |            |                                                        |                  |                     |        |        |        |        |                                                                           |  |  |  |  |  |  |  |  |  |  |  |  |  |                                                                                        |  |  |  |  |  |  |  |  |  |  |  |  |  |                       |                                |                            |                               |            |            |            |            |                  |                     |        |        |        |  |               |            |          |          |          |        |        |        |        |            |      |   |    |        |                |            |  |          |          |        |  |        |        |            |      |   |    |        |                        |            |  |  |        |  |  |  |  |  |  |  |  |  |                     |            |  |  |        |  |  |  |  |  |  |  |  |  |

(d) Stating the hypothesis:-  
 $H_0: B_d = 0$  and  $H_a: B_d \neq 0$   
where  $B_d$  will have slopes relating to  $x_1, x_2$  &  $x_3$   
From the SAS output we have  
 $h=3, p=2, V_H=h=3$ .  
 $V_E = n-p-1 = 21$ ,  $s = \min(p, h) = \min(2, 3) = 2$   
 $m = \frac{1}{2} (1h - p - 1) = \frac{1}{2} (13 - 2 - 1) = 0$ .  
 $N=9$ .

From class chap 6, we know that all four tests shows exact tests for  $H_0$  and rejects it with same probability.

In practice,  $\Lambda$  and  $V^{(s)}$  are often preferred but  $\Lambda$  is flexible and has historical precedence.

Therefore we test our hypothesis through Wilk's test.

Wilk's test :-

$$\Lambda = \frac{|E|}{|E| + |H|} = \prod_{i=1}^s \frac{1}{1+d_i}$$

Here,  $\Lambda = 0.5692$  and  $\Lambda_{0.05}(2,3,21) = 0.548$

Since  $\Lambda_{0.5}(2,3,21) < \Lambda \Rightarrow 0.548 < 0.5692$

$\Rightarrow$  We fail to reject  $H_0$ .

For  $\alpha = 0.05$  we see all the four test have results that are not significant with p-value greater than  $\alpha$ .

$\Rightarrow$  Even with the reduced model  $x_1, x_2$  &  $x_3$  are not significant predictors of other  $x$ 's.

**APPENDIX:**

This section will have the entire SAS code.

**# 10.14**

**Code:**

```
DATA work.DIABETES;  
  
INFILE "/folders/myfolders/data/T3_4_DIABETES.dat";  
INPUT NUM Y1 Y2 X1 X2 X3;  
  
TITLE "HW4 Q-10.14";  
  
PROC REG;  
MODEL Y1 Y2 = X1 X2 X3;  
OVERALL: MTEST /PRINT CANPRINT MSTAT = EXACT;  
PARTIAL_X1: MTEST X1/PRINT CANPRINT MSTAT = EXACT;  
PARTIAL_X2: MTEST X2/PRINT CANPRINT MSTAT = EXACT;  
PARTIAL_X3: MTEST X3/PRINT CANPRINT MSTAT = EXACT;  
RUN;
```

# 10.17

**Code:**

```
DATA work.SEISHU;

INFILE "/folders/myfolders/data/T7_1_SEISHU.dat";
INPUT Y1 Y2 X1 X2 X3 X4 X5 X6 X7 X8;

TITLE "HW4 Q-10.17";

PROC REG;
  MODEL Y1 Y2 = X1 X2 X3 X4 X5 X6 X7 X8;
  OVERALL: MTEST /PRINT CANPRINT MSTAT = EXACT;
  PARTIAL_X7_X8: MTEST X7, X8/PRINT CANPRINT MSTAT = EXACT;
  PARTIAL_X4_X5_X6: MTEST X4, X5, X6/PRINT CANPRINT MSTAT = EXACT;
  PARTIAL_X1_X2_X3: MTEST X1, X2, X3/PRINT CANPRINT MSTAT = EXACT;
RUN;
```