<u>STA 9705: HW4</u>

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10.14 (a)

HW4 Q-10.14

The REG Procedure
Model: MODEL1
Dependent Variable: Y1

Number of Observations Read	46
Number of Observations Used	46

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	0.18812	0.06271	4.88	0.0053
Error	42	0.54006	0.01286		
Corrected Total	45	0.72818			

Root MSE	0.11340	R-Square	0.2583
Dependent Mean	0.91783	Adj R-Sq	0.2054
Coeff Var	12.35484		

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	0.62639	0.17573	3.56	0.0009
X1	1	0.00090822	0.00052516	1.73	0.0911
X2	1	-0.00095571	0.00041408	-2.31	0.0260
X3	1	0.00149	0.00042503	3.51	0.0011

HW4 Q-10.14

The REG Procedure
Model: MODEL1
Dependent Variable: Y2

Number of Observations Read	46
Number of Observations Used	46

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	51.72482	17.24161	0.23	0.8737
Error	42	3123.42735	74.36732		
Corrected Total	45	3175.15217			

Root MSE	8.62365	R-Square	0.0163
Dependent Mean	90.41304	Adj R-Sq	-0.0540
Coeff Var	9.53806		

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	83.24254	13.36386	6.23	<.0001
X1	1	0.02870	0.03994	0.72	0.4763
X2	1	-0.01272	0.03149	-0.40	0.6882
X3	1	-0.00441	0.03232	-0.14	0.8922

<u>10.14</u> (a) We use the parameter estimates for Y_1 and Y_2 from our SAS output to get $\hat{B}$.

$$\hat{B} = \begin{pmatrix} 0.6264 & 83.2425 \\ 0.0009 & 0.0287 \\ -0.0010 & -0.0127 \\ 0.0015 & -0.0044 \end{pmatrix}$$

10.14 (b)

HW4 Q-10.14

The REG Procedure
Model: MODEL1
Multivariate Test: OVERALL

Error Matrix (E)	
0.5400626855	9.0162268852
9.0162268852	3123.4273529

Hypothesis Matrix (H)	
0.1881199232	0.7050774626
0.7050774626	51.724820999

					Eigenvalues of Inv(E)*H = CanRsq/(1-CanRsq)				Test of H0: The canonical correlations in the current row and all that follow are zero				
	Canonical Correlation	Adjusted Canonical Correlation	Approximate Standard Error	Squared Canonical Correlation	Eigenvalue	Difference	Proportion	Cumulative	Likelihood Ratio	Approximate F Value	Num DF	Den DF	Pr > F
1	0.514205	0.469242	0.109656	0.264407	0.3594	0.3434	0.9574	0.9574	0.72400963	2.39	6	82	0.0350
2	0.125488	0.047831	0.146724	0.015747	0.0160		0.0426	1.0000	0.98425273	0.34	2	42	0.7165

Multivariate Statistics		
S=2 M=0 N=19.5		
Statistic	Value	P-Value
Wilks' Lambda	0.72400963	0.0350
Pillai's Trace	0.28015406	0.0406
Hotelling-Lawley Trace	0.37544623	0.0311
Roy's Greatest Root	0.35944702	0.0205

(b) From the data — T3_4_DIABETES.dat we have,

$p = 2$, $q = 3$, $n = 46$, $V_H = q = 3$,

$V_E = n - q - 1 = 46 - 3 - 1 = 42$,

$s = \min(p, q) = \min(2, 3) = 2$

$m = \frac{1}{2}(|q - p| - 1) = \frac{1}{2}(|3 - 2| - 1) = 0$

$N = \frac{1}{2}(n - q - p - 2) = 0.5(46 - 3 - 2 - 2) = 19.5$

Stating the hypothesis :-

$H_0 : B_1 = 0$ and $H_a : B_1 \neq 0$.

For model 1 → overall the eigen values from SAS is :-

$\lambda_1 = 0.3594$ and $\lambda_2 = 0.0160$

Comparing all 4 test to check the significance :-

i) Wilk's test :-

$$\Lambda = \frac{|E|}{|E| + |H|} = \prod_{i=1}^{s} \frac{1}{1+\lambda_i} = \prod_{i=1}^{2} \frac{1}{1+\lambda_i}$$

$$= \frac{1}{(1 + 0.3594)} \times \frac{1}{(1 + 0.0160)} = 0.724$$

From table A.9 we look for $\Lambda_{(0.05)}(2,3,42)$

We see $\Lambda_{0.05}(2,3,42) \approx \Lambda_{0.05}(2,3,40)$

$\approx 0.730.$

We check for Critical Value

$\Lambda_{0.05}(2,3,42) > \Lambda_{0.05}(2,3,40) > \Lambda$

i.e. $0.730 > 0.724$

$\Rightarrow$ We reject H_0 in Wilk's test.

ii) Roy's test :-

$\theta = \frac{\lambda_1}{1+\lambda_1}$ where λ_1 is the largest eigen value of $E^{-1}H$.

$\Rightarrow \theta = \frac{0.3594}{1+0.3594} = 0.264$

From table A.10 we look for $\theta_{0.05}(2,0,19.5)$

We see $\theta_{0.05}(2,0,19.5) \approx \theta_{0.05}(2,0,20)$

≈ 0.221

We check for Critical value

$\theta > \theta_{(s,m,N)} \Rightarrow 0.264 > 0.221$

$\Rightarrow$ We reject H_0 in Roy's test.

iii) Pillai's test :-

$V^{(s)} = tr\left[H(E+H)^{-1}\right] = \sum_{i=1}^{s} \frac{\lambda_i}{1+\lambda_i}$

$V^{(2)} = \frac{0.3594}{1+0.3594} + \frac{0.0160}{1+0.0160} = 0.280$

From table A.11 we look for $V^{(2)}_{0.05}(2, 0, 19.5)$

We see $V^{(2)}_{0.05}(2, 0, 19.5) \simeq V^{(2)}(2, 0, 20)$

$\simeq 0.263$.

We check for critical value

$V^{(2)} > V^{(2)}_{0.05}(s, m, N)$

$\Rightarrow$ We reject H_0 in Pillai's test.

iv) <u>Lawley - Hotelling test :-</u>

$U^{(s)} = tr(E^{-1}H) = \sum_{i=1}^{s} d_i$

$U^{(2)} = 0.3594 + 0.0160 = 0.375$

$\therefore \frac{V_E}{V_H}(U^{(2)}) = \frac{42}{3}(0.375) = 5.25$

From table A.12 we look for $\tilde{U}^{(2)}_{0.05}(2, 3, 42)$ $\{p < V_H\}$

We see $\tilde{U}^{(2)}_{0.05}(2, 3, 42) \approx \tilde{U}^{(2)}_{0.05}(2, 3, 40)$

≈ 4.7424.

We check for critical value

$\frac{V_E}{V_H}(U^{(2)}) > \tilde{U}^{(2)}_{0.05}(p = 2, V_H = 3, V_E = 42) \Rightarrow 5.25 > 4.7424$

$\Rightarrow$ We reject H_0 in Lawley - Hotelling's test.

For $\alpha = 0.05$ all four test appear significant.

$\therefore$ There is a linear relationship between dependent variables y_1, y_2 and indepdent variables x_1, x_2, x_3.

10.14 (c)

(c) $d_1 = 0.3594 >>> d_2 = 0.0160$.

d_1 is 95.74% of the total share and is thus the only dominant eigen value.

$\Rightarrow$ The essential rank of $\hat{B}_1$ is 1.

The power ranking is $\Theta > U^{(s)} > \Lambda > V^{(s)}$.

10.14 (d)

HW4 Q-10.14

The REG Procedure
Model: MODEL1
Multivariate Test: PARTIAL_X1

Error Matrix (E)	
0.5400626855	9.0162268852
9.0162268852	3123.4273529

Hypothesis Matrix (H)	
0.0384591746	1.2153432557
1.2153432557	38.405900428

					Eigenvalues of Inv(E)*H = CanRsq/(1-CanRsq)				Test of H0: The canonical correlations in the current row and all that follow are zero				
	Canonical Correlation	Adjusted Canonical Correlation	Approximate Standard Error	Squared Canonical Correlation	Eigenvalue	Difference	Proportion	Cumulative	Likelihood Ratio	Approximate F Value	Num DF	Den DF	Pr > F
1	0.262634	0.224257	0.141980	0.068977	0.0741		1.0000	1.0000	0.93102343	1.52	2	41	0.2310

Note: The F statistic is exact.

Multivariate Statistics		
S=1 M=0 N=19.5		
Statistic	Value	P-Value
Wilks' Lambda	0.93102343	0.2310
Pillai's Trace	0.06897657	0.2310
Hotelling-Lawley Trace	0.07408682	0.2310
Roy's Greatest Root	0.07408682	0.2310

HW4 Q-10.14

The REG Procedure
Model: MODEL1
Multivariate Test: PARTIAL_X2

Error Matrix (E)	
0.5400626855	9.0162268852
9.0162268852	3123.4273529

Hypothesis Matrix (H)	
0.068497593	0.9119819959
0.9119819959	12.142195433

	Canonical Correlation	Adjusted Canonical Correlation	Approximate Standard Error	Squared Canonical Correlation	Eigenvalues of Inv(E)*H = CanRsq/(1-CanRsq)				Test of H0: The canonical correlations in the current row and all that follow are zero				
					Eigenvalue	Difference	Proportion	Cumulative	Likelihood Ratio	Approximate F Value	Num DF	Den DF	Pr > F
1	0.335804	0.308546	0.135302	0.112764	0.1271		1.0000	1.0000	0.88723586	2.61	2	41	0.0861

Note: The F statistic is exact.

Multivariate Statistics		
S=1 M=0 N=19.5		
Statistic	**Value**	**P-Value**
Wilks' Lambda	0.88723586	0.0861
Pillai's Trace	0.11276414	0.0861
Hotelling-Lawley Trace	0.12709601	0.0861
Roy's Greatest Root	0.12709601	0.0861

HW4 Q-10.14

The REG Procedure
Model: MODEL1
Multivariate Test: PARTIAL_X3

Error Matrix (E)	
0.5400626855	9.0162268852
9.0162268852	3123.4273529

Hypothesis Matrix (H)	
0.1579729491	-0.467207454
-0.467207454	1.3817733104

	Canonical Correlation	Adjusted Canonical Correlation	Approximate Standard Error	Squared Canonical Correlation	Eigenvalues of Inv(E)*H = CanRsq/(1-CanRsq)				Test of H0: The canonical correlations in the current row and all that follow are zero				
					Eigenvalue	Difference	Proportion	Cumulative	Likelihood Ratio	Approximate F Value	Num DF	Den DF	Pr > F
1	0.488266	0.474452	0.116142	0.238403	0.3130		1.0000	1.0000	0.76159667	6.42	2	41	0.0038

Note: The F statistic is exact.

Multivariate Statistics		
S=1 M=0 N=19.5		
Statistic	**Value**	**P-Value**
Wilks' Lambda	0.76159667	0.0038
Pillai's Trace	0.23840333	0.0038
Hotelling-Lawley Trace	0.31303096	0.0038
Roy's Greatest Root	0.31303096	0.0038

(d) When we run partial test the value of V_H is no more equal to q but $h = 1$.

In this case where $V_H = 1$, we can simply test any one of 4 MANOVA tests, all of which will give same results for α-value and equal F-transformat

For all three x's we have the following info

	Λ	F	p-Value
$x_1 \mid x_2, x_3$	0.9310	1.52	0.2310
$x_2 \mid x_1, x_3$	0.8872	2.61	0.0861
$x_3 \mid x_1, x_2$	0.7616	6.42	0.0038

From the table above we can observe that x_3 adjusted for other two x's shows significant result for $\alpha = 0.05$.

However for x_1 and x_2 when adjusted for other x's result insignificant results and thus we need reduced model.

10.17 (a)

HW4 Q-10.17

The REG Procedure
Model: MODEL1
Dependent Variable: Y1

Number of Observations Read	30
Number of Observations Used	30

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	8	1.41711	0.17714	1.17	0.3603
Error	21	3.17256	0.15107		
Corrected Total	29	4.58967			

Root MSE	0.38868	R-Square	0.3088
Dependent Mean	0.26333	Adj R-Sq	0.0454
Coeff Var	147.60102		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	-4.14039	4.78081	-0.87	0.3963
X1	1	1.10259	0.78372	1.41	0.1741
X2	1	0.23074	0.59447	0.39	0.7018
X3	1	1.17138	0.72314	1.62	0.1202
X4	1	0.11145	0.06278	1.78	0.0904
X5	1	0.61679	0.38692	1.59	0.1259
X6	1	0.26683	0.38342	0.70	0.4941
X7	1	-0.26258	0.20081	-1.31	0.2052
X8	1	-0.00432	0.00398	-1.09	0.2901

HW4 Q-10.17

The REG Procedure
Model: MODEL1
Dependent Variable: Y2

Number of Observations Read	30
Number of Observations Used	30

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	8	1.40951	0.17619	0.89	0.5440
Error	21	4.17349	0.19874		
Corrected Total	29	5.58300			

Root MSE	0.44580	R-Square	0.2525
Dependent Mean	0.07000	Adj R-Sq	-0.0323
Coeff Var	636.85715		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	4.93526	5.48336	0.90	0.3783
X1	1	-0.95500	0.89888	-1.06	0.3001
X2	1	-0.22174	0.68183	-0.33	0.7482
X3	1	1.77275	0.82941	2.14	0.0445
X4	1	0.04793	0.07201	0.67	0.5129
X5	1	-0.05787	0.44378	-0.13	0.8975
X6	1	0.48537	0.43976	1.10	0.2822
X7	1	-0.20928	0.23032	-0.91	0.3739
X8	1	-0.00407	0.00456	-0.89	0.3824

10.17 (a) We use the parameter estimates for Y_1 and Y_2 from our SAS output to get $\hat{B}$

$$\hat{B} = \begin{pmatrix} -4.1404 & 4.9353 \\ 1.1026 & -0.9550 \\ 0.2370 & -0.2217 \\ 1.1714 & 1.7728 \\ 0.1115 & 0.0479 \\ 0.6168 & -0.0579 \\ 0.2668 & 0.4854 \\ -0.2626 & -0.2093 \\ -0.0043 & -0.0041 \end{pmatrix}$$

HW4 Q-10.17

The REG Procedure
Model: MODEL1
Multivariate Test: OVERALL

Error Matrix (E)	
3.1725587246	2.2483661643
2.2483661643	4.173490596

Hypothesis Matrix (H)	
1.417107942	0.5786338357
0.5786338357	1.409509404

					Eigenvalues of Inv(E)*H = CanRsq/(1-CanRsq)				Test of H0: The canonical correlations in the current row and all that follow are zero				
	Canonical Correlation	Adjusted Canonical Correlation	Approximate Standard Error	Squared Canonical Correlation	Eigenvalue	Difference	Proportion	Cumulative	Likelihood Ratio	Approximate F Value	Num DF	Den DF	Pr > F
1	0.620790	0.454318	0.114132	0.385380	0.6270	0.3031	0.6594	0.6594	0.46423605	1.17	16	40	0.3321
2	0.494650	0.399406	0.140260	0.244678	0.3239		0.3406	1.0000	0.75532164	0.97	7	21	0.4766

Multivariate Statistics		
S=2 M=2.5 N=9		
Statistic	Value	P-Value
Wilks' Lambda	0.46423605	0.3321
Pillai's Trace	0.63005801	0.2973
Hotelling-Lawley Trace	0.95095995	0.3598

From the data → T7_1_SEISHU.DAT we have,

$p = 2$, $q = 8$, $n = 30$, $V_H = q = 8$,

$V_E = n - q - 1 = 30 - 8 - 1 = 21$,

$S = \min(p, q) = \min(2, 8) = 2$

$m = \frac{1}{2}(|q - p| - 1) = \frac{1}{2}(|8 - 2| - 1) = 2.5$

$N = 9$

Stating the hypothesis:-

$H_0: B_1 = 0$ and $H_a: B_1 \neq 0$

Now, compare all 4 test to check for significance:-

Wilk's test, $\Lambda = \frac{1}{(1 + 0.6270)} \times \frac{1}{(1 + 0.3239)} = 0.4642$

and $\Lambda_{0.05(2, 8, 21)} = 0.322$ for table A.9

$\Rightarrow \quad \Lambda_{0.5}(2,8,21) < \Lambda \Rightarrow 0.322 < 0.464$

$\Rightarrow$ we cannot reject H_0 in Wilk's test

ii) Roy's test :-

$\theta = \frac{\lambda_1}{1+\lambda_1}$ where λ_1 is the largest eigen value of $E^{-1}H$

$$\Rightarrow \theta = \frac{0.6270}{1+0.6270} = 0.3854$$

From table A.10 we look for $\theta_{0.05}(2, 2.5, 9)$

we see $\theta_{0.05}(2, 2.5, 9) \approx \theta_{0.05}(2, 2, 10)$

$= 0.514.$

we check for critical value and

$\theta < \theta_{(s,m,N)} \Rightarrow 0.3854 < 0.514.$

$\Rightarrow$ we fail to reject Ho in Roy's test.

(iii) Pillai's Test :-

$$V^{(s)} = \mathrm{tr}\left[H(E+H)^{-1}\right] = \sum_{i=1}^{s} \frac{\lambda_i}{1+\lambda_i}$$

$$V^{(2)} = \frac{0.6270}{1+0.6270} + \frac{0.3239}{1+0.3239} = 0.6301$$

From table A.11 we look for $V^{(2)}_{0.05}(2, 2.5, 9)$

we see $V^{(2)}_{0.05}(2, 2.5, 9) \approx V^{(2)}_{0.05}(2, 2, 9)$

$= 0.772$

We check for Critical values:-

$$V^{(2)}_{0.05}(2, 2.5, 9) > V^{(2)}_{0.05}(2, 2, 9) > V^{(2)}$$

$$\Rightarrow 0.772 > 0.6301$$

$\Rightarrow$ We fail to reject H_0 in Pillai's test.

(iv) Lawley - Hotelling test:-

$$U^{(s)} = tr(E^{-1}H) = \sum_{i=1}^{s} \lambda_i$$

$$U^{(2)} = 0.6270 + 0.3239 = 0.9509$$

$$\therefore \frac{V_E}{V_H} U^{(2)} = \frac{21}{8}(0.9509) = 2.496.$$

From table A.12 we look for $\tilde{U}^{(2)}_{0.05}(2, 8, 21)$

we see $\tilde{U}^{(2)}_{0.05}(2, 8, 21) > \tilde{U}^{(2)}_{0.05}(2, 8, 25)$

$= 4.147.$

We check for Critical value

$$\frac{V_E}{V_H} U^{(2)} < \tilde{U}^{(2)}_{0.5(2,8,25)} \Rightarrow 2.496 < 4.147$$

$\Rightarrow$ We fail to reject H_0 in Lawley – Hotelling's test.

$\therefore$ For $\alpha = 0.05$ all four tests are insignificant and there is no linear relationship between (y_1, y_2) and all other x's.

10.17 (b)

HW4 Q-10.17

The REG Procedure
Model: MODEL1
Multivariate Test: PARTIAL_X7_X8

Error Matrix (E)	
3.1725587246	2.2483661643
2.2483661643	4.173490596

Hypothesis Matrix (H)	
0.5165001532	0.4433479905
0.4433479905	0.3825011352

					Eigenvalues of Inv(E)*H = CanRsq/(1-CanRsq)				Test of H0: The canonical correlations in the current row and all that follow are zero				
	Canonical Correlation	Adjusted Canonical Correlation	Approximate Standard Error	Squared Canonical Correlation	Eigenvalue	Difference	Proportion	Cumulative	Likelihood Ratio	Approximate F Value	Num DF	Den DF	Pr > F
1	0.378587	0.286820	0.178629	0.143328	0.1673	0.1666	0.9956	0.9956	0.85604439	0.81	4	40	0.5274
2	0.027072	.	0.208362	0.000733	0.0007		0.0044	1.0000	0.99926709	0.02	1	21	0.9024

Multivariate Statistics		
S=2 M=-0.5 N=9		
Statistic	**Value**	**P-Value**
Wilks' Lambda	0.85604439	0.5274
Pillai's Trace	0.14406066	0.5316
Hotelling-Lawley Trace	0.16804101	0.5213
Roy's Greatest Root	0.16730756	0.4651

(b) Stating the hypothesis:-

$H_0: B_d = 0$ and $H_0: B_d \neq 0$.

where B_d will have slopes relative to x_7 & x_8.

From the SAS output we have.

$h = 2$, $p = 2$, $V_H = h = 2$,

$V_E = n - q - 1 = 30 - 8 - 1 = 21$

$s = \min(p, h) = \min(2, 2) = 2.$

$m = \frac{1}{2}(|h - p| - 1) = \frac{1}{2}(|2 - 2| - 1) = -0.5.$

$N = 9.$

(i) Wilk's Test:-

$$\Lambda = \frac{1}{(1+0.1673)} \times \frac{1}{(1+0.0007)} = 0.856$$

and $\Lambda_{0.05}(2,2,21) = 0.629$ from table A.9

$\Rightarrow \Lambda_{0.05}(2,2,21) < \Lambda \Rightarrow 0.629 < 0.856$.

$\Rightarrow$ we fail to reject H_0 in Wilk's test.

ii) Roy's test:-

$\theta = \dfrac{\lambda_1}{1+\lambda_1}$ where λ_1 is the largest eigen Value of $E^{-1}H$.

$$\Rightarrow \theta = \frac{0.1673}{1+0.1673} = 0.143$$

From table A.10 we look for $\theta_{0.05}(2,-0.5,9)$

We see $\theta_{0.05}(2,-0.5,9) > \theta_{0.05}(2,0,10)$

$= 0.374$

We check for critical value and

$\Theta < \Theta_{0.05}(2, 0, 10) \Rightarrow 0.143 < 0.374$

$\Rightarrow$ we fail to reject H_0 in Roy's test.

iii) Pillai's Test :-

$\left[\frac{0.1673}{1+0.1673} + \frac{0.0007}{1+0.0007}\right]$

$V^{(2)} = 0.144$ and $V_{0.05}(2, -0.5, 9) > V_{0.05}(2, 0, 9)$

From table A.11 we find $V_{0.05}(2, 0, 9) = 0.485$.

$\Rightarrow V^{(2)} < V_{0.05}(2, 0; 9) < V_{0.05}(2, -0.5, 9)$

i.e. $0.144 < 0.485$

$\Rightarrow$ we fail to reject H_0 in Pillai's test.

iv) Lawley - Hotelling test :-

$U^{(2)} = 0.1673 + 0.0007 = 0.168$

$\frac{V_E}{V_H} U^{(2)} = \frac{21}{2}(0.168) = 1.764.$

From table A.12 we have for $U^{(2)}_{0.05}(2, 2, 21)$

We see $\tilde{U}^{(2)}_{0.05}(2, 2, 21) > \tilde{U}^{(2)}(2, 2, 25) = 5.724.$

$\Rightarrow \frac{V_E}{V_H} U^{(2)} < \frac{V_E}{V_H} U_{0.05}(2, 2, 25) = \tilde{U}^{(2)}_{0.05}(2, 2, 25)$

$\Rightarrow 1.764 < 5.724.$

$\Rightarrow$ we fail to reject H_0 in Lawley-Hotelling test

For $\alpha = 0.05$ we see all the four test have results that are not significant with p-value being greater than α.

$\Rightarrow$ Even with the reduced model x_7 & x_8 are not significant predictors of other x's.

10.17 (c)

HW4 Q-10.17

The REG Procedure
Model: MODEL1
Multivariate Test: PARTIAL_X4_X5_X6

Error Matrix (E)	
3.1725587246	2.2483661643
2.2483661643	4.173490596

Hypothesis Matrix (H)	
0.9774657385	0.3758022424
0.3758022424	0.4143043703

					Eigenvalues of Inv(E)*H = CanRsq/(1-CanRsq)				Test of H0: The canonical correlations in the current row and all that follow are zero				
	Canonical Correlation	Adjusted Canonical Correlation	Approximate Standard Error	Squared Canonical Correlation	Eigenvalue	Difference	Proportion	Cumulative	Likelihood Ratio	Approximate F Value	Num DF	Den DF	Pr > F
1	0.516577	0.416761	0.149653	0.266852	0.3640	0.2755	0.8044	0.8044	0.67352570	1.46	6	40	0.2178
2	0.285173	0.266653	0.187524	0.081324	0.0885		0.1956	1.0000	0.91867645	0.93	2	21	0.4104

Multivariate Statistics S=2 M=0 N=9		
Statistic	Value	P-Value
Wilks' Lambda	0.67352570	0.2178
Pillai's Trace	0.34817566	0.2034
Hotelling-Lawley Trace	0.45250380	0.2264
Roy's Greatest Root	0.36398128	0.2529

(c) Stating the hypothesis:-

$H_0: B_d = 0$ and $H_a: B_d \neq 0$.

where B_d will have slopes relative to x_4, x_5 & x_6.

From the SAS output we have:-

$h = 3,\ p = 2,\ V_H = h = 3,\ N = 9$

$V_E = n - q - 1 = 21$, $s = \min(p, h) = \min(2, 3) = 2$

$m = \frac{1}{2}(|h-p| - 1) = \frac{1}{2}(|3-2| - 1) = 0.$

(i) Wilk's Test.

$$\Lambda = \frac{1}{(1+0.3640)} \times \frac{1}{(1+0.0885)} = 0.674$$

and $\Lambda_{0.05}(2,3,21) = 0.548$ from table A.9

$\Rightarrow \Lambda_{0.05}(2,3,21) < \Lambda \Rightarrow 0.548 < 0.674$.

$\Rightarrow$ We fail to reject H_0 in Wilk's test.

(ii) Ray's Test :-

$$\theta = \frac{0.3640}{1+0.3640} = 0.267$$ when $\lambda = 0.3640$ (largest eigen Value)

$\theta_{0.05}(2,0,9)$ from table A.10 is 0.374

$\Rightarrow \theta < \theta_{0.05}(2,0,9) \Rightarrow 0.267 < 0.374$

$\Rightarrow$ we fail to reject H_0 in Ray's test

iii) Pillai's Test

$V^{(2)} = 0.348$ and $V^{(2)}_{0.05}(2,0,9)$ from table A.11 gives us 0.485.

$\Rightarrow V^{(2)} < V^{(2)}_{0.05}(2,0,9) \Rightarrow 0.348 < 0.485$

$\Rightarrow$ We reject H_0 in Pillai's test.

iv) Lawley-Hotelling test:-

$U^{(2)} = 0.4525.$ and $\frac{V_E}{V_H} U^{(2)} = \frac{21}{3} \times (0.4525) = 3.171$

From table A.12 we have for $\tilde{U}^{(2)}_{0.05}(2,3,21)$

We see $\tilde{U}^{(2)}_{0.05}(2,3,21) > \tilde{U}^{(2)}_{0.05}(2,3,25)$

$= 5.1237$

$\Rightarrow \frac{V_E}{V_H} U^{(2)} < \frac{V_E}{V_H} U_{0.05}(2,3,25) = \tilde{U}^{(2)}_{0.05}(2,3,25)$

$\Rightarrow 3.171 < 5.1237.$

$\Rightarrow$ We fail to reject H_0 in Lawley-Hotelling test

For $\alpha = 0.05$ we see all the four test have results that are not significant with p-value greater than α.

$\Rightarrow$ Even with the reduced model x_4, x_5 & x_6 are not significant predictor of other x's.

10.17 (d)

HW4 Q-10.17

The REG Procedure
Model: MODEL1
Multivariate Test: PARTIAL_X1_X2_X3

Error Matrix (E)	
3.1725587246	2.2483661643
2.2483661643	4.173490596

Hypothesis Matrix (H)	
1.0505812926	0.5054338805
0.5054338805	1.0273346775

					Eigenvalues of Inv(E)*H = CanRsq/(1-CanRsq)				Test of H0: The canonical correlations in the current row and all that follow are zero				
	Canonical Correlation	Adjusted Canonical Correlation	Approximate Standard Error	Squared Canonical Correlation	Eigenvalue	Difference	Proportion	Cumulative	Likelihood Ratio	Approximate F Value	Num DF	Den DF	Pr > F
1	0.540038	0.386148	0.144593	0.291641	0.4117	0.1673	0.6275	0.6275	0.56921187	2.17	6	40	0.0663
2	0.443211	.	0.164027	0.196436	0.2445		0.3725	1.0000	0.80356412	2.57	2	21	0.1006

Multivariate Statistics		
S=2 M=0 N=9		
Statistic	Value	P-Value
Wilks' Lambda	0.56921187	0.0663
Pillai's Trace	0.48807689	0.0495
Hotelling-Lawley Trace	0.65616933	0.0836
Roy's Greatest Root	0.41171357	0.1955

(d) Stating the hypothesis:-

$H_0: B_d = 0$ and $H_a: B_d \neq 0$

where B_d will have slopes relative to x_1, x_2 & x_3

From the SAS output we have

$h = 3, \quad p = 2, \quad V_H = h = 3.$

$V_E = n - q - 1 = 21, \quad s = \min(p, h) = \min(2, 3) = 2$

$m = \frac{1}{2}(|h-p|-1) = \frac{1}{2}(|3-2|-1) = 0.$

$N = 9.$

(i) Wilk's Test :-

$\Lambda = 0.569$ $\quad \Lambda_{0.05}(2,3,21) = 0.548$ (from part c)

$\Rightarrow \Lambda_{0.05}(2,3,21) < \Lambda$

$\Rightarrow$ We fail to reject H_0 in Wilk's test.

(ii) Roy's test.

$$\theta = \frac{0.412}{1+0.412} = 0.292 \quad \text{where } \lambda = 0.412$$

(largest eigen value)

$\theta_{0.05}(2,0,9) = 0.374$ from part c.

$\Rightarrow \theta < \theta_{0.05}(2,0,9)$

$\Rightarrow$ We fail to reject H_0 in Roy's test.

iii) Pillai's Test

$V^{(2)} = 0.488$ and $V^{(2)}_{0.05}(2,0,9) = 0.485$ (part c)

$\Rightarrow V^{(2)} > V^{(2)}_{0.05}(2,0,9) \Rightarrow 0.488 > 0.485$

$\Rightarrow$ We reject Ho in in Pillai's test

iv) Lawley - Hotelling Test

$U^{(2)} = 0.656$ and $\frac{V_E}{V_H} \times U^{(2)} = \frac{21}{3} \times (0.656)$

$= 4.592$

$\Rightarrow \frac{V_E}{V_H} U^{(2)} < \tilde{U}^{(2)}_{0.05}(2,3,25)$ (from part c)

$\Rightarrow 4.592 < 5.1237$

$\Rightarrow$ We fail to reject Ho in Lawley-Hotelling test.

$\therefore$ For $\alpha = 0.05$ we see except Pillai all other tests have p-value greater than α

Also, for Pillai it is very close enough to ≈ 0.05.

$\Rightarrow$ We can conclude the reduced model x_1, x_2 & x_3 are not significant predictors of other x's.

APPENDIX:

This section will have the entire SAS code.

 # 10.14

Code:

```
DATA work.DIABETES;

  INFILE "/folders/myfolders/data/T3_4_DIABETES.dat";
  INPUT NUM Y1 Y2 X1 X2 X3;

TITLE "HW4 Q-10.14";

PROC REG;
  MODEL Y1 Y2 = X1 X2 X3;
  OVERALL: MTEST /PRINT CANPRINT MSTAT = EXACT;
  PARTIAL_X1: MTEST X1/PRINT CANPRINT MSTAT = EXACT;
  PARTIAL_X2: MTEST X2/PRINT CANPRINT MSTAT = EXACT;
  PARTIAL_X3: MTEST X3/PRINT CANPRINT MSTAT = EXACT;
RUN;
```

 # 10.17

Code:

```
DATA work.SEISHU;

  INFILE "/folders/myfolders/data/T7_1_SEISHU.dat";
  INPUT Y1 Y2 X1 X2 X3 X4 X5 X6 X7 X8;

TITLE "HW4 Q-10.17";

PROC REG;
  MODEL Y1 Y2 = X1 X2 X3 X4 X5 X6 X7 X8;
  OVERALL: MTEST /PRINT CANPRINT MSTAT = EXACT;
  PARTIAL_X7_X8: MTEST X7, X8/PRINT CANPRINT MSTAT = EXACT;
  PARTIAL_X4_X5_X6: MTEST X4, X5, X6/PRINT CANPRINT MSTAT = EXACT;
  PARTIAL_X1_X2_X3: MTEST X1, X2, X3/PRINT CANPRINT MSTAT = EXACT;
RUN;
```