STA 9705

#HW1
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4.10 Given y is $N_3(\mu, \Sigma)$, where

$$\mu = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 6 & 1 & -2 \\ 1 & 13 & 4 \\ -2 & 4 & 4 \end{pmatrix}$$

(c) Distribution of y_2

We know that if y is $N_p(\mu, \Sigma)$, then y_j is $N(\mu_j, \sigma_{jj})$ for $j = 1, 2, \ldots p$.

Therefore distribution of y_2 is $N(1, 13)$

(d) Joint distribution of y_1 and y_3

With joint distributions we include the co-variances too, that is where y_1 and μ_1 are $r \times 1$ and Σ_{11} is $r \times r$.

then y_1 is $N_r(\mu_1, \Sigma_{11})$ if y is $N_p(\mu, \Sigma)$

↳ This is taken from the book section: "Normality of marginal distribution"

Hence, for y_1 and y_3 it is given as:-

$$\begin{pmatrix} y_1 \\ y_3 \end{pmatrix} \text{ is } N_2\left[\begin{pmatrix} 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 6 & -2 \\ -2 & 4 \end{pmatrix}\right]$$

4.12 Using the logic in previous question:-

Given in que:-

$$\mu = \begin{pmatrix} -2 \\ 3 \\ -1 \\ 5 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 11 & -8 & 3 & 9 \\ -8 & 9 & -3 & 6 \\ 3 & -3 & 2 & 3 \\ 9 & 6 & 3 & 9 \end{pmatrix}$$

(d) Distribution of y_3 is givens as:-

y_3 is $N(-1, 2)$

(e) Joint distribution of y_2 and y_4 is:-

$$\begin{pmatrix} y_2 \\ y_4 \end{pmatrix} \text{ is } N_2\left[\begin{pmatrix} 3 \\ 5 \end{pmatrix}, \begin{pmatrix} 9 & 6 \\ 6 & 9 \end{pmatrix}\right]$$

4.14 Given y is $N_3(\mu, \Sigma)$ with

$$\mu = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 4 & -3 & 0 \\ -3 & 6 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

(a) y_1 and y_2

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \text{ is } N_2\left[\begin{pmatrix} 2 \\ -3 \end{pmatrix}, \begin{pmatrix} 4 & -3 \\ -3 & 6 \end{pmatrix}\right]$$

Since $\text{cov}(y_1, y_2) \neq 0 \rightarrow$ they are not independent.

(b) y_1 and y_3

$$\begin{pmatrix} y_1 \\ y_3 \end{pmatrix} \text{ is } N_2\left[\begin{pmatrix} 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 & 0 \\ 0 & 5 \end{pmatrix}\right]$$

since $\text{cov}(y_1, y_3) = 0 \rightarrow$ they are independent

(c) y_2 and y_3

$$\begin{pmatrix} y_2 \\ y_3 \end{pmatrix} \text{ is } N_2\left[\begin{pmatrix} -3 \\ 4 \end{pmatrix}, \begin{pmatrix} 6 & 0 \\ 0 & 5 \end{pmatrix}\right]$$

since $\text{cov}(y_2, y_3) = 0 \rightarrow$ they are indepent

(d) $(y_1 y_2)$ and y_3.

Let's find co-variance of this combination:-

$$\text{cov}\left(\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, y_3\right) = \begin{pmatrix} \text{cov}(y_1, y_3) \\ \text{cov}(y_2, y_3) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \{\text{from (b) and (c)}\}$$

hence they are independent.

(e) (y_1, y_3) and y_2

Let's find co-variance of this combination:-

$$\text{cov}\left(\begin{pmatrix} y_1 \\ y_3 \end{pmatrix}, y_2\right) = \begin{pmatrix} \text{cov}(y_1, y_2) \\ \text{cov}(y_3, y_2) \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \quad \{\text{from (a) and (c)}\}$$

hence they are not independent

Final ans :- (b), (c) and (d) are cases where the random variables are independent