STA 9705: HW3

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 # 6.6

6.6 For $s = 1 \rightarrow \theta = \frac{\lambda_1}{(1+\lambda_1)}$ $\rightarrow$ (1)

$$\Rightarrow \lambda_1 = \theta(1+\lambda_1)$$

$$\Rightarrow \lambda_1 = \theta + \theta\lambda_1$$

$$\Rightarrow \lambda_1(1-\theta) = \theta$$

$$\Rightarrow \lambda_1 = \frac{\theta}{1-\theta} \rightarrow (2)$$

$\rightarrow$ This is for Roy's test.

$\rightarrow$ Now, for Wilk's test :-

$$\Lambda = \prod_{i=1}^{1} \frac{1}{1+\lambda_i} = \frac{1}{1+\lambda_1}$$

$$= \frac{1}{1+\frac{\theta}{1-\theta}} \quad \{\text{using } (2)\}$$

$$= \frac{1-\theta}{1-\cancel{\theta}+\cancel{\theta}}$$

$$\Rightarrow \Lambda = 1-\theta.$$

$\rightarrow$ Now, for pillai's test :-

$$V^{(s)} = \sum_{i=1}^{s} \frac{\lambda_i}{1+\lambda_i} = \sum_{i=1}^{1} \frac{\lambda_i}{1+\lambda_i} \quad \{\text{given } s=1\}$$

$$\Rightarrow V^{(1)} = \frac{\lambda_1}{1+\lambda_1}$$

$$= \theta \quad \{\text{using } (1)\}$$

$\rightarrow$ Last, for Lawley-Hotelling test :-

$$U^{(s)} = \sum_{i=1}^{s} \lambda_i = \sum_{i=1}^{1} \lambda_i \Rightarrow U^{(1)} = \lambda_1 = \frac{\theta}{1-\theta} \quad \{\text{using } (2)$$

6.27 (a)

H = Type III SSCP Matrix for METHOD				
	AROMA	FLAVOR	TEXTURE	MOISTURE
AROMA	1.0505555556	2.1733333333	-1.375555556	-0.760277778
FLAVOR	2.1733333333	4.88	-2.373333333	-1.256666667
TEXTURE	-1.375555556	-2.373333333	2.3822222222	1.3844444444
MOISTURE	-0.760277778	-1.256666667	1.3844444444	0.8105555556

Characteristic Roots and Vectors of: E Inverse * H, where H = Type III SSCP Matrix for METHOD E = Error SSCP Matrix					
		Characteristic Vector V'EV=1			
Characteristic Root	Percent	AROMA	FLAVOR	TEXTURE	MOISTURE
2.95147543	95.86	0.02070610	0.53343536	-0.34683549	-0.13507923
0.12732437	4.14	-0.31733855	0.29837224	0.24313963	-0.02626254
0.00000000	0.00	0.01442826	-0.02423642	-0.24987896	0.40275575
0.00000000	0.00	0.27340509	-0.08726609	0.07093056	0.00000000

MANOVA Test Criteria and F Approximations for the Hypothesis of No Overall METHOD Effect H = Type III SSCP Matrix for METHOD E = Error SSCP Matrix S=2 M=0.5 N=14					
Statistic	Value	F Value	Num DF	Den DF	Pr > F
Wilks' Lambda	0.22448732	8.33	8	60	<.0001
Pillai's Trace	0.85987383	5.84	8	62	<.0001
Hotelling-Lawley Trace	3.07879980	11.33	8	40.602	<.0001
Roy's Greatest Root	2.95147543	22.87	4	31	<.0001
NOTE: F Statistic for Roy's Greatest Root is an upper bound.					
NOTE: F Statistic for Wilks' Lambda is exact.					

6.27

(a) From the SAS output we can see that two of the characteristic roots are:-

$$\lambda_1 = 2.951 \text{ and } \lambda_2 = 0.127 \rightarrow (3)$$

These are our eigen-values as rest are zero

From the data set we have.

- p = no. of variables = 4
- $V_H = K-1$ ∴ degrees of freedom for hypothesis $= 2$.
- $V_E = K(n-1)$ ∴ degrees of freedom for error $= 3(12-1) = 33$.

Now, let's define the hypothesis test:-

For given three methods.

$H_0 : \mu_1 = \mu_2 = \mu_3$.

$H_a : \mu_1 \neq \mu_2$ or $\mu_2 \neq \mu_3$ or $\mu_1 \neq \mu_3$

i.e. at least one inequality in H_0.

Comparing all 4 MANOVA test one by one :-

i) Wilk's test :-

$$\Lambda = \frac{|E|}{|E|+|H|} = \prod_{i=1}^{s} \frac{1}{1+\lambda_i}$$

Using eqn ③ :-

$$\Lambda = \frac{1}{(1+2.951)} \cdot \frac{1}{(1+(0.127))} = 0.224$$

From table A.9 we look for $\Lambda_{(0.05)(4,2,33)}$

we see $\Lambda_{0.05(4,2,33)} \approx \Lambda_{0.05(4,2,30)}$

$$= 0.580$$

we check for critical value.

$$\Lambda_{(0.05)(4,2,33)} > \Lambda_{(0.05)(4,2,30)} > \Lambda.$$

i.e. $0.580 > 0.224$.

$\Rightarrow$ we reject H_0 in Wilk's test.

ii) Roy's test :-

$\theta = \dfrac{\lambda_1}{1+\lambda_1}$ where λ_1 is the largest eigen value of $E^{-1}H$

From eqn ③ $\rightarrow$ $\lambda_1 = 2.951$

$$\Rightarrow \quad \theta = \frac{2.951}{(1+2.951)} = 0.747$$

Now, $s = \min(V_H, p) = \min(4,2) = 2$

$$m = \frac{1}{2}(|V_H - p| - 1) = \frac{1}{2}(|2-4|-1) = \frac{1}{2}.$$

$$N = \frac{1}{2}(V_E - p - 1) = \frac{1}{2}(33-4-1) = 14$$

From table A.10 we look for $\theta_{0.05}(2, 0.5, 14)$

We see $\theta_{0.05}(2,1,10) \approx \theta_{0.05}(2, 0.5, 14)$

$$= 0.455.$$

We check for critical value

$$\theta > \theta_\alpha(s,m,N) \Rightarrow 0.747 > 0.455$$

$\Rightarrow$ We reject H_0 in Roy's test.

iii) Pillai's Test :-

$$V^{(s)} = tr\left[H(E+H)^{-1}\right] = \sum_{i=1}^{s} \frac{d_i}{1+d_i}$$

Here, $S = 2$, $m = 1/2$, $N = 14$ $\rightarrow$ Same as Roy's test.

$$V^{(s)} = V^{(2)} = \frac{2.951}{1+2.951} + \frac{0.127}{1+0.127}$$

$$= 0.860$$

From table A.11 we look for $V^{(2)}_{0.05(2,0.5,14)}$

We see $V^{(2)}_{0.05}(2,0.5,14) \simeq V^{(2)}_{0.05}(2,1,10)$

$= 0.604$

We check for critical value

$V^{(2)} > V^{(2)}_{0.05}(s,m,N)$

$\Rightarrow$ We reject Ho in Pillai's test.

iv) Lawley - Hotelling test :-

$U^{(s)} = tr(E^{-1}H) = \sum_{i=1}^{s} \lambda_i$

$U^{(2)} = 2.951 + 0.127 = 3.078.$

$\frac{V_E}{V_H}(U^{(2)}) = \frac{33}{2}(3.078) = 50.787$

From table A.12 we look for $\tilde{U}^{(2)}_{0.05}(4,2,33)$

but, we have $p=4$ and $V_H = 2 \rightarrow p > V_H$.

so we use $\tilde{U}^{(2)}_{0.05}(2,4,31)$ for $\tilde{U}^{(2)}_{0.05}(4,2,33)$

We see $\tilde{U}^{(2)}_{0.05}(2,4,31) \simeq \tilde{U}^{(2)}_{0.05}(2,4,30)$

$= 4.6040$

We check for critical value

$\frac{V_E}{V_H}(U^{(2)}) > \tilde{U}^{(2)}_{0.05}(V_H, p, V_E + V_H - p) \Rightarrow 50.787 > 4.6040$

$\Rightarrow$ We reject Ho in Lawley Hotelling's test.

Since, all the 4 MANOVA test reject H_0 this means at least one of the equality in μ_1, μ_2, μ_3 are not equal.

$\Rightarrow$ All three methods for given features prepare the fish differently.

6.27 (c)

6.27 (c) $\lambda_1 = 2.951 >>> \lambda_2 = 4.14$.

λ_1 is 95.86% of the total share and is thus the only dominant eigen value.

$\Rightarrow$ Dimensionality of the space containing mean vectors is 1.

6.27 (d)

H = Contrast SSCP Matrix for One - Two Vs Three				
	AROMA	FLAVOR	TEXTURE	MOISTURE
AROMA	0.9568055556	1.7983333333	-1.475555556	-0.841527778
FLAVOR	1.7983333333	3.38	-2.773333333	-1.581666667
TEXTURE	-1.475555556	-2.773333333	2.2755555556	1.2977777778
MOISTURE	-0.841527778	-1.581666667	1.2977777778	0.7401388889

Characteristic Roots and Vectors of: E Inverse * H, where H = Contrast SSCP Matrix for One - Two Vs Three E = Error SSCP Matrix					
		Characteristic Vector V'EV=1			
Characteristic Root	Percent	AROMA	FLAVOR	TEXTURE	MOISTURE
2.70206732	100.00	-0.04113410	-0.51307665	0.36179773	0.13310371
0.00000000	0.00	0.04161438	-0.05283886	-0.26797149	0.40426689
0.00000000	0.00	0.26813661	-0.08175769	0.07422765	0.00000000
0.00000000	0.00	-0.31744291	0.33023951	0.19663752	0.00000000

MANOVA Test Criteria and Exact F Statistics for the Hypothesis of No Overall One - Two Vs Three Effect H = Contrast SSCP Matrix for One - Two Vs Three E = Error SSCP Matrix S=1 M=1 N=14					
Statistic	Value	F Value	Num DF	Den DF	Pr > F
Wilks' Lambda	0.27011934	20.27	4	30	<.0001
Pillai's Trace	0.72988066	20.27	4	30	<.0001
Hotelling-Lawley Trace	2.70206732	20.27	4	30	<.0001
Roy's Greatest Root	2.70206732	20.27	4	30	<.0001

6.27 (d) Using contrasts : methods 1 and 2 v/s 3.

$H_{01} = \frac{\mu_1 + \mu_2}{2} = \mu_3$ and $H_{a1} : \frac{\mu_1 + \mu_2}{2} \neq \mu_3$

From class chapter 6 we have studied :-

All four MANOVA tests are exact tests i.e. when H_0 is true each test rejects H_0 with same probability α (Type I error)

In practice, Λ and $V^{(s)}$ are often prepared but Λ is flexible and has historical precendence.

Therefore we test our hypothesis using Wilk's test

From the SAS output we have:-

Wilk's Test, $\Lambda = 0.270$

and the value of p is $< .001$ and thus we reject the H_{01}.

Though we don't have to check for other test we can still verify however that for

Roy's test, $\theta = \dfrac{2.702}{1+2.702} = 0.730$,

Pillai's test, $V^{(s)} = 0.730$ and,

Lawley Hotelling's Test, $U^{(s)} = 2.702$

all have p-value less than $< .001$ and hence significant.

H = Contrast SSCP Matrix for One vs Two				
	AROMA	FLAVOR	TEXTURE	MOISTURE
AROMA	0.09375	0.375	0.1	0.08125
FLAVOR	0.375	1.5	0.4	0.325
TEXTURE	0.1	0.4	0.1066666667	0.0866666667
MOISTURE	0.08125	0.325	0.0866666667	0.0704166667

Characteristic Roots and Vectors of: E Inverse * H, where H = Contrast SSCP Matrix for One vs Two E = Error SSCP Matrix					
		Characteristic Vector V'EV=1			
Characteristic Root	Percent	AROMA	FLAVOR	TEXTURE	MOISTURE
0.37673248	100.00	-0.15892782	0.60932989	-0.15353075	-0.12693576
0.00000000	0.00	0.29751928	-0.08379193	0.02111276	0.01745556
0.00000000	0.00	-0.03454000	-0.02672039	-0.19718713	0.40587059
0.00000000	0.00	-0.24722910	-0.05257444	0.42893143	0.00000000

MANOVA Test Criteria and Exact F Statistics for the Hypothesis of No Overall One vs Two Effect H = Contrast SSCP Matrix for One vs Two E = Error SSCP Matrix S=1 M=1 N=14					
Statistic	Value	F Value	Num DF	Den DF	Pr > F
Wilks' Lambda	0.72635753	2.83	4	30	0.0422
Pillai's Trace	0.27364247	2.83	4	30	0.0422
Hotelling-Lawley Trace	0.37673248	2.83	4	30	0.0422
Roy's Greatest Root	0.37673248	2.83	4	30	0.0422

→ Using contrasts : method 1 v/s method 2.

$H_{02}: \mu_1 = \mu_2$ and $H_{02}: \mu_1 \neq \mu_2$

From the SAS output we have :-

Wilk's test, $\Lambda = 0.726$ has a p-value that is extremely small and less than 0.001.

Therefore we reject H_{02} as the Values are significant.

The same is proven through

Roy's test, $\Theta = \frac{0.377}{1+0.377} = 0.274$

Pillai's test, $V^{(s)} = 0.274$ and

Lawley - Hotelling's Test, $U^{(s)} = 0.377$

all have p-value that is < 0.001 and thus null hypothesis is rejected.

⇒ All Values are significant

For conclusion,

We can state that from 1st contrast that method 1 & 2 are different from method 3.

and,

We can also state that from 2nd contrast that method 1 and method 2 are different.

6.28 (a)

6.28 (a) Three null hypotheses:-

$H_{0A}: \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$ (Main effect of Sowing)

$H_{0B}: \beta_1 = \beta_2 = \beta_3 = 0$ (Main effect of Variety)

$H_{0AB}: \gamma_{11} = \gamma_{12} = \cdots = \gamma_{43}$ (Interaction effect b/w sowing and Variety)

H_a: One inequality in H_{0A} or H_{0B} or H_{0AB}.

MANOVA Tests for the Hypothesis of No Overall SOWING Effect
H = Type III SSCP Matrix for SOWING
E = Error SSCP Matrix

S=3 M=0 N=21.5

Statistic	Value	P-Value
Wilks' Lambda	0.00064500	<.0001
Pillai's Trace	2.35677028	<.0001
Hotelling-Lawley Trace	142.30423395	<.0001
Roy's Greatest Root	137.16775571	<.0001

→ Sowing effect:-

From the SAS output we have:-

Wilk's Test, $\Lambda = 0.0065$

Roy's test, $\theta = \dfrac{137.168}{1+137.168} = 0.993$

Pillais test, $V^{(s)} = 2.357$

Lawley-Hotelling, $U^{(s)} = 142.304$

We can see that p-values for all test are $< .001$ and we can therefore reject H_0.

$\Rightarrow$ All sowing effect are significant.

MANOVA Tests for the Hypothesis of No Overall VARIETY Effect H = Type III SSCP Matrix for VARIETY E = Error SSCP Matrix S=2 M=0.5 N=21.5		
Statistic	**Value**	**P-Value**
Wilks' Lambda	0.06530009	<.0001
Pillai's Trace	1.10700655	<.0001
Hotelling-Lawley Trace	11.67522513	<.0001
Roy's Greatest Root	11.44466449	<.0001

→ Variety effect :-

From the SAS output we have :-

Wilk's Test, $\Lambda = 0.065$

Roy's Test, $\theta = \frac{11.445}{1+11.445} = 0.9196$

Pillai's Test, $V^{(s)} = 1.107$

Lawley - Hotelling's Test, $U^{(s)} = 11.675$

We can see that p-value for all test are < 0.001 and we can therefore reject H_0

⇒ All Variety effect are significant.

MANOVA Tests for the Hypothesis of No Overall SOWING*VARIETY Effect H = Type III SSCP Matrix for SOWING*VARIETY E = Error SSCP Matrix S=4 M=0.5 N=21.5		
Statistic	**Value**	**P-Value**
Wilks' Lambda	0.13794739	<.0001
Pillai's Trace	1.32129866	<.0001
Hotelling-Lawley Trace	3.45046384	<.0001
Roy's Greatest Root	2.64879665	<.0001

→ Interaction Effect :-

From the SAS output we have :-

Wilk's Test, $\Lambda = 0.138$

Roy's Test, $\theta = \frac{2.649}{1+2.649} = 0.726$

Pillai's Test, $V^{(s)} = 1.321$

Lawley-Hotelling's Test, $U^{(s)} = 3.450$.

We can again see that p-value for all test are <0.0

and we therefore reject Ho.

⇒ All interaction effect for Sowing * Variety are significant.

NOTE:- If we reject H_{0AB}, we conclude both factors Sowing and Variety are significant and hence no need to test H_{0A} and H_{0B}.

APPENDIX:

This section will have the entire SAS code.

 # 6.27 (a), (c) and (d)

Code:

```
DATA work.FISH;

  INFILE "/folders/myfolders/data/T6_17_FISH.dat";
  INPUT METHOD AROMA FLAVOR TEXTURE MOISTURE;

TITLE "HW3 Q-6.27 a), c), d)";

PROC GLM;
  CLASS METHOD;
  MODEL AROMA FLAVOR TEXTURE MOISTURE = METHOD;
  CONTRAST 'One - Two Vs Three'
    METHOD .5 .5 -1;
  CONTRAST 'One vs Two'
    METHOD 1 -1 0;
  MANOVA H=METHOD/PRINTE PRINTH;
RUN;
```

 # 6.28 (a)

Code:

```
DATA work.SNAPBEAN;

  INFILE "/folders/myfolders/data/T6_18_SNAPBEAN.dat";
  INPUT SOWING VARIETY SNAPBEAN YIELD SLA TOTALYIELD AVGSLA;

TITLE "HW3 Q-6.28 a)";

PROC GLM;
  CLASS SOWING VARIETY;
  MODEL YIELD SLA TOTALYIELD AVGSLA = SOWING VARIETY SOWING*VARIETY;
  MANOVA H=_ALL_/PRINTH PRINTE MSTAT=EXACT;
RUN;
```