

5.11

Explanation:

5.11 $\bar{y}' = (6, 11.25)$
 $H_0 = \mu' = (6, 11)$
 $H_a = \mu' \neq (6, 11)$
Test statistics, $T^2 = n(\bar{y} - \mu_0)' S^{-1}(\bar{y} - \mu_0) = 0.0610329$
Rejection region: $T^2 > T_{\alpha(2,3)}^2$ or
 $T^2 > 57.00$ (as seen from table)
Conclusion:- We fail to reject H_0 because as
seen above $T^2 \leq T_{\alpha(2,3)}^2$ for $\alpha = 0.05$.

Output:

HW2 Q-5.11

X 4 rows 2 cols (numeric)

3	10
6	12
5	14
10	9

N 1 row 1 col (numeric)

4

MU 2 rows 1 col (numeric)

6
11

XBAR 2 rows 1 col (numeric)

6
11.25

S 2 rows 2 cols (numeric)

8.666667	-2.666667
-2.666667	4.916667

T2 1 row 1 col (numeric)

0.0610329

Code:

```
TITLE "HW2 Q-5.11";
```

```
PROC IML;
```

```
  RESET PRINT;
```

```
  X = {3 10, 6 12, 5 14, 10 9};
```

```
  N = NROW(X);
```

```
  MU = {6, 11};
```

```
  XBAR = 1/N*X`*J(N,1);
```

```
  S = 1/(N-1)*X`*(I(N)-1/N*J(N))*X;
```

```
  T2 = N*(XBAR-MU)`*INV(S)*(XBAR-MU);
```

```
RUN;
```

5.14 a

Explanation:

5.14(a) $\bar{Y}' = (48.655, 49.625, 50.57, 51.455)$

$$H_0 = \mu' = (48, 49, 50, 51)$$

$$H_a = \mu' \neq (48, 49, 50, 51)$$

Test statistics, $T^2 = n(\bar{y} - \mu_0)' S^{-1} (\bar{y} - \mu_0) = 1.8197716$

Rejection region: $T^2 \geq T_{\alpha(4,19)}^2$ or.

$$T^2 \geq 14.283 \quad (\text{as seen from table})$$

Conclusion:- We fail to reject H_0 because as seen above $T^2 < T_{\alpha(4,19)}^2$ for $\alpha = 0.05$

Output:

HW2 Q-5.14 a)

MU 4 rows 1 col (numeric)

48
49
50
51

XBAR 4 rows 1 col (numeric)

48.655
49.625
50.57
51.445

S 4 rows 4 cols (numeric)

6.3299737	6.1890789	5.777	5.5347632
6.1890789	6.4493421	6.1534211	5.9056579
5.777	6.1534211	6.918	6.9266842
5.5347632	5.9056579	6.9266842	7.4331316

T2 1 row 1 col (numeric)

1.8197716

Code:

```
DATA work.BONE;

  INFILE "/folders/myfolders/data/T3_6_BONE.dat";

  INPUT OBS Y1 Y2 Y3 Y4;


TITLE "HW2 Q-5.14 a)";


PROC IML;

  USE work.BONE;

  READ ALL VAR{Y1 Y2 Y3 Y4} INTO X;

  N = NROW(X);

  RESET PRINT;

  MU = {48, 49, 50, 51};

  XBAR = 1/N*X`*J(N,1);

  S = 1/(N-1)*X`*(I(N)-1/N*J(N))*X;

  T2 = N*(XBAR-MU)`*INV(S)*(XBAR-MU);

RUN;
```

5.16 a

Explanation:

5.16(a) $\bar{Y}_1 = [194.47, 267.05, 137.37, 185.95]$
 $\bar{Y}_2 = [179.55, 290.8, 157.2, 209.25]$

$H_0: \mu_{Y_1} = \mu_{Y_2}$
 $H_a: \mu_{Y_1} \neq \mu_{Y_2}$

$T^2 = \frac{n_1 n_2}{n_1 + n_2} (\bar{Y}_1 - \bar{Y}_2)' S_{PL}^{-1} (\bar{Y}_1 - \bar{Y}_2) = 133.4873$

Rejection region: $T^2 > T_{\alpha}^2(p, n_1 + n_2 - 2)$ or
 $T^2 > 11.674$ ($\approx T_{\alpha}^2(4, 35)$)
as seen from table

Conclusion:- We reject the null hypothesis
because $T^2 > T_{\alpha}^2(4, 35)$ for $\alpha = 0.05$

Output:

HW2 Q-5.16 a)

N1 1 row 1 col (numeric)

19

N2 1 row 1 col (numeric)

20

X1BAR 4 rows 1 col (numeric)

194.47368
267.05263
137.36842
185.94737

X2BAR 4 rows 1 col (numeric)

179.55
290.8
157.2
209.25

S1 4 rows 4 cols (numeric)

187.59649	176.86257	48.371345	113.58187
176.86257	345.38596	75.979532	118.7807
48.371345	75.979532	66.356725	16.24269
113.58187	118.7807	16.24269	239.94152

S2 4 rows 4 cols (numeric)

101.83947	128.06316	36.989474	32.592105
128.06316	389.01053	165.35789	94.368421
36.989474	165.35789	167.53684	66.526316
32.592105	94.368421	66.526316	177.88158

Spl 4 rows 4 cols (numeric)

143.5591	151.80341	42.5266	71.992532
151.80341	367.78777	121.87653	106.24467
42.5266	121.87653	118.31408	42.064011
71.992532	106.24467	42.064011	208.0729

T2 1 row 1 col (numeric)

133.4873

Code:

```
DATA work.FBEETLES;
```

```
INFILE "/folders/myfolders/data/T5_5_FBEETLES.dat";
```

```
INPUT OBS SPEC Y1 Y2 Y3 Y4;
```

```
TITLE "HW2 Q-5.16 a)";
```

```
PROC IML;
```

```
USE work.FBEETLES;
```

```
READ ALL VAR {Y1 Y2 Y3 Y4} INTO X;
```

```
X1 = X[1:19,];
```

```
X2 = X[20:39,];
```

```
RESET PRINT;
```

```
N1 = NROW(X1);
```

```
N2 = NROW(X2);
```

```
X1BAR = 1/N1*X1`*J(N1,1);
```

```
X2BAR = 1/N2*X2`*J(N2,1);
```

```
S1 = 1/(N1-1)*X1`*(I(N1)-1/N1*J(N1))*X1;
```

```
S2 = 1/(N2-1)*X2`*(I(N2)-1/N2*J(N2))*X2;
```

```
Spl = 1/(N1+N2-2)*((N1-1)*S1+(N2-1)*S2);
```

```
T2 = N1*N2/(N1+N2)*(X1BAR-X2BAR)`*INV(Spl)*(X1BAR-X2BAR);
```

```
RUN;
```

5.22

Explanation:

5.22 $\bar{D}' = (49.5, 106.875)$
 $H_0 = \mu_d = 0$
 $H_a = \mu_d \neq 0$
 $T^2 = n(\bar{D} - \mu_0)' S^{-1} (\bar{D} - \mu_0) = 22.323833$
Rejection region: $T^2 \geq T^2_{\alpha}(2, 15)$ or
 $T^2 \geq 8.012$ (as seen from table)
Conclusion:- We ~~reject~~ reject the null hypothesis
because $T^2 > T^2_{\alpha}(2, 15)$ for $\alpha = 0.05$

Output:

HW2 Q-5.22

D 16 rows 2 cols (numeric)

9	41
327	405
-64	-4
352	392
198	74
24	66
-50	12
-26	26
125	95
-109	4
50	136
-50	0
154	186
38	111
-243	0
57	166

N 1 row 1 col (numeric)

16

MU 2 rows 1 col (numeric)

0
0

DBAR 2 rows 1 col (numeric)

49.5
106.875

S 2 rows 2 cols (numeric)

23915.067	17461.467
17461.467	16619.45

T2 1 row 1 col (numeric)

22.323833

Code:

```
DATA work.BRONCUS;

INFILE "/folders/myfolders/data/T5_10_BRONCUS.dat";
INPUT Y1 Y2 X1 X2;

TITLE "HW2 Q-5.22";

PROC IML;
  USE work.BRONCUS;
  READ ALL VAR {Y1 Y2} INTO Y;
  READ ALL VAR {X1 X2} INTO X;
  RESET PRINT;
  D = Y - X;
  N = NROW(D);
  MU = {0, 0};
  DBAR = 1/N*D`*J(N,1);
  S = 1/(N-1)*D`*(I(N)-1/N*J(N))*D;
  T2 = N*(DBAR-MU)`*INV(S)*(DBAR-MU);
RUN;
```