RO2102: Dynamics and Simulation

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Homework-5

Problem 14 **Due : 18-Feb-2025** **Time Spent : 1 Hour**

$$\vec{F} = m\vec{a}$$

$$\vec{r}_{P/c} \times \vec{F} = m\vec{r}_{P/c} \times \vec{a}$$

Guess : $\frac{d}{dt}\vec{H}_{/c} = \vec{r}_{/c} \times \vec{F}$

Candidate Def^{n}'s :

(a) $m\vec{r}_{P/C'} \times \vec{v}_{/C'}$: C' is a point coincident w/ C but not moving

(b) $m\vec{r}_{P/c} \times \vec{v}_{/F}$

(c) $m\vec{r}_{P/c} \times \vec{v}_{/c}$

Try (a), $\vec{H}_{/c} = m\frac{d}{dt}(\vec{r}_P - \vec{r}_{C'}) \times (\vec{v}_P - \vec{v}_{C'})$

$= m(\vec{v}_P - \vec{v}_{C'}) \times (\vec{a}_P - \vec{a}_{C'}) + m(\vec{v}_P - \vec{v}_{C'}) \times (\vec{v}_P - \vec{v}_{C'})$ ($\vec{a}_{C'} = \vec{0}$, $(\vec{v}_P - \vec{v}_{C'}) \times (\vec{v}_P - \vec{v}_{C'}) = \vec{0}$)

$= m\vec{r}_{P/c} \times \vec{a}_P$ ⟵ (a) is a good guess of $\vec{H}_{/c}$

Try (c), $\frac{d}{dt}\vec{H}_{/c} = m\frac{d}{dt}\left[(\vec{r}_P - \vec{r}_c) \times (\vec{v}_P - \vec{v}_c)\right]$

$= m(\vec{v}_P - \vec{v}_c) \times (\vec{a}_P - \vec{a}_c) + m(\vec{v}_P - \vec{v}_c) \times (\vec{v}_P - \vec{v}_c)$ ($(\vec{v}_P - \vec{v}_c) \times (\vec{v}_P - \vec{v}_c) = \vec{0}$)

$= m\vec{r}_{P/c} \times \vec{a}_P - m\vec{r}_{P/c} \times \vec{a}_c$ ⟵ Good when $\vec{r}_{P/c} \times \vec{a} = \vec{0}$ ⇒

- $\vec{a}_c = \vec{0}$
- $\vec{a}_c$ points to P from C
- P and C coincide

Try (b), $\frac{d}{dt}\vec{H}_{/c} = m\frac{d}{dt}(\vec{r}_{P/c} \times \vec{v}_{/F})$

$= m\left[\vec{v}_{P/c} \times \vec{v}_{/F} + \vec{r}_{P/c} \times \vec{a}_{/F}\right]$ ⟵ Good when $\vec{v}_{P/c} \times \vec{v}_{/F} = \vec{0}$ ⇒

- C is moving at the same speed as P
- C is at a fixed point
- P, C and origin are co-linear