RO2102: Dynamics and Simulation

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Homework-6

Problem 18 Due : 25-Feb-2025 Time Spent : 2 Hours

(a) $$\sum \vec{v}_i \cdot \vec{F}_i = \frac{d}{dt} \sum E_{Ri}$$

$$\Rightarrow \sum \vec{v}_i \cdot (\vec{F}_i^{int} + \vec{F}_i^{ext}) = \frac{d}{dt} \sum E_{Ri}$$

$$= \frac{1}{2} \frac{d}{dt} \sum m_i \vec{v}_i \cdot \vec{v}_i$$

$$E_{Rtot} = \sum \frac{1}{2} m_i \vec{v}_i \cdot \vec{v}_i$$

$$= \frac{1}{2} \sum m_i (\vec{v}_{i/G} + \vec{v}_{G/O}) \cdot (\vec{v}_{i/G} + \vec{v}_{G/O})$$

avg velocity of a stationary point

$$= \frac{1}{2} \sum m_i [\vec{v}_{i/G} \cdot \vec{v}_{i/G} + \cancel{2\vec{v}_{i/G} \cdot \vec{v}_G} + \vec{v}_G \cdot \vec{v}_G]$$

$$\boxed{E_{Rtot} = \frac{1}{2} m_{tot} \vec{v}_G \cdot \vec{v}_G + \frac{1}{2} \sum m_i \vec{v}_{i/G} \cdot \vec{v}_{i/G}}$$

(b) $$\sum \vec{F}_i \cdot \vec{v}_i = \frac{d}{dt} \sum E_{Ri}$$

$$\Rightarrow \left(\sum \vec{F}_i^{int} \cdot \vec{v}_i + \sum \vec{F}_i^{ext} \cdot \vec{v}_i\right) = \frac{d}{dt}\left(\frac{1}{2} m_{tot} \vec{v}_G \cdot \vec{v}_G + \frac{1}{2} \sum m_i \vec{v}_{i/G} \cdot \vec{v}_{i/G}\right)$$

$$\Rightarrow \sum \vec{F}_i^{int} \cdot (\vec{v}_{i/G} + \vec{v}_G) + \sum \vec{F}_i^{ext} \cdot (\vec{v}_{i/G} + \vec{v}_G) = \frac{d}{dt}\left(\frac{1}{2} m_{tot} \vec{v}_G \cdot \vec{v}_G + \frac{1}{2} \sum m_i \vec{v}_{i/G} \cdot \vec{v}_{i/G}\right)$$

$$\Rightarrow \cancelto{0}{\left(\sum \vec{F}_i^{int}\right)} \cdot \vec{v}_G + \sum \vec{F}_i^{int} \cdot \vec{v}_{i/G} + \sum \vec{F}_i^{ext} \cdot \vec{v}_{i/G} + \left(\sum \vec{F}_i^{ext}\right) \cdot \vec{v}_G = \frac{d}{dt}\left(\frac{1}{2} m_{tot} \vec{v}_G \cdot \vec{v}_G + \frac{1}{2} \sum m_i \vec{v}_{i/G} \cdot \vec{v}_{i/G}\right)$$

Cases for the condition to be satisfied:

1) $\sum \vec{F}_i^{int} \cdot \vec{v}_{i/G} + \sum \vec{F}_i^{ext} \cdot \vec{v}_{i/G} = \frac{d}{dt}\left(\frac{1}{2}\sum m_i \vec{v}_{i/G} \cdot \vec{v}_{i/G}\right)$

$\Rightarrow \cancelto{0}{\sum \vec{F}_i^{int}} + \sum F_i^{ext} = \sum \vec{m}_i \vec{a}_{i/G} \cdot \vec{v}_{i/G}$

This is satisfied iff $\vec{a}_G = \vec{0}$.

<u>OR</u>

2) $\sum \vec{F}_i^{ext} \cdot \vec{v}_{i/G} = 0$ and $\frac{d}{dt}\left(\frac{1}{2}\sum m_i \vec{v}_{i/G} \cdot \vec{v}_{i/G}\right) = 0$ and $\sum \vec{F}_i^{int} \cdot \vec{v}_{i/G} = 0$

(a) (i) $\vec{F}_i^{ext}$ is $\perp$ $\vec{v}_{i/G}$ $\forall$ i

OR

(ii) $\vec{F}_i^{ext} = \vec{0}$ $\forall$ i

OR

(iii) $\vec{v}_{i/G} = \vec{0}$ $\forall$ i

(b) (i) $\vec{v}_{i/G}$ is not changing $\forall$ i

OR

(ii) $\vec{v}_{i/G} = \vec{0}$

(c) (i) $\vec{F}_i^{int}$ is $\perp$ $\vec{v}_{i/G}$ $\forall$ i

OR

$\vec{F}_i^{int} = \vec{0}$ $\forall$ i

OR

$\vec{v}_{i/G} = \vec{0}$ $\forall$ i

$\therefore$ Either 1 <u>OR</u> any condition in 2(a) <u>AND</u> 2(b) <u>AND</u> 2(c) must be satisfied.

(c) As derived in part (b),

$$\sum \vec{F}_i^{int} \cdot \vec{v}_{i/G} + \sum \vec{F}_i^{ext} \cdot \vec{v}_{i/G} = \frac{d}{dt}\left(\frac{1}{2}\sum m_i \vec{v}_{i/G} \cdot \vec{v}_{i/G}\right)$$

The power of internal forces being equal to $\frac{d}{dt}\left(\frac{1}{2}\sum m_i \vec{v}_{i/G} \cdot \vec{v}_{i/G}\right)$ is satisfied when,

(i) $\vec{F}_i^{ext}$ is $\perp$ $\vec{v}_{i/G}$ $\forall$ i

<u>OR</u>

(ii) $\vec{F}_i^{ext} = \vec{0}$ $\forall$ i

<u>OR</u>

(iii) $\vec{v}_{i/G} = \vec{0}$ $\forall$ i