

Final - Project

Problem 53

Due : 15 - May - 2025

Time Spent : 5 Hours

Given:  $\vec{N}_A = \underline{\underline{I}} \cdot \vec{\omega} + \vec{\omega} \times (\underline{\underline{I}} \cdot \vec{\omega})$

To Find: Animation of object

Assume,  $\underline{\underline{I}} = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix}$ ,  $\vec{\omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$ ,  $\vec{\omega} = \begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{bmatrix}$

$$\underline{\underline{I}} \cdot \vec{\omega} = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \cdot \begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{bmatrix}$$

$$= [I_1 \dot{\omega}_1, I_2 \dot{\omega}_2, I_3 \dot{\omega}_3]'_B$$

$$\underline{\underline{I}} \cdot \vec{\omega} = [I_1 \omega_1, I_2 \omega_2, I_3 \omega_3]'_B$$

$$\vec{\omega} \times (\underline{\underline{I}} \cdot \vec{\omega}) = \begin{vmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \\ \omega_1 & \omega_2 & \omega_3 \\ I_1 \omega_1 & I_2 \omega_2 & I_3 \omega_3 \end{vmatrix}$$

$$= \omega_2 \omega_3 (I_3 - I_1) \hat{e}_1 - \omega_1 \omega_3 (I_3 - I_2) \hat{e}_2 + \omega_1 \omega_2 (I_2 - I_1) \hat{e}_3$$

$$\underline{\underline{I}} \cdot \vec{\omega} + \vec{\omega} \times (\underline{\underline{I}} \cdot \vec{\omega}) = \begin{bmatrix} I_1 \dot{\omega}_1 + \omega_2 \omega_3 (I_3 - I_1) \\ I_2 \dot{\omega}_2 - \omega_1 \omega_3 (I_3 - I_1) \\ I_3 \dot{\omega}_3 + \omega_1 \omega_2 (I_2 - I_1) \end{bmatrix}_B$$