Tanay Srinivasa

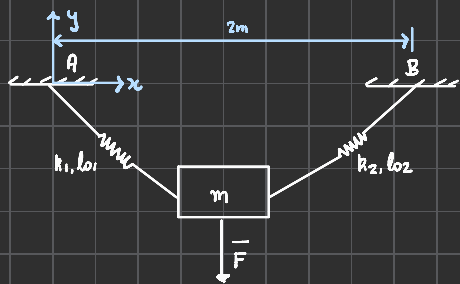
U20220086

RO2102: Dynamics and Simulation

Homework-1

Problem 1 **Due:** 16-Jan-2025 **Time Spent:** 2.5 Hours

Sketch:



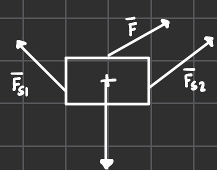
Given: $k_1 = 10\,N/m$; $k_2 = 20\,N/m$; $m = 5\,kg$; $r(0) = [1 \;\; -2]'\,m$; $\bar{r}_{B/A} = [2 \;\; 0]'\,m$

$l_{01} = 1\,m$; $l_{02} = 1.414\,m$; $g = 10\,ms^{-2}$; $v(0) = [0 \;\; 0]'\,ms^{-1}$; $\bar{F} = [5 \;\; 1]'\,N$

To Find: $r(t)$

Apply LMB,

FBD: Mass



$$\Sigma \bar{F}_i = m\bar{a}$$

$$\Rightarrow \boxed{\bar{F} + \bar{F}_{s1} + \bar{F}_{s2} - mg\hat{j} = m\bar{a}}$$

$$\bar{F}_{s1} = k_1\left(|\bar{r}_{A/M}| - l_{01}\right)\frac{(\bar{r}_{A/M})}{|\bar{r}_{A/M}|} = -k_1\left(|\bar{r}(t)| - l_{01}\right)\frac{\bar{r}(t)}{|\bar{r}(t)|}$$

$$\bar{F}_{s2} = k_2\left(|\bar{r}_{B/M}| - l_{02}\right)\frac{(\bar{r}_{B/M})}{|\bar{r}_{B/M}|} = -k_2\left(|\bar{r}(t) + \bar{r}_{A/B}| - l_{02}\right)\frac{(\bar{r}(t) + \bar{r}_{A/B})}{|\bar{r}(t) + \bar{r}_{A/B}|}$$

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Homework-1

Problem 2 Due : 16-Jan-2025 Time Spent : 2.5 Hours

Given: $\bar{r}_A, \bar{r}_B, \bar{r}_C, \bar{r}_D, \bar{F}$

(a) **To Find:** Shortest Distance between AB and CD.

Approach: We want to find a vector perpendicular to $\vec{r}_{AB}$ and $\vec{r}_{CD}$, and dot any vector between the two lines.

$$\hat{n} = \frac{\bar{r}_{B/A} \times \bar{r}_{D/C}}{|\bar{r}_{B/A} \times \bar{r}_{D/C}|}$$

Reference : Ruina-Pratap Sample 1.23

$$\boxed{d = \bar{r}_{C/A} \cdot \hat{n}}$$

(b) **To Find:** End Points of shortest line segment

Approach: start with two points: $\vec{r}_A, \vec{r}_C$

Define : $\bar{r}_1 = \bar{r}_A + x\hat{\lambda}_{AB}$
$\bar{r}_2 = \bar{r}_B + y\hat{\lambda}_{CD}$

Apply gradient descent on $d = |\bar{r}_1 - \bar{r}_2|$ and update x and y accordingly.

Code can be found in the MATLAB file.

(c) **To Find:** Volume of tetrahedron ABCD

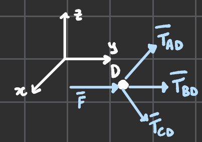
Volume of a parallelopiped $= \bar{r}_{A/D} \cdot (\bar{r}_{B/D} \times \bar{r}_{C/D}) = 6 \times$ Volume of Tetrahedron [Reference: Ruina-Pratap 1.3: Mixed Triple Product]

$$\Rightarrow \boxed{\text{Volume of ABCD} = \frac{1}{6}\, \bar{r}_{A/D} \times (\bar{r}_{B/D} \cdot \bar{r}_{C/D})}$$

(d) <u>To Find:</u> Tension in bar AD

Assume static equilibrium,

FBD : Joint D



$$\Sigma \bar{F} = \bar{0}$$

$$\Rightarrow \boxed{\bar{T}_{AD} \frac{(\bar{r}_{A/D})}{|\bar{r}_{A/D}|} = \bar{F} + T_{BD} \frac{\bar{r}_{D/B}}{|\bar{r}_{D/B}|} + T_{CD} \frac{\bar{r}_{D/C}}{|\bar{r}_{D/C}|}} \quad \text{— (1)}$$

From the above equation, 3 scalar equations are obtained. When solved simultaneously, the three unknowns : T_{AD}, T_{BD}, T_{CD} can be found.

Alternatively, to find it in a single scalar equation:

$$\hat{\lambda} = \frac{\bar{r}_{B/D} \times \bar{r}_{C/D}}{|\bar{r}_{B/D} \times \bar{r}_{C/D}|}$$

Now dot (1) with $\hat{\lambda}$,

$$\vec{T}_{AD} \cdot \hat{\lambda} = \vec{F} \cdot \hat{\lambda}$$

$$\Rightarrow \begin{vmatrix} T_{ADx} & T_{ADy} & T_{ADz} \\ T_{BDx} & T_{BDy} & T_{BDz} \\ T_{CDx} & T_{CDy} & T_{CDz} \end{vmatrix} = \begin{vmatrix} F_x & F_y & F_z \\ T_{BDx} & T_{BDy} & T_{BDz} \\ T_{CDx} & T_{CDy} & T_{CDz} \end{vmatrix}$$

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Homework-2

Problem 4 **Due** : 24 - Jan-2025 **Time Spent** : 3.5 Hours

(a) The cross product can be thought of as :

- Project b onto a vector orthogonal to $\bar{a}$ in the plane defined by $\bar{a}, \bar{b}$.
- Scale Projection by $|\bar{a}|$
- Rotate the scaled orthogonal vector positively about the axis defined by a by an angle $\pi/2$.

Given : $\bar{a}, \bar{b}, \bar{c}$

To Prove: $\bar{a} \times (\bar{b} + \bar{c}) = (\bar{a} \times \bar{b}) + (\bar{a} \times \bar{c})$

Define : $\bar{d} = \bar{b} + \bar{c}$

$\bar{d}' = |\bar{d}| \sin\theta_{ad} \, \hat{a}_{\perp d}$

$\bar{d}'' = |\bar{a}||\bar{d}| \sin\theta_{ad} \, \hat{a}_{\perp d}$

$\bar{d}''' = |\bar{a}||\bar{d}| \sin\theta_{ad} \, \hat{n}_{ad}$

$\bar{b}' = |\bar{b}| \sin\theta_{ab} \, \hat{a}_{\perp b}$

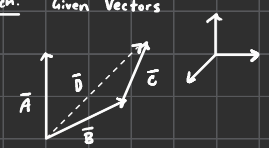
$\bar{b}'' = |\bar{a}||\bar{b}| \sin\theta_{ab} \, \hat{a}_{\perp b}$

$\bar{b}''' = |\bar{a}||\bar{b}| \sin\theta_{ab} \, \hat{n}_{ab}$

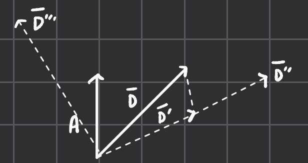
$\bar{c}' = |\bar{c}| \sin\theta_{ac} \, \hat{a}_{\perp c}$

$\bar{c}'' = |\bar{a}||\bar{c}| \sin\theta_{ac} \, \hat{a}_{\perp c}$

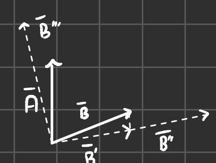
Sketch: Given Vectors



Sketch : $\bar{a} \times \bar{d}$

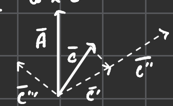


Sketch: $\bar{a} \times \bar{b}$



$$\bar{c}''' = |\bar{a}||\bar{c}| \sin\theta_{ac}\ \hat{n}_{ac}$$

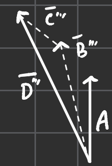
Sketch: $\bar{a}\times\bar{c}$



$$\bar{a}\times\bar{d} = \bar{d}'''$$

$$(\bar{a}\times\bar{b}) + (\bar{a}\times\bar{c}) = \bar{a}''' + \bar{d}'''$$

Sketch: $(\bar{a}\times\bar{b}) + (\bar{a}\times\bar{c})$



This visualization in 3-D can be seen in the matlab file.

(b) $\bar{a} = [a_x \quad a_y \quad a_z]'$

$\bar{b} = [b_x \quad b_y \quad b_z]'$

For X-Y Plane,

$$\bar{a}\times\bar{b} = a_x b_x \sin(0)\,\bar{0} + a_y b_y \sin(0)\,\bar{0} + a_x b_y \sin 90\ \hat{k} + a_y b_x \sin(-90)\,\hat{k}$$

$$= (a_x b_y - a_y b_x)\ \hat{k}$$

For Y-Z Plane,

$$\bar{a}\times\bar{b} = a_y b_y \sin(0)\,\bar{0} + a_z b_z \sin(0)\,\bar{0} + a_y b_z \sin(90)\,\hat{i} + a_z b_y \sin(-90)\,\hat{i}$$

$$= (a_y b_z - a_z b_y)\,\hat{i}$$

For X-Z plane,

$$\bar{a}\times\bar{b} = a_x b_x \sin(0)\,\bar{0} + a_z b_z \sin(0)\,\bar{0} + a_x b_z \sin(-90)\,\hat{j} + a_z b_x \sin 90(\hat{j})$$

$$= (a_z b_x - a_x b_z)\,\hat{j}$$

In 3-D,

$$\boxed{\bar{a}\times\bar{b} = (a_y b_z - a_z b_y)\,\hat{i} + (a_z b_x - a_x b_z)\,\hat{j} + (a_x b_y - a_y b_x)\ \hat{k}}$$

RO2102: Dynamics and Simulation

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Homework-2

Problem 5 **Due** : 24 - Jan-2025 **Time Spent** : 3.5 Hours

I have carefully done some of the reading assigned. That which I don't understand or agree with, I have asked Prof. Ruina. I haven't yet carefully read Chapter 3.

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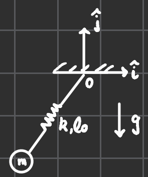
Homework-3

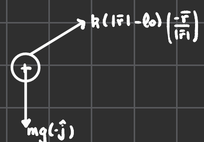
Problem 6 Due : 28 - Jan-2025 Time Spent : 1 Hour 15 Min



Given : $m, g, k, l_0, \bar{r}_0, \bar{v}_0$

To Find : Equations of motion

FBD: Mass



Apply LMB,

$$\sum \bar{F} = \dot{\bar{L}}$$

$$\Rightarrow k(|\bar{r}|-l_0)\left(\frac{-\bar{r}}{|\bar{r}|}\right) - mg\hat{j} = m\bar{a}$$

$$\Rightarrow \boxed{\ddot{\bar{r}} = -\frac{k(|\bar{r}|-l_0)\bar{r}}{m|\bar{r}|} - g\hat{j}}$$

(c) (i) $k = 0$

(ii) $x_0 = 0$, $V_{x0} = 0$

(iii) $g = 0$, $\bar{v}_0 = \bar{0}$

(iv) $L_0 = 0$: In this case for any $\bar{r}_0 \neq \bar{0}$ and $\bar{v}_0 \neq 0$, motion is observed.

(v) $|\bar{r}_0| = L_0$; $\bar{v}_0 = \vec{0}$

(vi) $|\bar{r}_0| = L_0$; $\bar{v}_0 \neq \vec{0}$

RO2102: Dynamics and Simulation

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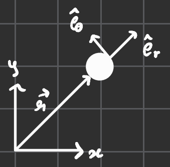
Homework-3

Problem 8 Due : 28-Jan-2025 Time Spent : 1.5 Hours

Sketch:



Given: $\bar{a} = 0$, $\bar{r}_0$, $\bar{v}_0$

To Find: Equations of Motion

$$\bar{r} = r\,\hat{e}_r$$

$$\bar{v} = \frac{d}{dt}\bar{r} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\bar{a} = \frac{d}{dt}\bar{v} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{e}_\theta = \vec{0}$$

$$\Rightarrow \ddot{r} = r\dot{\theta}^2$$

$$\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r}$$

Define: $z = \begin{bmatrix} r \\ \theta \\ v_r \\ \omega \end{bmatrix}$

$$\Rightarrow \dot{z} = \begin{bmatrix} \dot{r} \\ \dot{\theta} \\ \ddot{r} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} v_r \\ \omega \\ r\omega^2 \\ -\frac{2v_r\omega}{r} \end{bmatrix}$$

RO2102: Dynamics and Simulation

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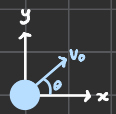
Homework-3

Problem 9   **Due** : 28 - Jan-2025   **Time Spent** : 2 Hours 30 Min

Sketch:

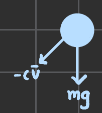


Given: m, c, g, θ_0, V_0

To Find: EoM

Apply LMB,

FBD:



$$\sum \bar{F} = \dot{\bar{L}}$$

$$\Rightarrow \boxed{-c\bar{V} - mg\hat{j} = m\bar{a}}$$

(e) $$x = \frac{m}{c} V_{x0} \left(1 - e^{-\frac{c}{m}t}\right)$$

$$= \frac{m}{c} V_0 \cos\theta_0 \left(1 - e^{-c/m\, t}\right)$$

$$y = \frac{m}{c}\left(V_0 \sin\theta_0 + \frac{mg}{c}\right) * \left(1 - e^{-c/m\, t}\right) - \left(\frac{mgt}{c}\right)$$

At the cannon touching the surface, $y = 0$

$$\Rightarrow \frac{\cancel{m} g t}{\cancel{c}} \times \frac{1}{\left(1 - e^{-c/m\, t}\right)} = \frac{\cancel{m}}{\cancel{c}}\left(V_0 \sin\theta_0 + \frac{mg}{c}\right)$$

$$\Rightarrow \sin\theta_0 = \left(\frac{gt}{\left(1 - e^{-c/m\, t}\right)} - \frac{mg}{c}\right) V_0$$

Now solve this equation to find t for a θ_0 and v_0.

However, as the equation is highly non-linear it needs to be solved numerically.

Solve for a range of theta$_0$ and choose that which gives highest x.

Could do it in matlab but too tedious

For $v_0 \to \infty$, θ should tend to 0

RO2102: Dynamics and Simulation

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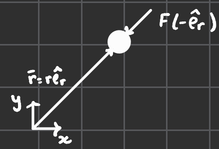
Homework-4

Problem 11 **Due** : 11 - Feb - 2025 **Time Spent** : 1 Hour 30 Min

Sketch :



FBD : Mass

$$\vec{r} = r\hat{e}_r$$

$$\Rightarrow \dot{\vec{r}} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

For circular motion, $\dot{r} = 0$

$$\Rightarrow \dot{\vec{r}} = r\dot{\theta}\hat{e}_\theta = v\hat{e}_\theta$$

$$\Rightarrow \ddot{\vec{r}} = r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r = \vec{a}$$

Apply LMB,

$$\Sigma\vec{F} = \dot{\vec{L}}$$

$$\Rightarrow F\hat{e}_r = m\vec{a}$$

$$F\hat{e}_r = mr\ddot{\theta}\hat{e}_\theta - mr\dot{\theta}^2\hat{e}_r$$

$$= mr\ddot{\theta}\hat{e}_\theta - m\dot{\theta}v\hat{e}_r \quad \text{——— (I)}$$

Dot (I) with $\hat{e}_\theta$,

$$mr\ddot{\theta} = 0$$

$$\Rightarrow \ddot{\theta} = 0$$

Dot (I) with $\hat{e}_r$,

$$\boxed{F = mv\dot{\theta}}$$

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Homework-4

Problem 13 Due : 11 - Feb - 2025 Time Spent : 1 Hour 30 Min

Apply LMB on FBD of mass,

$$\sum \vec{F} = \dot{\vec{L}}$$

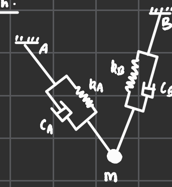
$$\Rightarrow \vec{F}_{SA} + \vec{F}_{SB} + \vec{F}_{CA} + \vec{F}_{CB} = m\vec{a}$$

$$\Rightarrow -k_A(r_A - l_{0A})\hat{r}_A - k_B(r_B - l_{0B})\hat{r}_B - c_A\vec{v}_A\cdot\vec{r}_A\hat{r}_A - c_B\vec{v}_B\cdot\vec{r}_A\hat{r}_B = m\vec{a}$$

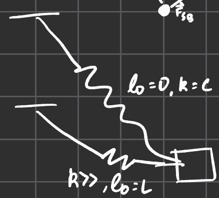
$$\Rightarrow \vec{a} = -\frac{k_A}{m}(r_A - l_{0A})\hat{r}_A - \frac{k_B}{m}(r_B - l_{0B})\hat{r}_B - \frac{c_A}{m}\vec{v}_A\cdot\vec{r}_A\hat{r}_A - \frac{c_B}{m}\vec{v}_B\cdot\vec{r}_A\hat{r}_B$$

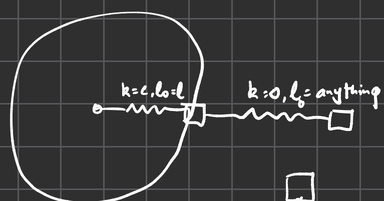
Where $\vec{a}$ is for the dynamical model.

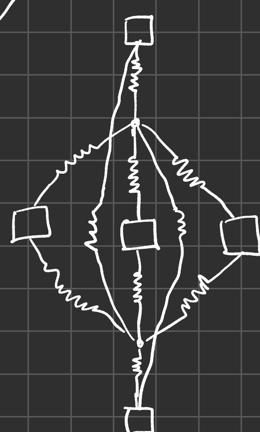
Sketch:



FBD: mass







RO2102: Dynamics and Simulation

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Homework-5

Problem 14 **Due** : 18-Feb-2025 **Time Spent** : 1 Hour

$$\vec{F} = m\vec{a}$$

$$\vec{r}_{P/c} \times \vec{F} = m\vec{r}_{P/c} \times \vec{a}$$

Guess : $\frac{d}{dt}\vec{H}_{P/c} = \vec{r}_{P/c} \times \vec{F}$

Candidate Defⁿ's :

(a) $m\vec{r}_{/c'} \times \vec{v}_{/c'}$: c' is a point coincident w/ c but not moving

(b) $m\vec{r}_{P/c} \times \vec{v}_{/F}$

(c) $m\vec{r}_{P/c} \times \vec{v}_{/c}$

Try (a),
$$\dot{\vec{H}}_{/c} = m\frac{d}{dt}(\vec{r}_p - \vec{r}_{c'}) \times (\vec{v}_p - \vec{v}_{c'})$$
$$= m(\vec{r}_p - \vec{r}_{c'}) \times (\vec{a}_p - \cancelto{0}{\vec{a}_{c'}}) + \cancelto{0}{m(\vec{v}_p - \vec{v}_{c'}) \times (\vec{v}_p - \vec{v}_{c'})}$$
$$= m\vec{r}_{P/c} \times \vec{a}_p \longleftarrow \text{(a) is a good guess of } \vec{H}_{/c}$$

Try (c),
$$\frac{d}{dt}\vec{H}_{/c} = m\frac{d}{dt}\left[(\vec{r}_p - \vec{r}_c) \times (\vec{v}_p - \vec{v}_c)\right]$$
$$= m(\vec{r}_p - \vec{r}_c) \times (\vec{a}_p - \vec{a}_c) + \cancelto{0}{m(\vec{v}_p - \vec{v}_c) \times (\vec{v}_p - \vec{v}_c)}$$
$$= m\vec{r}_{P/c} \times \vec{a}_p - m\vec{r}_{P/c} \times \vec{a}_c \longleftarrow \text{Good when } \vec{r}_{P/c} \times \vec{a} = \vec{0} \Rightarrow \begin{cases} \vec{a}_c = \vec{0} \\ \vec{a}_c \text{ points to } p \text{ from } c \\ p \text{ and } c \text{ coincide} \end{cases}$$

Try (b),
$$\frac{d}{dt}\vec{H}_{/c} = m\frac{d}{dt}(\vec{r}_{P/c} \times \vec{v}_{/F})$$
$$= m\left[\vec{v}_{P/c} \times \vec{v}_{/F} + \vec{r}_{P/c} \times \vec{a}_{/F}\right] \longleftarrow \text{Good when } \vec{v}_{P/c} \times \vec{v}_{/F} = \vec{0} \Rightarrow \begin{cases} c \text{ is moving at the same speed as } P \\ c \text{ is at a fixed point} \\ P, c \text{ and origin are co-linear} \end{cases}$$

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Homework-5

Problem 15 Due : 18-Feb-2025 Time Spent : 30 Min

$$\vec{F} = -\vec{\nabla} E_p$$

$$\Rightarrow E_p = -\int \vec{F} \cdot d\vec{r}$$

(a) $$\vec{F} = -mg\hat{j}$$

$$d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$E_p = -\int \vec{F} \cdot d\vec{r} = +\int mg\,dy$$

$$= mgy$$

(b) $$\vec{F} = -kr\hat{e}_r$$

$$d\vec{r} = dr\,\hat{e}_r + r\,d\theta\,\hat{e}_\theta + r\sin\theta\,d\phi\,\hat{e}_\phi$$

$$E_p = -\int \vec{F} \cdot d\vec{r} = \int kr\,dr$$

$$= \frac{kr^2}{2}$$

(c) $$\vec{F} = -k(l - l_0)\hat{e}_r = -k(r - l_0)\hat{e}_r$$

$$d\vec{r} = dr\,\hat{e}_r + r\,d\theta\,\hat{e}_\theta + r\sin\theta\,d\phi\,\hat{e}_\phi$$

$$E_p = -\int \vec{F} \cdot d\vec{r} = \int k(r - l_0)\,dr$$

$$= \frac{kr^2}{2} - kl_0 r = k\left(\frac{l^2}{2} - l_0 l\right)$$

(d) $\vec{F} = \frac{-C}{r^2}\hat{e}_r$

$E_p = -\int \vec{F}\cdot d\vec{r} = \int \frac{c}{r^2}dr$

$= \frac{-c}{r}$

(e) $\vec{F} = -f(r)\hat{e}_r$

$E_p = -\int \vec{F}(r')\cdot d\vec{r}' = \int_{r_0}^{r} f(r')\,dr'$

Where r_0 is chosen s.t the integral is not divergent.

(f) $\vec{F} = -k(r_2 - r_1 - l_0)\hat{r}$

$E_p = -\int \vec{F}\cdot d\vec{r} = \int k(r_2 - r_1 - l_0)\,dr$

$= \frac{kr^2}{2} - kl_0 r = \frac{kl_{12}^2}{2} - kl_0 l_{12}$

(g) $\vec{F} = \frac{-c}{r^2}\hat{\lambda}$

$E_p = -\int \vec{F}\cdot d\vec{r} = \int_{l_1}^{l_2} \frac{c}{r^2}dr$

$= \frac{-c}{l_{12}}$

(h) $\vec{F} = -C\hat{\lambda}$

$E_p = -\int \vec{F}\cdot d\vec{r} = \int C\hat{\lambda}\cdot d\vec{r}$

$= C\hat{\lambda}\cdot\vec{r}$

(i) $\vec{F} = -f(x)\hat{i}$

$E_p = -\int \vec{F}\cdot d\vec{r} = \int_{x_0}^{x} f(x')\,dx'$

Where x_0 is chosen s.t the integral is not divergent.

RO2102: Dynamics and Simulation

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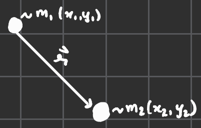
U20220086

Homework-5

Problem 16 **Due** : 18-Feb-2025 **Time Spent** : 2Hours 15Min

Sketch:



Apply LMB on FBD of m_1,

$$\sum \vec{F} = \dot{\vec{L}}$$

$$\Rightarrow \frac{Gm_1m_2}{r^2}\hat{r} = m_1\vec{a}_1$$

$$\Rightarrow \vec{a}_1 = \frac{Gm_2}{r^2}\hat{r}$$

FBD: m_1

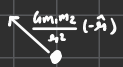
Apply LMB on FBD of m_2,

$$\sum \vec{F} = \dot{\vec{L}}$$

$$\Rightarrow \frac{Gm_1m_2}{r^2}(-\hat{r}) = m_2\vec{a}_2$$

$$\Rightarrow \vec{a}_2 = \frac{-Gm_1\hat{r}}{r^2}$$

FBD: m_2



To find time period,

Assume motion to be circular:

$$\Rightarrow F_g = \underbrace{F_c}_{\text{centripial force}}$$

$$\Rightarrow \frac{Gm_1m_2}{d^2} = m_2\omega^2\frac{m_1}{m_1+m_2}d$$

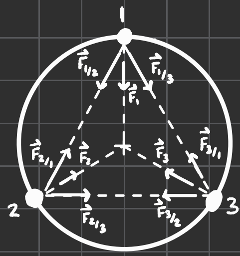
$$\Rightarrow \omega^2 = \frac{G(m_1+m_2)}{d^3}$$

$$\Rightarrow \omega = \sqrt{\frac{G(m_1+m_2)}{d^3}} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{d^3}{G(m_1+m_2)}}}$$

For three particles,

Assume position of particles form an equilateral triangle,

Sketch:



$\Rightarrow$ Effective force of gravity acts through the center of the circle.

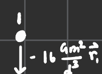
$$\Rightarrow \vec{F}_1 = 2F_g\,\hat{r} = \frac{16Gm^2}{d^3}\vec{r}_1$$

$$\Rightarrow \vec{F}_2 = \frac{16Gm^2}{d^3}\vec{r}_2$$

$$\Rightarrow \vec{F}_3 = \frac{16Gm^2}{d^3}\vec{r}_3$$

Apply LMB on FBD 1,

FBD: 1



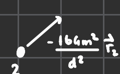
$$\sum \vec{F} = \dot{\vec{L}}$$

$$\Rightarrow \frac{-16Gm^2\vec{r}_1}{d^3} = m\vec{a}_1$$

$$\Rightarrow \vec{a}_1 = \frac{-16Gm}{d^3}\vec{r}_1$$

Apply LMB on FBD 2,

FBD : 2



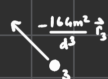
$$\sum \vec{F} = \dot{\vec{L}}$$

$$\Rightarrow \frac{-16Gm^2\vec{r}_2}{d^3} = m\vec{a}_2$$

$$\Rightarrow \vec{a}_2 = \frac{-16Gm}{d^3}\vec{r}_2$$

Apply LMB on FBD 2,

FBD : 3



$$\sum \vec{F} = \dot{\vec{L}}$$

$$\Rightarrow \frac{-16Gm^2\vec{r}_3}{d^3} = m\vec{a}_3$$

$$\Rightarrow \vec{a}_3 = \frac{-16Gm}{d^3}\vec{r}_3$$

From $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$, in general:

$$\ddot{x} = \frac{-16Gm}{d^3} x$$

$$\ddot{y} = \frac{-16Gm}{d^3} y$$

$$\Rightarrow x = c_1 e^{i\sqrt{\frac{16Gm}{d^3}}t}$$

$$y = c_2 e^{i\sqrt{\frac{16Gm}{d^3}}t}$$

$$\Rightarrow \omega = \sqrt{\frac{16Gm}{d^3}}$$

$$\Rightarrow \boxed{T = 2\pi\sqrt{\frac{d^3}{16Gm}}}$$

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Homework-6

Problem 18 **Due** : 25 - Feb - 2025 **Time Spent** : 2 Hours

(a) $$\sum \vec{v}_i \cdot \vec{F}_i = \frac{d}{dt} \sum E_{Ki}$$

$$\Rightarrow \sum \vec{v}_i \cdot (\vec{F}_i^{int} + \vec{F}_i^{ext}) = \frac{d}{dt} \sum E_{Ki}$$

$$= \frac{1}{2} \frac{d}{dt} \sum m_i \vec{v}_i \cdot \vec{v}_i$$

$$E_{Ktot} = \sum \frac{1}{2} m_i \vec{v}_i \cdot \vec{v}_i$$

$$= \frac{1}{2} \sum m_i (\vec{v}_{i/G} + \vec{v}_{G/O}) \cdot (\vec{v}_{i/G} + \vec{v}_{G/O})$$

avg velocity of a stationary point

$$= \frac{1}{2} \sum m_i [\vec{v}_{i/G} \cdot \vec{v}_{i/G} + \cancel{2\vec{v}_{i/G} \cdot \vec{v}_G} + \vec{v}_G \cdot \vec{v}_G]$$

$$\boxed{E_{Ktot} = \frac{1}{2} m_{tot} \vec{v}_G \cdot \vec{v}_G + \frac{1}{2} \sum m_i \vec{v}_{i/G} \cdot \vec{v}_{i/G}}$$

(b) $$\sum \vec{F}_i \cdot \vec{v}_i = \frac{d}{dt} \sum E_{Ki}$$

$$\Rightarrow \left(\sum \vec{F}_i^{int} \cdot \vec{v}_i + \sum \vec{F}_i^{ext} \cdot \vec{v}_i\right) = \frac{d}{dt}\left(\frac{1}{2} m_{tot} \vec{v}_G \cdot \vec{v}_G + \frac{1}{2} \sum m_i \vec{v}_{i/G} \cdot \vec{v}_{i/G}\right)$$

$$\Rightarrow \sum \vec{F}_i^{int} \cdot (\vec{v}_{i/G} + \vec{v}_G) + \sum \vec{F}_i^{ext} \cdot (\vec{v}_{i/G} + \vec{v}_G) = \frac{d}{dt}\left(\frac{1}{2} m_{tot} \vec{v}_G \cdot \vec{v}_G + \frac{1}{2} \sum m_i \vec{v}_{i/G} \cdot \vec{v}_{i/G}\right)$$

$$\Rightarrow \cancelto{0}{\left(\sum \vec{F}_i^{int}\right)} \cdot \vec{v}_G + \sum \vec{F}_i^{int} \cdot \vec{v}_{i/G} + \sum \vec{F}_i^{ext} \cdot \vec{v}_{i/G} + \left(\sum \vec{F}_i^{ext}\right) \cdot \vec{v}_G = \frac{d}{dt}\left(\frac{1}{2} m_{tot} \vec{v}_G \cdot \vec{v}_G + \frac{1}{2} \sum m_i \vec{v}_{i/G} \cdot \vec{v}_{i/G}\right)$$

Cases for the condition to be satisfied:

1) $\sum \vec{F}_i^{int} \cdot \vec{v}_{i/G} + \sum \vec{F}_i^{ext} \cdot \vec{v}_{i/G} = \frac{d}{dt}\left(\frac{1}{2}\sum m_i \vec{v}_{i/G} \cdot \vec{v}_{i/G}\right)$

$\Rightarrow \cancelto{0}{\sum \vec{F}_i^{int}} + \sum \vec{F}_i^{ext} = \sum \vec{m}_i \vec{a}_{i/G} \cdot \vec{v}_{i/G}$

This is satisfied iff $\vec{a}_G = \vec{0}$.

<u>OR</u>

2) $\sum \vec{F}_i^{ext} \cdot \vec{v}_{i/G} = 0$ and $\frac{d}{dt}\left(\frac{1}{2}\sum m_i \vec{v}_{i/G} \cdot \vec{v}_{i/G}\right) = 0$ and $\sum \vec{F}_i^{int} \cdot \vec{v}_{i/G} = 0$

(a) (i) $\vec{F}_i^{ext}$ is $\perp$ $\vec{v}_{i/G}$ $\forall\ i$

OR

(ii) $\vec{F}_i^{ext} = \vec{0}$ $\forall\ i$

OR

(iii) $\vec{v}_{i/G} = \vec{0}$ $\forall\ i$

(b) (i) $\vec{v}_{i/G}$ is not changing $\forall\ i$

OR

(ii) $\vec{v}_{i/G} = \vec{0}$

(c) (i) $\vec{F}_i^{int}$ is $\perp$ $\vec{v}_{i/G}$ $\forall\ i$

OR

$\vec{F}_i^{int} = \vec{0}$ $\forall\ i$

OR

$\vec{v}_{i/G} = \vec{0}$ $\forall\ i$

$\boxed{\therefore \text{ Either 1 } \underline{\text{OR}} \text{ any condition in 2(a) } \underline{\text{AND}} \text{ 2(b) } \underline{\text{AND}} \text{ 2(c) must be satisfied.}}$

(c) As derived in part (b),

$$\sum \vec{F}_i^{int} \cdot \vec{v}_{i/G} + \sum \vec{F}_i^{ext} \cdot \vec{v}_{i/G} = \frac{d}{dt}\left(\frac{1}{2}\sum m_i \vec{v}_{i/G} \cdot \vec{v}_{i/G}\right)$$

The power of internal forces being equal to $\frac{d}{dt}\left(\frac{1}{2}\sum m_i \vec{v}_{i/G} \cdot \vec{v}_{i/G}\right)$ is satisfied when,

$$\boxed{\begin{array}{ll} \text{(i)} & \vec{F}_i^{ext} \text{ is } \perp \ \vec{v}_{i/G} \quad \forall\ i \\ & \underline{\text{OR}} \\ \text{(ii)} & \vec{F}_i^{ext} = \vec{0} \quad \forall\ i \\ & \underline{\text{OR}} \\ \text{(iii)} & \vec{v}_{i/G} = \vec{0} \quad \forall\ i \end{array}}$$

RO2102: Dynamics and Simulation

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Homework-6

Problem 19 **Due**: 25-Feb-2025 **Time Spent**: 1 Hour 15 Minutes

For one particle,

$$\vec{F} = m\vec{a}$$

$$\Rightarrow \vec{r}_{P/c} \times \vec{F} = m\,\vec{r}_{P/c} \times a$$

For many particles,

$$\Sigma \vec{F}_i = \Sigma m_i \vec{a}_i$$

$$\Rightarrow \Sigma \vec{r}_{i/c} \times \vec{F}_i = \Sigma \vec{r}_{i/c} \times m_i \vec{a}_i$$

Guess: $\frac{d}{dt}\vec{H}_{/c} = \Sigma \vec{r}_{i/c} \times \vec{F}_i$

Candidate Def^n's of $\vec{H}_{P/c}$: (a) $\vec{H}_{/c} = \Sigma \vec{r}_{i/c'} \times \vec{v}_{i/c'} m_i$

where, C' is a fixed point coinciding with C.

(b) $\vec{H}_{/c} = \Sigma \vec{r}_{i/c} \times \vec{v}_{i/o} m_i$

(c) $\vec{H}_{/c} = \Sigma \vec{r}_{i/c} \times \vec{v}_{i/c} m_i$

Try Def^n (a),

$$\frac{d}{dt}\vec{H}_{/c} = \frac{d}{dt}\sum \vec{r}_{i/c'} \times \vec{v}_{i/c'} m_i = \sum \frac{d}{dt}(\vec{r}_{i/c'} \times \vec{v}_{i/c'}) m_i$$

$$= \sum \frac{d}{dt}\left[(\vec{r}_i - \vec{r}_{c'}) \times (\vec{v}_i - \vec{v}_{c'}) m_i\right] = \sum (\vec{r}_i - \vec{r}_{c'}) \times (\vec{a}_i - \cancelto{0}{\vec{a}_{c'}}) m_i + \cancelto{0}{(\vec{v}_i - \vec{v}_{c'}) \times (\vec{v}_i - \vec{v}_{c'}) m_i}$$

$$= \sum (\vec{r}_i - \vec{r}_{c'}) \times \vec{a}_i m_i \qquad = \sum (\vec{r}_i - \vec{r}_{c'}) \times \vec{F}_i$$

$$= \sum \vec{r}_{i/c'} \times \vec{F}_i \qquad = \sum \vec{r}_{i/c} \times \vec{F}_i$$

$\therefore$ (a) is a good definition.

Try Def^n (b),

$$\frac{d}{dt} \vec{H}_{/c} = \frac{d}{dt} \sum \vec{r}_{i/c} \times \vec{v}_{i/o} m_i = \sum \frac{d}{dt} \left[\vec{r}_{i/c} \times \vec{v}_{i/o} \right] m_i$$

$$= \sum \frac{d}{dt} \left[(\vec{r}_i - \vec{r}_c) \times (\vec{v}_i - \cancelto{0}{\vec{v}_o}) m_i \right] = \sum \left[(\vec{r}_i - \vec{r}_c) \times \vec{a}_i m_i + (\vec{v}_i - \vec{v}_c) \times \vec{v}_i m_i \right]$$

$$= \sum \vec{r}_{i/c} \times \vec{a}_i m_i + \vec{v}_c \times \vec{v}_i m_i$$

$\therefore$ (b) is only a good def^n when $\vec{v}_c = \vec{0}$ or $\vec{v}_i = \vec{0} \; \forall i$ or $\vec{v}_c \perp \vec{v}_i \; \forall i$

Try Def^n (c),

$$\frac{d}{dt} \vec{H}_{/c} = \frac{d}{dt} \sum \vec{r}_{i/c} \times \vec{v}_{i/c} m_i = \sum \frac{d}{dt} \left[\vec{r}_{i/c} \times \vec{v}_{i/c} m_i \right]$$

$$= \sum \frac{d}{dt} (\vec{r}_i - \vec{r}_c) \times (\vec{v}_i - \vec{v}_c) m_i = \sum m_i \vec{r}_{i/c} \times (\vec{a}_i - \vec{a}_c) + \cancelto{0}{(\vec{v}_i - \vec{v}_c) \times (\vec{v}_i - \vec{v}_c)} m_i$$

$$= \sum \vec{r}_{i/c} \times m_i \vec{a}_i + \sum \vec{r}_{i/c} \times m_i \vec{a}_c$$

$\therefore$ (c) is only good when

- $\rightarrow$ $\vec{r}_{i/c} \perp \vec{a}_c \; \forall i$: all particles lie on a surface perpendicular to $\vec{a}_c$
- $\rightarrow$ $\vec{a}_c = \vec{0}$: C is fixed
- $\rightarrow$ $\vec{r}_{i/c} = \vec{0} \; \forall i$: All particles are concentrated at C
- $\rightarrow$ $\sum m_i \vec{r}_{i/c} = 0$: C is COM