

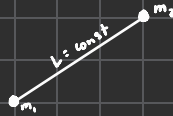
Homework-7

Problem 25

Due : 22 - Mar - 2025

Time Spent : 45 Minutes

Sketch:



Given: $m_1 = m_2 = m$, L , $g = 0$

To Find: 5 DAEs

Apply LMB, mass m_1

$$\Rightarrow \sum \vec{F} = m \ddot{\vec{z}}$$

$$\sum \vec{F} \cdot \hat{i} \Rightarrow T \cos \theta = m \ddot{x}_1 \quad (1)$$

$$\sum \vec{F} \cdot \hat{j} \Rightarrow T \sin \theta = m \ddot{y}_1 \quad (2)$$

Apply LMB, Mass m_2

$$\Rightarrow \sum \vec{F} = m \ddot{\vec{z}}$$

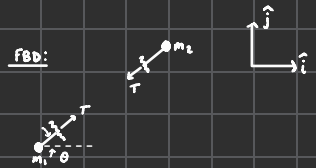
$$\sum \vec{F} \cdot \hat{i} \Rightarrow -T \cos \theta = m \ddot{x}_2 \quad (3)$$

$$\sum \vec{F} \cdot \hat{j} \Rightarrow -T \sin \theta = m \ddot{y}_2 \quad (4)$$

Apply Constraint,

$$\text{Define, } x = x_2 - x_1$$

$$y = y_2 - y_1$$



$$\{ \dot{x}^2 + \dot{y}^2 = L = \text{const.} \}$$

$$\Rightarrow \frac{d}{dt} \xi \Rightarrow x \dot{x} + y \dot{y} = 0$$

$$\Rightarrow \frac{d}{dt} \left[\frac{d}{dt} \xi \right] \Rightarrow \dot{x}^2 + x \ddot{x} + \dot{y}^2 + y \ddot{y} = 0$$

$$\Rightarrow x \ddot{x} + y \ddot{y} = -(\dot{x}^2 + \dot{y}^2)$$

$$\Rightarrow (x_1 - x_2)(\ddot{x}_1 - \ddot{x}_2) + (y_1 - y_2)(\ddot{y}_1 - \ddot{y}_2) = -((\dot{x}_1 - \dot{x}_2)^2 + (\dot{y}_1 - \dot{y}_2)^2)$$

$$\Rightarrow -(x_1 - x_2)\ddot{x}_1 + (x_1 - x_2)\ddot{x}_2 - (y_1 - y_2)\ddot{y}_1 + (y_1 - y_2)\ddot{y}_2 + 0T = -((\dot{x}_1 - \dot{x}_2)^2 + (\dot{y}_1 - \dot{y}_2)^2) \quad (5)$$

$$\begin{bmatrix} m & 0 & 0 & 0 & \frac{-x}{\sqrt{x^2+y^2}} \\ 0 & m & 0 & 0 & \frac{-y}{\sqrt{x^2+y^2}} \\ 0 & 0 & m & 0 & \frac{x}{\sqrt{x^2+y^2}} \\ 0 & 0 & 0 & m & \frac{y}{\sqrt{x^2+y^2}} \\ -x & x & -y & y & 0 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \\ \ddot{x}_2 \\ \ddot{y}_2 \\ T \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -(\dot{x}^2 + \dot{y}^2) \end{bmatrix}$$

$$A w = b$$

$$w = A \backslash b$$

$$\ddot{x}_1 = w(1)$$

$$\ddot{y}_1 = w(2)$$

$$\ddot{x}_2 = w(3)$$

$$\ddot{y}_2 = w(4)$$