RO2102: Dynamics and Simulation

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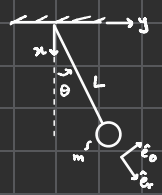
Homework-7

Problem 23 Due : 22-Mar-2025 Time Spent : 45 Minutes

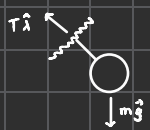
Sketch:



Given: m, L, g

To Find: $\ddot{\theta}$

FBD:



Apply LMB,

$$\sum \vec{F} = m\vec{a}$$

$$\Rightarrow -T\hat{e}_r + mg\hat{\imath} = m(\ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath})$$

Constraint : $\{x^2 + y^2 = L = \text{const.}\}$

$$\Rightarrow \frac{d}{dt}\{\} \Rightarrow x\dot{x} + y\dot{y} = 0$$

$$\Rightarrow \frac{d}{dt}\left[\frac{d}{dt}\{\}\right] \Rightarrow \dot{x}^2 + x\ddot{x} + \dot{y}^2 + y\ddot{y} = 0$$

$$\Rightarrow x\ddot{x} + y\ddot{y} = -(\dot{x}^2 + \dot{y}^2)$$

$z = [x, y, \dot{x}, \dot{y}]'$

$$\hat{\imath} \cdot \{\sum \vec{F} = m\vec{a}\} \Rightarrow m\ddot{x} + 0\ddot{y} + \cos\theta T = mg$$

$$\hat{\jmath} \cdot \{\sum \vec{F} = m\vec{a}\} \Rightarrow 0\ddot{x} + m\ddot{y} + \sin\theta T = 0$$

$$x\ddot{x} + y\ddot{y} + 0T = -\dot{x}^2 - \dot{y}^2$$

$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}, \quad \sin\theta = \frac{y}{\sqrt{x^2+y^2}}$$

$$\begin{bmatrix} m & 0 & \frac{x}{\sqrt{x^2+y^2}} \\ 0 & m & \frac{y}{\sqrt{x^2+y^2}} \\ x & y & 0 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ T \end{bmatrix} = \begin{bmatrix} mg \\ 0 \\ -(\dot{x}^2+\dot{y}^2) \end{bmatrix}$$

$$\Rightarrow \quad A w = b$$

$$\Rightarrow \quad w = A \backslash b$$

$$\ddot{x} = w(1)$$

$$\ddot{y} = w(2)$$