RO2102: Dynamics and Simulation

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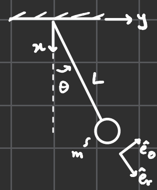
Homework-7

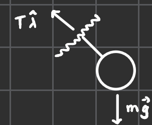
Problem 22 Due : 22-Mar-2025 Time Spent : 1 Hour 30 Min

Sketch:



Given: m, L, g

FBD:



To Find: $\ddot{\theta}$ using :
(a) LMB
(b) LMB, dot with $\hat{e}_\theta$
(c) LMB, cross with $\vec{r}$
(d) AMB
(e) Conservation of Energy
(f) Power Balance
(g) Lagrange Equations

(a) $\sum \vec{F} = m\vec{a}$

$\Rightarrow \{ -T\hat{e}_r - mg\hat{j} = m(L\ddot{\theta}\hat{e}_\theta - L\dot{\theta}^2\hat{e}_r) \}$

$\{\} \cdot \hat{i}$

$\Rightarrow -T\cos\theta = m(-\sin\theta L\ddot{\theta} - L\dot{\theta}^2\cos\theta)$

$\Rightarrow T = m(L\dot{\theta}^2 + L\ddot{\theta}\tan\theta)$

$\{\} \cdot \hat{j}$

$\Rightarrow -T\sin\theta - mg = m(L\ddot{\theta}\cos\theta + L\dot{\theta}^2\sin\theta)$

$$\Rightarrow -m(L\ddot{\theta}\tan\theta + L\dot{\theta}^2)\sin\theta - mg = m(L\ddot{\theta}\cos\theta - L\dot{\theta}^2\sin\theta)$$

$$\Rightarrow \cancel{-L\dot{\theta}^2\sin\theta} - g = L\ddot{\theta}\left(\cos\theta + \frac{\sin^2\theta}{\cos\theta}\right) \cancel{- L\dot{\theta}^2\sin\theta}$$

$$\Rightarrow \boxed{\ddot{\theta} = \frac{-g}{L}\sin\theta}$$

(b) $\Sigma\vec{F} = m\vec{a}$

$$\Rightarrow \{-T\hat{e}_r - mg\hat{j} = m(L\ddot{\theta}\hat{e}_\theta - L\dot{\theta}^2\hat{e}_r)\} \cdot \hat{e}_\theta$$

$$\Rightarrow -mg\sin\theta = mL\ddot{\theta}$$

$$\Rightarrow \boxed{\ddot{\theta} = -\frac{g}{L}\sin\theta}$$

(c) $\vec{r} \times \{\Sigma\vec{F} = m\vec{a}\}$

$$\Rightarrow \{L\hat{e}_r \times (-mg\hat{i}) = L\hat{e}_r \times m(L\ddot{\theta}\hat{e}_\theta - L\dot{\theta}^2\hat{e}_r)\} \cdot \hat{k}$$

$$\Rightarrow -Lmg\sin\theta = mL^2\ddot{\theta}$$

$$\Rightarrow \boxed{\ddot{\theta} = -\frac{g}{L}\sin\theta}$$

(d) $\Sigma\vec{M}_{/O} = \vec{r} \times m\vec{a}$

$$\Rightarrow \vec{r} \times mg\hat{i} = L\hat{e}_r \times m(L\ddot{\theta}\hat{e}_\theta - L\dot{\theta}^2\hat{e}_r)$$

$$\Rightarrow \{L\hat{e}_r \times mg\hat{i} = mL^2\ddot{\theta}\hat{k}\} \cdot \hat{k}$$

$$\Rightarrow -mgL\sin\theta = mL^2\ddot{\theta}$$

$$\Rightarrow \boxed{\ddot{\theta} = -\frac{g}{L}\sin\theta}$$

(e) $E_k + E_p = \text{const}$

$$\left\{ \frac{1}{2} m (L\dot{\theta})^2 - mgL\cos\theta = \text{const} \right\}$$

$$\frac{d}{dt}\{\} = mL^2\ddot{\theta}\dot{\theta} + mgL\sin\theta\,\dot{\theta} = 0$$

$$\Rightarrow \boxed{\ddot{\theta} = -\frac{g}{L}\sin\theta} \quad [\text{Iff } \dot{\theta} = 0]$$

(f) $P = \dot{E}_k$

$$\Rightarrow \vec{v}\cdot\vec{F} = \frac{d}{dt}(mL^2\dot{\theta}^2)$$

$$\Rightarrow L\dot{\theta}\hat{e}_\theta \cdot mg\hat{i} = mL^2\dot{\theta}\ddot{\theta}$$

$$\Rightarrow -mgL\dot{\theta}\sin\theta = mL^2\dot{\theta}\ddot{\theta}$$

$$\Rightarrow \ddot{\theta}\dot{\theta} = -\frac{g}{L}(\sin\theta)\dot{\theta}$$

$$\overset{\dot{\theta}=0}{\Rightarrow} \boxed{\ddot{\theta} = -\frac{g}{L}\sin(\theta)}$$

(g) $\mathcal{L} = E_k - E_p$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) - \frac{\partial \mathcal{L}}{\partial \theta} = 0$$

$$E_k = \frac{1}{2} m\ell^2\dot{\theta}^2$$

$$E_p = -\ell\cos\theta \cdot mg$$

$$\mathcal{L} = \frac{1}{2} m (L\dot{\theta})^2 - [-mgL\cos\theta]$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = -mgL\sin\theta$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = mL\dot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) - \frac{\partial \mathcal{L}}{\partial \theta} = 0$$

$$\Rightarrow \quad mL\ddot{\theta} + mgL\sin\theta = 0$$

$$\Rightarrow \quad \boxed{\ddot{\theta} = \frac{-g}{L}\sin\theta}$$