RO2102: Dynamics and Simulation

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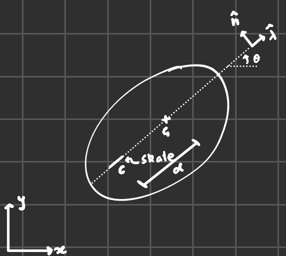
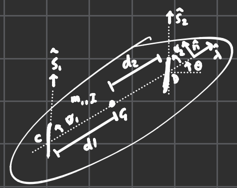
Final - Project

Problem 39 Due : 15 - May - 2025 **Time Spent** : 5 Hours

Sketch: Single Skate



Sketch: Two Skates



To Find: EoM using : (a) Single Skate

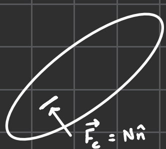
(b) Two Skates

(c) Two Skates using Full Lagrange

(d) Simple Solution

(a) Use DAE

FBD:



LMB:

$$\Sigma \vec{F} = m\vec{a}$$

$$\Rightarrow \{ N\hat{n} = m(\ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}) \}$$

$\{\} \cdot \hat{\imath}$

$$\Rightarrow m\ddot{x} = -N\sin\theta \quad \text{——— (i)}$$

$\{\} \cdot \hat{\jmath}$

$$\Rightarrow m\ddot{y} = N\cos\theta \quad \text{——— (ii)}$$

AMB

$$\left\{ \sum \vec{M}_{/G} = I\dot{\omega}\hat{k} \right\}$$

$$\{\} \cdot \hat{k}$$

$$\Rightarrow \quad -Nd = I\dot{\omega} \quad \text{———} \quad (iii)$$

3 Eq^{n}'s 4 Unknowns : $\ddot{x}$, $\ddot{y}$, $\dot{\omega}$, N

Constraint:

$$\vec{V}_c \cdot \hat{n} = 0$$

$$\Rightarrow (\vec{V}_G + \vec{V}_{c/G}) \cdot \hat{n} = 0$$

$$\Rightarrow [(\dot{x}_G\hat{i} + \dot{y}_G\hat{j}) + (-d\dot{\theta}\hat{n})] \cdot \hat{n} = 0$$

$$\Rightarrow \{-\sin\theta\,\dot{x}_G + \cos\theta\,\dot{y}_G - d\dot{\theta} = 0\}$$

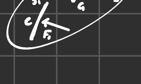
$$\frac{d}{dt}\{\} \quad \Rightarrow \quad \ddot{y}\cos\theta - \dot{\theta}(\dot{x}\cos\theta - \dot{y}\sin\theta) - d\ddot{\theta} - \ddot{x}\sin\theta = 0 \quad \text{————} \quad (iv)$$

$$\underbrace{\begin{bmatrix} m & 0 & 0 & s\theta \\ 0 & m & 0 & -c\theta \\ 0 & 0 & I & d \\ -\sin\theta & \cos\theta & \ddot{\theta} & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \dot{\omega} \\ N \end{bmatrix}}_{\omega} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \dot{\theta}(\dot{x}\cos\theta - \dot{y}\sin\theta) \end{bmatrix}}_{b}$$

$$\omega = A\backslash b$$

(b) Apply LMB,

FBD:



$$F_1\hat{f}_1 + F_2\hat{f}_2 = m\vec{a}_G$$

$$F_1(-\sin(\theta+\phi_1)\hat{i} + \cos(\theta+\phi_2)\hat{j}) + F_2(-\sin(\theta+\phi_2)\hat{i} + \cos(\theta+\phi_3)\hat{j}) = m\vec{a}_G \quad \text{———} \quad (1)$$

Apply $AMB_{/G}$,

$$\sum \vec{M}_{/G} = (-d_1 \hat{\lambda} \times F_1 \hat{f}_1) + d_2 \hat{\lambda} \times F_2 \hat{f}_2 = I \ddot{\theta} \hat{k} \quad \text{—— (2)}$$

① and ② are 3 eq^{n}'s in 5 unknowns : $\ddot{x}_G, \ddot{y}_G, \ddot{\theta}, F_1, F_2$

Constraints : $\vec{V}_C \cdot \hat{f}_1 = 0 \quad *$

$\vec{V}_D \cdot \hat{f}_2 = 0 \quad **$

$$\vec{V}_C = \vec{V}_G + \vec{V}_{C/G}$$
$$= \vec{V}_G + \dot{\theta}\hat{k} \times \vec{r}_{C/G}$$
$$= \dot{x}_G \hat{i} + \dot{y}_G \hat{j} + \dot{\theta}\hat{k} \times -d_1 \hat{\lambda}$$

$$\vec{V}_D = \dot{x}_G \hat{i} + \dot{y}_G \hat{j} + \dot{\theta}\hat{k} \times d_2 \hat{\lambda}$$
$$= \dot{x}_G \hat{i} + \dot{y}_G \hat{j} + d_2 \dot{\theta} \hat{n}$$

(c) Minimal Co-ordinates: x, y, θ

Full Lagrange Eq^n:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i$$

where, Q_i : $dW = Q_i \, \delta q_i$

$$Q_x = -(F_1 \sin(\theta + \phi_1) + F_2 \sin(\theta + \phi_2))$$

$$Q_y = F_1 \cos(\theta + \phi_1) + F_2 \sin(\theta + \phi_2)$$

$$Q_\theta = F_1 d_1 + F_2 d_2$$

Constraint : $V_C \cdot \hat{f}_1 = 0$

$$\Rightarrow \{ (\dot{x}\hat{i} + \dot{y}\hat{j} + (\dot{\theta}\hat{k} \times -d_1 \hat{\lambda})) \cdot \hat{f}_1 \} = 0$$

$$\Rightarrow \quad \frac{d}{dt}\{\} = 0$$

$$\vec{V}_D \cdot \hat{j}_2 = 0$$

$$\Rightarrow \quad \{ (\dot{x}\,\hat{i} + \dot{y}\,\hat{j} + (\dot{\theta}\hat{k} \times d_2\hat{\lambda})) \cdot \hat{j}_2 \} = 0$$

$$\Rightarrow \quad \frac{d}{dt}\{\} = 0$$

(d)