RO2102: Dynamics and Simulation

Tanay Srinivasa

U20220086

Homework-2

Problem 4 Due : 24 - Jan-2025 Time Spent : 3·5 Hours

(a) The cross product can be thought of as :

→ Project b onto a vector orthogonal to $\bar{a}$ in the plane defined by $\bar{a},\bar{b}$.

→ Scale Projection by $|\bar{a}|$

→ Rotate the scaled orthogonal vector positively about the axis defined by a by an angle $\pi/2$.

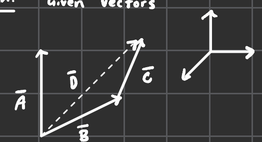
Given : $\bar{a}, \bar{b}, \bar{c}$

To Prove: $\bar{a} \times (\bar{b} + \bar{c}) = (\bar{a} \times \bar{b}) + (\bar{a} \times \bar{c})$

For proof, refer to MATLAB Script.

Aternatively,

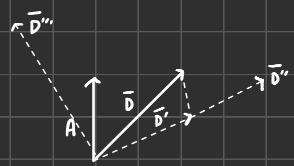
Sketch: Given Vectors



Define : $\bar{d} = \bar{b} + \bar{c}$

$\bar{d}' = |\bar{d}| \sin\theta_{ad} \, \hat{a}_{\perp d}$

Sketch : $\bar{a} \times \bar{d}$



$\bar{d}'' = |\bar{a}||\bar{d}| \sin\theta_{ad} \, \hat{a}_{\perp d}$

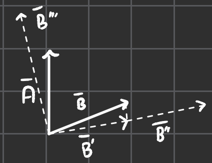
$\bar{d}''' = |\bar{a}||\bar{d}| \sin\theta_{ad} \, \hat{n}_{ad}$

$$\bar{b}' = |\bar{b}| \sin\theta_{ab} \, \hat{a}_{\perp b}$$

$$\bar{b}'' = |\bar{a}||\bar{b}| \sin\theta_{ab} \, \hat{a}_{\perp b}$$

$$\bar{b}''' = |\bar{a}||\bar{b}| \sin\theta_{ab} \, \hat{n}_{ab}$$

Sketch: $\bar{a} \times \bar{b}$

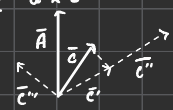


$$\bar{c}' = |\bar{c}| \sin\theta_{ac} \, \hat{a}_{\perp c}$$

$$\bar{c}'' = |\bar{a}||\bar{c}| \sin\theta_{ac} \, \hat{a}_{\perp c}$$

$$\bar{c}''' = |\bar{a}||\bar{c}| \sin\theta_{ac} \, \hat{n}_{ac}$$

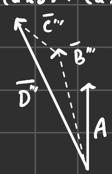
Sketch: $\bar{a} \times \bar{c}$



$$\bar{a} \times \bar{d} = \bar{d}'''$$

$$(\bar{a} \times \bar{b}) + (\bar{a} \times \bar{c}) = \bar{a}''' + \bar{d}'''$$

Sketch: $(\bar{a} \times \bar{b}) + (\bar{a} \times \bar{c})$



(b) $\bar{a} = [a_x \quad a_y \quad a_z]'$

$\bar{b} = [b_x \quad b_y \quad b_z]'$

For X-Y Plane,

$$\bar{a} \times \bar{b} = a_x b_x \sin(0)\bar{0} + a_y b_y \sin(0)\bar{0} + a_x b_y \sin 90\ \hat{k} + a_y b_x \sin(-90)\hat{k}$$

$$= (a_x b_y - a_y b_x)\ \hat{k}$$

For Y-Z Plane,

$$\bar{a} \times \bar{b} = a_y b_y \sin(0)\bar{0} + a_z b_z \sin(0)\bar{0} + a_y b_z \sin(90)\hat{i} + a_z b_y \sin(-90)\hat{i}$$

$$= (a_y b_z - a_z b_y)\hat{i}$$

For X-Z plane,

$$\bar{a} \times \bar{b} = a_x b_x \sin(0)\bar{0} + a_z b_z \sin(0)\bar{0} + a_x b_z \sin(-90)\hat{j} + a_z b_x \sin 90(\hat{j})$$

$$= (a_z b_x - a_x b_z)\hat{j}$$

In 3-D,

$$\boxed{\bar{a} \times \bar{b} = (a_y b_z - a_z b_y)\hat{i} + (a_z b_x - a_x b_z)\hat{j} + (a_x b_y - a_y b_x)\ \hat{k}}$$