

Final - Project

Problem 41

Due : 15 - May - 2025

Time Spent : 1 Hour

Given:  $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ ,  $a = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ ,  $b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

To Find: (a)  $c = a_i b_i$  (b)  $c = \delta_{ii}$  (c)  $c_i = A_{ij} b_j$  (d)  $c_i = b_j A_{ij}$   
 (e)  $c_i = A_{ji} b_j$  (f)  $c = A_{ij} B_{ij}$  (g)  $c = A_{ij} \delta_{ij}$  (h)  $c_k = B_{jn} b_n$   
 (i)  $c_i = A_{ij} B_{jk}$  (j)  $c_{ik} = A_{ij} B_{jk}$  (k)  $\delta_{ij} \delta_{jk} \delta_{ki} \delta_{ii}$  (l)  $c_k = \epsilon_{ijk} a_i b_j$

(a)  $c = a_i b_i = 1+0+2 = 3$

(b)  $c = \delta_{ii} = 1+1+1 = 3$

(c)  $c_i = A_{ij} b_j$

$[c] = [A][b]$   
 $= [6 \ 15 \ 24]'$

(d)  $c_i = b_j A_{ij}$

$= A_{ij} b_j$

$[c] = [6 \ 15 \ 24]'$

(e)  $c_i = A_{ji} b_j$

$[c] = [A^T][b]$  or  $[b^T][A]$   
 $= [12 \ 15 \ 18]'$   
 or  
 $[12 \ 15 \ 18]$

(f)  $c = A_{ij} B_{ij}$   
 $= \sum_i \sum_j a_{ij} b_{ij}$   
 $= 26$

(g)  $c = A_{ij} \delta_{ij}$   
 $= a_{11} + a_{22} + a_{33}$   
 $= 15$

(h)  $c_k = B_{jn} b_n$   
 Does not make sense,  
 $j$  is the free index

(i)  $c_{ik} = A_{ij} B_{jk}$   
 Does not make sense  
 $k$  is also a free index.

(j)  $c_{ik} = A_{ij} B_{jk}$   
 $[c] = [A] \times [B]$   
 $= \begin{bmatrix} 4 & 0 & 6 \\ 10 & 0 & 15 \\ 16 & 0 & 24 \end{bmatrix}$

(k)  $\delta_{ij} \delta_{jk} \delta_{ki} \delta_{ii}$   
 $= \delta_{ij} \delta_{ji} \delta_{ii}$   
 $= \delta_{ii} \delta_{ii}$   
 $= 3 \times 3$   
 $= 9$

(l)  $c_k = \epsilon_{ijk} a_i b_j$   
 $[c] = \vec{a} \times \vec{b}$