RO2102: Dynamics and Simulation

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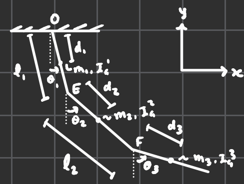
Final - Project

Problem 43   **Due** : 15 - May - 2025   **Time Spent** : 4 Hours

Sketch:



Given: $m_1, I_G^1, m_2, I_G^2, d_1, d_2, \ell_1, \ell_2, \vec{z}(0)$

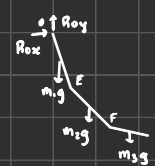
To Find: a) EoM for triple pendulum using : i) N-E Minimal

ii) Lagrange Equations

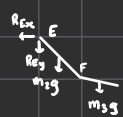
iii) DAE

b) EoM for four-bar linkage

FBD: system



FBD: Link 2-3



a) i) Apply $AMB_{/O}$ on FBD system

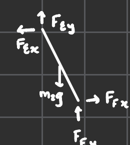
Apply $AMB_{/E}$ on FBD link 2-3

Apply $AMB_{/F}$ on FBD link 3

ii) $$E_K = \frac{1}{2} m_1 (\vec{v}_1 \cdot \vec{v}_1) + \frac{1}{2} m_2 (\vec{v}_2 \cdot \vec{v}_2) + \frac{1}{2} m_3 (\vec{v}_3 \cdot \vec{v}_3) + \frac{1}{2} I_G^1 \dot{\theta}_1^2 + \frac{1}{2} I_G^2 \dot{\theta}_2^2 + \frac{1}{2} I_G^3 \dot{\theta}_3^2$$

FBD: Link 1

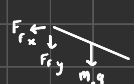
FBD: Link 2



$$E_P = m_1 g (\vec{r}_{G1/O} \cdot \hat{j}) + m_2 g (\vec{r}_{G2/O} \cdot \hat{j}) + m_3 g (\vec{r}_{G3/O} \cdot \hat{j})$$

$$\mathcal{L} = E_K - E_P$$

FBD: Link 3



Choose minimal co-ordinates: θ_1, θ_2

Solve: $$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} = 0$$

iii) Apply LMB and AMB$_{/G1}$ on FBD link1.

Apply LMB and AMB$_{/G2}$ on FBD link2.

Apply LMB and AMB$_{/G3}$ on FBD link3.

Use constraints:

$$\vec{V}_O = \vec{V}_O$$

$$\Rightarrow \vec{V}_{O/G1} + \vec{V}_{G1/\mathcal{I}} - \vec{V}_{O/\mathcal{I}} = \vec{0}$$

$$\Rightarrow \{ -d_1 \dot{\theta}_1 \hat{e}_{\theta_1} + \dot{x}_1 \hat{i} + \dot{y}_1 \hat{j} - \vec{0} = \vec{0} \}$$

$$\frac{d}{dt}\{\} \Rightarrow -d_1 \ddot{\theta}_1 \hat{e}_{\theta_1} + d_1 \dot{\theta}_1^2 \hat{e}_{r_1} + \ddot{x}_1 \hat{i} + \ddot{y}_1 \hat{j} = \vec{0}$$

$$\vec{V}_E = \vec{V}_E$$

$$\Rightarrow \vec{V}_{E/G2} + \vec{V}_{G2/\mathcal{I}} - \vec{V}_{E/\mathcal{I}} = \vec{0}$$

$$\Rightarrow \{ -d_2 \dot{\theta}_2 \hat{e}_{\theta_2} + \dot{x}_2 \hat{i} + \dot{y}_2 \hat{j} - (l_1 \dot{\theta}_1 \hat{e}_{\theta_1}) = \vec{0} \}$$

$$\frac{d}{dt}\{\} \Rightarrow \ddot{x}_2 \hat{i} + \ddot{y}_2 \hat{j} + d_2 \dot{\theta}_2^2 \hat{e}_{r_2} + l_1 \dot{\theta}_1^2 \hat{e}_{r_1} - d_2 \ddot{\theta}_2 \hat{e}_{\theta_2} - l_1 \ddot{\theta}_1 \hat{e}_{\theta_1} = \vec{0}$$

$$\vec{V}_F = \vec{V}_F$$

$$\Rightarrow \vec{V}_{F/G3} + \vec{V}_{G3/\mathcal{I}} - \vec{V}_{F/\mathcal{I}} = \vec{0}$$

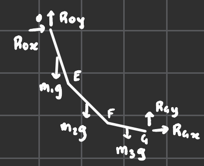
$$\Rightarrow \{ d_3 \dot{\theta}_3 \hat{e}_{\theta_3} + \dot{x}_3 \hat{i} + \dot{y}_3 \hat{j} - (l_1 \dot{\theta}_1 \hat{e}_{\theta_1} + l_2 \dot{\theta}_2 \hat{e}_{\theta_2}) = \vec{0} \}$$

$$\frac{d}{dt}\{\} \Rightarrow \ddot{x}_3 \hat{i} + \ddot{y}_3 \hat{j} + l_1 \dot{\theta}_1^2 \hat{e}_{r_1} + l_2 \dot{\theta}_2^2 \hat{e}_{r_2} + d_3 \dot{\theta}_3^2 \hat{e}_{r_3} - l_1 \ddot{\theta}_1 \hat{e}_{\theta_1} - l_2 \ddot{\theta}_2 \hat{e}_{\theta_2} - d_3 \ddot{\theta}_3 \hat{e}_{\theta_3} = \vec{0}$$

With the 9 dynamics equations and 6 constraints, solve.

b) For a 4-bar linkage, the free-end of link 3 is constrained.

FBD: system



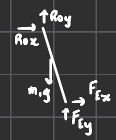
Choose DAE Approach to solve.

Apply LMB and $AMB_{/G_1}$ on FBD link 1.

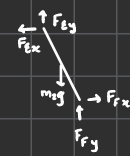
Apply LMB and $AMB_{/G_2}$ on FBD link 2.

Apply LMB and $AMB_{/G_3}$ on FBD link 3.

FBD: Link 1



FBD: Link 2

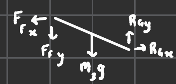


Use constraints as in part (a) with additional

$$\vec{v}_G = \vec{v}_G$$

$$\Rightarrow \vec{v}_{G/G_3} + \vec{v}_{G_3/I} - \vec{v}_{G/I} = \vec{0}$$

FBD: Link 3



$$\Rightarrow \{(l_3-d_3)\dot{\theta}_3\hat{e}_{\theta_3} + \dot{x}_3\hat{i} + \dot{y}_3\hat{j} - \vec{0} = \vec{0}\}$$

$$\frac{d}{dt}\{\} \Rightarrow \ddot{x}_3\hat{i} + \ddot{y}_3\hat{j} - (l_3-d_3)\dot{\theta}_3^2\hat{e}_{r_3} + (l_3-d_3)\ddot{\theta}_3\hat{e}_{r_3} = \vec{0}$$

With the additional 2 constraints, the two extra reactions of G can be found.