RO2102: Dynamics and Simulation

Tanay Srinivasa
U20220086

Homework-5

Problem 16 **Due : 18-Feb-2025** **Time Spent : 2Hours 15Min**

Apply LMB on FBD of m_1,

$$\sum \vec{F} = \dot{\vec{L}}$$

$$\Rightarrow \frac{Gm_1m_2}{s_1^2}\hat{s_1} = m_1\vec{a}_1$$

$$\Rightarrow \vec{a}_1 = \frac{Gm_2}{s_1^2}\hat{s_1}$$

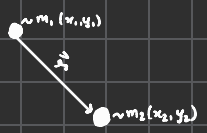
Apply LMB on FBD of m_2,

$$\sum \vec{F} = \dot{\vec{L}}$$

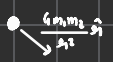
$$\Rightarrow \frac{Gm_1m_2}{s_1^2}(-\hat{s_1}) = m_2\vec{a}_2$$

$$\Rightarrow \vec{a}_2 = -\frac{Gm_1\hat{s_1}}{s_1^2}$$

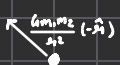
Sketch:



FBD: m_1



FBD: m_2



To find time period,

Assume motion to be circular:

$$\Rightarrow F_g = F_c \quad \text{(centripital force)}$$

$$\Rightarrow \frac{Gm_1m_2}{s_1^2} = m_2\omega^2 s_1 \qquad (\because m_2 > m_1)$$

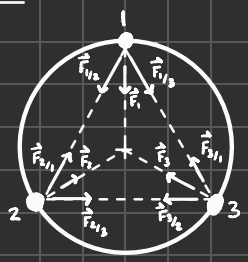
$$\Rightarrow \omega^2 = \frac{Gm_1}{s_1^3}$$

$$\Rightarrow \omega = \sqrt{\frac{Gm_1}{s_1^3}} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{r^3}{Gm_1}}}$$

For three particles,

Assume position of particles form an equilateral triangle,

Sketch:



$\Rightarrow$ Effective force of gravity acts through the center of the circle.

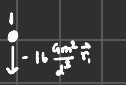
$$\Rightarrow \vec{F}_1 = 2F_3\hat{r} = \frac{16Gm^2}{d^3}\vec{r}_1$$

$$\Rightarrow \vec{F}_2 = \frac{16Gm^2}{d^3}\vec{r}_2$$

$$\Rightarrow \vec{F}_3 = \frac{16Gm^2}{d^3}\vec{r}_3$$

Apply LMB on FBD 1,

FBD: 1



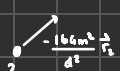
$$\Sigma \vec{F} = \dot{\vec{L}}$$

$$\Rightarrow \frac{-16Gm^2}{d^3}\vec{r}_1 = m\vec{a}_1$$

$$\Rightarrow \vec{a}_1 = \frac{-16Gm}{d^2}\vec{r}_1$$

Apply LMB on FBD 2,

FBD: 2



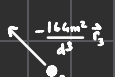
$$\Sigma \vec{F} = \dot{\vec{L}}$$

$$\Rightarrow \frac{-16Gm^2}{d^3}\vec{r}_2 = m\vec{a}_2$$

$$\Rightarrow \vec{a}_2 = \frac{-16Gm}{d^2}\vec{r}_2$$

Apply LMB on FBD 2,

FBD: 3



$$\Sigma \vec{F} = \dot{\vec{L}}$$

$$\Rightarrow \frac{-16Gm^2}{d^3}\vec{r}_3 = m\vec{a}_3$$

$$\Rightarrow \vec{a}_3 = \frac{-16Gm}{d^3}\vec{r}_3$$

From $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$, in general:

$$\ddot{x} = \frac{-16Gm}{d^3}x$$

$$\ddot{y} = \frac{-16Gm}{d^3}y$$

$$\Rightarrow \quad x = c_1 e^{i\sqrt{\frac{16Gm}{d^3}}t}$$

$$y = c_2 e^{i\sqrt{\frac{16Gm}{d^3}}t}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{16Gm}{d^3}}$$

$$\Rightarrow \quad \boxed{T = 2\pi\sqrt{\frac{d^3}{16Gm}}}$$