RO2102: Dynamics and Simulation

Tanay Srinivasa
U20220086

Homework-5

Problem 15 Due : 18-Feb-2025 Time Spent : 30 Min

$$\vec{F} = -\vec{\nabla} E_p$$

$$\Rightarrow E_p = -\int \vec{F} \cdot d\vec{r}$$

(a) $\vec{F} = -mg\hat{j}$

$$d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$E_p = -\int \vec{F} \cdot d\vec{r} = +\int mg\,dy$$

$$= mgy$$

(b) $\vec{F} = -kr\hat{e}_r$

$$d\vec{r} = dr\hat{e}_r + r\,d\theta\hat{e}_\theta + r\sin\theta\,d\phi\hat{e}_\phi$$

$$E_p = -\int \vec{F} \cdot d\vec{r} = \int kr\,dr$$

$$= \frac{kr^2}{2}$$

(c) $\vec{F} = -k(l-l_0)\hat{e}_r = -k(r-l_0)\hat{e}_r$

$$d\vec{r} = dr\hat{e}_r + r\,d\theta\hat{e}_\theta + r\sin\theta\,d\phi\hat{e}_\phi$$

$$E_p = -\int \vec{F} \cdot d\vec{r} = \int k(r-l_0)\,dr$$

$$= \frac{kr^2}{2} - kl_0 r = k\left(\frac{l^2}{2} - l_0 l\right)$$

(d) $\vec{F} = \frac{-C}{r^2}\hat{e}_r$

$E_p = -\int \vec{F}\cdot d\vec{r} = \int \frac{C}{r^2} dr$

$= \frac{-C}{r}$

(e) $\vec{F} = -f(r)\hat{e}_r$

$E_p = -\int \vec{F}(r)\cdot d\vec{r}' = \int_{r_0}^{r} f(r')\,dr'$

Where r_0 is chosen s.t the integral is not divergent.

(f) $\vec{F} = -k(x_2 - x_1 - l_0)\hat{x}$

$E_p = -\int \vec{F}\cdot d\vec{r} = \int k(x_2 - x_1 - l_0)\,dx$

$= \frac{kx^2}{2} - kl_0 x = \frac{kl_{12}^2}{2} - kl_0 l_{12}$

(g) $\vec{F} = \frac{-C}{x^2}\hat{\lambda}$

$E_p = -\int \vec{F}\cdot d\vec{r} = \int_{l_1}^{l_2} \frac{C}{x^2} dx$

$= \frac{-C}{l_{12}}$

(h) $\vec{F} = -C\hat{\lambda}$

$E_p = -\int \vec{F}\cdot d\vec{r} = \int C\hat{\lambda}\cdot d\vec{r}$

$= C\hat{\lambda}\cdot\vec{r}$

(i) $\vec{F} = -f(x)\hat{i}$

$E_p = -\int \vec{F}\cdot d\vec{r} = \int_{x_0}^{x} f(x')\,dx'$

Where x_0 is chosen s.t the integral is not divergent.