Phase 2 - Lecture 3

Some problems and elementary stochastic calculus

What we derived in last class:

$$S_{i+1} = S_i \left(1 + \mu\,\delta t + \sigma \phi\,\delta t^{1/2} \right).$$

Problem

The current value of the stock of company ABC is Rs. 100. The risk free return stands at 5% pa and the volatility is at 30%pa. Simulate the behavior of the stock for next 4 days.

Take $\varepsilon_{1,2,3,4}$ = 0.4423,-0.1170,0.0291,0.6872

Problem - Do on python

A share has an expected return of 12% per annum (with continuous compounding) and a volatility of 20% per annum. Changes in the share price satisfy $dS = \mu S\, dt + \sigma S\, dX$. Simulate the movement of the share price, currently $100, over a year, using a time interval of one week.

Elementary Stochastic Calculus

A coin example

Toss a coin. Every time you throw a head I give you $1, every time you throw a tail you give me $1

E[Ri] = 0, E[Ri^2] = 1 and E[RiRj] = 0

$$S_i = \sum_{j=1}^{i} R_j. \quad E[S_i] = 0 \quad \text{and} \quad E[S_i^2] = E[R_1^2 + 2R_1R_2 + \cdots] = i.$$

Markov Property and Martingale

The expected value of the random variable Si conditional upon all of the past events only depends on the previous value Si-1. This is the Markov property.

The conditional expectation of your winnings at any time in the future is just the amount you already hold - Martingale

Quadratic Variation

$$\sum_{j=1}^{i} (S_j - S_{j-1})^2 = i.$$

Brownian Motion

I am going to change the rules of my coin-tossing experiment. First of all I am going to restrict the time allowed for the six tosses to a period t, so each toss will take a time t/6. Second, the size of the bet will not be $1 but (t/6)^0.5.

$$\sum_{j=1}^{6}(S_j - S_{j-1})^2 = 6 \times \left(\sqrt{\frac{t}{6}}\right)^2 = t.$$

Brownian Motion

Properties:

Finitiness, Continuity, Markov, Martingale, Quadratic Variation, Normality