Phase 2 - Lecture 2
CTE

$$dS = \mu S \, dt + \sigma S \, dX.$$

Wall Street Club

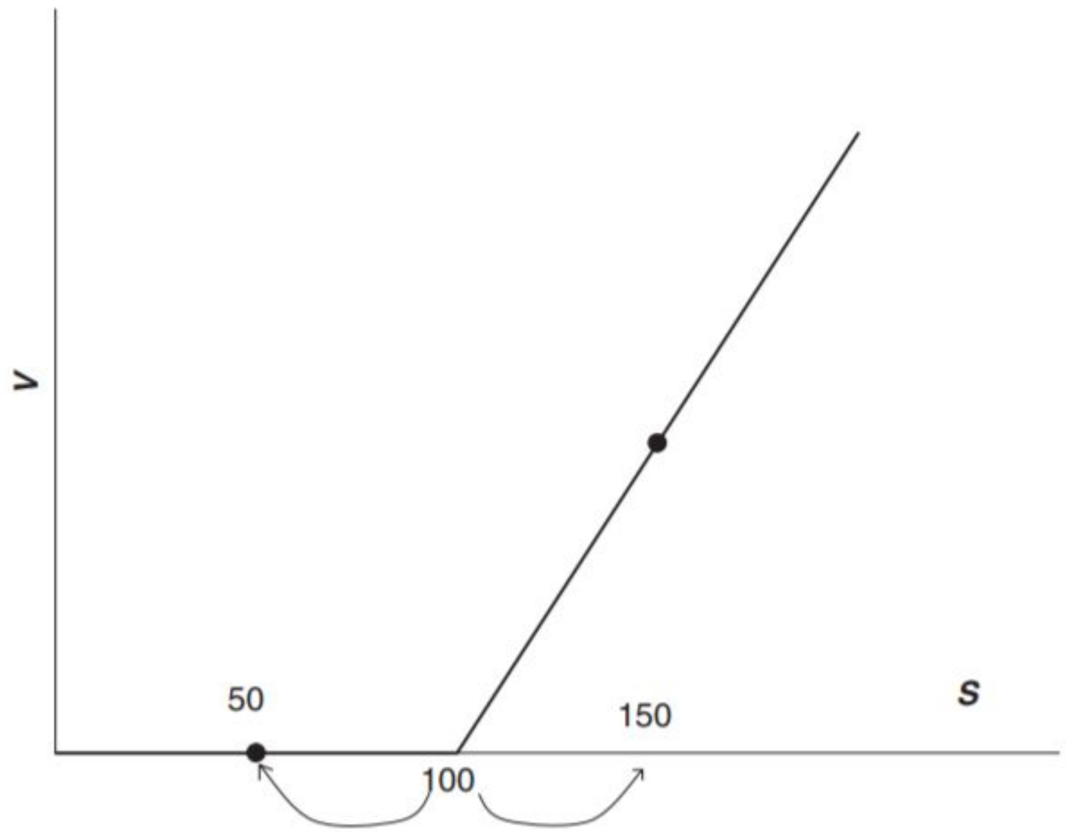

Figure 4.1 Future scenarios.

Modelling returns

$$\frac{S_{i+1} - S_i}{S_i} = R_i.$$

Returns

$$\overline{R} = \frac{1}{M}\sum_{i=1}^{M} R_i$$

Returns' mean

$$\sqrt{\frac{1}{M-1}\sum_{i=1}^{M}(R_i - \overline{R})^2},$$

Returns' standard deviation

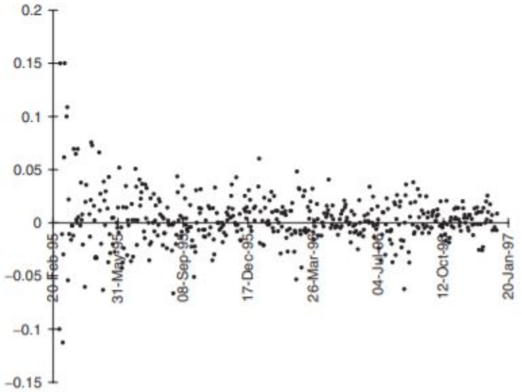

Figure 4.4 Daily returns of Perez Companc.

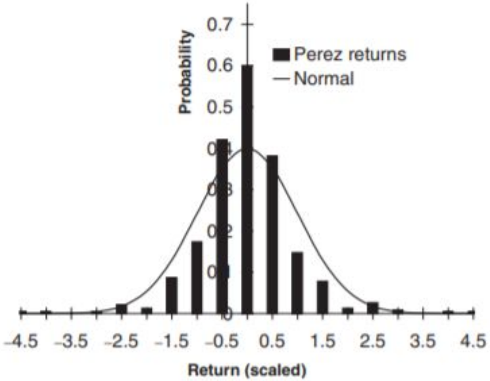

Figure 4.5 Normalized frequency distribution of Perez Companc and the standardized Normal distribution.

$$R_i = \frac{S_{i+1} - S_i}{S_i} = \text{mean} + \text{standard deviation} \times \phi.$$

Modelling using timescales

$$\text{mean} = \mu\,\delta t,$$

$$\frac{S_{i+1} - S_i}{S_i} = \mu\,\delta t.$$

$$S_M = S_0\,(1 + \mu\,\delta t)^M = S_0 e^{M \log(1+\mu\,\delta t)} \approx S_0 e^{\mu M\,\delta t} = S_0 e^{\mu T}.$$

Standard deviation

$$\sqrt{\frac{1}{M-1}\sum_{i=1}^{M}(R_i - \overline{R})^2},$$

In order for the standard deviation to remain finite as we let δt tend to zero, the individual terms in the expression must each be of O(δt). Since each term is a square of a return, the standard deviation of the asset return over a time step δt must be O(δt 1/2):

$$\text{standard deviation} = \sigma\ \delta t^{1/2},$$

Final discretized equation that I get

$$S_{i+1} - S_i = \mu S_i \,\delta t + \sigma S_i \phi \,\delta t^{1/2}.$$