

Assignment 3

Tanay Yadav - AI20BTECH11026

Download the python codes from

<https://github.com/tanayyadav28/Assignments/blob/main/Assignment%203/code/assignment3.py>

and latex-tikz codes from

<https://github.com/tanayyadav28/Assignments/blob/main/Assignment%203/assignment3.tex>

1 PROBLEM

(GATE EC, Q. 25) A fair coin is tossed till a head appears for the first time. The probability that the number of required tosses is odd, is

- (A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

2 SOLUTION

Let X be the Bernoulli random variable such that $X = \{0, 1\}$ denotes the outcome of the given experiment.

$X = 0$ denotes the outcome that odd number of tries are required to get the first head.

$X = 1$ denotes all the other outcomes.

Probability that head appears on the fair coin is $\frac{1}{2}$.

$$\begin{aligned} \Pr(X = 0) &= \frac{1}{2} + \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \\ &\quad + \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \\ &\quad + \dots \end{aligned} \quad (2.0.1)$$

Hence, $\Pr(X = 0)$ is an infinite sum of a Geometric Progression with $a = \frac{1}{2}$ and $r = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$.

$$\Pr(X = 0) = \frac{\frac{1}{2}}{1 - \frac{1}{4}} \quad (2.0.2)$$

$$\Pr(X = 0) = \frac{\left(\frac{1}{2}\right)}{\left(\frac{3}{4}\right)} \quad (2.0.3)$$

$$\Pr(X = 0) = \frac{1}{2} \times \frac{4}{3} \quad (2.0.4)$$

$$\therefore \Pr(X = 0) = \frac{2}{3} \quad (2.0.5)$$