SNR Coverage Probability Analysis of RIS-Aided Communication Systems

Tanay Yadav - AI20BTECH11026

May 17, 2021

Important Terminology

Re-configurable Intelligent Surface (RIS)

A re-configurable intelligent surface (RIS) is a two-dimensional surface of engineered material whose properties are re-configurable rather than static. The scattering, absorption, reflection, and diffraction properties can be changed with time and controlled by software.

Coverage Probability

The coverage probability of a technique for calculating a confidence interval is the proportion of the time that the interval contains the true value of interest

Fading Channel

A fading channel is a communication channel that experiences fading, i.e. variation of the attenuation of a signal with various variables

Important Terminology

Rayleigh Fading

Rayleigh fading is a statistical model for the effect of a propagation environment on a radio signal, such as that used by wireless devices.

Signal to Noise ratio (SNR)

Signal-to-noise ratio is a measure used in science and engineering that compares the level of a desired signal to the level of background noise. SNR is defined as the ratio of signal power to the noise power, often expressed in decibels.

TS and SR links

TS Link: Link between the signal transmitter and the RIS.
SR Link: Link between the RIS and the signal receiver.

System Model

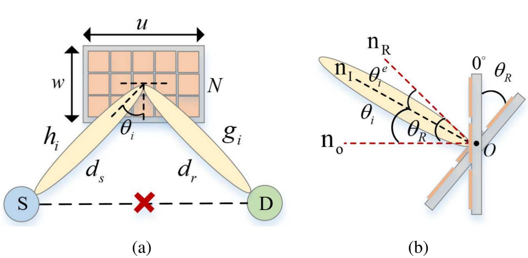

Figure: System Model, (a) an RIS-aided communication system with N elements, (b) an illustration of the rotation of RIS plane with an angle of θ_R.

1. Here, we consider the far-field transmission for the transceivers, which means $d_s, d_r \geq \dfrac{2(\max\{u, w\})^2}{\lambda}$ where, λ is the wavelength.
2. The angle of incidence is denoted as $\theta_i = \langle n_O, n_I \rangle$ and the angle of rotation as $\theta_R = \langle n_O, n_R \rangle$.

System Model

(A) Channel Model

1. The realistic large-scale path loss model used in the far-field is:

$$P_l(d_s, d_r, \theta_i^e) = \frac{G_s G_r}{(4\pi)^2} \left(\frac{uw}{d_s d_r}\right)^2 \cos(\theta_i^e) \tag{1}$$

where, G_s and G_r are the antenna gains of transmitter and receiver, respectively. Also, $\theta_i^e = |\theta_i - \theta_R|$

2. The amplitudes of small-scale fading of TS and SR links are defined as α_i and β_i, where i is the index of the element in the RIS. They both follow Rayleigh distribution with PDF given by:

$$f_\alpha(\alpha) = \left(\frac{\alpha}{\sigma^2}\right) \exp\left(-\frac{\alpha^2}{2\sigma^2}\right) \tag{2}$$

where, σ represents the fading coefficient of the channel.

System Model

(B) Signal to Noise Ratio

1. Assuming quasi-static Rayleigh Fading of the TS and SR links, the received signal is expressed as:

$$y = \sqrt{P_s P_l}\left[\sum_{i=1}^{N} h_i \chi_i g_i\right] x + n_0 \tag{3}$$

Here,

1. x is transmit signal with power of P_s.
2. $\chi_i = \varrho_i(\phi_i) e^{(j\phi_i)}$ is the reflection coefficient produced by the i^{th} element.
3. $\varrho_i(\phi_i) = 1, \forall i = 1, 2, \ldots, N$ for ideal condition.
4. $h_i = \alpha_i e^{-j\vartheta_i}$ and $g_i = \beta_i e^{-j\varphi_i}$ are the channel gains.
5. n_0 is additive white Gaussian noise following $N(0, \sigma_n^2)$.

System Model

2. The Signal to Noise Ratio is now given as:

$$\gamma = \frac{\left|\sum_{i=1}^{N} \alpha_i \beta_i e^{j(\phi_i - \vartheta_i - \varphi_i)}\right|^2 P_s G_s G_r (uw \times \cos(\theta_i^e))^2}{(4\pi \sigma_n d_s d_r)^2} \tag{4}$$

3. To obtain the maximum value of γ, the phase shift considered is $\phi_i = \vartheta_i + \varphi_i$

$$\therefore \gamma_{max} = \frac{A^2 \bar{\gamma}}{d_s^2 d_r^2} \tag{5}$$

where,

1. $A = \sum_{i=1}^{N} \alpha_i \beta_i$
2. Average SNR is: $\bar{\gamma} = \frac{P_s G_s G_r (uw \times \cos(\theta_i^e))^2}{16\pi^2 \sigma_n^2}$, here $uw \times \cos(\theta_i^e)$ represents the total effective area of the beam on the RIS from source.

SNR Coverage Probability Analysis

Here, we first determine the distribution of A^2 for $N = 1$ and $N \geq 1$ and the **SNR Coverage Probability** is obtained with the CDF of A^2

(A) SNR Coverage Probability for N = 1

1. PDF of $\alpha\beta$: α and β are two Independent and Identically Distributed(i.i.d.) Rayleigh Random Variables with fading coefficients σ_1 and σ_2. The PDF of $\eta = \alpha\beta$ is given as:

$$f_\eta(\eta) = \frac{\eta}{a^2}\mathcal{K}_0\left(\frac{\eta}{a}\right) \tag{6}$$

   here, $a = \sigma_1\sigma_2$ and $\mathcal{K}_0(.)$ is the zeroth order of modified Bessel function of the second kind.

2. CDF of $\alpha\beta$:

$$F_\eta(\eta) = 1 - \frac{\eta}{a}\mathcal{K}_1\left(\frac{\eta}{a}\right) \tag{7}$$

SNR Coverage Probability Analysis

here, $\mathcal{K}_1(.)$ is the first order of modified Bessel function of the second kind.
We can also obtain the CDF of η^2 by $F_{\eta^2}(\eta) = F_\eta(\sqrt{\eta})$ and thus, derive the SNR Coverage Probability for $N = 1$.

3. The SNR coverage probability is defined as the probability that the SNR is large than a specific threshold, which can be expressed as:

$$\begin{aligned} P_{cov}(\gamma_{th}) &= Pr(\gamma \geq \gamma_{th}) \quad (8) \\ &= 1 - Pr(\gamma \leq \gamma_{th}) \quad (9) \\ &= 1 - F_{\eta^2}\left(\frac{\gamma_{th}}{\bar{\gamma}} d_s^2 d_r^2\right) \quad (10) \end{aligned}$$

$$\therefore P_{cov}(\gamma_{th}) = \frac{d_s d_r}{\sigma_1 \sigma_2}\sqrt{\frac{\gamma_{th}}{\bar{\gamma}}}\mathcal{K}_1\left(\frac{d_s d_r}{\sigma_1 \sigma_2}\sqrt{\frac{\gamma_{th}}{\bar{\gamma}}}\right) \quad (11)$$

SNR Coverage Probability Analysis

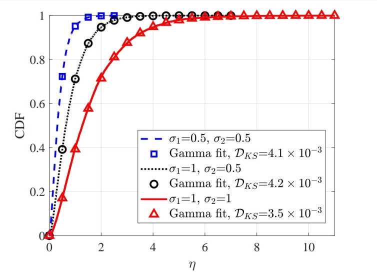

Figure: Actual CDFs of η with Gamma fits for different fading coefficients.

(B) Approximating SNR Coverage Probability for Arbitrary N

The distribution of η is in accordance to K-distribution $K(b, \nu)$ with $b = a = \sigma_1\sigma_2$ and $\nu = 0$. The K-distribution is complex and intractable. So, we use a Gamma distribution to approximate it.

SNR Coverage Probability Analysis

1. The distribution of η can be approximated as Gamma distribution:

$$\eta \sim Ga\left(\frac{\pi^2}{16-\pi^2}, \frac{16-\pi^2}{2\pi}\sigma_1\sigma_2\right) \tag{12}$$

   where,

   1. $Ga(k, \theta)$ represents Gamma distribution,
   2. $k = \dfrac{\pi^2}{16-\pi^2}$ is the shape parameter,
   3. $\theta = \dfrac{16-\pi^2}{2\pi}\sigma_1\sigma_2$ is the scale parameter,

2. The statistic $\mathcal{D}_{KS}$ is used to assess the accuracy of approximation as it finds the maximum divergence of 2 CDFs.

$$\mathcal{D}_{KS} = \max|F_{\text{approx}}(t) - F_{\text{actual}}(t)| \tag{13}$$

3. The Gamma function is denoted as $\Gamma(.)$. and it is highly accurate as the magnitude of $\mathcal{D}_{KS}$ is around 10^{-3}.

SNR Coverage Probability Analysis

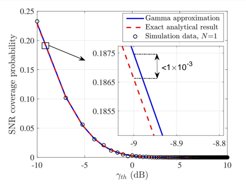

Figure: Results of the exact analysis, Gamma approximation, and simulation

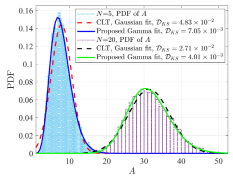

Figure: Comparison of proposed Gamma and CLT-based Gaussian fitting

SNR Coverage Probability Analysis

4. Now, for N elements in the RIS, $A \sim Ga(Nk, \theta)$. For A^2 a generalized gamma distribution is obtained with the parameters $p = 1$, $d = \frac{Nk}{2}$ and $q = \theta^2$ and the CDF is expressed as:

$$F_{A^2}\left(z, \frac{1}{2}, \frac{Nk}{2}, \theta^2\right) = \frac{\zeta(Nk, \frac{\sqrt{z}}{\theta})}{\Gamma(Nk)} \tag{14}$$

   here $\zeta(.,.)$ denotes lower incomplete gamma function.

5. For arbitrary N, given the threshold γ_{th}, the general form of SNR coverage probability is expressed as:

$$P_{cov}(\gamma_{th}) = \frac{\Gamma(Nk, s)}{\Gamma(Nk)} \tag{15}$$

   here, $s = \frac{d_s d_r}{\theta}\left(\frac{\gamma_{th}}{\bar{\gamma}}\right)^{\frac{1}{2}}$ and $\Gamma(.,.)$ is upper incomplete gamma function. Also, $\zeta(Nk, s) = \Gamma(Nk) - \Gamma(Nk, s)$

SNR Coverage Probability Analysis

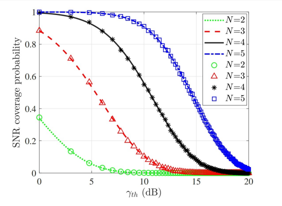

Figure: Results of Gamma approximations (lines) and simulations (markers).

(C) Optimal number of elements of RIS

For infinite N, $P_{cov} = 1$, but we need to have a optimal(minimum) number of RIS elements.

$$N^* = min\left\{N|\Gamma(Nk, s) = \Gamma(Nk)\right\}, \tag{16}$$

where N^* is optimal number of RIS elements and $N \in \mathbb{N}$

Simulation Results

It is seen that both the exact result and Gamma approximation are perfectly consistent with the simulation result.

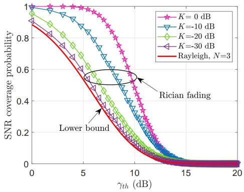

Figure: SNR coverage probability under Rayleigh vs. Rician fading.

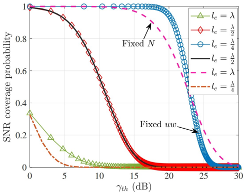

Figure: SNR coverage probability vs. different sizes of elements.

Simulation Results

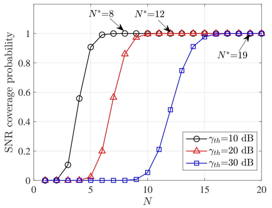

Figure: SNR coverage probability vs. N for different SNR thresholds.

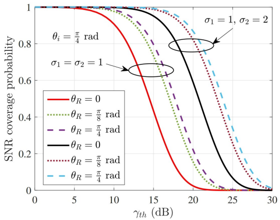

Figure: SNR coverage probability vs. fading coefficients and incident angles.