

Assignment 3

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Download the python codes from

<https://github.com/tanayyadav28/Assignments/blob/main/Assignment%203/code/assignment3.py>

and latex-tikz codes from

<https://github.com/tanayyadav28/Assignments/blob/main/Assignment%203/assignment3.tex>

1 PROBLEM

(GATE EC, Q. 25) A fair coin is tossed till a head appears for the first time. The probability that the number of required tosses is odd, is

- (A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

2 SOLUTION

Let X be the Bernoulli random variable such that $X \sim B(1, 0)$ denotes the outcome of the given experiment.

$X = 0$ denotes the outcome that odd number of tries are required to get the first head.

$X = 1$ denotes all the other outcomes.

Probability that head appears on the fair coin is $\frac{1}{2}$.

Probability that a head appears on an odd try is

$$\left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)^{(2n+1)-1}$$

$$\therefore \Pr(X = 0) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right) \times \left(\frac{1}{4}\right)^n \quad (2.0.1)$$

$$\Pr(X = 0) = \frac{\frac{1}{2}}{1 - \frac{1}{4}} \quad (2.0.2)$$

$$\Pr(X = 0) = \frac{\left(\frac{1}{2}\right)}{\left(\frac{3}{4}\right)} \quad (2.0.3)$$

$$\Pr(X = 0) = \frac{1}{2} \times \frac{4}{3} \quad (2.0.4)$$

$$\therefore \Pr(X = 0) = \frac{2}{3} \quad (2.0.5)$$

Hence, the correct option is (C).