Assignment 3

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Download the python codes from

https://github.com/tanayyadav28/Assignments/blob/main/Assignment%203/code/assignment3.py

and latex-tikz codes from

https://github.com/tanayyadav28/Assignments/blob/main/Assignment%203/assignment3.tex

1 Problem

(GATE EC, Q. 25) A fair coin is tossed till a head appears for the first time. The probability that the number of requried tosses is odd,is
(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

2 Solution

Let $Y \in \mathbb{N}$ denote the number of times the experiment is performed. $Y = k$ represents $k-1$ failures before getting 1 success. $p = \frac{1}{2}$

$$\therefore p_Y(k) = (1-p)^{k-1} \times p \tag{2.0.1}$$

For getting first success on an odd try, $k = 2n+1$. Let X be the Bernoulli Random Variable $X \sim B(1,0)$. $X = 1$ denotes the outcome that odd number of tries are required to get the first head, $X = 0$ denotes all other outcomes. The probability $p_Y(k)$ is mutually exclusive for all odd tries. Hence, by the rule of sum,

$$\Pr(X=1) = \sum_{n=0}^{\infty} p_Y(2n+1) \tag{2.0.2}$$

$$\Pr(X=1) = \sum_{n=0}^{\infty} (1-p)^{2n} \times p \tag{2.0.3}$$

$$\Pr(X=1) = \frac{p}{1-(p^2-2p+1)} \tag{2.0.4}$$

$$\Pr(X=1) = \frac{1}{2-p} \tag{2.0.5}$$

$$\therefore \Pr(X=1) = \frac{2}{3} \tag{2.0.6}$$

Hence, the correct option is (C).