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Assignment 3

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Download the python codes from

https://github.com/tanayyadav28/Assignments/blob/main/Assignment%203/code/assignment3.py

and latex-tikz codes from

https://github.com/tanayyadav28/Assignments/blob/main/Assignment%203/assignment3.tex

1 Problem

(GATE EC, Q. 25) A fair coin is tossed till a head appears for the first time. The probability that the number of requried tosses is odd,is

(A)
$$\frac{1}{3}$$

(B)
$$\frac{1}{2}$$

(C)
$$\frac{2}{3}$$

(D)
$$\frac{3}{4}$$

2 Solution

Let X be the Bernoulli random variable such that $X \sim B(1,0)$ denotes the outcome of the given experiment.

X = 0 denotes the outcome that odd number of tries are required to get the first head.

X = 1 denotes all the other outcomes.

Probability that head appears on the fair coin is $\frac{1}{2}$. Probability that a head appears on an odd try is

$$\left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)^{(2n+1)-1}$$

$$\therefore \Pr(X=0) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right) \times \left(\frac{1}{4}\right)^n \tag{2.0.1}$$

$$\Pr\left(X=0\right) = \frac{\frac{1}{2}}{1-\frac{1}{4}} \tag{2.0.2}$$

$$\Pr(X = 0) = \frac{\left(\frac{1}{2}\right)}{\left(\frac{3}{4}\right)}$$
 (2.0.3)

$$\Pr(X = 0) = \frac{1}{2} \times \frac{4}{3}$$
 (2.0.4)

$$\therefore \Pr(X = 0) = \frac{2}{3} \tag{2.0.5}$$

Hence, the correct option is (C).

