Assignment 4

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Download all the latex-tikz codes from

https://github.com/tanayyadav28/Assignments/blob/main/Assignment%204/assignment4.tex

1 Problem

Let X_1, X_2 and X_3 be independent and identically distributed random variables with $E(X_1) = 0$ and $E\left(X_1^2\right) = \frac{15}{4}$. If $\psi : (0, \infty) \to (0, \infty)$ is defined through the conditional expectation

$$\psi(t) = E\left(X_1^2 | X_1^2 + X_2^2 + X_3^2 = t\right), t > 0$$

Then, $E(\psi((X_1 + X_2)^2))$ is equal to,

2 Solution

It is given that X_1, X_2 and X_3 are independent and identically distributed random variables.

$$\begin{aligned} E\left(X_1^2 | X_1^2 + X_2^2 + X_3^2 = t\right) &= E\left(X_2^2 | X_1^2 + X_2^2 + X_3^2 = t\right) \\ &= E\left(X_3^2 | X_1^2 + X_2^2 + X_3^2 = t\right) \end{aligned} \tag{2.0.1}$$

Now,

$$\sum_{n=1}^{3} E\left(X_n^2 | X_1^2 + X_2^2 + X_3^2 = t\right)$$

$$= E\left(X_1^2 + X_2^2 + X_3^2 | X_1^2 + X_2^2 + X_3^2 = t\right) \tag{2.0.2}$$

$$= t \tag{2.0.3}$$

Hence, from (2.0.1).

$$E\left(X_1^2 | X_1^2 + X_2^2 + X_3^2 = t\right) = \frac{t}{3} \tag{2.0.4}$$

$$\therefore \psi(t) = \frac{t}{3} \tag{2.0.5}$$

Hence, from (2.0.5),

$$E(\psi((X_1 + X_2)^2)) = E\left(\frac{(X_1 + X_2)^2)}{3}\right) \tag{2.0.6}$$

$$= E\left(\frac{X_1^2 + X_2^2 + 2X_1 \times X_2}{3}\right) \tag{2.0.7}$$

$$= \frac{E(X_1^2) + E(X_2^2) + 2 \times E(X_1) \times E(X_2)}{3} \tag{2.0.8}$$

$$= \frac{\frac{15}{4} + \frac{15}{4} + 2 \times 0 \times 0}{3} \tag{2.0.9}$$

$$= \frac{15}{6} \tag{2.0.10}$$

$$\therefore E(\psi((X_1 + X_2)^2)) = 2.5 \tag{2.0.11}$$