

# Assignment 3

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Download the python codes from

<https://github.com/tanayyadav28/Assignments/blob/main/Assignment%203/code/assignment3.py>

and latex-tikz codes from

<https://github.com/tanayyadav28/Assignments/blob/main/Assignment%203/assignment3.tex>

## 1 PROBLEM

(GATE EC, Q. 25) A fair coin is tossed till a head appears for the first time. The probability that the number of required tosses is odd, is

- (A)  $\frac{1}{3}$       (B)  $\frac{1}{2}$       (C)  $\frac{2}{3}$       (D)  $\frac{3}{4}$

## 2 SOLUTION

Let  $Y \in \mathbb{N}$  denote the number of times the experiment is performed.  $Y = k$  represents  $k - 1$  failures before getting 1 success. Let  $p$  be the probability of success.

$$\therefore p_Y k = (1 - p)^k \times p \quad k \in \mathbb{N}$$

For getting first success on an odd try,  $k = 2n + 1$ . Let  $X$  be the Bernoulli Random Variable  $X \sim B(1, 0)$ , which denotes the outcome of the given experiment.  $X = 1$  denotes the outcome that odd number of tries are required to get the first head,  $X = 0$  denotes all

other outcomes. Here,  $p = \frac{1}{2}$

$$\therefore \Pr(X = 1) = p_Y(1) + p_Y(3) + p_Y(5) + \dots \quad (2.0.1)$$

$$\Pr(X = 1) = \sum_{n=0}^{\infty} p_Y(2n + 1) \quad (2.0.2)$$

$$\Pr(X = 1) = \sum_{n=0}^{\infty} (1 - p)^{(2n+1)-1} \times p \quad (2.0.3)$$

$$\Pr(X = 1) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n} \times \frac{1}{2} \quad (2.0.4)$$

$$\Pr(X = 1) = \frac{\frac{1}{2}}{1 - \frac{1}{4}} \quad (2.0.5)$$

$$\Pr(X = 1) = \frac{\left(\frac{1}{2}\right)}{\left(\frac{3}{4}\right)} \quad (2.0.6)$$

$$\therefore \Pr(X = 1) = \frac{2}{3} \quad (2.0.7)$$

Hence, the correct option is (C).

