Assignment 3

Tanay Yadav - AI20BTECH11026

Download the python codes from:

https://github.com/tanayyadav28/EE3900–Assignments/blob/main/Assignment_3/code/Assignment_3.py

Download the latex-tikz codes from:

https://github.com/tanayyadav28/EE3900–Assignments/blob/main/Assignment_3/Assignment_3.tex

1 Problem

[Construction S2; Q8] Can you construct a quadrilateral MIST where $MI = 3.5$, $IS = 6.5$, $\angle M = 100°$, $\angle I = 105°$, and $\angle S = 120°$.

2 Solution

Given,

$$\|\mathbf{MI}\| = 3.5 \tag{2.0.1}$$
$$\|\mathbf{IS}\| = 6.5 \tag{2.0.2}$$
$$\angle TMI = 100° \tag{2.0.3}$$
$$\angle MIS = 105° \tag{2.0.4}$$
$$\angle IST = 120° \tag{2.0.5}$$
$$\tag{2.0.6}$$

If quadrilateral MIST is possible,

$$\therefore \angle STM = 360 - 100 - 105 - 120 \tag{2.0.7}$$
$$\angle STM = 35° \tag{2.0.8}$$
$$\text{Let, } \|\mathbf{ST}\| = x \tag{2.0.9}$$
$$\|\mathbf{TM}\| = y \tag{2.0.10}$$

Considering $\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ to be the midpoint of **IS**.

$$\therefore \|\mathbf{IO}\| = 3.25 \tag{2.0.11}$$
$$\|\mathbf{OS}\| = 3.25 \tag{2.0.12}$$
$$\tag{2.0.13}$$

The vectors are along the x-axis. Hence the coordinates are:

$$\therefore \mathbf{I} = \begin{pmatrix} -3.25 \\ 0 \end{pmatrix} \tag{2.0.14}$$
$$\mathbf{S} = \begin{pmatrix} 3.25 \\ 0 \end{pmatrix} \tag{2.0.15}$$

$$\because \angle MIS = 105° \tag{2.0.16}$$
$$\therefore \mathbf{M} = \begin{pmatrix} -3.25 + \|\mathbf{MI}\| \cos(\angle MIS) \\ 0 + \|\mathbf{MI}\| \sin(\angle MIS) \end{pmatrix} \tag{2.0.17}$$
$$\therefore \mathbf{M} = \begin{pmatrix} -4.15 \\ 3.38 \end{pmatrix} \tag{2.0.18}$$

Now,

$$\mathbf{T} = \mathbf{S} + x \begin{pmatrix} \cos(180 - \angle IST) \\ \sin(180 - \angle IST) \end{pmatrix}. \tag{2.0.19}$$

Now, the angle made by **MI** with negative x-axis is $180 - 105 = 75°$.
$\therefore$ The angle made by **TM** with x-axis: $\alpha = (75 - 35) = 40°$. Hence,

$$\mathbf{T} = \mathbf{M} + y \begin{pmatrix} \cos(180 - \alpha) \\ \sin(180 - \alpha) \end{pmatrix} \tag{2.0.20}$$
$$\therefore \mathbf{S} + x \begin{pmatrix} \cos(60) \\ \sin(60) \end{pmatrix} = \mathbf{M} + y \begin{pmatrix} \cos(140) \\ \sin(140) \end{pmatrix} \tag{2.0.21}$$
$$\therefore \begin{pmatrix} 3.25 \\ 0 \end{pmatrix} + x \begin{pmatrix} 0.5 \\ 0.86 \end{pmatrix} = \begin{pmatrix} -4.15 \\ 3.38 \end{pmatrix} + y \begin{pmatrix} -0.76 \\ 0.64 \end{pmatrix} \tag{2.0.22}$$
$$\begin{pmatrix} 0.5 & 0.76 \\ 0.86 & -0.64 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -7.4 \\ 3.38 \end{pmatrix} \tag{2.0.23}$$

Let

$$\mathbf{A} = \begin{pmatrix} 0.5 & 0.76 \\ 0.86 & -0.64 \end{pmatrix} \tag{2.0.24}$$
$$\left|\mathbf{A}\right| = -0.97 \tag{2.0.25}$$

$\therefore \mathbf{A}^{-1}$ exists.
Calculating $\mathbf{A}^{-1}$ by adjoint method,

$$\mathbf{A}^{-1} = \frac{1}{|\mathbf{A}|}(adjoint(\mathbf{A})) \tag{2.0.26}$$

$$\therefore \mathbf{A}^{-1} = \begin{pmatrix} 0.66 & 0.78 \\ 0.88 & -0.51 \end{pmatrix} \tag{2.0.27}$$

Pre-multiplying $\mathbf{A}^{-1}$ to (2.0.23),

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.66 & 0.78 \\ 0.88 & -0.51 \end{pmatrix} \times \begin{pmatrix} -7.4 \\ 3.38 \end{pmatrix} \tag{2.0.28}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2.24 \\ -8.23 \end{pmatrix} \tag{2.0.29}$$

$$\therefore x = -2.24 \tag{2.0.30}$$

$$y = -8.23 \tag{2.0.31}$$

But, x and y are magnitudes of **ST**, **TM** and hence are always positive.
Hence, a quadrilateral cannot be constructed using the given parameters.
The following python plot shows that this quadrilateral cannot be constructed.

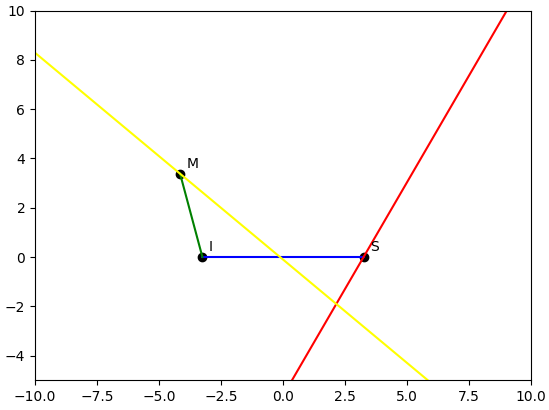

Fig. 0: Plot for Quadrilateral MIST