Construction : Q2.8

Tanay Yadav - AI20BTECH11026

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Question

Construction Q2.8

Can you construct a quadrilateral MIST where $MI = 3.5$, $IS = 6.5$, $\angle M = 100^\circ$, $\angle I = 105^\circ$, and $\angle S = 120^\circ$.

Lemma 1

1. Any Vector X can be expressed as:

$$X = A + xH \tag{1}$$

where,

1. A is the tail of the vector X,
2. x is the magnitude of the required vector X,
3. H is the unit vector in the direction of the vector X.

2. H is given as

$$H = \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} \tag{2}$$

where, θ is the angle made by the vector X with the positive x-axis.

Lemma 1

Addition of two such vectors (x, y) can be written as:

$$\mathrm{x} = \mathrm{A} + x\mathrm{H} \tag{3}$$

$$\mathrm{y} = \mathrm{B} + y\mathrm{K} \tag{4}$$

$$\therefore \mathrm{x} + \mathrm{y} = \mathrm{A} + \mathrm{B} + x\mathrm{H} + y\mathrm{K} \tag{5}$$

$$\therefore \mathrm{x} + \mathrm{y} = (\mathrm{A} + \mathrm{B}) + \begin{pmatrix} \mathrm{H} & \mathrm{K} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{6}$$

Solution

Given

$$\|\mathrm{MI}\| = 3.5 \tag{7}$$
$$\|\mathrm{IS}\| = 6.5 \tag{8}$$
$$\angle TMI = 100^\circ \tag{9}$$
$$\angle MIS = 105^\circ \tag{10}$$
$$\angle IST = 120^\circ \tag{11}$$

If Quadrilateral MIST is possible

$$\therefore \angle STM = 360 - 100 - 105 - 120 \tag{12}$$
$$\angle STM = 35^\circ \tag{13}$$
$$\text{Let, } \|\mathrm{ST}\| = x \tag{14}$$
$$\|\mathrm{TM}\| = y \tag{15}$$

Solution

Considering $O = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ to be the midpoint of IS.

$$\therefore \|IO\| = 3.25 \tag{16}$$

$$\|OS\| = 3.25 \tag{17}$$

$$\tag{18}$$

The vectors are along the x-axis. Hence the coordinates are:

$$\therefore I = \begin{pmatrix} -3.25 \\ 0 \end{pmatrix} \tag{19}$$

$$S = \begin{pmatrix} 3.25 \\ 0 \end{pmatrix} \tag{20}$$

Using Lemma 1,

$$\because \angle MIS = 105^\circ \tag{21}$$

Solution

$$\therefore \mathrm{M} = \begin{pmatrix} -3.25 + \|\mathrm{MI}\| \cos(\angle MIS) \\ 0 + \|\mathrm{MI}\| \sin(\angle MIS) \end{pmatrix} \tag{22}$$

$$\therefore \mathrm{M} = \begin{pmatrix} -4.15 \\ 3.38 \end{pmatrix} \tag{23}$$

Now by Lemma 1,

$$\mathrm{T} = \mathrm{S} + x \begin{pmatrix} \cos(180 - \angle IST) \\ \sin(180 - \angle IST) \end{pmatrix}. \tag{24}$$

Now, the angle made by MI with negative x-axis is $180 - 105 = 75°$.
$\therefore$ The angle made by TM with x-axis: $\alpha = (75 - 35) = 40°$. Hence,

$$\mathrm{T} = \mathrm{M} + y \begin{pmatrix} \cos(180 - \alpha) \\ \sin(180 - \alpha) \end{pmatrix} \tag{25}$$

Solution

$$\therefore \mathrm{S} + x\begin{pmatrix}\cos(60)\\ \sin(60)\end{pmatrix} = \mathrm{M} + y\begin{pmatrix}\cos(140)\\ \sin(140)\end{pmatrix} \tag{26}$$

$$\therefore \begin{pmatrix}3.25\\ 0\end{pmatrix} + x\begin{pmatrix}0.5\\ 0.86\end{pmatrix} = \begin{pmatrix}-4.15\\ 3.38\end{pmatrix} + y\begin{pmatrix}-0.76\\ 0.64\end{pmatrix} \tag{27}$$

Using Lemma 1,

$$\begin{pmatrix}0.5 & 0.76\\ 0.86 & -0.64\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}-7.4\\ 3.38\end{pmatrix} \tag{28}$$

Let

$$\mathrm{A} = \begin{pmatrix}0.5 & 0.76\\ 0.86 & -0.64\end{pmatrix} \tag{29}$$

$$\left|\mathrm{A}\right| = -0.97 \tag{30}$$

$\therefore \mathrm{A}^{-1}$ exists.

Solution

Calculating A^{-1} by adjoint method,

$$\mathrm{A}^{-1} = \frac{1}{|\mathrm{A}|}(adjoint(\mathrm{A})) \tag{31}$$

$$\therefore \mathrm{A}^{-1} = \begin{pmatrix} 0.66 & 0.78 \\ 0.88 & -0.51 \end{pmatrix} \tag{32}$$

Pre-multiplying A^{-1} to (28),

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.66 & 0.78 \\ 0.88 & -0.51 \end{pmatrix} \begin{pmatrix} -7.4 \\ 3.38 \end{pmatrix} \tag{33}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2.24 \\ -8.23 \end{pmatrix} \tag{34}$$

$$\therefore x = -2.24 \tag{35}$$

$$y = -8.23 \tag{36}$$

But, x and y are magnitudes of ST, TM and hence are always positive. Hence, a quadrilateral cannot be constructed using the given parameters. The adjacent python plot shows that this quadrilateral cannot be constructed.

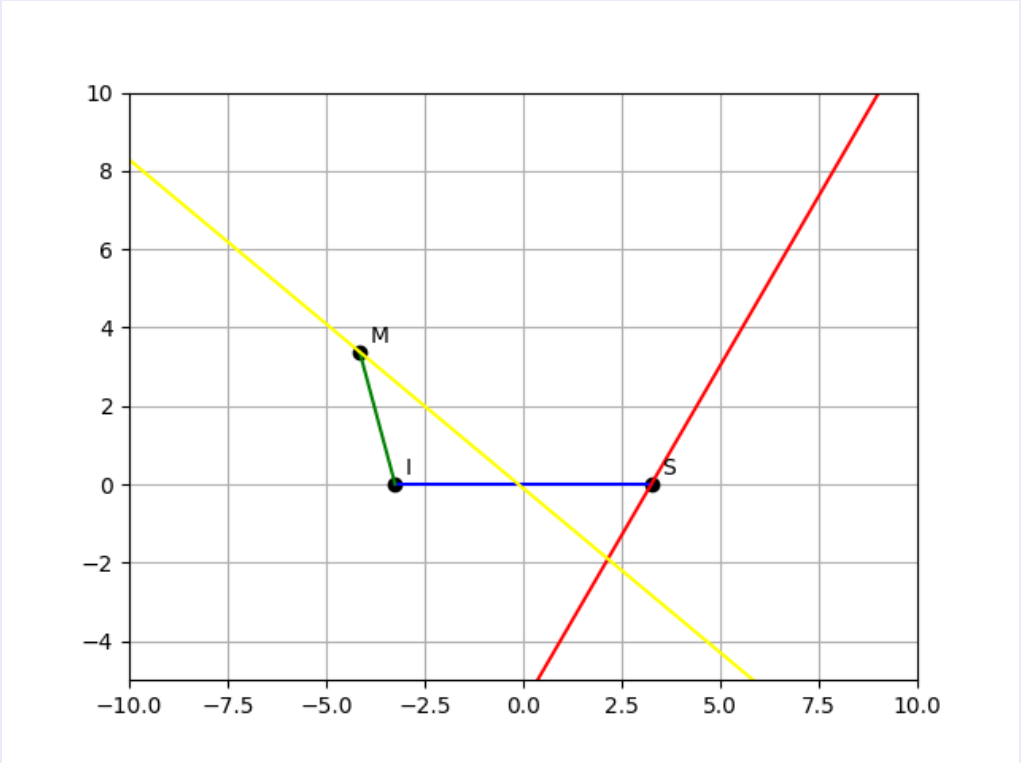

Figure: Plot for Quadrilateral MIST