Test 2

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Download the python codes from:

https://github.com/tanayyadav28/EE3900–Assignments/blob/main/Test_2/Test_2.py

Download the latex-tikz codes from:

https://github.com/tanayyadav28/EE3900–Assignments/blob/main/Test_2/Test_2.tex

1 Problem

[Q3.19 (a)]
For each of the following pairs of input z-transform $X[z]$ and system function $H[z]$, determine the ROC and the output z-transform $Y[z]$.

$$X(z) = \frac{1}{1+\frac{1}{2}z^{-1}}, \qquad |z| > \frac{1}{2}$$

$$H(z) = \frac{1}{1-\frac{1}{4}z^{-1}}, \qquad |z| > \frac{1}{4}$$

2 Solution

$$X(z) = \frac{1}{1+\frac{1}{2}z^{-1}}, |z| > \frac{1}{2} \tag{2.0.1}$$

$$H(z) = \frac{1}{1-\frac{1}{4}z^{-1}}, |z| > \frac{1}{4} \tag{2.0.2}$$

System transfer function is given as:

$$H(z) = \frac{Y(z)}{X(z)} \tag{2.0.3}$$

$$\therefore Y(z) = X(z)H(z) \tag{2.0.4}$$

$$= \left(\frac{1}{1+\frac{1}{2}z^{-1}}\right)\left(\frac{1}{1-\frac{1}{4}z^{-1}}\right) \tag{2.0.5}$$

$$\therefore Y(z) = \frac{1}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}} \tag{2.0.6}$$

Now,

$$Y(z) = \frac{z^2}{z^2+\frac{1}{4}z-\frac{1}{8}} \tag{2.0.7}$$

$$\therefore Y(z) = \frac{z^2}{\left(z+\frac{1}{2}\right)\left(z-\frac{1}{4}\right)} \tag{2.0.8}$$

The ROC of $Y(z)$ is the intersection of ROCs of $X(z)$ and $H(z)$.
Therefore, the poles of $Y(z)$ are $\left\{-\frac{1}{2}, \frac{1}{4}\right\}$, the zero is at 0 and the ROC is $|z| > \frac{1}{2}$.

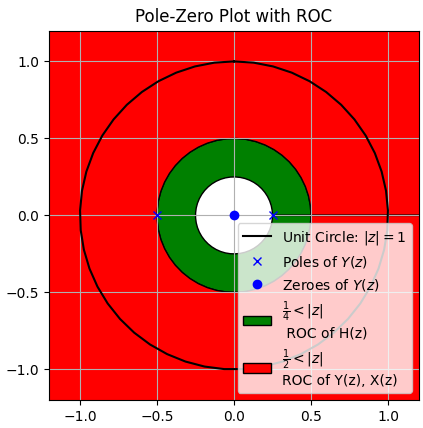

Fig. 0: Poles, Zeros and ROC of $Y(z)$