Assignment 3

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Download the python codes from:

https://github.com/tanayyadav28/EE3900-Assignments/blob/main/Assignment_3/code/Assignment_3.py

Download the latex-tikz codes from:

https://github.com/tanayyadav28/EE3900-Assignments/blob/main/Assignment_3/Assignment_3.tex

1 Problem

[Construction S2; Q8] Can you construct a quadrilateral MIST where $MI = 3.5$, $IS = 6.5$, $\angle M = 100°$, $\angle I = 105°$, and $\angle S = 120°$.

2 Solution

Lemma 2.1. *Any Vector* $\mathbf{X}$ *can be expressed as:*

$$\mathbf{X} = \mathbf{A} + x\mathbf{H} \tag{2.0.1}$$

where,
$\mathbf{A}$ *is the tail of the vector* $\mathbf{X}$*,*
x *is the magnitude of the required vector* $\mathbf{X}$*,*
$\mathbf{H}$ *is the unit vector in the direction of the vector* $\mathbf{X}$*,*
$\mathbf{H}$ *is given as*

$$\mathbf{H} = \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} \tag{2.0.2}$$

where, θ *is the angle made by the vector* $\mathbf{X}$ *with the positive x-axis.*
Addition of two such vectors ($\mathbf{x}$*,* $\mathbf{y}$*) can be written as:*

$$\mathbf{x} = \mathbf{A} + x\mathbf{H} \tag{2.0.3}$$

$$\mathbf{y} = \mathbf{B} + y\mathbf{K} \tag{2.0.4}$$

$$\therefore \mathbf{x} + \mathbf{y} = \mathbf{A} + \mathbf{B} + x\mathbf{H} + y\mathbf{K} \tag{2.0.5}$$

$$\therefore \mathbf{x} + \mathbf{y} = \begin{pmatrix} \mathbf{A} + \mathbf{B} \end{pmatrix} + \begin{pmatrix} \mathbf{H} & \mathbf{K} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{2.0.6}$$

Given,

$$\|\mathbf{MI}\| = 3.5 \tag{2.0.7}$$

$$\|\mathbf{IS}\| = 6.5 \tag{2.0.8}$$

$$\angle TMI = 100° \tag{2.0.9}$$

$$\angle MIS = 105° \tag{2.0.10}$$

$$\angle IST = 120° \tag{2.0.11}$$

$$\tag{2.0.12}$$

If quadrilateral MIST is possible,

$$\therefore \angle STM = 360 - 100 - 105 - 120 \tag{2.0.13}$$

$$\angle STM = 35° \tag{2.0.14}$$

$$\text{Let, } \|\mathbf{ST}\| = x \tag{2.0.15}$$

$$\|\mathbf{TM}\| = y \tag{2.0.16}$$

Considering $\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ to be the midpoint of $\mathbf{IS}$.

$$\therefore \|\mathbf{IO}\| = 3.25 \tag{2.0.17}$$

$$\|\mathbf{OS}\| = 3.25 \tag{2.0.18}$$

$$\tag{2.0.19}$$

The vectors are along the x-axis. Hence the coordinates are:

$$\therefore \mathbf{I} = \begin{pmatrix} -3.25 \\ 0 \end{pmatrix} \tag{2.0.20}$$

$$\mathbf{S} = \begin{pmatrix} 3.25 \\ 0 \end{pmatrix} \tag{2.0.21}$$

Using Lemma (2.1),

$$\because \angle MIS = 105° \tag{2.0.22}$$

$$\therefore \mathbf{M} = \begin{pmatrix} -3.25 + \|\mathbf{MI}\| \cos(\angle MIS) \\ 0 + \|\mathbf{MI}\| \sin(\angle MIS) \end{pmatrix} \tag{2.0.23}$$

$$\therefore \mathbf{M} = \begin{pmatrix} -4.15 \\ 3.38 \end{pmatrix} \tag{2.0.24}$$

Now by Lemma (2.1),

$$\mathbf{T} = \mathbf{S} + x \begin{pmatrix} \cos(180 - \angle IST) \\ \sin(180 - \angle IST) \end{pmatrix}. \tag{2.0.25}$$

Now, the angle made by **MI** with negative x-axis is $180 - 105 = 75°$.

$\therefore$ The angle made by **TM** with x-axis: $\alpha = (75 - 35) = 40°$. Hence,

$$\mathbf{T} = \mathbf{M} + y\begin{pmatrix}\cos(180-\alpha)\\ \sin(180-\alpha)\end{pmatrix} \tag{2.0.26}$$

$$\therefore \mathbf{S} + x\begin{pmatrix}\cos(60)\\ \sin(60)\end{pmatrix} = \mathbf{M} + y\begin{pmatrix}\cos(140)\\ \sin(140)\end{pmatrix} \tag{2.0.27}$$

$$\therefore \begin{pmatrix}3.25\\ 0\end{pmatrix} + x\begin{pmatrix}0.5\\ 0.86\end{pmatrix} = \begin{pmatrix}-4.15\\ 3.38\end{pmatrix} + y\begin{pmatrix}-0.76\\ 0.64\end{pmatrix} \tag{2.0.28}$$

Using Lemma (2.1)

$$\begin{pmatrix}0.5 & 0.76\\ 0.86 & -0.64\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}-7.4\\ 3.38\end{pmatrix} \tag{2.0.29}$$

Let

$$\mathbf{A} = \begin{pmatrix}0.5 & 0.76\\ 0.86 & -0.64\end{pmatrix} \tag{2.0.30}$$

$$\left|\mathbf{A}\right| = -0.97 \tag{2.0.31}$$

$\therefore \mathbf{A}^{-1}$ exists.
Calculating $\mathbf{A}^{-1}$ by adjoint method,

$$\mathbf{A}^{-1} = \frac{1}{\left|\mathbf{A}\right|}(adjoint(\mathbf{A})) \tag{2.0.32}$$

$$\therefore \mathbf{A}^{-1} = \begin{pmatrix}0.66 & 0.78\\ 0.88 & -0.51\end{pmatrix} \tag{2.0.33}$$

Pre-multiplying $\mathbf{A}^{-1}$ to (2.0.29),

$$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}0.66 & 0.78\\ 0.88 & -0.51\end{pmatrix}\begin{pmatrix}-7.4\\ 3.38\end{pmatrix} \tag{2.0.34}$$

$$\therefore \begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}-2.24\\ -8.23\end{pmatrix} \tag{2.0.35}$$

$$\therefore x = -2.24 \tag{2.0.36}$$

$$y = -8.23 \tag{2.0.37}$$

But, x and y are magnitudes of **ST**, **TM** and hence are always positive.
Hence, a quadrilateral cannot be constructed using the given parameters.
The adjacent python plot shows that this quadrilateral cannot be constructed.

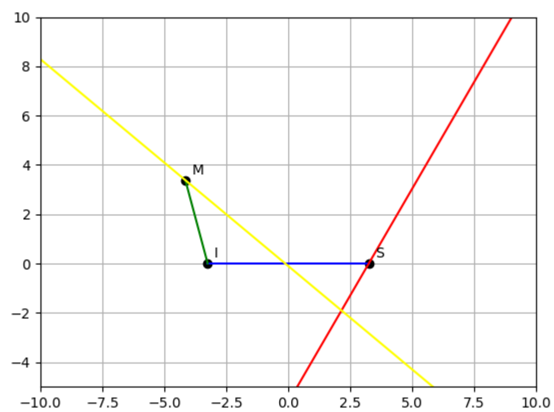

Fig. 0: Plot for Quadrilateral MIST