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Test 2

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Download the python codes from:

https://github.com/tanayyadav28/EE3900— Assignments/blob/main/Test 2/Test 2.py

Download the latex-tikz codes from:

https://github.com/tanayyadav28/EE3900— Assignments/blob/main/Test 2/Test 2.tex

1 Problem

[Q3.19 (a)]

For each of the following pairs of input z-transform X[z] and system function H[z], determine the ROC and the output z-transform Y[z].

$$X(z) = \frac{1}{1 + \frac{1}{2}z^{-1}}, \qquad |z| > \frac{1}{2}$$

$$H(z) = \frac{1}{1 - \frac{1}{4}z^{-1}}, \qquad |z| > \frac{1}{4}$$

2 Solution

$$X(z) = \frac{1}{1 + \frac{1}{2}z^{-1}}, |z| > \frac{1}{2}$$
 (2.0.1)

$$H(z) = \frac{1}{1 - \frac{1}{4}z^{-1}}, |z| > \frac{1}{4}$$
 (2.0.2)

System transfer function is given as:

$$H(z) = \frac{Y(z)}{X(z)}$$
 (2.0.3)

$$\therefore Y(z) = X(z)H(z) \tag{2.0.4}$$

$$= \left(\frac{1}{1 + \frac{1}{2}z^{-1}}\right) \left(\frac{1}{1 - \frac{1}{4}z^{-1}}\right) \tag{2.0.5}$$

$$\therefore Y(z) = \frac{1}{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$
 (2.0.6)

Now,

$$Y(z) = \frac{z^2}{z^2 + \frac{1}{4}z - \frac{1}{8}}$$
 (2.0.7)

$$\therefore Y(z) = \frac{z^2}{\left(z + \frac{1}{2}\right)\left(z - \frac{1}{4}\right)}$$
 (2.0.8)

The ROC of Y(z) is the intersection of ROCs of X(z) and H(z).

Therefore, the poles of Y(z) are $\left\{-\frac{1}{2}, \frac{1}{4}\right\}$, the zero is at 0 and the ROC is $|z| > \frac{1}{2}$.

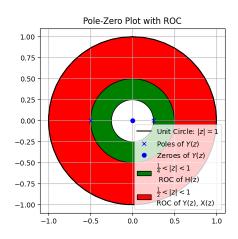


Fig. 0: Poles, Zeros and ROC of Y(z)