

Assignment 2

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Download the python codes from:

https://github.com/tanayyadav28/EE3900-Assignments/blob/main/Assignment_3/code/Assignment_3.py

Download the latex-tikz codes from:

https://github.com/tanayyadav28/EE3900-Assignments/blob/main/Assignment_3/Assignment_3.tex

1 PROBLEM

[Construction S2; Q8] Can you construct a quadrilateral MIST where $MI = 3.5$, $IS = 6.5$, $\angle M = 100^\circ$, $\angle I = 105^\circ$, and $\angle S = 120^\circ$.

2 SOLUTION

Given,

$$\|\mathbf{MI}\| = 3.5 \quad (2.0.1)$$

$$\|\mathbf{IS}\| = 6.5 \quad (2.0.2)$$

$$\angle TMI = 100^\circ \quad (2.0.3)$$

$$\angle MIS = 105^\circ \quad (2.0.4)$$

$$\angle IST = 120^\circ \quad (2.0.5)$$

$$(2.0.6)$$

If quadrilateral MIST is possible,

$$\therefore \angle STM = 360 - 100 - 105 - 120 \quad (2.0.7)$$

$$\angle STM = 35^\circ \quad (2.0.8)$$

$$\text{Let, } \|\mathbf{ST}\| = x \quad (2.0.9)$$

$$\|\mathbf{TM}\| = y \quad (2.0.10)$$

Considering $\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ to be the midpoint of \mathbf{IS} .

$$\therefore \|\mathbf{IO}\| = 3.25 \quad (2.0.11)$$

$$\|\mathbf{OS}\| = 3.25 \quad (2.0.12)$$

$$(2.0.13)$$

The vectors are along the x-axis. Hence the coordinates are:

$$\therefore \mathbf{I} = \begin{pmatrix} -3.25 \\ 0 \end{pmatrix} \quad (2.0.14)$$

$$\mathbf{S} = \begin{pmatrix} 3.25 \\ 0 \end{pmatrix} \quad (2.0.15)$$

$$\therefore \angle MIS = 105^\circ \quad (2.0.16)$$

$$\therefore \mathbf{M} = \begin{pmatrix} -3.25 + \|\mathbf{MI}\| \cos(\angle MIS) \\ 0 + \|\mathbf{MI}\| \sin(\angle MIS) \end{pmatrix} \quad (2.0.17)$$

$$\therefore \mathbf{M} = \begin{pmatrix} -4.15 \\ 3.38 \end{pmatrix} \quad (2.0.18)$$

Now,

$$\mathbf{T} = \mathbf{S} + x \begin{pmatrix} \cos(180 - \angle IST) \\ \sin(180 - \angle IST) \end{pmatrix}. \quad (2.0.19)$$

Now, the angle made by \mathbf{MI} with negative x-axis is $180 - 105 = 75^\circ$.

\therefore The angle made by \mathbf{TM} with x-axis: $\alpha = (75 - 35) = 40^\circ$. Hence,

$$\mathbf{T} = \mathbf{M} + y \begin{pmatrix} \cos(180 - \alpha) \\ \sin(180 - \alpha) \end{pmatrix} \quad (2.0.20)$$

$$\therefore \mathbf{S} + x \begin{pmatrix} \cos(60) \\ \sin(60) \end{pmatrix} = \mathbf{M} + y \begin{pmatrix} \cos(140) \\ \sin(140) \end{pmatrix} \quad (2.0.21)$$

$$\therefore \begin{pmatrix} 3.25 \\ 0 \end{pmatrix} + x \begin{pmatrix} 0.5 \\ 0.86 \end{pmatrix} = \begin{pmatrix} -4.15 \\ 3.38 \end{pmatrix} + y \begin{pmatrix} -0.76 \\ 0.64 \end{pmatrix} \quad (2.0.22)$$

$$\begin{pmatrix} 0.5 & 0.76 \\ 0.86 & -0.64 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -7.4 \\ 3.38 \end{pmatrix} \quad (2.0.23)$$

Let

$$\mathbf{A} = \begin{pmatrix} 0.5 & 0.76 \\ 0.86 & -0.64 \end{pmatrix} \quad (2.0.24)$$

$$|\mathbf{A}| = -0.97 \quad (2.0.25)$$

$\therefore \mathbf{A}^{-1}$ exists.

Calculating \mathbf{A}^{-1} by adjoint method,

$$\mathbf{A}^{-1} = \frac{1}{|\mathbf{A}|}(\text{adjoint}(\mathbf{A})) \quad (2.0.26)$$

$$\therefore \mathbf{A}^{-1} = \begin{pmatrix} 0.66 & 0.78 \\ 0.88 & -0.51 \end{pmatrix} \quad (2.0.27)$$

Pre-multiplying \mathbf{A}^{-1} to (2.0.23),

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.66 & 0.78 \\ 0.88 & -0.51 \end{pmatrix} \times \begin{pmatrix} -7.4 \\ 3.38 \end{pmatrix} \quad (2.0.28)$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2.24 \\ -8.23 \end{pmatrix} \quad (2.0.29)$$

$$\therefore x = -2.24 \quad (2.0.30)$$

$$y = -8.23 \quad (2.0.31)$$

But, x and y are magnitudes of **ST**, **TM** and hence are always positive.

Hence, a quadrilateral cannot be constructed using the given parameters.

The following python plot shows that this quadrilateral cannot be constructed.

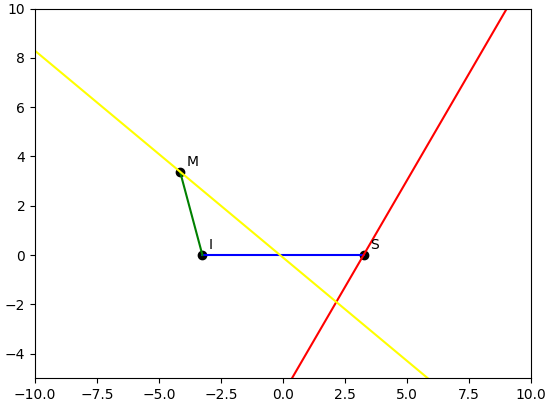


Fig. 0: Plot for Quadrilateral MIST