# Assignment 5

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Download the python codes from:

https://github.com/tanayyadav28/EE3900-Assignments/blob/main/Assignment_5/code/Assignment_5.py

Download the latex-tikz codes from:

https://github.com/tanayyadav28/EE3900-Assignments/blob/main/Assignment_5/Assignment_5.tex

## 1 Problem

[Quadratic Forms 2.22] Solve:

$$x^2 + x + 1 = 0 \tag{1.0.1}$$

## 2 Solution

Let

$$y = x^2 + x + 1 = 0 \tag{2.0.1}$$

Representing $y$ in vector form,

$$y = \mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} \frac{1}{2} & 0 \end{pmatrix} \mathbf{x} + 1 \tag{2.0.2}$$

where,

$$\mathbf{x} = \begin{pmatrix} x \\ 0 \end{pmatrix} \tag{2.0.3}$$

Putting $y = 0$ we get,

$$x^2 + x + 1 = 0 \tag{2.0.4}$$

$$x^2 + 2\left(\frac{1}{2}\right)x + \frac{1}{4} + \frac{3}{4} = 0 \tag{2.0.5}$$

$$\left(x + \frac{1}{2}\right)^2 + \frac{3}{4} = 0 \tag{2.0.6}$$

$$\left(x + \frac{1}{2}\right)^2 = -\frac{3}{4} \tag{2.0.7}$$

A square of a real number can never be negative. Therefore, the given equation has no real roots which is verified by the python plot.
Obtaining the Affine Transformation,

$$V = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{2.0.8}$$

$$\mathbf{u} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \tag{2.0.9}$$

$$f = 1 \tag{2.0.10}$$

The equation for Affine transformation is:

$$\mathbf{x} = \mathbf{P}\mathbf{y} + \mathbf{c} \tag{2.0.11}$$

The Eigenvalues of $V$ are:

$$\lambda_1 = 1 \tag{2.0.12}$$

$$\lambda_2 = 0 \tag{2.0.13}$$

$$\mathbf{D} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{2.0.14}$$

The Eigen vectors of $V$ are:

$$\mathbf{p_1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.15}$$

$$\mathbf{p_2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.16}$$

$$\therefore \mathbf{P} = \begin{pmatrix} \mathbf{p_1} & \mathbf{p_2} \end{pmatrix} \tag{2.0.17}$$

$$\therefore \mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2.0.18}$$

Since, $\left|V\right| = 0$

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p}_1^T \\ V \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p_1} - \mathbf{u} \end{pmatrix} \tag{2.0.19}$$

$$\eta = \mathbf{u^T p_1} \tag{2.0.20}$$

$$\therefore \eta = -\frac{1}{2} \tag{2.0.21}$$

$$\begin{pmatrix} \frac{1}{2} & -1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -1 \\ -\frac{1}{2} \\ 0 \end{pmatrix} \tag{2.0.22}$$

$$\mathbf{c} = \begin{pmatrix} -\frac{1}{2} \\ \frac{3}{4} \end{pmatrix} \tag{2.0.23}$$

The quadratic equation does not have real roots if:

$$(\mathbf{p_1}^T\mathbf{c})(\mathbf{p_2}^T\mathbf{V}\mathbf{p_2}) > 0 \tag{2.0.24}$$

Substituting the values,

$$\left(\frac{3}{4}\right)(1) = \frac{3}{4} > 0 \tag{2.0.25}$$

Hence, the given equation has no real roots.

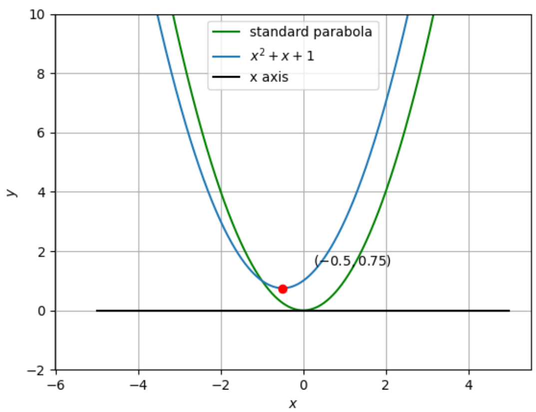


Fig. 0: Plot from Python Code.