CONTROL SYSTEM ASSIGNMENT -1

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CONTROL SYSTEMS
ASSIGNMENT – 1.

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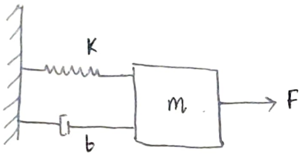
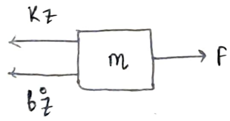


Spring constant = K
Damping constant = b
Position of mass = z
Speed of mass = $\dot{z}$
External Force = F

Parameters

$m = 4.493 kg$.
$K = 2.943 N/m$
$b = 0.499 N\text{-}s/m$



* HOMEWORK D.2

a) Kinetic energy Expression

$$K = \frac{1}{2} m \|v\|^2 \qquad v = [\dot{z}\ 0\ 0]^T$$

$$= \frac{1}{2} m v^T v$$

$$= \frac{1}{2} m [\dot{z}\ 0\ 0] \begin{bmatrix} \dot{z} \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{K = \frac{1}{2} m \dot{z}^2}$$

b) Matlab Code

* HOMEWORK D.3

a) Potential Energy

$$\boxed{P = \frac{1}{2} K z^2}$$

b) Generalized Coordinates

$$q = (z)^T = z$$

c) Generalised Forces & Damping Forces

Force: $\tau = (F)^T = F$

Damping force: $-B\dot{q} = -(b)\dot{z} = -b\dot{z}$

d) Euler Lagrange Equation

Euler Lagrange equation is given by

$$L(q,\dot{q}) = K - P = \frac{1}{2} m \dot{q}^2 - \frac{1}{2} K q^2$$

$$L(z,\dot{z}) = K - P = \frac{1}{2} m \dot{z}^2 - \frac{1}{2} K z^2$$

General form of Euler Lagrange Equation

$$\frac{d}{dt}\left(\frac{dL}{d\dot{z}}\right) - \frac{dL}{dz} = F - b\dot{z}$$

$$\boxed{m\ddot{z} + Kz = F - b\dot{z}} \quad \text{---} (1)$$

* HOMEWORK D.4

a) Equilibria of system

Defining $x = \begin{pmatrix} z \\ \dot{z} \end{pmatrix}$, we get

$$\dot{x} = \begin{pmatrix} \dot{z} \\ \ddot{z} \end{pmatrix}$$

From (1)

$$m\ddot{z} + kz = F - b\dot{z}$$

$$\ddot{z} = \frac{F - b\dot{z} - kz}{m}$$

$$\dot{x} = \begin{pmatrix} \dot{z} \\ \frac{F}{m} - \frac{b\dot{z}}{m} - \frac{kz}{m} \end{pmatrix} \triangleq f(x,u)$$

The equilibrium at $f(x_e, u_e) = 0$.

z_e = anything

$\dot{z}_e = 0$, $\ddot{z}_e = 0$

$$\therefore \boxed{F_e = kz_e}$$

b) Linearize system using Jacobian linearization

- Linearize quantities

$$\tilde{z} \triangleq z - z_e$$

$$\dot{\tilde{z}} \triangleq \dot{z} - \dot{z}_e = \dot{z}$$

$$\ddot{\tilde{z}} \triangleq \ddot{z} - \ddot{z}_e = \ddot{z}$$

$$\tilde{F} \triangleq F - F_e$$

$$m\ddot{z} = m\ddot{z}_e + m \left.\frac{d\ddot{z}}{d\ddot{z}}\right|_e (\ddot{z} - \ddot{z}_e) = m\ddot{\tilde{z}}$$

$$Kz = Kz_e + K\left.\frac{dz}{dz}\right|_e (z - z_e) = Kz_e + K\tilde{z}$$

$$F = F_e + \left.\frac{dF}{dF}\right|_e (F - F_e) = F_e + \tilde{F}$$

$$b\dot{z} = b\dot{z}_e + b\left.\frac{d\dot{z}}{d\dot{z}}\right|_e (\dot{z} - \dot{z}_e) = b\dot{\tilde{z}}$$

Substituting the values obtained in (1)

$$m\ddot{z} + Kz = F - b\dot{z}$$

$$m\ddot{\tilde{z}} + K(\tilde{z} + z_e) = (\tilde{F} + F_e) - b\dot{\tilde{z}}$$

At equilibrium

$$F_e = Kz_e$$

$$m\ddot{\tilde{z}} + K\tilde{z} + \cancel{Kz_e} = \tilde{F} + \cancel{Kz_e} - b\dot{\tilde{z}}$$

$\therefore$ Jacobian linearization

$$\boxed{m\ddot{\tilde{z}} + K\tilde{z} = \tilde{F} - b\dot{\tilde{z}}} \quad (2)$$

c) Feedback linearization

Assume $g(z, \dot{z})$ be the non linear function of z & $\dot{z}$

and u be the input signal.

then in eq^n

$$m\ddot{z} + b\dot{z} + kz = F$$

$$g(z, \dot{z}) = 0 \text{ and } F = u$$

In feedback linearization

$$u = g(z, \dot{z}) + \tilde{u}$$

$$F = 0 + \tilde{u}$$

the resulting equation of motion is

$$\boxed{m\ddot{z} + b\dot{z} + kz = \tilde{u}} \quad -(3)$$

* HOMEWORK D.5

a) Laplace transform

$$\frac{d}{dt} x(t) \xrightarrow{\mathcal{L}} s\,X(s)$$

$$\frac{d^n}{dt^n} x(t) \xrightarrow{\mathcal{L}} s^n X(s)$$

so, the required laplace transform is

$$m s^2 Z(s) + b s Z(s) + k Z(s) = F(s)$$

For linearized equation

$$m\ddot{\tilde{z}} + b\dot{\tilde{z}} + k\tilde{z} = \tilde{F}$$

laplace transform is

$$\boxed{m s^2 \ddot{\tilde{Z}}(s) + b s \dot{\tilde{Z}}(s + k \tilde{Z}(s) = \tilde{F}(s)} \quad -(4)$$

b) Transfer function

Given force $\tilde{F}(s)$ to be input and position $Z(s)$ to be output

From laplace transform $x(t) * h(t) \xrightarrow{\mathcal{L}} X(s)\,H(s)$

$$\Rightarrow Y(s) = X(s)\,H(s)$$

$$\Rightarrow \frac{Y(s)}{X(s)} = H(s)$$

So, $(ms^2 + bs + k)\,\tilde{Z}(s) = \tilde{F}(s)$

$$\boxed{H(s) = \frac{\tilde{Z}(s)}{\tilde{F}(s)} = \frac{1/m}{s^2 + b/m + k/m}} \quad \text{— (5)}$$

c) Block diagram

$$\tilde{F}(s) \longrightarrow \boxed{\frac{1/m}{s^2 + b/m + k/m}} \longrightarrow \tilde{Z}(s)$$

* HOMEWORK D.6

Linear state space Equation

$$x = (Z, \dot{Z})^T \qquad u = F$$

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

We have $m\ddot{z} + bz + kz = F$

$x = (z, \dot{z})^T \quad , u = F$

$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$$\dot{x} = \begin{pmatrix} \dot{z} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} \dot{z} \\ \frac{F}{m} - \frac{b}{m}\dot{z} - \frac{k}{m}\dot{z} \end{pmatrix} = \begin{pmatrix} x_2 \\ \frac{1}{m}u - \frac{b}{m}x_2 + \frac{k}{m}x_1 \end{pmatrix}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} u$$

$$\boxed{\dot{x} = Ax + Bu} \qquad \boxed{A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1/m \end{bmatrix}}$$

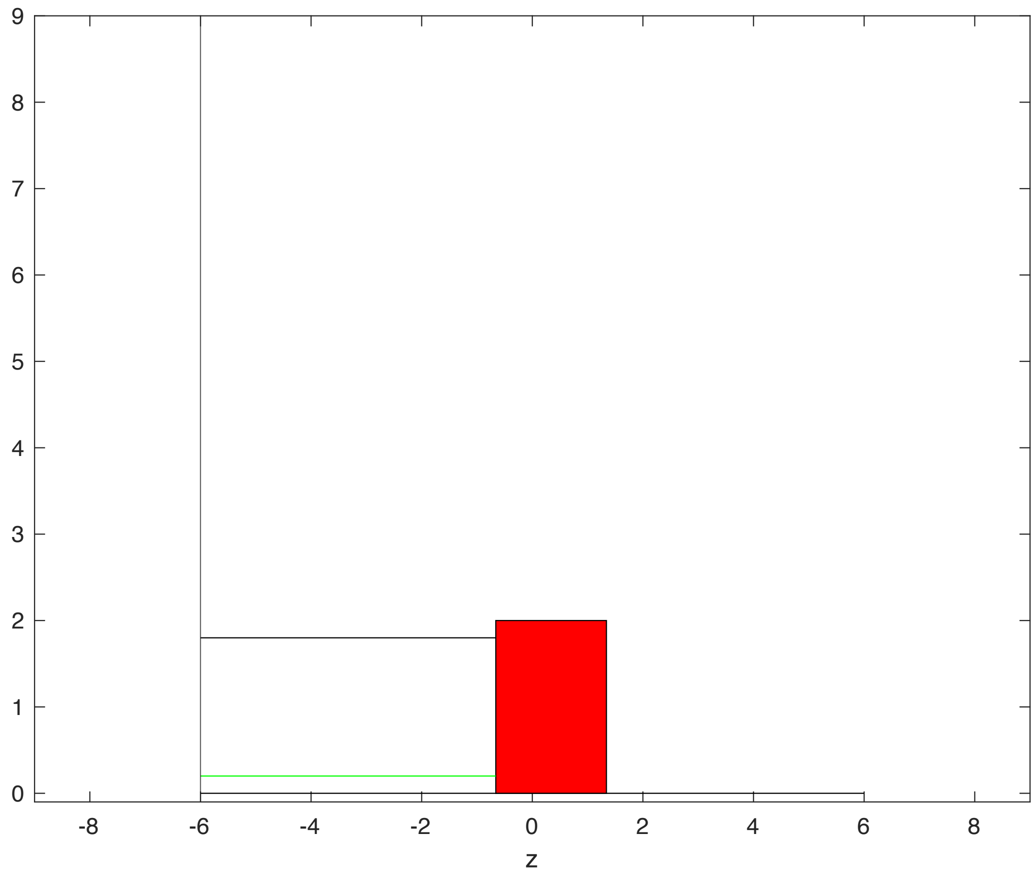
Now $y = z$

$$y = z = x_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]u$$

$$\boxed{y = Cx + Du} \qquad \boxed{C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D = [0]}$$

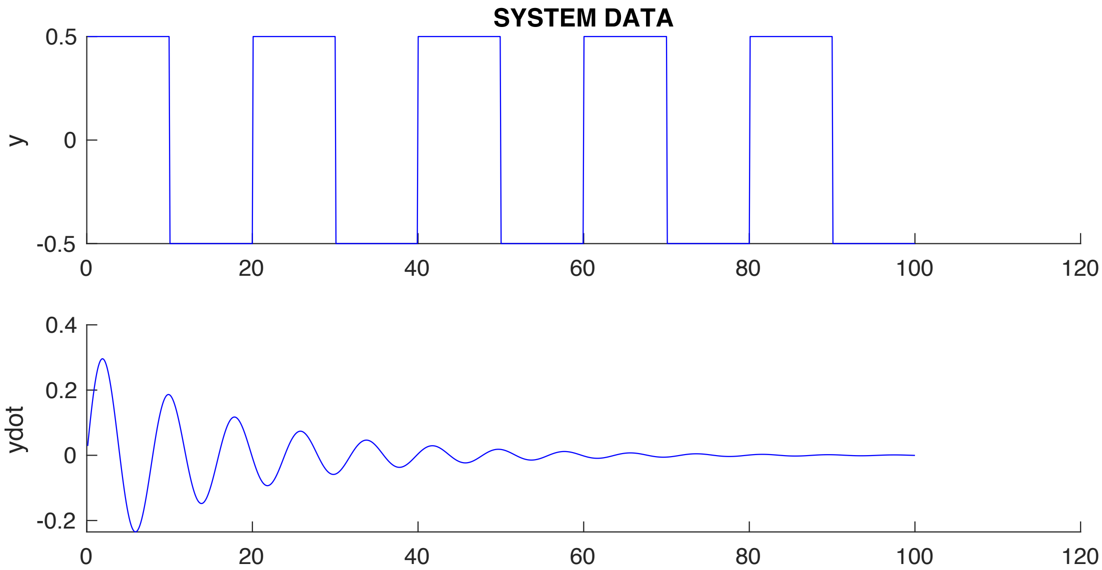
ANIMATION FOR SPRING MASS DAMPER:



SIMULATION OF SPRING MASS DAMPER :

STOP TIME: 100s

A) STEP INPUT FORCE



B) SINE WAVE FORCE: