RDSML-Day-21 👉 Basic Probability Theory

1. The classic definition of probability
2. Relative frequency of occurrence
3. Breaking down the probability terminologies: Experiment/trial, Event or Outcome, Sample Space
4. Union and intersection
5. Mutually exclusive events, independent events, and complementary events

---

The classic definition of probability

Probability is the measure of the **likelihood** that a particular event will occur. It quantifies uncertainty and is expressed as a number between **0 and 1**, where:

- **0** means the event is **impossible**
- **1** means the event is **certain**
- Values between 0 and 1 indicate varying degrees of likelihood

Formal Definition:

If an experiment has **n** equally likely outcomes, and **f** of them are favorable to an event **A**, then the probability of **A** is given by:

$$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}} = \frac{f}{n}$$

Example:

If you toss a fair coin, the probability of getting heads is:

$$P(\text{Heads}) = \frac{1}{2}$$

Key Points:

- Probability is denoted by **P(Event)**
- The sum of probabilities of all possible outcomes in a sample space is **1**
- Used in statistics, science, engineering, finance, and many real-world applications

Relative Frequency of Occurrence

Definition:
The **relative frequency of occurrence** of an event is the **ratio** of the number of times the event occurs to the **total number of trials** or observations.

$$\text{Relative Frequency} = \frac{\text{Number of times the event occurred}}{\text{Total number of trials}}$$

It's a way to estimate **probability based on experimental data**, especially when theoretical probability is difficult to calculate.

Example:

Suppose you roll a die 100 times, and you get the number **4** exactly **18 times**.

$$\text{Relative Frequency of getting a } 4 = \frac{18}{100} = 0.18$$

This suggests the probability of rolling a 4 is approximately **0.18**, based on actual data.

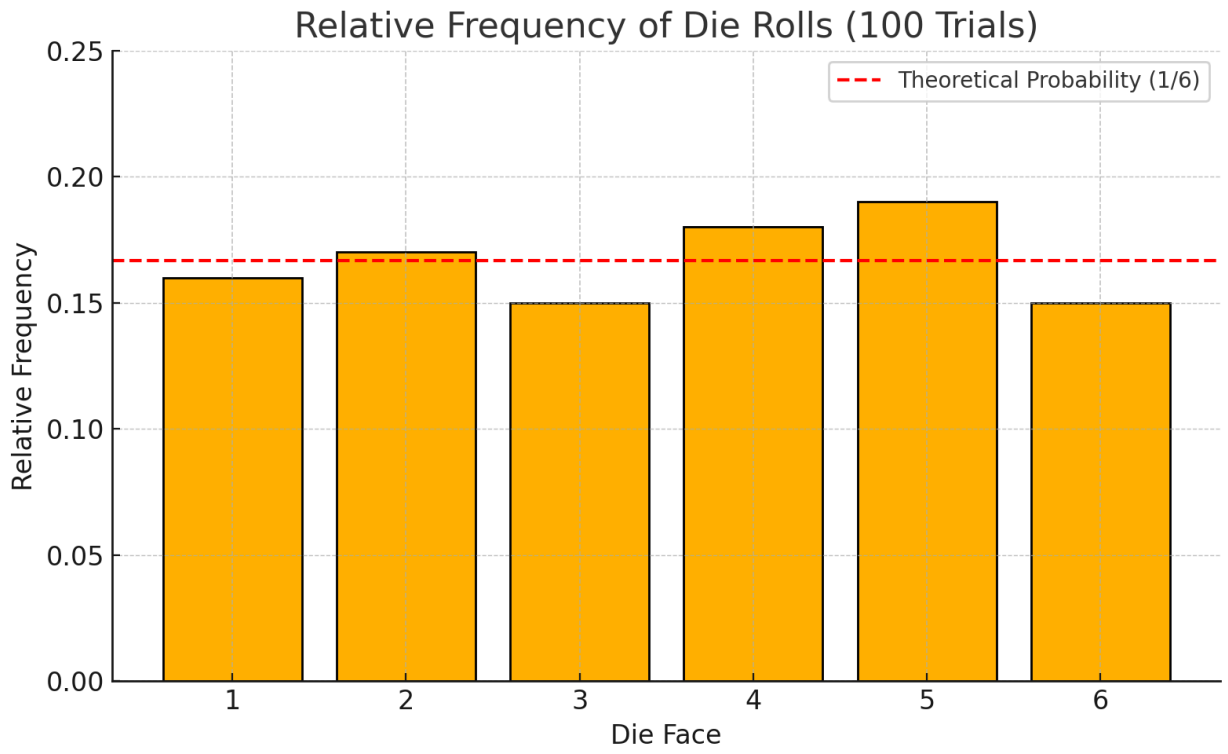
Visual Explanation:

Let's say we roll a die repeatedly and track how often each number occurs:

Die Face	Count	Relative Frequency
1	16	0.16
2	17	0.17
3	15	0.15
4	18	0.18
5	19	0.19

6	15	0.15
Total	100	**1.00**

The relative frequency can be **visualized in a bar chart** to see how close it is to the theoretical probability (which is 1/6 ≈ 0.167 for each face).



Here is the bar chart showing the **relative frequency** of each die face from 100 trials, along with the red dashed line representing the **theoretical probability** (1/6 ≈ 0.167). This visual helps compare experimental results with theoretical expectations.

Breaking down the probability terminologies

1. Experiment / Trial

- **Definition:**
  An **experiment** (or **trial**) is a process or action that produces a set of results or outcomes.

- **Examples:**
  - Tossing a coin once → one trial
  - Rolling a die
  - Drawing a card from a deck

2. Outcome

- **Definition:**
  An **outcome** is a **single possible result** of an experiment.

- **Examples:**
  - Tossing a coin → "Heads" is one outcome, "Tails" is another
  - Rolling a die → "4" is one outcome

3. Event

- **Definition:**
  An **event** is a **set of one or more outcomes** from an experiment.

- **Types of Events:**

  - **Simple Event:** Involves a single outcome (e.g., rolling a 6)

  - **Compound Event:** Involves multiple outcomes (e.g., rolling an even number → 2, 4, or 6)

- **Example:**
  Event A: Rolling a number less than 4 → outcomes: {1, 2, 3}

4. Sample Space (S)

- **Definition:**
  The **sample space** is the **set of all possible outcomes** of an experiment.

- **Notation:** Usually denoted by **S**

- **Examples:**

  - Tossing a coin:
    S = {Heads, Tails}
  - Rolling a die:
    S = {1,2,3,4,5,6}
  - Drawing a card from a deck:
    S = {52 cards}

Summary Table:

Term	Definition	Example
Experiment/Trial	A process that yields outcomes	Tossing a coin
Outcome	A single result of an experiment	"Tails" when tossing a coin
Event	One or more outcomes	Rolling an even number → {2, 4, 6}
Sample Space	All possible outcomes	Die roll → {1, 2, 3, 4, 5, 6}

Union and Intersection in Probability

These are two fundamental concepts used to combine **events** from a sample space.

1. Union (A ∪ B)

- **Definition:**
  The **union** of two events A and B is the event that **either A occurs, B occurs, or both occur**.

- **Symbol:** $A \cup B$
- **Formula:**
  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- **Example:**
  Let's roll a die:
    - Event A: getting an even number → {2, 4, 6}
    - Event B: getting a number greater than 4 → {5, 6}
    - $A \cup B$ = {2, 4, 5, 6}

2. Intersection ($A \cap B$)

- **Definition:**
  The **intersection** of two events A and B is the event that **both A and B occur at the same time**.

- **Symbol:** $A \cap B$
- **Formula:**
  If A and B are **independent**, $P(A \cap B) = P(A) \times P(B)$
- **Example:**
  Using the same die roll:
    - A = {2, 4, 6}
    - B = {5, 6}

- A ∩ B = {6} (Only 6 is common in both)

Mutually exclusive events, independent events, and complementary events

1. Mutually Exclusive Events (Disjoint Events)

- **Definition:**
  Two events are **mutually exclusive** if **they cannot occur at the same time**.

2. Independent Events

- **Definition:**
  Two events are **independent** if the **occurrence of one does not affect the occurrence** of the other.

3. Complementary Events

- **Definition:**
  The **complement** of an event A is the event that A **does not happen**.