a) After 1st iteration $i = 2 \cdot 2$

After 2nd iteration $i = 4$

After 3rd iteration $i = 16$

4th $((2)^2)^2)^2$

5th $(((2^2)^2)^2)^2$

$\sum_{i=2}^{\log(\log n)} \rightarrow$ loop time

$\Theta(1) = \Theta(\log(\log n))$

The runtime would be $\log(\log n)$

b)

The outer loop increments by one each time so it will iterate $n$ times.

The if statement will be triggered $i / \sqrt{n} == 0$

so runtime of the statement $= \sqrt{n}$, the inside loop will iterate $i^3$ times in the inner loop.

$T(n) = \Sigma_{i=0}^{n} \Theta(1) + \Sigma_{i=1}^{\sqrt{n}} (1 + \Sigma_{i=1}^{i^3} \Theta(1))$

$= \Theta(n) + \Sigma_{i=1}^{\sqrt{n}} (\Theta(1) + \Theta(n^3))$

$= \Theta(n^{\frac{7}{2}})$

c). The outer loop incurements by one each time so it will iterate n times.

- same as the first inner loop.
- Since given that the content of A[ ] array does not change, the run-time analysis does not change. so the inner loop will run $\Theta(\log n)$ because increasing at exponential speed.

$T(n) = \sum_{i=0}^{n} \sum_{k=0}^{n} \Theta(1) + \Theta(\log n)$

$= \Theta(n^2)$

d)

| 1 | 2 | 3 | ... | k |
|---|---|---|---|---|
| 10 | $10(\frac{3}{2})$ | $10(\frac{3}{2})^2$ | .. | $10(\frac{3}{2})^{k-1}$ |

$n = 10\left(\frac{3}{2}\right)^{k-1}$

$\log_{\frac{3}{2}}\left(\frac{n}{10}\right) = k-1$

$k = 1 + \log_{\frac{3}{2}}\left(\frac{n}{10}\right) = \Theta\left(\log_{\frac{3}{2}}\left(\frac{n}{10}\right)\right)$

If statement would be trigger (i==size).

The outer loop $= \sum_{n}^{n} \Theta(1) = \Theta(n)$ first / second line $\Theta(1) \cdot 2$

Total runtime $= \Theta(n) + \Theta(1) + \sum_{k=1}^{\log_{\frac{3}{2}}(\frac{n}{10})} \Theta\left(10\left(\frac{3}{2}\right)^{k-1}\right)$

$= \Theta(n) + \Theta(1) + \Theta\left(10 \cdot \left(\frac{3}{2}\right)^{\log_{\frac{3}{2}}(\frac{n}{10}) - 1}\right)$

$= \Theta(n)$