1. $C(7,5) = 7!/(5!\cdot 2!) = 21$ unique subsets

$P(7,5) = 7!/2! = 2520$ different strings

2.

\# ways of 5 card with 2 pair

$= \binom{13}{2}\binom{4}{2}^2\cdot\binom{11}{1}\binom{4}{1} = 123552$

\# ways of 5 card with 2 same color pair

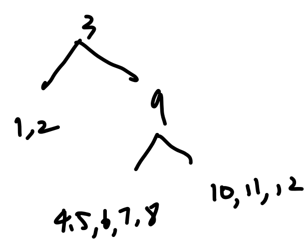
$= \binom{13}{2}\binom{2}{1}\binom{2}{1}\cdot(52-8) = 13728$

3.

\# ways $= 2^{11}\times 2\times 1$ after A pick the team, B has only one option $= 4096$

4. \# ways $= \binom{16-1-1}{7-1-1} = 2002$

5.



\# ways for 1,2 $= 2$

for 4,5,6,7,8 $= 42$

for 10,11,12 $= 5$

$\Rightarrow 2\cdot 42\cdot 5 = 420.$

6. case 1: on a break.

\# patient $= \binom{3-1}{2} + \binom{4-1}{2} + \cdots + \binom{10-1}{2} = 120$

case 2: not on break

\# patient $= \binom{4-1}{3} + \cdots + \binom{10-1}{3} = 210$

$\Rightarrow$ \# $= 120 + 210 = 330$

1. $p = \frac{21!}{21^{13}(21-13)!} = 0.0082$

2. # from $100 - 1000 = 5 \cdot 4 \cdot 5 = 100$

$1000 - 10000 = 5 \cdot 4 \cdot 7 \cdot 5 = 700$

$10000 - 99999 = 5 \cdot 4 \cdot 7 \cdot 6 \cdot 5 = 4200$

$P(\text{get desired number}) = \frac{5000}{10^5} = 0.05$

$P = \binom{10}{7}(0.05)^7 \cdot (1-0.05)^3 = 8.04 \cdot 10^{-8}$

3. $P(A) = \frac{3 + \binom{6}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)}{6^3} = 0.0156$

$P(B) = \frac{6}{6^3} = 0.0278$

$P(A \cap B) = \frac{3}{6^3} = 0.000064$

$P(A) \cdot P(B) = 0.000437 \neq P(A \cap B)$

$\Rightarrow P(A \cap B) \neq P(A)P(B)$

$\Rightarrow$ not independent.

4. # straights $= 10 \cdot 4^5$

hands $= \binom{52}{5}$

$\Rightarrow p = 0.00394$

$E(x) = \frac{1}{p} \approx 254$

5. $P(\text{superstar} \mid \text{win } 3/5)$

$= \frac{P(\text{win } 3/5 \mid \text{superstar})\, P(\text{superstar})}{P(\text{win } 3/5)}$

$= \frac{\binom{5}{3}(0.75)^3(0.25)^2 \cdot 0.65}{0.65 \cdot \binom{5}{3}(0.75)^3(0.25)^2 + 0.35\binom{5}{3}(0.4)^3(0.6)^2} = 0.68$