Total points: 100 points

Name: ____________________

Number: ____________________

AMS 161–F20, HW Assignment 3

Due Date: Friday 09/25/2020
You MUST show all your work in order to get full credits

Question:	1	2	3	4	5	6	7	Total
Points:	26	10	12	18	8	16	10	100
Score:								

In this assignment, you will **modify the provided codes** in this file on the Jupyter notebook. You may download the file and work on it using `Anaconda Distribution` (local host) or `CoCalc`.

In order to be accepted, you **MUST** submit the two following files:

a. A single PDF file showing all your supporting work with the name:

Number-FirstName-LastName-AMS161-HW3.pdf

b. A file on Jupyter notebook including all coding assignments with the name:

Number-FirstName-LastName-AMS161-HW3.ipynb

In this HW, "Hand" stands for solving by hand and "Jupy" stands for solving by `Jupyter Notebook.`

1. (Hand) **Substitution Method**

   (a) Evaluate the following integrals. (10)

   i. $\displaystyle\int x^3(2+x^4)^5\,dx$

   ii. $\displaystyle\int \frac{dt}{(1-6t)^4}$

   iii. $\displaystyle\int \frac{\sec^2(1/x)}{x^2}\,dx$

   iv. $\displaystyle\int (3t+2)^{2.4}\,dt$

   v. $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

   vi. $\displaystyle\int \sec 2\theta\tan 2\theta\,d\theta$

   vii. $\displaystyle\int \frac{\tan^{-1}x}{1+x^2}\,dx$

   viii. $\displaystyle\int \frac{\sin(\ln x)}{x}\,dx$

   ix. $\displaystyle\int \frac{\sin x}{1+\cos^2 x}\,dx$

   x. $\displaystyle\int \frac{x}{1+x^4}\,dx$

   (b) Evaluate the definite integrals. (16)

   i. $\displaystyle\int_0^1 (3t-1)^{50}\,dt$

   ii. $\displaystyle\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx$

   iii. $\displaystyle\int_{1/6}^{1/2} \csc\pi t\cot\pi t\,dt$

   iv. $\displaystyle\int_0^{\pi/2} \cos x\sin(\sin x)\,dx$

   v. $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{x^2\sin x}{1+x^6}\,dx$

   vi. $\displaystyle\int_0^a x\sqrt{a^2-x^2}\,dx$

   vii. $\displaystyle\int_0^{1/2} \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

   viii. $\displaystyle\int_0^{T/2} \sin(2\pi t/T-\alpha)\,dt$

   You may write a Python code to test your answers.

2. **Theoretical Problems**

   (a) If f is continuous and $\int_0^9 f(x)\,dx = 4$, find $\int_0^3 xf(x^2)\,dx$. (2)

   (b) If f is continuous, prove that (2)

$$\int_0^{\pi/2} f(\cos x)\,dx = \int_0^{\pi/2} f(\sin x)\,dx.$$

   (c) Use part (b) to evaluate $\int_0^{\pi/2}\cos^2 x\,dx$ and $\int_0^{\pi/2}\sin^2 x\,dx$. (2)

   (d) Use part (b) to evaluate $\displaystyle\int_0^{\pi/2}\frac{\sin x}{\sin x+\cos x}\,dx$ and $\displaystyle\int_0^{\pi/2}\frac{\cos x}{\sin x+\cos x}\,dx$. (2)

   (e) Find possible formulas for $f(x)$ and $g(x)$ given that (2)

$$\int f(g(x))g(x)\cdot\frac{1}{\sqrt{x}}dx = \sin\left(e^{\sqrt{x}}\right)+C.$$

3. (Hand) **Substitution Method** (12)

Evaluate the following integrals. Your answer should not contain f, which is a differentiable function with the following values:

x	0	1	$\pi/2$	e	3
$f(x)$	5	7	8	10	11
$f'(x)$	2	4	6	9	12

(a) $\displaystyle\int_0^1 f'(x)\sin f(x)dx$

(b) $\displaystyle\int_1^3 f'(x)e^{f(x)}dx$

(c) $\displaystyle\int_1^3 \frac{f'(x)}{f(x)}dx$

(d) $\displaystyle\int_0^1 e^x f'(e^x)dx$

(e) $\displaystyle\int_1^e \frac{f'(\ln x)}{x}dx$

(f) $\displaystyle\int_0^1 f'(x)(f(x))^2dx$

4. (Hand) **Integration By Parts** (18)

Evaluate the following integrals

(a) $\displaystyle\int xe^{-x}\,dx$

(b) $\displaystyle\int e^{\sqrt{x}}\,dx$

Hint: Use the substitution first $w=\sqrt{x}$.

(c) $\displaystyle\int \sin^{-1}x\,dx$

(d) $\displaystyle\int t^3e^{t^2}\,dt$

(e) $\displaystyle\int \theta^5\cos\theta^3\,d\theta$

(f) $\displaystyle\int_1^{\sqrt{3}} \arctan(1/x)\,dx$

You may write a Python code to test your answers.

5. (Hand) **Integration By Parts**

(a) Find possible formulas for $f(x)$ and $g(x)$ given that (4)

$$\int x^3g'(x)dx = f(x)g(x) - \int x^2\cos x dx.$$

(b) Suppose that $f(1)=2, f(4)=7, f'(1)=5, f'(4)=3$, and f'' is continuous. Find the value of $\displaystyle\int_1^4 xf''(x)\,dx$. (4)

6. (Hand: 50% & Jupy: 50%) **Applied Problems**

   (a) A model for the basal metabolism rate, in kcal/h, of a young man is (4)

   $$R(t) = 85 - 0.18\cos(\pi t/12)$$

   where t is the time in hours measured from 5:00 AM. What is the total basal metabolism of this man, $\int_0^{24} R(t)\, dt$, over a 24-hour time period?

   (b) Alabama instruments Company has set up a production line to manufacture a new calculator. The rate of production of these calculators after t weeks is (4)

   $$\frac{dx}{dt} = 5000\left(1 - \frac{100}{(t+10)^2}\right) \text{ calculators/week}$$

   (Notice that production approaches 5000 per week as time goes on, but the initial production is lower because of the workers' unfamiliarity with the new techniques.) Find the number of calculators produced from the beginning of the third week to the end of the fourth week.

   (c) A particle that moves along a straight line has velocity $v(t) = t^2e^{-t}$ meters per second after t seconds. How far will it travel during the first 2 seconds? (4)

   (d) A rocket accelerates by burning its onboard fuel, so its mass decreases with time. Suppose the initial mass of the rocket at liftoff (including its fuel) is m, the fuel is consumed at rate r, and the exhaust gases are ejected with constant velocity v_e (relative to the rocket). A model for the velocity of the rocket at time t is given by the equation (4)

   $$v(t) = -gt - v_e \ln\frac{m - rt}{m}$$

   where g is the acceleration due to gravity and t is not too large. If $g = 9.8$ m/s^2, $m = 30,000$ kg, $r = 160$ kg/s, and $v_e = 3000$ m/s, find the height of the rocket one minute after liftoff.

7. **Challenging Problem**

   (a) Use integration by parts to prove the reduction formula. (4)

   $$\int (\ln x)^n\, dx = x(\ln x)^n - n\int (\ln x)^{n-1}\, dx$$

   (b) Use part (a) to find $\int (\ln x)^3\, dx$. (6)

   (c) **(+4 Extra Credits)** Write a code to find $\int (\ln x)^n\, dx$ and use it to find $\int (\ln x)^{10}\, dx$ using the recursive formula in part (a).