Total points: 100 points

Name: ____________________

Number: ____________________

AMS 161–F20, HW Assignment 4

Due Date: Tuesday 10/20/2020
You MUST show all your work in order to get full credits

Question:	1	2	3	4	5	6	7	Total
Points:	22	10	10	13	17	20	8	100
Score:								

In this assignment, you will **modify the provided codes** in this file on the Jupyter notebook. You may download the file and work on it using `Anaconda Distribution` (local host) or `CoCalc`.

In order to be accepted, you **MUST** submit the two following files:

a. A single PDF file showing all your supporting work with the name:

Number-FirstName-LastName-AMS161-HW4.pdf

b. A file on Jupyter notebook including all coding assignments with the name:

Number-FirstName-LastName-AMS161-HW4.ipynb

In this HW, "Hand" stands for solving by hand and "Jupy" stands for solving by `Jupyter Notebook.`

For problems asking you to estimate the errors in approximating integral rules, you may need Theorem 2.8.3 and Theorem 2.8.7 in **Lecture 9**.

Error Bounds in the Trapezoidal and Midpoint Rules

Suppose $|f''(x)| \leq K$ for $a \leq x \leq b$. If E_T and E_M are the errors in the Trapezoidal and Midpoint Rules, then

$$|E_T| \leq \frac{K(b-a)^3}{12n^2} \quad \text{and} \quad |E_M| \leq \frac{K(b-a)^3}{24n^2}$$

Error Bound for Simpson's Rule

Suppose that $|f^{(4)}(x)| \leq K$ for $a \leq x \leq b$. If E_S is the error involved in using Simpson's Rule, then

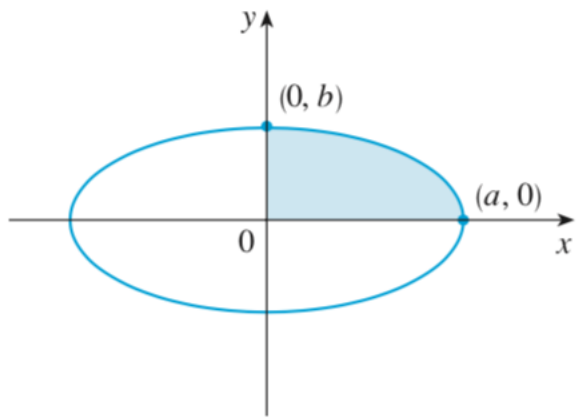
$$|E_S| \leq \frac{K(b-a)^5}{180n^4}.$$

1. (a) (Hand) Make a substitution to express the integrand as a rational function and then evaluate the integral.

i. $\displaystyle\int_9^{16} \frac{\sqrt{x}}{x-4}\,dx$ (4)

ii. $\displaystyle\int \frac{dx}{2\sqrt{x+3}+x}$ (4)

(b) (Hand) Find the area enclosed by the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ (see the below figure). (4)



Hint: You may solve the equation of the ellipse for y to get $y=\pm\frac{b}{a}\sqrt{a^2-x^2}$.

- Because the ellipse is symmetric with respect to both axes, the total area A is four times the area in the first quadrant.

- Thus the total area $A=\dfrac{4b}{a}\displaystyle\int_0^a \sqrt{a^2-x^2}\,dx$.

(c) (Hand) A rumor is spread in a school. For $0<a<1$ and $b>0$, the time t at which a fraction p of the school population has heard the rumor is given by

$$t(p)=\int_a^p \frac{b}{x(1-x)}\,dx$$

i. Evaluate the integral to find an explicit formula for $t(p)$. Write your answer so it has only one ln term. (4)

ii. At time $t=0$ one percent of the school population ($p=0.01$) has heard the rumor. What is a? (2)

iii. At time $t=1$ half the school population ($p=0.5$) has heard the rumor. What is b? (2)

iv. At what time has 90% of the school population ($p=0.9$) heard the rumor? (2)

2. Draw the graph of $f(x) = \sin\left(\frac{1}{2}x^2\right)$ in the viewing rectangle $[0,1]$ by $[0,0.5]$ and let $I = \int_0^1 f(x)\,dx$.

   (a) (Hand & Jupy) Use the graph to decide whether L_2, R_2, M_2, and T_2 underestimate or overestimate. Please record your answers by filling out the following table. (4)

	L_2	R_2	M_2	T_2
Overestimates				
Underestimates				

(Check "X" if applicable)

   (b) (Hand) For any value of n, list the numbers L_n, R_n, M_n, T_n, and I in increasing order. (2)

   (c) (Hand & Jupy) Compute L_5, R_5, M_5, and T_5. From the graph, which do you think gives the best estimate of I? (4)

3. (Jupy) Use

   - the Trapezoidal Rule
   - the Midpoint Rule, and
   - the Simpson's Rule

   to approximate the given integral with the specified value of n. (Round your answers to six decimal places.)

   (a) $\displaystyle\int_0^{1/2} \sin(x^2)\,dx, \quad n = 4$ (2)

   (b) $\displaystyle\int_0^3 \frac{dt}{1+t^2+t^4}, \quad n = 6$ (2)

   (c) $\displaystyle\int_0^4 \sqrt{1+\sqrt{x}}\,dx, \quad n = 8$ (2)

   (d) $\displaystyle\int_0^4 \cos\sqrt{x}\,dx, \quad n = 10$ (2)

   (e) $\displaystyle\int_4^6 \ln(x^3+2)\,dx \quad n = 10$ (2)

4. (a) (Jupy) Find the approximation T_8 and M_8 for the integral $\int_0^1 \cos(x^2)\,dx$. (2)

   (b) (Hand & Jupy) Estimate the errors in the approximations of part (a). (2)

   (c) (Jupy) How large do we have to choose n so that the approximations T_n and M_n to the integral in part (a) are accurate to within

      i. 0.01? (3)

      ii. 0.001? (3)

      iii. 0.0001? (3)

5. (a) (Jupy) Find the approximation T_{10}, M_{10}, and S_{10} for $\int_0^\pi \sin x \; dx$ and the corresponding errors E_T, E_M, and E_S. (6)

   (b) (Hand & Jupy) Compare the actual errors in part (a) with the error estimates given by Theorem 2.8.3 and Theorem 2.8.7 in **Lecture 9**. (2)

   (c) (Jupy) How large do we have to choose n so that the approximations T_n, M_n, and S_n to the integral in part (a) are accurate to within

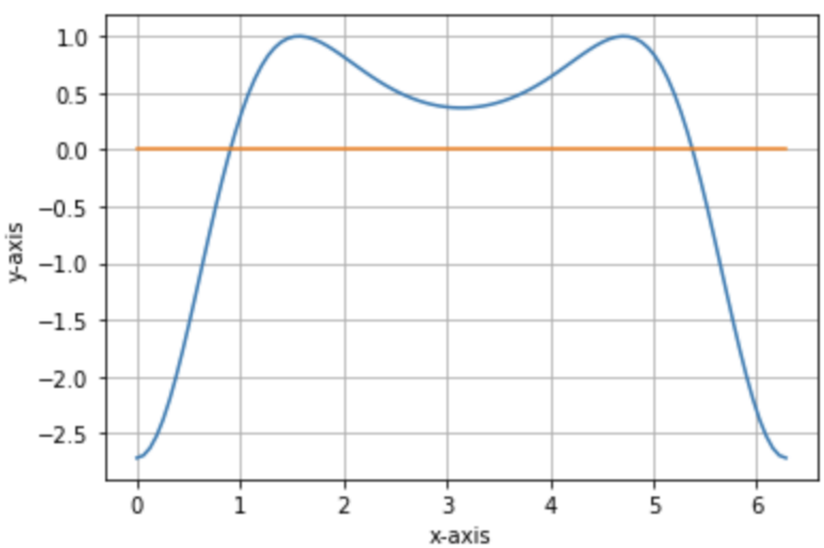
      i. 0.001? (3)

      ii. 0.0001? (3)

      iii. 0.00001? (3)

6. (Hand & Jupy) *In this question, you have to modify the code in Jupy and write down your answers in your own papers. There is no need to include the graphs in the papers.*
   The trouble with the error estimates is that it is often very difficult to compute four derivatives and obtain a good upper bound K for $|f^{(4)}(x)|$ by hand. But `Python` has no problem computing $f^{(4)}$ and graphing it, so we can easily find a value of K. This exercise deals with approximations to the integral $I = \int_0^{2\pi} f(x)\, dx$, where $f(x) = e^{\cos x}$.

   (a) Use a graph to get a good upper bound for $|f''(x)|$. (2)
   Running the code in Jupy gives us the following graph.



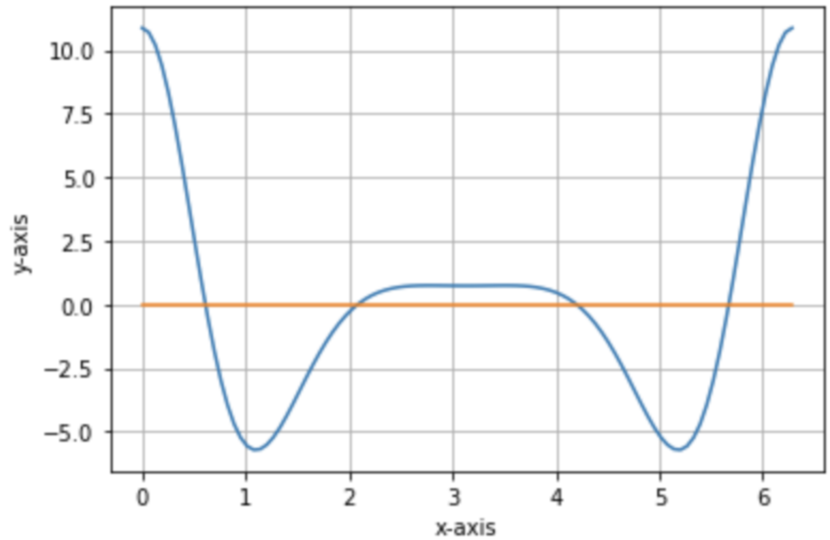
   (b) Use M_{10} to approximate I. (2)

   (c) Use part (a) to estimate the error in part (b). (2)

   (d) Use the built-in numerical integration in `Python` to approximate I. (2)

   (e) How does the actual error compare with the error estimate in part (c)? (2)

   (f) Use a graph to get a good upper bound for $|f^{(4)}(x)|$. (2)
   If your code runs successfully, you will get the following graph.



(g) Use S_{10} to approximate I. (2)

(h) Use part (f) to estimate the error in part (g). (2)

(i) How does the actual error compare with the error estimate in part (h)? (2)

(j) How large should n be to guarantee that the size of the error in using S_n is less than 0.0001? (2)

7. (Hand & Jupy) The table (supplied by San Diego Gas and Electric) gives the power consumption P in megawatts in San Diego Country from midnight to $6:00$ AM on a day in December. Use Simpson's Rule to estimate the energy used during that time period. (Use the fact that the power is the derivative of energy.) (8)

t	P	t	P
0:00	1814	3:30	1611
0:30	1735	4:00	1621
1:00	1686	4:30	1666
1:30	1646	5:00	1745
2:00	1637	5:30	1886
2:30	1609	6:00	2052
3:00	1604		