Total points: 100 points

Name: ____________________________

Number: ____________________________

AMS 161–F20, HW Assignment 2

Due Date: Friday 09/18/2020
You MUST show all your work in order to get full credits

Question:	1	2	3	4	5	6	7	Total
Points:	20	18	12	12	16	10	12	100
Score:								

In this assignment, you will **modify the provided codes** in this file on the Jupyter notebook. You may download the file and work on it using `Anaconda Distribution` (local host) or `CoCalc`.

In order to be accepted, you **MUST** submit the two following files:

a. A single PDF file showing all your supporting work with the name:

Number-FirstName-LastName-AMS161-HW2.pdf

b. A file on Jupyter notebook including all coding assignments with the name:

Number-FirstName-LastName-AMS161-HW2.ipynb

In this HW, "Hand" stands for solving by hand and "Jupy" stands for solving by `Jupyter Notebook.`

1. **Understanding Evaluating Definite Integrals**

   The **Evaluation Theorem** says that:

   If f is continuous on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

   where F is any antiderivative of f, that is $F' = f$.

   (a) (Hand) Explain the importance of the Evaluation Theorem. (2)

   (b) (Hand) Evaluate the following integrals using the Evaluation Theorem.

   i. $\displaystyle\int_0^2 \left(x^4 - \frac{3}{4}x^2 + \frac{2}{3}x - 1\right)\,dx$ (2)

   ii. $\displaystyle\int_0^1 x\left(\sqrt[3]{x} + \sqrt[4]{x}\right)\,dx$ (2)

   iii. $\displaystyle\int_0^{\pi/3} \frac{\sin\theta + \sin\theta\tan^2\theta}{\sec^2\theta}\,d\theta$ (2)

   iv. $\displaystyle\int_0^2 |2x - 1|\,dx$ (4)

   v. $\displaystyle\int_0^{3\pi/2} |\sin x|\,dx$ (4)

   vi. $\displaystyle\int_1^8 \left(f^{-1}(x^{-1}) + f'(x)\right)dx,$ (4)

   where $f(x) = 8x^{-3}$ and f^{-1} denotes the inverse function of f.

2. **Indefinite Integral/General Antiderivative**

   (a) (Jupy & Hand) Verify by differentiation that the formula is correct.

   i. $\displaystyle\int \cos^3 x\,dx = \sin x - \frac{1}{3}\sin^3 x + C$ (2)

   ii. $\displaystyle\int x\cos x\,dx = x\sin x + \cos x + C$ (2)

   (b) (Jupy & Hand) Find the general indefinite integral.

   i. $\displaystyle\int v(v^2 + 2)\,dv$ (3)

   ii. $\displaystyle\int \frac{\sin x}{1 - \sin^2 x}\,dx$ (3)

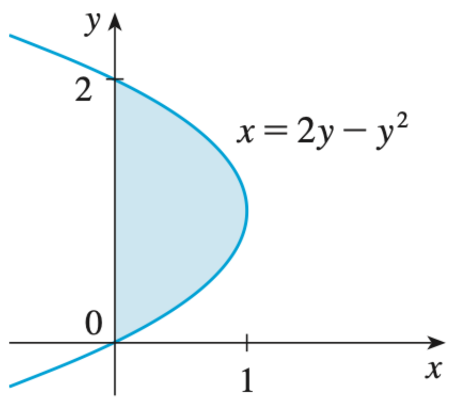
   (c) (Jupy & Hand) Find the general indefinite integral. Illustrate by graphing several members of the family on the same screen.

   i. $\displaystyle\int \left(\cos x + \frac{1}{2}x\right)\,dx$ (4)

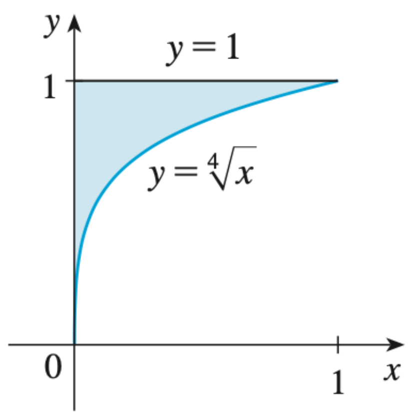
   ii. $\displaystyle\int \left(e^x - 2x^2\right)\,dx$ (4)

3. (Hand) **Applications of Definite Integrals**

(a) The area of the region that lies to the right of the y-axis and to the left of the parabola $x = 2y - y^2$ (the shaded region in the figure) is given by the integral $\int_0^2 (2y - y^2)\,dy$. (Turn your head clockwise and think of the region as lying below the curve $x = 2y - y^2$ from $y = 0$ to $y = 2$). Find the area of the region. (4)



(b) The boundaries of the shaded region are the y-axis, the line $y = 1$, and the curve $y = \sqrt[4]{x}$. Find the area of this region by writing x as a function of y and integrating with respect to y (as in part (a)). (4)

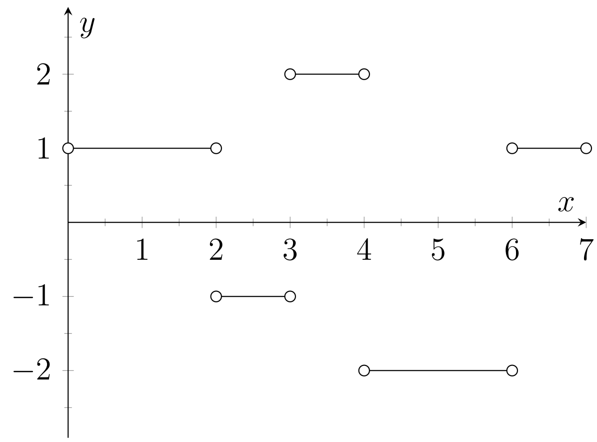


(c) The marginal cost of manufacturing x yards of a certain fabric is (4)

$$C'(x) = 3 - 0.01x + 0.000006x^2$$

(in dollars per yard). Find the increase in cost if the production level is raised from 2000 yards to 4000 yards.

4. (Hand) **Visualizing Antiderivative**
   Assume that f' is given by the graph in the below figure. Suppose f is continuous and that $f(3) = 0$.



   (a) Find $f(0), f(2), f(3), f(4), f(6)$, and $f(7)$. (6)

   (b) Sketch a graph of f. (4)

   (c) Find $\int_0^7 f'(x)\,dx$ in two different ways. (2)

5. (Hand) **Understanding the Fundamental Theorem of Calculus**

   (a) Find the derivative of the following funcions.

   i. $g(x) = \int_1^x \frac{1}{t^3+1}\,dt$ (2)

   ii. $h(y) = \int_2^{1/y} \arctan t\,dt$ (2)

   iii. $g(x) = \int_{2x}^{3x} \frac{u^2-1}{u^2+1}\,du$ (3)

   iv. $g(x) = \int_1^x \cos^2(t)\,dt + \int_1^{x^2} \frac{\sin^2(\sqrt{t})}{2\sqrt{t}}\,dt$, where x is positive. (3)

   (b) (Hand) The **error function**

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt$$

   is used in probability, statistics, and engineering.

   i. Show that $\int_a^b e^{-t^2}\,dt = \frac{1}{2}\sqrt{\pi}[\operatorname{erf}(b) - \operatorname{erf}(a)]$. (2)

   ii. Show that the function $y = e^{x^2}\operatorname{erf}(x)$ satisfies the differential equation (4)

$$y' = 2xy + 2/\sqrt{\pi}.$$

6. The **sine integral function**[1]

$$\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}\, dt$$

is important in electrical engineering.[The integrand $f(t) = \sin t/t$ is not defined when $t = 0$, but we know that its limit is 1 when $t \to 0$. So we define $f(0) = 1$ and this makes f a continuous function everywhere.]

(a) (Jupy) Draw the graph of Si. (Just run the code in `Jupyter Notebook`.) (2)

(b) (Hand & Jupy) At what values of x does this function have local maximum values? Find this local maximum value in `Jupyter Notebook`. (2)

(c) (Hand) Find the coordinates of the first inflection point to the right of the origin. (2)

(d) (Hand) Does this function have horizontal asymptotes? (2)

(e) (Hand & Jupy) Solve the following equation correct to one decimal place: (2)

$$\int_0^x \frac{\sin t}{t} = 1.$$

7. (Hand & Jupy) Let

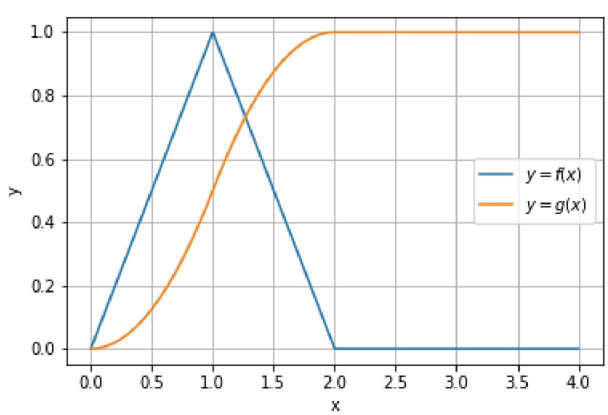
$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \le x \le 1 \\ 2 - x & \text{if } 1 < x \le 2 \\ 0 & \text{if } x > 2 \end{cases}$$

and

$$g(x) = \int_0^x f(t)\, dt.$$

(a) (Hand) Find an expression for $g(x)$ similar to the one for $f(x)$. (6)

(b) (Jupy) Sketch the graphs of f and g. If your code runs successfully, you will get the following graph. (4)



(c) (Hand) Where is f differentiable? Where is g differentiable? (2)

[1] See more about trigonometric integral functions here.