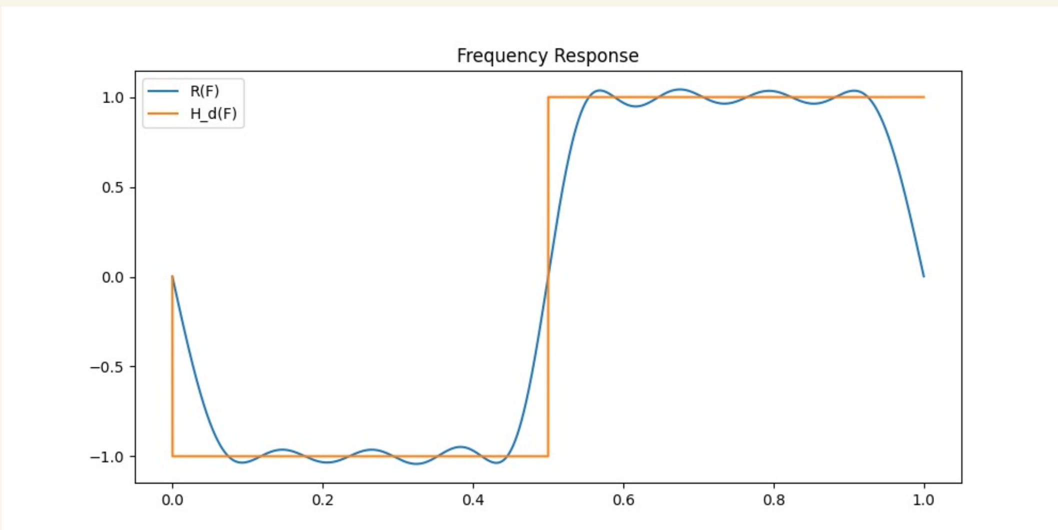
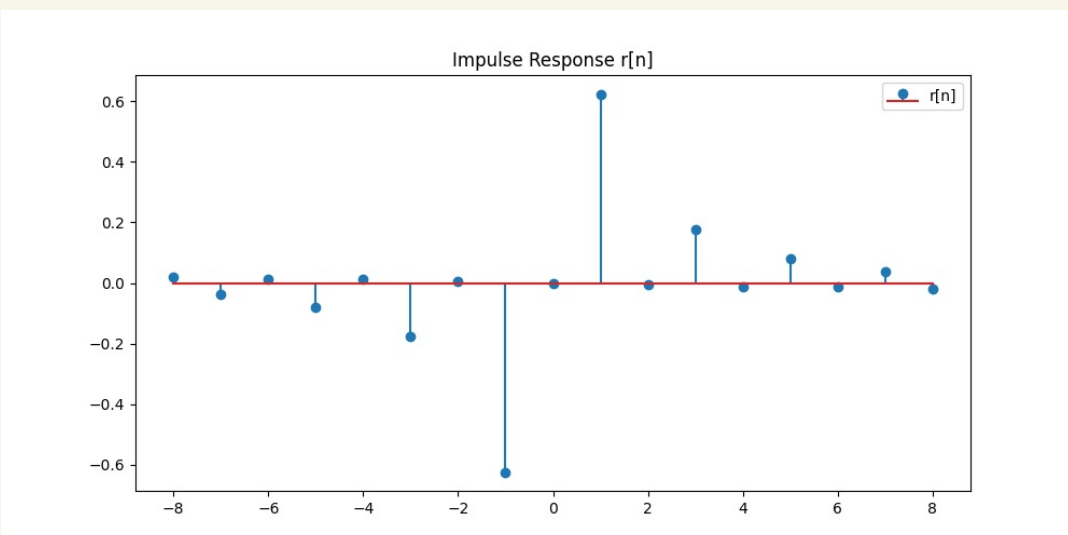
(1) Write a Matlab or Python code that uses the frequency sampling method to design a (2*k*+1)-point discrete Hilbert transform filter (*k* is an input parameter and can be any integer). (25 scores)

The transition band is assigned to reduce the error (unnecessary to optimize). (i) The impulse response and (ii) the imaginary part of the frequency response (DTFT of *r*[*n*], see pages 113 and 114) of the designed filter should be shown in the homework. The code should be handed out by NTU Cool.

set $k=8$, transition band: $\frac{1}{2k+1}$ $\frac{k}{2k+1}$ $\frac{k+1}{2k+1}$ $\frac{2k}{2k+1}$

$-0.9i$ $-0.7i$ $0.7i$ $0.9i$





(2) Estimate <u>the length of the digital filter</u> if both the passband ripple and the stopband ripple are smaller than 0.01, the sampling interval $\Delta_t = 0.00005$, and the transition band is from 5000Hz to 6000Hz. (10 scores)

$\Delta t = 0.00005 \Rightarrow f_s = 20000$, $\delta_1 = \delta_2 = 0.01$

$\Delta F = \dfrac{f_1 - f_2}{f_s} = \dfrac{6000 - 5000}{20000} = 0.05$

$N = \dfrac{2}{3} \dfrac{1}{\Delta f} \log_{10} \dfrac{1}{10 \delta_1 \delta_2} = \underline{40}$ #

(3) Why it is improper to use the method of $y[n] = \mathrm{IDFT}(\mathrm{DFT}(x[n])H[m])$ for FIR filter design? (5 scores)

在 frequency domain 做 sampling 会因為 window effect 產生 Gibbs phenomenon，使得 frequency response 不会跟設計的一樣

(4) Derive the way to use the algorithm on page 58-61 to implement an odd symmetric filter with even length (i.e., type 4 on page 90). (10 scores)

type 4 :

(5) Suppose that $x[n] = 1+\sin(n)$. (a) What is the Hilbert transform of $x[n]$? (b) What is the analytic function corresponding to $x[n]$? (10 scores)

$$x[n] = 1+\sin(n)$$

$$X(F) = \delta(F) - \frac{j}{2}\left[\delta\left(F-\frac{1}{2\pi}\right) - \delta\left(F+\frac{1}{2\pi}\right)\right]$$

$$X_H(F) = \delta(F)\cdot 0 - \frac{j}{2}\left(\delta\left(F-\frac{1}{2\pi}\right)(-j) - \delta\left(F+\frac{1}{2\pi}\right)j\right)$$

$$= -\frac{1}{2}\delta\left(F-\frac{1}{2\pi}\right) - \frac{1}{2}\delta\left(F+\frac{1}{2\pi}\right)$$

$$x_H(n) = -\frac{1}{2}\exp(jn) - \frac{1}{2}\exp(-jn) = -\cos(n)$$

$$x_a(n) = 1+\sin(n) + j(-\cos(n)) = 1+\sin(n) - j\cos(n)$$

(6) Among the following filters: (i) the Notch filter (ii) the Hilbert transform, (iii) the matched filter, (iv) the difference, (v) the Kalman filter, (vi) the particle filter, and (vii) the Wiener filter,

(a) Which filters are suitable for <u>edge detection</u>? (b) Which filters are suitable for <u>prediction</u>? (10 scores)

(a)

(ii) (iii) (iv)

(b) (v) (vi) (vii)

(7) (a) What are the <u>two main advantages</u> of the minimum phase filter? (b) Compared to the equalizer, what are the <u>two main advantages</u> of the cepstrum to deal with the multipath problem? (10 scores)

(a) 1、minimal phase filter 跟他的 inversion 都是 stable & causal

2、能量会集中在 n=0 ⇒ n很大能量会接近 0

(b) 1、不需觀察不同路徑的延遲，直接看不同 path 在 cepstrum 上的效果並濾掉就好

2、cepstrum 不会是 unstable

(8) If the z-transform of $h[n]$ is $H(z)=\dfrac{1+z^{-1}-1.5z^{-2}+z^{-3}}{1-0.3z^{-1}-0.4z^{-2}}$

(a) Determine the cepstrum of $h[n]$.

(b) Convert the IIR filter into the minimum phase filter. (20 scores)

(a)

$$H(z)=\frac{1+z^{-1}-1.5z^{-2}+z^{-3}}{1-0.3z^{-1}-0.4z^{-2}}=\frac{(1+2z^{-1})(1-(0.5+0.5j)z^{-1})(1-(0.5-0.5j)z^{-1})}{(1-0.4z^{-1})(1+0.1z^{-1})}$$

$$=\frac{2z^{-1}(1+0.5z^{-1})(1-(0.5+0.5j)z^{-1})(1-(0.5-0.5j)z^{-1})}{(1-0.4z^{-1})(1+0.4z^{-1})}$$

$$\hat{x}(n)=\begin{cases}\log(2), & n=0\\ \dfrac{(-0.5)^n}{n}, & n<0\\ -\dfrac{(0.5+0.5j)^n}{n}-\dfrac{(0.5-0.5j)^n}{n}+\dfrac{(0.4)^n}{n}+\dfrac{(-0.1)^n}{n}, & n>0\end{cases}$$

(b)