(2) Suppose that the probabilities of Chinese characters can be modeled as

$$P[n] = (\exp(0.002) - 1)\exp(-0.002n) \qquad n = 1, 2, 3, \cdots\cdots, 80000$$

(a) Determine the entropy of the Chinese characters. (b) Estimate the range of the coding length if we use the <u>Huffman code</u> to encode 10^5 Chinese characters using binary numbers. (c) Estimate the range of the coding length if we use the <u>arithmetic code</u> to encode 10^5 Chinese characters using binary numbers.

(15 scores)

(a) $\text{entropy} = \sum_{n=1}^{80000} P[n] \ln \frac{1}{P[n]} \doteq 7.2146$

(b) $\lfloor N \frac{\text{entropy}}{\ln k} \rfloor \le \text{mean len} \le \lceil N \cdot \frac{\text{entropy}}{\ln k} + N \rceil$

$\Rightarrow \lfloor 10^5 \frac{7.2146}{\ln 2} \rfloor \le \text{mean len} \le \lceil 10^5 \frac{7.2146}{\ln 2} + 10^5 \rceil$

$\Rightarrow 1040847 \le \text{mean len} \le 1140846$

(c) $\lfloor N \frac{\text{entropy}}{\ln k} \rfloor \le \text{mean len} \le \lceil N \frac{\text{entropy}}{\ln k} + \log_k 2 + 1 \rceil$

$\Rightarrow 1040847 \le \text{mean len} \le 1040848$

(4) What is the complexity of the $M\times N\times P$-point 3D DFT? The deriving process should be given. (10 scores)

1D DFT $\Rightarrow O(L\log L)$

1st dim
- 1D DFT $\Rightarrow N\cdot P$
- each 1D DFT $\Rightarrow O(M\log M)$
- total $\Rightarrow N\cdot P\, O(M\log M)$

2nd dim $\Rightarrow M\cdot P\cdot O(N\log N)$

3rd dim $\Rightarrow M\cdot N\, O(P\log P)$

All $\Rightarrow O(MNP\log MNP)$

(3) Suppose that x is a complex number. What are the constraints of θ such that the multiplication of x and $\exp(j\,\theta)$ required only 2 real multiplications? (10 scores)

$x = a + bj\ ,\ e^{j\theta} = \cos\theta + j\sin\theta = c + dj$

$$\begin{bmatrix} c & -d \\ d & c \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \underbrace{\begin{bmatrix} c & c \\ c & c \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}}_{1\text{ multi}} + \underbrace{\begin{bmatrix} 0 & -c-d \\ d-c & 0 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}}_{\text{must } 1\text{ multi}} = \begin{bmatrix} f \\ g \end{bmatrix}$$

$$\Rightarrow \begin{cases} -c-d=0 \text{ or } d-c=0 \\ f=0 \text{ or } g=0 \end{cases}$$

$\Rightarrow \theta = 0$ or $\dfrac{(2n+1)\pi}{4}$

(5) How do we implement the 4-point DST-I with the least number of nontrivial multiplications? The number of real multiplications should also be shown.

$$X[m] = \sum_{n=1}^{4} \sin\left(\frac{\pi}{5} mn\right) x[n] \qquad \begin{matrix} m = 1, 2, 3, 4 \\ n = 1, 2, 3, 4 \end{matrix} \qquad \text{(15 scores)}$$

$$\begin{bmatrix} X[1] \\ X[2] \\ X[3] \\ X[4] \end{bmatrix} = \begin{bmatrix} a & b & b & a \\ b & a & -a & -b \\ b & -a & -a & b \\ a & -b & b & -a \end{bmatrix} \begin{bmatrix} x[1] \\ x[2] \\ x[3] \\ x[4] \end{bmatrix} \qquad a = 0.5878, \quad b = 0.9511$$

(Hint: we can convert it into two 2x2 matrices.)

let $z_1 = ax(1) + bx[3]$, $z_3 = bx[1] - ax[3]$

$z_2 = bx(2) + ax(4)$, $z_4 = ax[2] - bx[4]$

$X(1) = ax(1) + bx[2] + bx[3] + ax[4] = z_1 + z_2$

$X[2] = bx[1] + ax[2] - ax(3) - bx[4) = z_3 + z_4$

$X[3) = bx(1] - ax[2) - ax[3] + bx[4] = z_3 - z_4$

$X[4) = ax[1] - bx(2) + bx[3] - ax[4) = z_1 - z_2$

$$\begin{bmatrix} z_1 \\ z_3 \end{bmatrix} = \underbrace{\begin{bmatrix} a & b \\ b & -a \end{bmatrix} \begin{bmatrix} x(1) \\ x[3] \end{bmatrix}}_{3}, \quad \begin{bmatrix} z_3 \\ z_4 \end{bmatrix} = \underbrace{\begin{bmatrix} b & a \\ a & -b \end{bmatrix} \begin{bmatrix} x(2) \\ x(4) \end{bmatrix}}_{3}$$

$\Rightarrow 6$

(6) Determining the numbers of real multiplications for the (a) 143-point DFT, (b) 195-point DFT, and the (c) 196-point DFT. (15 scores)

(a) $143 = 11 \times 13 \Rightarrow mul_{143} = 13 \cdot mul_{11} + 11\, mul_{13}$

$$= 13 \times 40 + 11 \times 52 = 1092$$

(b) $195 = 15 \times 13 = 3 \times 5 \times 13 \Rightarrow 13 \cdot (5 \cdot mul_3 + 3 \cdot mul_5) + 15\, mul_{13}$

$$= 1300$$

(c) $196 = 4 \times 49$

$$\Rightarrow 49 \cdot \underbrace{mul_4}_{0} + 4 \cdot mul_{49} = 4 \cdot (7 \cdot mul_7 + 7\, mul_7 + 3(7-1)(7-1))$$

$$= 1328$$

(7) Derive the transform matrices of the (a) forward and (b) inverse 5-point NTTs where the prime number M is 11 and the value of α should be as small as possible. (15 scores)