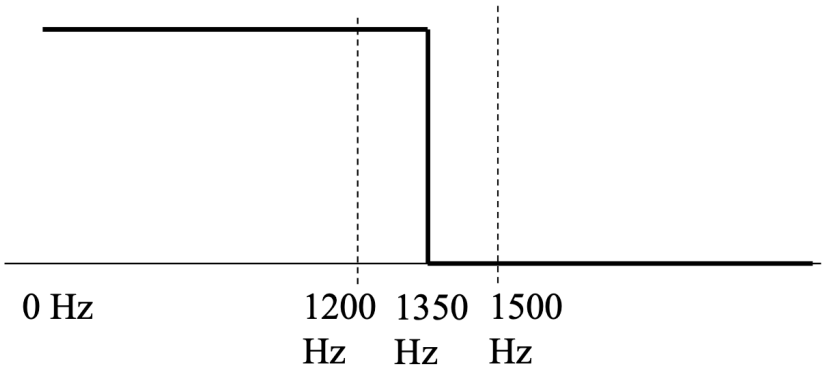
(1) Design a Mini-max **lowpass** FIR filter such that (40 scores)

① Filter length = 17, ② Sampling frequency f_s = 6000Hz,

③ Pass Band 0~1200Hz ④ Transition band: 1200~1500 Hz,

⑤ Weighting function: $W(F) = 1$ for passband, $W(F) = 0.6$ for stop band .

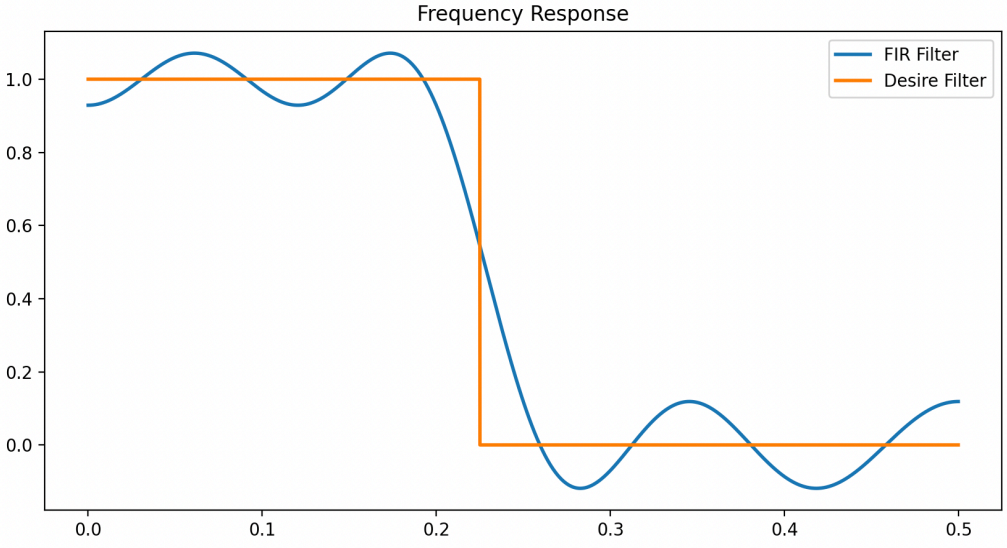
⑥ Set $\Delta = 0.0001$ in Step 5.



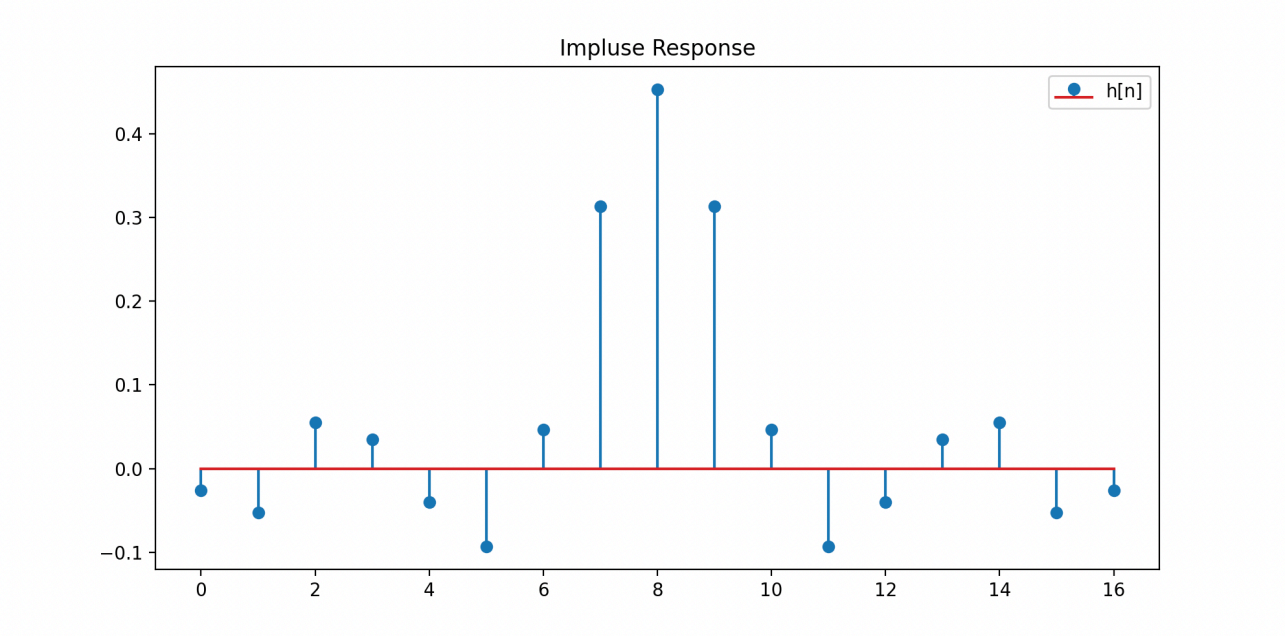
※ The code should be handed out by NTUCool, too.

Show (a) the frequency response, (b) the impulse response $h[n]$, and (c) the maximal error for each iteration.

(a) Frequency response



(b) Impulse response



(c) Maximum error for each iteration

1. 1.2362984401511334
2. 0.7704144356675191
3. 0.7127611617996479
4. 0.7120728523467945
5. 0.7120728523467945

(2) How do we implement $y[n] = x[n] * (0.8^n u[n] + 0.5^n u[n])$ efficiently where * means convolution and $u[n]$ is the unit step function? (10 scores)

$$H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} [0.8^n u[n] - 0.5^n u[n]] z^{-n}$$

$$= \sum_{n=0}^{\infty} (0.8^n - 0.5^n) z^{-n}$$

$$= \frac{1}{1-0.8z^{-1}} - \frac{1}{1-0.5z^{-1}} = \frac{1-\frac{0.5}{z} - (1-\frac{0.8}{z})}{(1-\frac{0.8}{z})(1-\frac{0.5}{z})} = \frac{\frac{0.3}{z}}{1-\frac{1.3}{z}+\frac{0.4}{z^2}}$$

$$Y(z) = X(z) \cdot H(z) = X(z) \cdot \boxed{\quad}$$

$$Y(z) = X(z) + \left[1 - \frac{1 - 1.3z^{-1} + 0.4z^{-2}}{0.3z^{-1}}\right] Y(z)$$

$$= X(z) + \left[\frac{0.3z^{-1} + 1.3z^{-1} - 0.4z^{-2}}{0.3z^{-1}}\right] Y(z)$$

$$= X(z) + \frac{-0.4z^{-2} + 1.6z^{-1} - 1}{0.3z^{-1}} Y(z)$$

$$\Rightarrow 0.3z^{-1}Y(z) = 0.3z^{-1}X(z) - Y(z) + 1.6z^{-1}Y(z) - 0.4z^{-2}Y(z)$$

$$\Rightarrow Y(z) = 0.3z^{-1}X(z) + 1.3z^{-1}Y(z) - 0.4z^{-2}Y(z)$$

$\Rightarrow y[n] = 0.3x[n-1] + 1.3y[n-1] - 0.4y[n-2]$

(3) (a) What are the two main advantages of the Fourier transform (FT)? (b) What are the two main problems to implement the FT? (10 scores)

(a)
1. sinusoidal functions are easy to analyze
2. FT has FFT algorithm to implement

(b)
…

(4) Suppose that $x[n] = y(0.002n)$ and the length of $x[n]$ is 2000. If $X[m]$ is the FFT of $x[n]$, which frequencies do (a) $X[200]$ and (b) $X[1600]$ correspond to? (10 scores)

$f_s = \frac{1}{0.002} = 500\,Hz$, $N = 2000$

(a) $f = 200 \cdot \frac{500}{2000} = 50\,Hz$

(b) since $1600 > \frac{N}{2} = 1000$, $f = 1600 \cdot \frac{500}{2000} - 500 = -100\,Hz$

(5) Why (a) the step invariance method and (b) the bilinear transform can reduce or avoid the aliasing effect in IIR filter design? (10 scores)

(a) step invariance 用積分將高頻的能量壓下來，所以能降低高頻的 aliasing

(b) bilinear 把 $(-\infty, \infty)$ mapping 到 $(-\frac{f_s}{2}, \frac{f_s}{2})$ 所以完全避免了 aliasing

(6) (a) Which of the following filters are usually even? (b) Which of the following filters are usually odd? (i) Notch filter; (ii) highpass filter; (iii) edge detector; (iv) integral; (v) differentiation 4 times; (vi) particle filter; (vii) matched filter. (10 scores)

(a) i、ii、iii、v

(b) iv、vi、vii