EE6222: MACHINE VISION

Project Report (Assignment 2)

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Question 1

Find the focal length *f* of your hand phone (in pixels). You may use a real person or printed figure, and include one figure of the settings in your report. Make sure you turn the camera's "zooming/auto-focusing" off. (40 marks)

We used two crosses with a midpoint (in pink tape) as denoted below. Pictures were taken at a varying distance from the phone camera (**O**) to the physical midpoint of the two crosses, denoted by the distance **OL**.

The diagram below depicts how the experiment was set up. We vary the distance **OL** between 1240mm and 1800mm, and measure the pixel distance ($\mathbf{x_a x_b}$) between the two crosses in the resulting image plane. We are trying to find the focal length $\boldsymbol{f_x}$ in terms of its pixel value. The physical distance between the two crosses (**AB**) is fixed at 1200mm.

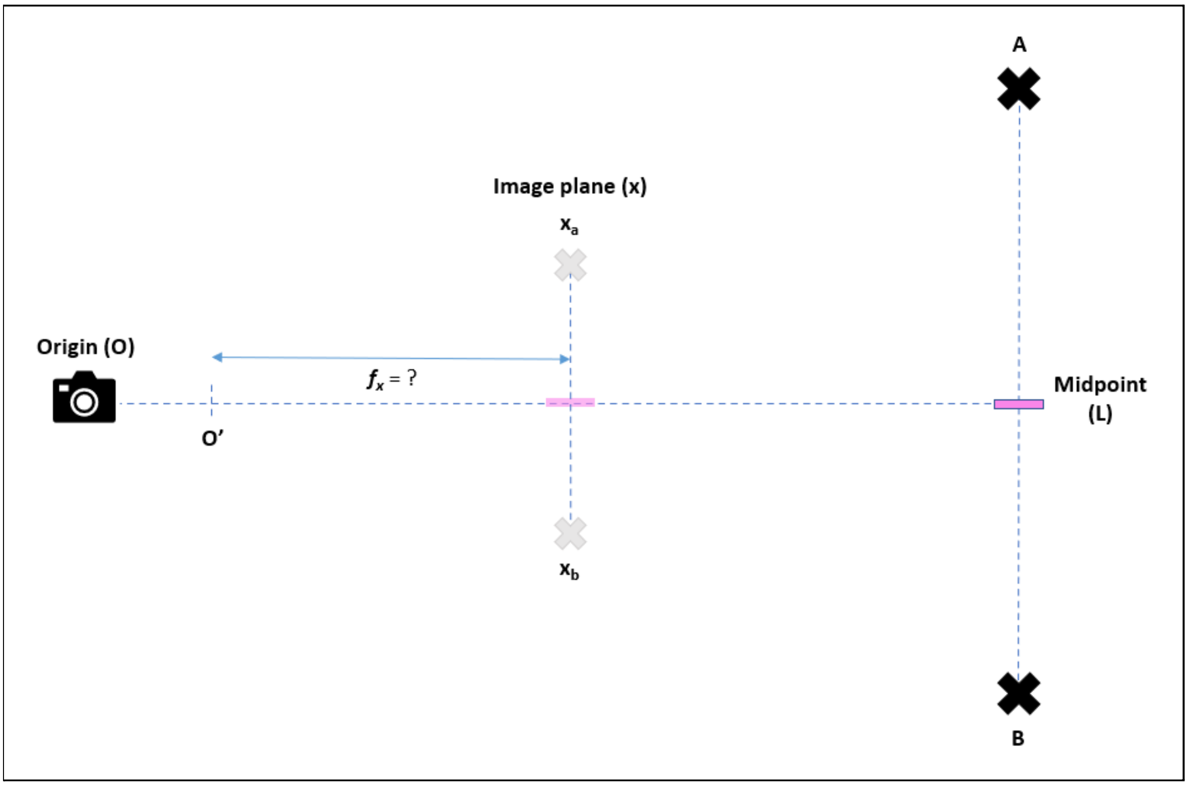


Figure 1: Experimental setup

The settings used for the phone camera as well as the resulting images are listed below:

Phone model	Samsung Galaxy S21+
Manufacturer's focal length	26mm
Zoom level	1.0x (no zoom)
Focus mode	Manual focus
HDR mode	Off
Camera type	Main (Not telephoto/selfie)

Figure 2: Two images at different distances

By projective geometry, we can see two similar triangles as shown below:

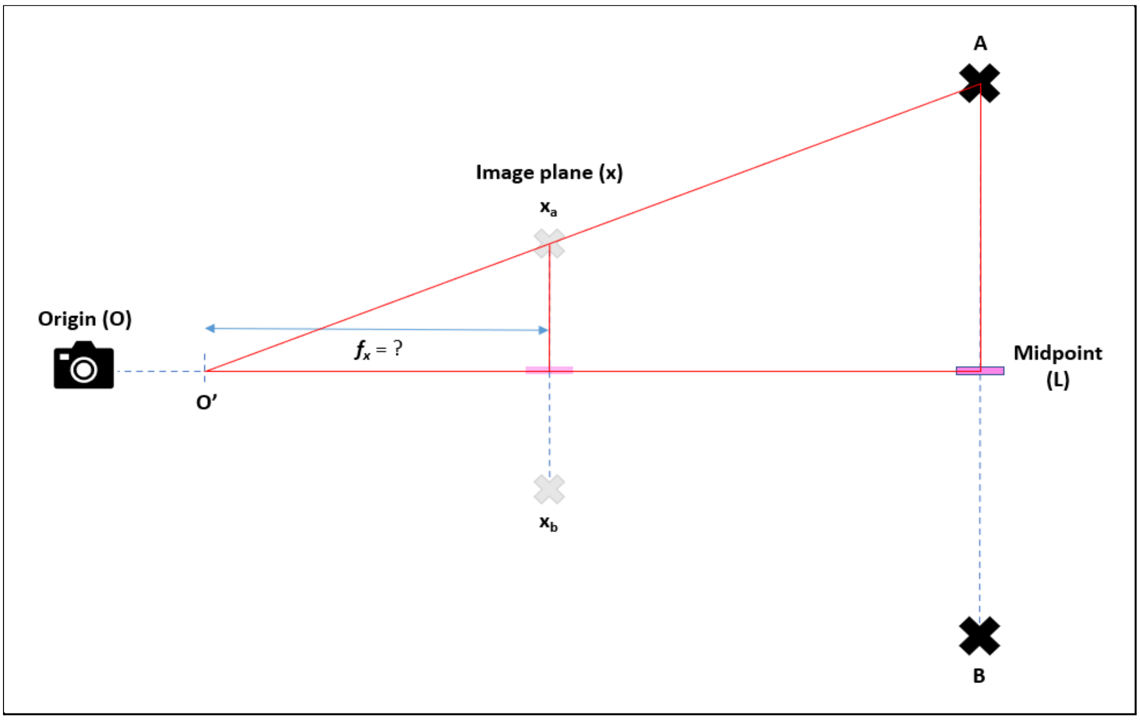


Figure 3: Projective geometry

We refer to Slide 4 of [2] 3D vision 1.pdf lecture slides for the equation to resolve both the pixel focal length $\boldsymbol{f_x}$, and the actual focal length ***OO'***

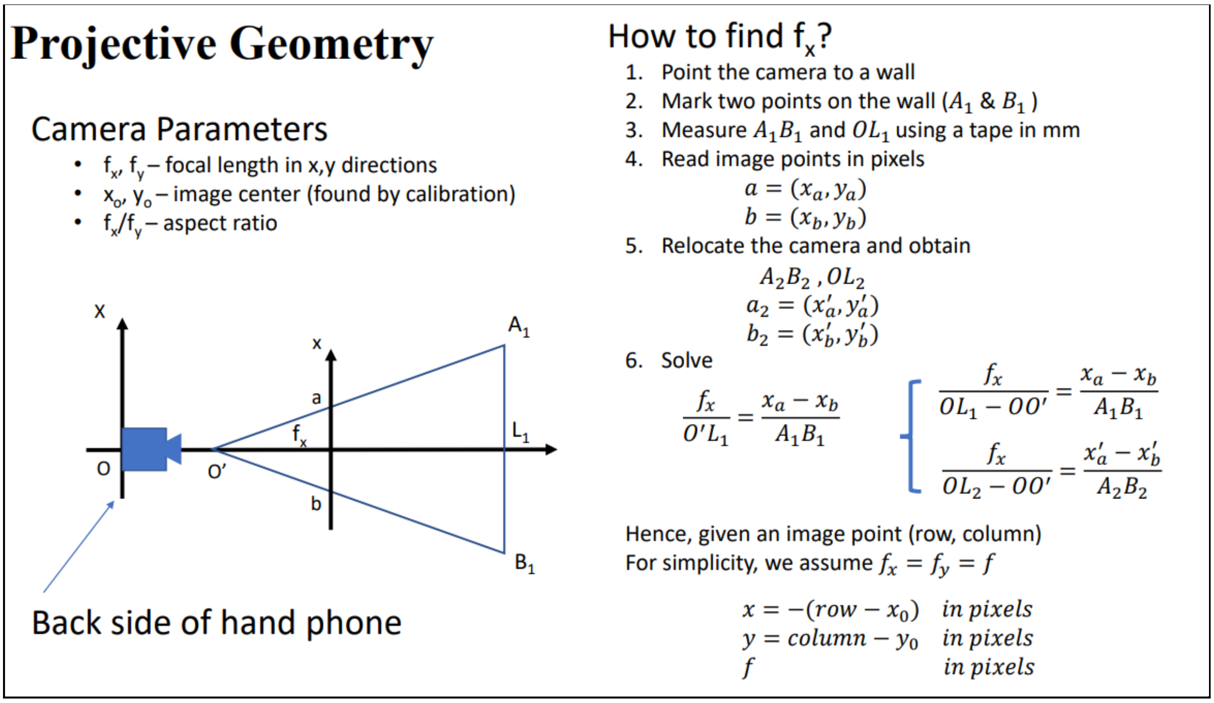


Figure 4: Projective Geometry equation from lecture slides

Using two images taken at different distances ***OL***, we are able to derive two sets of equations relating $\boldsymbol{f_x}$ and ***OO'***, therefore we can solve for both variables.

$$\frac{f_x}{OL_1 - OO'} = \frac{x_{a1} - x_{b1}}{A_1B_1}$$

$$f_x = \frac{x_{a1} - x_{b1}}{A_1B_1}(OL_1 - OO')$$

$$f_x = \frac{x_{a1} - x_{b1}}{A_1B_1}OL_1 - \frac{x_{a1} - x_{b1}}{A_1B_1}OO'$$

$$\frac{f_x}{OL_2 - OO'} = \frac{x_{a2} - x_{b2}}{A_2B_2}$$

$$f_x = \frac{x_{a2} - x_{b2}}{A_2B_2}(OL_2 - OO')$$

$$f_x = \frac{x_{a2} - x_{b2}}{A_2B_2}OL_2 - \frac{x_{a2} - x_{b2}}{A_2B_2}OO'$$

$$\left(\frac{x_{a1} - x_{b1}}{A_1B_1} - \frac{x_{a2} - x_{b2}}{A_2B_2}\right)OO' = \frac{x_{a1} - x_{b1}}{A_1B_1}OL_1 - \frac{x_{a2} - x_{b2}}{A_2B_2}OL_2$$

$$where\ A_1B_1 = A_2B_2 = AB,$$

$$OO' = \frac{(x_{a1} - x_{b1})OL_1 - (x_{a2} - x_{b2})OL_2}{(x_{a1} - x_{b1}) - (x_{a2} - x_{b2})}$$

$$f_x = \frac{x_{a1} - x_{b1}}{AB}OL_1 - \frac{x_{a1} - x_{b1}}{AB}OO' \quad or \quad f_x = \frac{x_{a2} - x_{b2}}{AB}OL_2 - \frac{x_{a2} - x_{b2}}{AB}OO'$$

Figure 5: Formula derivation for $\boldsymbol{f_x}$ *and* ***OO'***

In our case, we have the following values after measurement from both images:

	Image 1	**Image 2**
OL	1240mm	1800mm
AB	1200mm	1200mm
$\mathbf{x_a}$	1111	925
$\mathbf{x_b}$	157	273

Figure 6: Experimental values

This allows us to solve the value of ***OO'*** and $\boldsymbol{f_x}$ using the derivations in Figure 5

$$Focal\ Length = OO' = \frac{(1111 - 157)1240 - (925 - 273)1800}{(1111 - 157) - (925 - 273)} = 30.99mm$$

$$Pixel\ Focal\ Length = f_x = \frac{1111 - 157}{1200}1240 - \frac{1111 - 157}{1200}30.99 = 961.16px$$

The focal length discovered is similar to the manufacturer's specification of 26mm with around a 6mm difference.

Alternatively, we may make use of a simpler equation to determine the pixel focal length from only one image. This operates on the assumption that the distance ***OO'*** is negligible, that is, the distance of the lens to the focal point is negligible.

$$\frac{x_a - x_b}{AB} = \frac{f_x}{OL} \quad \rightarrow \quad f_x = \frac{x_a - x_b}{AB} OL$$

*Figure 7: Simplified formula with **OO'** being negligible*

Using the simplified equation, we can plug in the value of each image respectively, ending up with the following results:

$$Pixel\ Focal\ Length\ (Image\ 1) = f_{x1} = \frac{1111 - 157}{1200}1240 = 985.8px$$

$$Pixel\ Focal\ Length\ (Image\ 2) = f_{x2} = \frac{925 - 273}{1200}1800 = 978px$$

These values are within +25px of the value found using two images. This suggests that it is possible to just approximate ***OO'*** as zero to simplify the calculations of the pixel focal length.

Question 2

Take two snaps of an outdoor scene, with a 5 to 10 degrees angle difference. You need to keep the angle as ground truth. (0 marks)

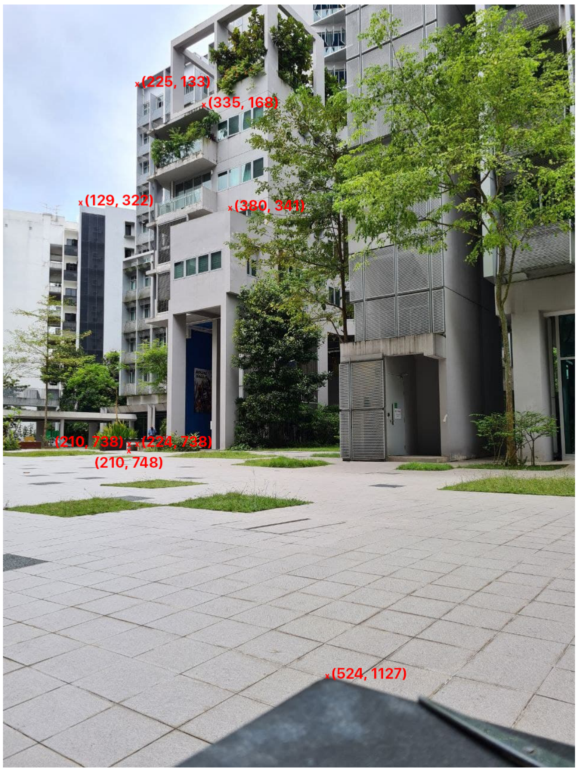
0 degrees | **9 degrees**

Figure 8: Two images at 9 degree angle difference

Question 3

Hand pick 8 points or more from one image, and find the matching points on the other image. These points should not be coplanar. You need to turn these points into n-vectors, and submit them into the equation for calculation. (10 marks)

The 8 points selected in each image have been marked as red crosses with their corresponding pixel coordinates below:



0 degrees

**Coords = [(129, 322), (544, 1127), (225, 133), (335, 168), (380, 341), (210, 738), (210, 748), (224, 738)
]**

9 degrees

**Coords = [(304, 318), (725, 1106), (390, 114), (499, 137), (544, 309), (385, 708), (386, 717), (399, 708)
]**

Figure 9: Choosing 8 random matching points in the two images

To turn these points into n-vectors, we will first homogenize these points. The pixel locations for each point in the image are a representation of a vector of shape (3,1) in world space (3D) which represents the x, y and z locations for each point. Since the points only exist as 2D vectors in the image, we will use the pixel focal length calculated in question 1 as the z-coordinate across all points to homogenize them.

We then adjust each point's coordinates for the image center (i.e. make the image center the origin) to have a unified origin for our normalization process. One can make the center point of the image the origin by calculating the relative vectors from the center point as such:

$$x = x_{location} - x_0$$

$$y = y_{location} - y_0$$

Here, (x_0, y_0) denotes the x and y coordinates of the center of the image. Now that we have an origin, we can normalize the vectors using their Euclidean norms.

```
% Focal length in pixels
F=961;

% Eight corresponding points of two images
pts1 = [[129 322]; [544 1127]; [225 133]; [335 168]; [380 341]; [210 738];
[210 748]; [224 738];];
pts2 = [[304 318]; [725 1106]; [390 114]; [499 137]; [544 309]; [385 708];
[386 717]; [399 708];];

% Center coordinates of image
x_center = 960/2;
y_center = 1280/2;

% Obtaining N-vectors
image1_points = [pts1 ones(8,1)*F];
image1_points(: ,1) = image1_points (: ,1) - X0 ;
image1_points (: ,2) = image1_points (: ,2) - Y0 ;
image1_norm = vecnorm (image1_points , 2 , 2) ;
image1_points = image1_points ./ image1_norm ;

image2_points = [pts2 ones(8,1)*F];
image2_points (: ,1) = image2_points (: ,1) - X0 ;
image2_points (: ,2) = image2_points (: ,2) - Y0 ;
image2_norm = vecnorm (image2_points , 2 , 2) ;
image2_points = image2_points ./ image2_norm ;
```

Figure 10: Obtaining N-vectors of the points

The final n-vectors for each image are as follows:

image1_points =	image2_points =
-0.3276 -0.2968 0.8970	-0.1711 -0.3130 0.9342
0.0593 0.4512 0.8904	0.2236 0.4253 0.8770
-0.2285 -0.4543 0.8611	-0.0819 -0.4785 0.8743
-0.1342 -0.4369 0.8895	0.0175 -0.4637 0.8858
-0.0989 -0.2956 0.9502	0.0628 -0.3250 0.9436
-0.2692 0.0977 0.9581	-0.0981 0.0702 0.9927
-0.2689 0.1076 0.9571	-0.0970 0.0795 0.9921
-0.2562 0.0981 0.9616	-0.0838 0.0703 0.9940

Question 4

Calculate the rotation angle from the matched points using the quaternion approach (pp 14 in [4]), or the SVD(in [3]). (40 marks)

We are opting for the SVD approach in [3].

A 3x3 matrix W is obtained where $W = \sum_{i=1}^{N} x_i^T p_i \in R\ 3x3$. x and p are the N-vectors relative to the center of the images respectively, corresponding to *image1_points* and *image2_points* respectively.

```
% Creating W matrix
weight_matrix = zeros (3 ,3) ;
for i = 1:8
temp = transpose ( image1_points (i ,:) ) * image2_points (i ,:) ;
weight_matrix = weight_matrix + temp ;
end
```

Figure 11: Computing the W matrix

W can be further factorized into 3 matrices $U\Sigma V^T$ via singular value decomposition. Any invertible linear transformation can be broken down into a rotation/reflection, scaling and another rotation/reflection. As such, the rotational matrix R of the linear transformation can be computed via the following formula:

$$R = UV^T$$

```
% SVD approach
[U , S , V ] = svd( W ) ;
rotational_matrix = U * transpose ( V ) ;
```

Figure 12: Performing SVD of the W matrix to compute the Rotational matrix

Using the rotational matrix R, we can calculate the rotation angle Ω as follows:

$$\Omega = cos^{-1}(\frac{trace(R) - 1}{2})$$

This gives us a rotation angle of 10.2375°, which is close to the ground truth of 9°.

```
angleInRadians = acos((trace(rotational_matrix)-1)/2);
angleInDeg = rad2deg(angleInRadians);
% angleInDeg = 10.2375 degrees;
```

Figure 13: Obtaining rotational angle in degrees

We can further double check the validity of our rotational matrix by calculating the N-vector of the rotational axis *l*. We obtain a N-vector that is approximately [0, -1, 0], indicating that we only have rotation about the y axis. This is accurate as we only rotated the phone 9° around a single axis.

```
l = transpose ([ rotational_matrix(3 ,2)-rotational_matrix(2 ,3)
rotational_matrix(1 ,3)-rotational_matrix(3 ,1) rotational_matrix(2
,1)-rotational_matrix(1 ,2)]) ;
l = l ./ vecnorm(l);
% N vector of rotational axis, [-0.1640 -0.9831 0.0814 ]
```

Figure 14: Obtaining the rotational axis