International Trade PhD
Homework 1

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Question 1 - Krugman (1980) and the gravity equation

Redding and Venables (2004) is one of the early papers showing how to obtain a structural gravity equation from the CES-monopolistic competition setup. According to the structural gravity theory, the bilateral trade equation is given as:

$$X_{ni} = \frac{Y_i}{\Omega_i}\frac{X_n}{\Phi_n}\phi_{ni} \tag{1}$$

where i is the origin, n the destination, $Y_i = \sum_n X_{ni}$ the value of production, $X_n = \sum_i X_{ni}$ the value of expenditure, and Ω_i and Φ_n are "multilateral resistance" terms defined as

$$\Phi_n = \sum_l \frac{\phi_{nl}Y_l}{\Omega_l} \quad \text{and} \quad \Omega_i = \sum_l \frac{\phi_{li}X_l}{\Phi} \tag{2}$$

Redding and Venables (2004) summarized in their equation (9) the structural gravity equation from the CES-monopolistic competition setup, where there is constant elasticity of substitution ($\sigma > 1$) between pairs of products:

$$n_i p_i x_{ij} = n_i p_i^{1-\sigma}(T_{ij})^{1-\sigma}E_j G_j^{\sigma-1} \tag{3}$$

where $n_i p_i x_{ij}$ is the value of the goods demanded by country j produced by country i, $n_i p_i^{1-\sigma}$ measures the supply capacity of exporting country i, $(T_{ij})^{1-\sigma}$ measures bilateral transport costs between countries and $E_j G_j^{\sigma-1}$ is the market capacity of importing country j.
It is equivalent to:

$$X_{ni} = s_i p_i^{1-\sigma}(\tau_{ni})^{1-\sigma}X_n P_n^{\sigma-1} \tag{4}$$

$X_n P_n^{\sigma-1}$ measures the market capacity of importing country n and $s_i p_i^{1-\sigma}$ measures the supply capacity of exporting country i. τ_{ni} is an iceberg transport cost factor.

a) **Introduce in this setup vertical differentiation such that the varieties from each origin have a specific utility (quality) shifter. How is the bilateral trade equation changed?**
This refers to the case of CES National Product Differentiation, first discussed by

Armington (1969) and Anderson (1979).
Suppose that consumers in country n consume q_{ni} units of product from country i. The utility function can be written as:

$$U_n = \left(\sum_i (A_i q_{ni})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma-1}{\sigma}} \tag{5}$$

where A_i is the utility shifter that indicates the quality of country i's product. We denote the price of labor production factor as wages w_i. Hence the price in country n of consuming one unit from country i is $p_{ni} = w_i \tau_{ni}$.
We can solve for the representative consumer's utility maximization problem, subject to the budget constraint:

$$\max_{X_{in}} \left(\sum_i (A_i q_{ni})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma-1}{\sigma}} \quad \text{s.t} \quad \sum q_{ni} w_i \tau_{ni} \leq X_n \tag{6}$$

It yields the optimal demand for each variety:

$$q_{ni} = A_i^{\sigma-1} w_i^{-\sigma} \tau_{ni}^{-\sigma} X_n P_n^{(\sigma-1)} \tag{7}$$

It implies that quantity consumed in n of a good produced in i depends negatively on the price w_i and the trade cost τ_{ni}, and positively on the quality level A_i.
Since the value of total trade equal to the price times quantity, $X_{ni} = p_{ni} q_{ni}$, we obtain the bilateral trade equation:

$$X_{ni} = A_i^{\sigma-1} w_i^{1-\sigma} \tau_{ni}^{1-\sigma} X_n P_n^{(\sigma-1)} \tag{8}$$

Therefore, the components in the structural gravity with vertical differentiation are: $S_i = A_i^{\sigma-1} w_i^{1-\sigma}$, and $\phi_{ni} = \tau_{ni}^{1-\sigma}$

b) **Introduce *bilateral* preferences in this setup, such that now quality is assessed differently in each country. How is the bilateral trade equation changed? Can we separately identify bilateral preferences from bilateral trade costs using bilateral friction variables (distance, common language etc.)?**
In this case, the utility function of consumers can be written as:

$$u_n = \ln(\psi_{ni} q_{ni}) \tag{9}$$

where ψ_{ni} represents the idiosyncratic preference shock and q_{ni} is the quantity consumed. The price of product varieties from country i for consumers in country n is $p_{ni} = w_i \tau_{ni}$. There are N_i varieties of product. Each consumer chooses the product giving highest utility.
Similar to what we did in question a, we denote A_i as an origin-specific quality shifter, and moreever, a_{ni} as the bilateral preference parameter which also shifts utility upwards.
We use methods inspired by Eaton and Kortum (2002) by assuming that the heterogeneity is Fréchet distributed with a cumulative distribution function:

$$\exp\left\{ - \left(\frac{\psi}{A_i a_{ni}} \right)^{-\theta} \right\} \tag{10}$$

Since the Fréchet distribution is an extreme value distribution, we can derive the probability of varieties produced in country i being the highest valuation and thus being chosen by consumers in n.
In this case, the bilateral trade equation becomes:

$$X_{ni} = N_i w_i^{-\theta} A_i^{\theta} \tau_{ni}^{-\theta} a_{ni}^{\theta} X_n P_n^{\theta} \tag{11}$$

Therefore, the components in the structural gravity with bilateral preferences are: $S_i = N_i w_i^{-\theta} A_i^{-\theta}$, and $\phi_{ni} = \tau_{ni}^{-\theta} a_{ni}^{\theta}$.
Thanks to this setup, we are now able to distinguish bilateral preference from bilateral trade cost. As we can see from the accessibility term ϕ_{ni}, it is composed of two factors, τ_{ni} representing traditional bilateral trade cost like tariff and freight cost and a_{ni} representing preference. Friction variables, such as language and distance, affect trade not only through bilateral trade cost, but also through bilateral preferences.

c) Does the gravity equation hold when introducing additive trade costs? Show why (/why not).

It does not hold. Indeed, starting from Equation (3) in Redding and Venables we have country j's demand for each product as:

$$x_{ij} = p_{ij}^{-\sigma} E_j G_j^{\sigma-1} \tag{12}$$

The setting used in the paper represents trade costs through a multiplicative factor on the original price. Hence $p_{ij} = p_i T_{ij}$. Adding additive trade costs on top of that we have:

$$p_{ij} = (p_i T_{ij} + C_{ij}) \tag{13}$$

Plugging this structure into (12):

$$x_{ij} = (p_i T_{ij} + C_{ij})^{-\sigma} E_j G_j \tag{14}$$

Multiplying both sides by $n_i p_i$ we find :

$$n_i p_i x_{ij} = n_i p_i (p_i T_{ij} + C_{ij})^{-\sigma} E_j G_j \tag{15}$$

Hence additive trade costs don't allow us to properly estimate the gravity equation because of the persistence of the additive structure. Indeed taking logs won't allow us to split the right hand-side of the equation.

d) Does the gravity equation hold when firms are Cournot oligopolists? Show why (/why not).

Let us show that gravity holds when firms are Cournot oligopolists, that is they compete on quantities.
Equation (4) in Redding and Venables (2004) gives the profit of a firm in country i:

$$\pi_i = \sum_j^R \frac{p_{ij} x_{ij}}{T_{ij}} - G_i^{\alpha} w_i^{\beta} v_i^{\gamma} c_i [F + x_i] \tag{16}$$

Let $p_i = \frac{p_{ij}}{T_{ij}}$ as in the paper. The firm of country i solves for each country j to which it sells:

$$\max_{x_{ij}} \pi_{ij} = p_i x_{ij} - G_i^{\alpha} w_i^{\beta} v_i^{\gamma} c_i [F + x_i] \tag{17}$$

The first-order condition with respect to x_{ij} yields:

$$p_i + x_{ij}\frac{\partial p_i}{\partial x_{ij}} = G_i^{\alpha} w_i^{\beta} v_i^{\gamma} c_i \tag{18}$$

$$\implies p_i\left(1 + \frac{x_{ij}}{p_i}\frac{\partial p_i}{\partial x_{ij}}\right) = G_i^{\alpha} w_i^{\beta} v_i^{\gamma} c_i \tag{19}$$

Let us recall the formula of the price elasticity of demand σ:

$$\sigma = -\frac{\partial x_{ij}}{\partial p_i}\frac{p_i}{x_{ij}} \tag{20}$$

Thus we can rewrite:

$$p_i\left(1 - \frac{1}{\sigma}\right) = G_i^{\alpha} w_i^{\beta} v_i^{\gamma} c_i \tag{21}$$

We recover equation (5) of Redding and Venables (2004), that is the price p_i is a constant markup over marginal cost:

$$p_i = \frac{G_i^{\alpha} w_i^{\beta} v_i^{\gamma} c_i \sigma}{\sigma - 1} \tag{22}$$

which is the optimal price.
Then:

$$p_{ij} = \frac{G_i^{\alpha} w_i^{\beta} v_i^{\gamma} c_i \sigma T_{ij}}{\sigma - 1} \tag{23}$$

We can plug this expression into equation (3) of the paper:

$$x_{ij} = \left(\frac{G_i^{\alpha} w_i^{\beta} v_i^{\gamma} c_i \sigma T_{ij}}{\sigma - 1}\right)^{-\sigma} E_j G_j^{\sigma-1} \tag{24}$$

Finally, we obtain the gravity equation:

$$x_{ij} n_i p_i = \left(\frac{G_i^{\alpha} w_i^{\beta} v_i^{\gamma} c_i \sigma T_{ij}}{\sigma - 1}\right)^{-\sigma} E_j G_j^{\sigma-1} n_i \frac{G_i^{\alpha} w_i^{\beta} v_i^{\gamma} c_i \sigma}{\sigma - 1} \tag{25}$$

$$= \left(\frac{G_i^{\alpha} w_i^{\beta} v_i^{\gamma} c_i \sigma}{\sigma - 1}\right)^{1-\sigma} T_{ij}^{-\sigma} E_j G_j^{\sigma-1} n_i \tag{26}$$

It holds because we can properly estimate the coefficients by taking logs.

e) **Krugman (1980) is a homogeneous firms model. Keeping with CES demand and monopolistic competition, what is the main additional condition needed for gravity to hold if firms' productivity is heterogeneous? Show why.**
The condition needed for gravity to hold if firms' productivity is heterogeneous is increasing returns to scale, that is, the more is produced, the lower the average cost of production.
To show that, let us recall the model in Krugman (1980).
There is a compact set of R countries: i represents the origin country and j the destination. τ_{ij} is an iceberg trade cost from i to j. We assume there are a mass M_i

of firms in country i and firms must incur a fixed cost $f_{ij} > 0$ to export to country j.
To model the heterogeneity of firms' productivity, let us assume that each firm in i's productivity φ is drawn from a cumulative distributive function $G_i(\varphi)$: it costs $\frac{1}{\varphi}$ units of labor to produce one unit of its differentiated variety.
Since consumers have CES preferences, a representative agent in country j gets utility U_j from the consumption of goods imported Ω_{ij} such as

$$U_j = \left(\sum_i^R \int_{\Omega_{ij}} (q_{ij}(z))^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}} \tag{27}$$

where $q_{ij}(z)$ is the quantity consumed in country j of variety z and $\sigma - 1$ is the elasticity of substitution.
Let us now recall the equilibrium of the model.
The optimal quantity demanded of z by a consumer in country j is:

$$q_{ij}(z) = p_{ij}(z)^{-\sigma} Y_j P_j^{\sigma-1} \tag{28}$$

CES demand will yield a Dixit-Stiglitz price index:

$$P_j = \left(\sum_i^R \int_{\Omega_i} p_{ij}(z)^{1-\sigma} dz \right)^{\frac{1}{\sigma-1}} \tag{29}$$

and

$$x_{ij}(z) = p_{ij}(z)^{1-\sigma} Y_j P_j^{\sigma-1} \tag{30}$$

Aggregating over all firms in country i yields the total bilateral trade flows:

$$X_{ij} = \int_{\Omega_{ij}} x_{ij}(z) dz = Y_j P_j^{\sigma-1} \int_{\Omega_i} p_{ij}(z)^{1-\sigma} dz \tag{31}$$

Firms with different levels of productivity will charge different prices.
Next, let us determine the equilibrium prices set by a firm with productivity φ. The firm solves, with:

$$\max_{q_{ij}(\varphi)_j} \sum_j \left(p_{ij}(\varphi) q_{ij}(\varphi) - \frac{w_i}{\varphi} \tau_{ij} q_{ij}(\varphi) - f_{ij} \right) \tag{32}$$

such that

$$q_{ij}(\varphi) = p_{ij}(\varphi)^{-\sigma} Y_j P_j^{\sigma-1} \tag{33}$$

with w_i being the wage costs of the firm.
Substituting the constraint into the problem yields:

$$\max_{q_{ij}(\varphi)_j} \sum_j \left(p_{ij}(\varphi)^{1-\sigma} Y_j P_j^{\sigma-1} - \frac{w_i}{\varphi} \tau_{ij} p_{ij}(\varphi)^{-\sigma} Y_j P_j^{\sigma-1} - f_{ij} \right) \tag{34}$$

From the first-order condition, we get:

$$(1-\sigma) p_{ij}(\varphi)^{-\sigma} + \sigma \frac{w_i}{\varphi} \tau_{ij} p_{ij}(\varphi)^{-\sigma-1} = 0 \tag{35}$$

Hence:

$$\frac{\sigma}{\sigma-1}\frac{w_i}{\varphi}\tau_{ij} = \frac{p_{ij}(\sigma)^{-\sigma}}{p_{ij}(\varphi)^{-\sigma-1}} \tag{36}$$

It implies that a firm from country i with productivity φ, conditional on selling to country j will charge the following price:

$$p_{ij}(\varphi) = \frac{\sigma}{\sigma-1}\frac{w_i}{\varphi}\tau_{ij} \tag{37}$$

Combining that optimal price with the optimal demand gives the total revenue of the firm

$$X_{ij}(\varphi) = p_{ij}(\varphi)q_{ij}(\varphi) = \left(\frac{\sigma}{\sigma-1}\frac{w_i}{\varphi}\tau_{ij}\right)^{1-\sigma} Y_j P_j^{\sigma-1} \tag{38}$$

and the profit function without the fixed costs:

$$\begin{aligned}
\pi_{ij}(\varphi) &= \left(p_{ij}(\varphi) - \frac{w_i}{\varphi}\tau_{ij}\right) q_{ij}(\varphi) && (39)\\
&= \left(\frac{\sigma}{\sigma-1}\frac{w_i}{\varphi}\tau_{ij} - \frac{w_i}{\varphi}\tau_{ij}\right)\left(\frac{\sigma}{\sigma-1}\frac{w_i}{\varphi}\tau_{ij}\right)^{-\sigma} Y_j P_j^{\sigma-1} && (40)\\
&= \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\frac{w_i}{\varphi}\tau_{ij}\right)^{1-\sigma} Y_j P_j^{\sigma-1} && (41)\\
&= \frac{1}{\sigma}x_{ij}(\varphi) && (42)
\end{aligned}$$

In words, both revenue and profits are increasing in the firm's productivity.
Next, let us look at the aggregate level.
Let $\mu_{ij}(\varphi)$ be the (equilibrium) probability density function of the productivity of firms from country i that sell to country j and let M_{ij} be the (equilibrium) measure of firms exporting from i to j.
Then we can write the average prices charged by all firms in i selling to j as:

$$\begin{aligned}
\int_{\Omega_i} p_{ij}(z)^{1-\sigma}dz &= \int_0^\infty M_{ij}\left(\frac{\sigma}{\sigma-1}\frac{w_i}{\varphi}\tau_{ij}\right)^{1-\sigma}\mu_{ij}(\varphi)d\varphi && (43)\\
&= M_{ij}\left(\frac{\sigma}{\sigma-1}w_i\tau_{ij}\right)^{1-\sigma}\int_0^\infty \varphi^{\sigma-1}\mu_{ij}(\varphi)d\varphi && (44)\\
&= M_{ij}\left(\frac{\sigma}{\sigma-1}w_i\tau_{ij}\right)^{1-\sigma}(\tilde{\varphi}_{ij})^{\sigma-1} && (45)
\end{aligned}$$

where $\tilde{\varphi}_{ij} = (\int_0^\infty \varphi^{\sigma-1}\mu_{ij}(\varphi)d\varphi)^{\frac{1}{\sigma-1}}$ captures the "average" productivity of producers from i selling to j.
This allows us to write the gravity equation as:

$$X_{ij} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} w_i^{1-\sigma}\tau_{ij}^{1-\sigma}M_{ij}(\tilde{\varphi}_{ij})^{\sigma-1}Y_j P_j^{\sigma-1} \tag{46}$$

A firm from country i with productivity φ will thus export to j if and only if:

$$\pi_{ij}(\varphi) \geq f_{ij} \tag{47}$$

From the formula of $\pi_{ij}(\varphi)$, this can be rewritten as:

$$\frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{w_i}{\varphi}\tau_{ij}\right)^{1-\sigma} Y_j P_j^{\sigma-1} \geq f_{ij} \tag{48}$$

$$\varphi \geq \varphi_{ij}^* = \left(\frac{\sigma f_{ij}\left(\frac{\sigma}{\sigma-1}w_i\tau_{ij}\right)^{\sigma-1}}{Y_j P_j^{\sigma-1}}\right)^{\frac{1}{\sigma-1}} \tag{49}$$

Therefore, only firms that are sufficiently productive will find it profitable to incur the fixed cost of exporting to destination j. In other words, only firms with increasing returns to scale will trade.

Question 2 - Empirical exercise: market potential and development

The goal is to replicate Redding and Venables (2004) regressions and graphs for a recent set of years. The dataset is an updated version of the one in Head and Mayer (2011), with international trade only (no self trade), so that the Foreign Market Potential (FMP) can be calculated.
Comments are provided throughout the code. Please note that there is a separate code for the estimation of potential fixed effects.

a) **Calculate FMP of each country in 2016 and regress it on GDP per capital (in log-log).**
We want to estimate the foreign market potential of each country in 2016, that is equation (17) from Redding and Venables (2004):

$$F\hat{M}A_i = \sum_{j\neq i}(\exp(\text{ptn})_j))^{\hat{\lambda}_j}\text{dist}_{ij}^{\hat{\delta}_1}\text{bord}_{ij}^{\hat{\delta}_2} \tag{50}$$

where, in the database given, ptn = 1 (since we focus on partners), dist = distw and bord = contig.
First, in order to get the estimates $\hat{\lambda}_j$, $\hat{\delta}_1$ and $\hat{\delta}_2$, we need to estimate equation (16) of the paper:

$$\ln(X_{ij}) = \theta + \mu_i\text{cty}_i + \lambda_j\text{ptn}_j + \delta_1\ln(\text{dist}_{ij}) + \delta_2\text{bord}_{ij} + u_{ij} \tag{51}$$

Using our database, this is equivalent to regressing contig, iso_o and iso_d ln(flow) on ln(dist).
Please note that, in line with footnote 8 from Redding and Venables (2004), we add 1 to the value of bilateral flows equal to 0 before taking the logs. We are aware that the data source is unlikely to be the same, i.e. COMTRADE since Head and Mayer (2011) use CEPII data. However, as there are some NA values for other variables such as GDP, we consider those 0s as true 0s as in the paper.
Table 1 below presents the results of the first regression (country-specific coefficients are not displayed as there are hundreds of them). Our results are significant and are broadly consistent with Redding and Venables (2004), that is distance is inversely proportional to flows (that is, trade decreases with distance). As expected, the reverse is true for contiguity.
Then, using a simple procedure, we obtain the FMP (the table is not reproduced here but can be accessed in the code under the name biltrade2016_FMP) and Table

2 presents the results of the regression of ln(FMP) on ln(GDP per capita). Our results are similar to the paper's too: a 1% increase in the FMP means that the GDP per capita increases by 0.37%.
Finally, Figure 1 replicates the same figure 1 in the paper. The relationship between the GDP per capita and the foreign market potential is very clear. Contrary to the paper and due to our different database, we have less obvious outliers.

b) Replicate the exercise with all years since 2004. Which fixed effects can you now introduce in the second step? Does it change results?

The replication is available in the code for all years since 2004. The first column of Table 3 gives the coefficient of the regression, which is consistent and very similar with the results for 2016: a 1% increase in the FMP means that the GDP per capita increases by 0.387%. Adding time fixed effects doesn't change the results as the coefficient merely loses half a basis point (0.381%). This result is consistent with our expectations since countries' FMPs are relatively stable over time. Moreover, adverse shocks tend to be symmetric both on FMP and GDP, thus preserving the relationship between the two variables.
Adding country-specific fixed effects the coefficient on FMP experiences a sharp drop to 0.143. This means that countries have important unobserved characteristics aside from FMP that explain their respective GDPs. These being partially correlated with FMP they lower the effect previously attributed to FMP.
Combining both country and time fixed effects, the effect of FMP on GDP per capita changes direction and greatly loses magnitude. Significance of the coefficient is also reduced while remaining two standard errors away from the null. Hence,removing the combined cyclical and local effects, the effect of FMP on GDP seems rather limited.

As a possible extension, we can introduce other covariates in the second step in order to isolate the effect of Foreign Market Potential on GDP. Redding and Venables (2004) include physical geography as well as institutional variables. We have chosen the ease of doing business, the status of tax haven, and time specific effects through the cost of oil. The latter is used as a proxy for the business cycle. In this case the magnitude of the effect of FMP on GDP falls while remaining significant. As shown in Table 4, one percent increase in FMP only yields a 0.1 percent increase in GDP per capita.

Table 1: Regression 1

	Dependent variable:
	log(flows)
log(distance)	-2.241^{***}
	(0.030)
Contiguity	1.017^{***}
	(0.153)
Observations	31,150
R^2	0.679
Adjusted R^2	0.675
Root MSE	2.281
Residual Std. Error	3.302
F Statistic	163.573^{***}
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 2: Regression 2

	Dependent variable:
	log(GDP)
log(FMP)	0.373^{***}
	(0.056)
Observations	138
R^2	0.247
Adjusted R^2	0.242
Residual Std. Error	1.313
F Statistic	44.629^{***}
Note:	*p<0.1; **p<0.05; ***p<0.01

Figure 1: GDP per capita and Foreign Market Potential

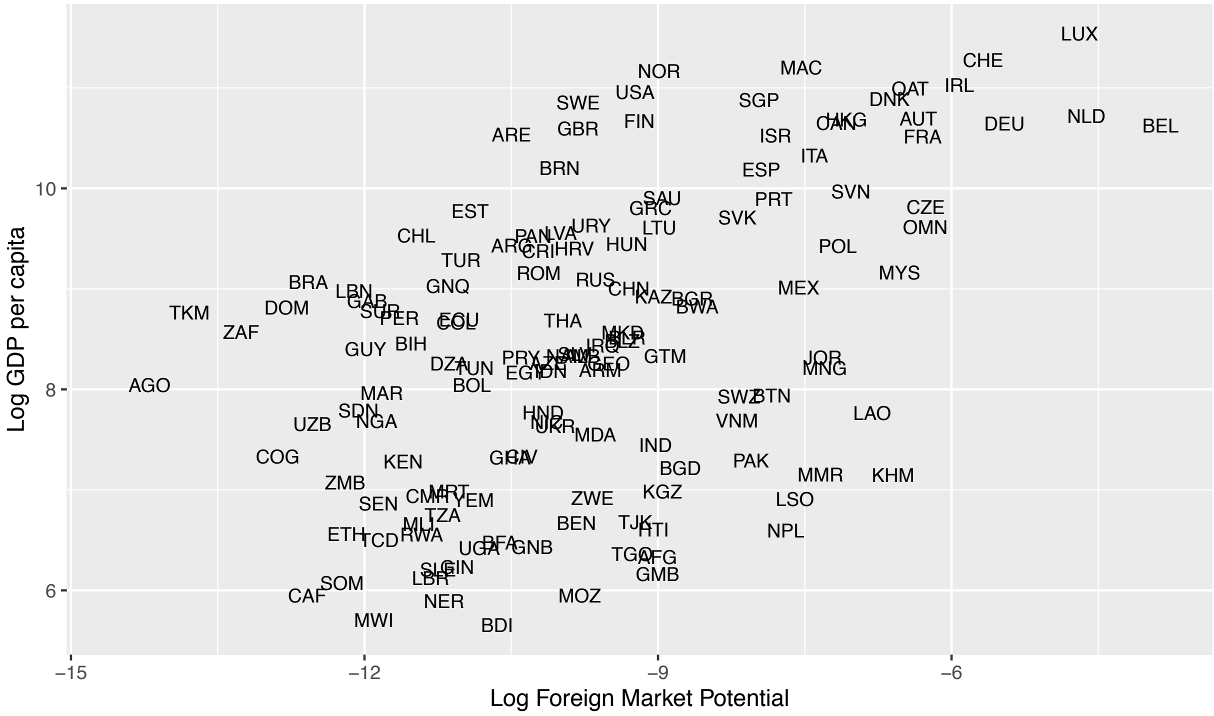

Table 3: Regression 3

	Dependent variable:			
	log(GDP)			
	(1)	(2)	(3)	(4)
log(FMP)	0.387***	0.381***	0.143***	−0.027**
	(0.016)	(0.016)	(0.016)	(0.012)
Year fixed effect	No	Yes	No	Yes
Country fixed effect	No	No	Yes	Yes
Observations	1,850	1,850	1,850	1,850
R^2	0.239	0.249	0.967	0.984
Adjusted R^2	0.239	0.248	0.965	0.982
Residual Std. Error	1.395	1.387	0.300	0.212
F Statistic	580.798***	305.602***	349.358***	707.700***
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 4: Regression 4

	Dependent variable:
	log(GDP)
log(FMP)	0.098***
	(0.014)
Doing Business Index	0.083***
	(0.002)
Tax haven status	0.562***
	(0.081)
Average Brent Price	0.007***
	(0.001)
Observations	1,823
R^2	0.606
Adjusted R^2	0.605
Residual Std. Error	1.005
F Statistic	697.779***
Note:	*p<0.1; **p<0.05; ***p<0.01

Sources:
Doing Business Index (World Bank)
List of tax havens (Oxfam)
Brent Price Index (USEIA)