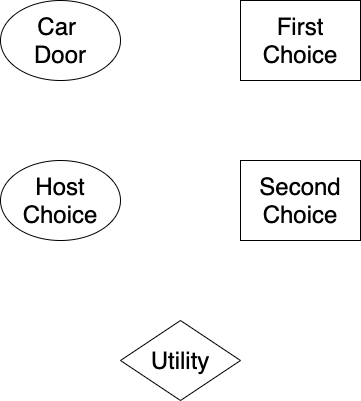
1.a)



|  |  |
| --- | --- |
| Car door | Prob |
| 1 | p1 |
| 2 | p2 |
| 3 | 1-p1-p2 |

|  |  |  |  |
| --- | --- | --- | --- |
| FirstChoice | CarDoor | HostChoice | P(HostChoice | FirstChoice ^ CarDoor) |
| 1 | 1 | Smaller | 0.5 |
| 1 | 1 | Bigger | 0.5 |
| 1 | 2 | Smaller | 0 |
| 1 | 2 | Bigger | 1 |
| 1 | 3 | Smaller | 1 |
| 1 | 3 | Bigger | 0 |
| 2 | 1 | Smaller | 0 |
| 2 | 1 | Bigger | 1 |
| 2 | 2 | Smaller | 0.5 |
| 2 | 2 | Bigger | 0.5 |
| 2 | 3 | Smaller | 1 |
| 2 | 3 | Bigger | 0 |
| 3 | 1 | Smaller | 0 |
| 3 | 1 | Bigger | 1 |
| 3 | 2 | Smaller | 1 |
| 3 | 2 | Bigger | 0 |
| 3 | 3 | Smaller | 0.5 |
| 3 | 3 | Bigger | 0.5 |

|  |  |  |  |
| --- | --- | --- | --- |
| CarDoor | FirstChoice | SecondChoice | Utility |
| 1 | 1 | Stay | 1 |
| 1 | 1 | Switch | 0 |
| 1 | 2 | Stay | 0 |
| 1 | 2 | Switch | 1 |
| 1 | 3 | Stay | 0 |
| 1 | 3 | Switch | 1 |
| 2 | 1 | Stay | 0 |
| 2 | 1 | Switch | 1 |
| 2 | 2 | Stay | 1 |
| 2 | 2 | Switch | 0 |
| 2 | 3 | Stay | 0 |
| 2 | 3 | Switch | 1 |
| 3 | 1 | Stay | 0 |
| 3 | 1 | Switch | 1 |
| 3 | 2 | Stay | 0 |
| 3 | 2 | Switch | 1 |
| 3 | 3 | Stay | 1 |
| 3 | 3 | Switch | 0 |

b) Let the tables above represent the factor tables f1, f2, f3 respectively such that p1 = 1/3, p2 = 1/3.

1. Multipy f1 and f2 to get f4

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| CarDoor | FirstChoice | SecondChoice | HostChoice | Value |
| 1 | 1 | Stay | Smaller | 0.167 |
| 1 | 2 | Stay | Smaller | 0 |
| 1 | 3 | Stay | Smaller | 0 |
| 1 | 1 | Switch | Smaller | 0 |
| 1 | 2 | Switch | Smaller | 0 |
| 1 | 3 | Switch | Smaller | 0 |
| 1 | 1 | Stay | Bigger | 0.167 |
| 1 | 2 | Stay | Bigger | 0 |
| 1 | 3 | Stay | Bigger | 0 |
| 1 | 1 | Switch | Bigger | 0 |
| 1 | 2 | Switch | Bigger | 0.333 |
| 1 | 3 | Switch | Bigger | 0.333 |
| 2 | 1 | Stay | Smaller | 0 |
| 2 | 2 | Stay | Smaller | 0.167 |
| 2 | 3 | Stay | Smaller | 0 |
| 2 | 1 | Switch | Smaller | 0 |
| 2 | 2 | Switch | Smaller | 0 |
| 2 | 3 | Switch | Smaller | 0.333 |
| 2 | 1 | Stay | Bigger | 0 |
| 2 | 2 | Stay | Bigger | 0.167 |
| 2 | 3 | Stay | Bigger | 0 |
| 2 | 1 | Switch | Bigger | 0.333 |
| 2 | 2 | Switch | Bigger | 0 |
| 2 | 3 | Switch | Bigger | 0 |
| 3 | 1 | Stay | Smaller | 0 |
| 3 | 2 | Stay | Smaller | 0 |
| 3 | 3 | Stay | Smaller | 0.167 |
| 3 | 1 | Switch | Smaller | 0.333 |
| 3 | 2 | Switch | Smaller | 0.333 |
| 3 | 3 | Switch | Smaller | 0 |
| 3 | 1 | Stay | Bigger | 0 |
| 3 | 2 | Stay | Bigger | 0 |
| 3 | 3 | Stay | Bigger | 0.167 |
| 3 | 1 | Switch | Bigger | 0 |
| 3 | 2 | Switch | Bigger | 0 |
| 3 | 3 | Switch | Bigger | 0 |

2. Sum out CarDoor since it is not apparent of any of the decision nodes to get f5

|  |  |  |  |
| --- | --- | --- | --- |
| FirstChoice | SecondChoice | HostChoice | Value |
| 1 | Stay | Smaller | 0.167 |
| 2 | Stay | Smaller | 0.167 |
| 3 | Stay | Smaller | 0.167 |
| 1 | Switch | Smaller | 0.333 |
| 2 | Switch | Smaller | 0.333 |
| 3 | Switch | Smaller | 0.333 |
| 1 | Stay | Bigger | 0.167 |
| 2 | Stay | Bigger | 0.167 |
| 3 | Stay | Bigger | 0.167 |
| 1 | Switch | Bigger | 0.333 |
| 2 | Switch | Bigger | 0.333 |
| 3 | Switch | Bigger | 0.333 |

3. Sum out SecondChoice by restricting it in f5 to produce f6. Optimal policy for SecondChoice is Switch

|  |  |  |
| --- | --- | --- |
| FirstChoice | HostChoice | Value |
| 1 | Smaller | 0.333 |
| 1 | Bigger | 0.333 |
| 2 | Smaller | 0.333 |
| 2 | Bigger | 0.333 |
| 3 | Smaller | 0.333 |
| 3 | Bigger | 0.333 |

4. Sum out HostChoice from f6 to produce f7

|  |  |
| --- | --- |
| FirstChoice | Value |
| 1 | 0.667 |
| 2 | 0.667 |
| 3 | 0.667 |

5. Restrict f7 to get optimal policy for FirstChoice

|  |
| --- |
| Value |
| 0.667 |

6. Sum out the result to get expected utility of 0.667. Optimal policy for FirstChoice is to pick any door since they all have the same value. Optimal policy for SecondChoice is to switch. Expected utility of the optimal policy is 0.667.

c)

|  |  |
| --- | --- |
| **Car door** | **Prob** |
| 1 | 0.6 |
| 2 | 0.25 |
| 3 | 0.15 |

Let the table above represent the factor table f1. f2 and f3 are the same as in part 2.

1. Multipy f1 and f2 to get f4

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **CarDoor** | **FirstChoice** | **SecondChoice** | **HostChoice** | **Value** |
| 1 | 1 | Stay | Smaller | 0.3 |
| 1 | 2 | Stay | Smaller | 0 |
| 1 | 3 | Stay | Smaller | 0 |
| 1 | 1 | Switch | Smaller | 0 |
| 1 | 2 | Switch | Smaller | 0 |
| 1 | 3 | Switch | Smaller | 0 |
| 1 | 1 | Stay | Bigger | 0.3 |
| 1 | 2 | Stay | Bigger | 0 |
| 1 | 3 | Stay | Bigger | 0 |
| 1 | 1 | Switch | Bigger | 0 |
| 1 | 2 | Switch | Bigger | 0.6 |
| 1 | 3 | Switch | Bigger | 0.6 |
| 2 | 1 | Stay | Smaller | 0 |
| 2 | 2 | Stay | Smaller | 0.125 |
| 2 | 3 | Stay | Smaller | 0 |
| 2 | 1 | Switch | Smaller | 0 |
| 2 | 2 | Switch | Smaller | 0 |
| 2 | 3 | Switch | Smaller | 0.25 |
| 2 | 1 | Stay | Bigger | 0 |
| 2 | 2 | Stay | Bigger | 0.125 |
| 2 | 3 | Stay | Bigger | 0 |
| 2 | 1 | Switch | Bigger | 0.25 |
| 2 | 2 | Switch | Bigger | 0 |
| 2 | 3 | Switch | Bigger | 0 |
| 3 | 1 | Stay | Smaller | 0 |
| 3 | 2 | Stay | Smaller | 0 |
| 3 | 3 | Stay | Smaller | 0.075 |
| 3 | 1 | Switch | Smaller | 0.15 |
| 3 | 2 | Switch | Smaller | 0.15 |
| 3 | 3 | Switch | Smaller | 0 |
| 3 | 1 | Stay | Bigger | 0 |
| 3 | 2 | Stay | Bigger | 0 |
| 3 | 3 | Stay | Bigger | 0.075 |
| 3 | 1 | Switch | Bigger | 0 |
| 3 | 2 | Switch | Bigger | 0 |
| 3 | 3 | Switch | Bigger | 0 |

2. Sum out CarDoor since it is not apparent of any of the decision nodes to get f5

|  |  |  |  |
| --- | --- | --- | --- |
| FirstChoice | SecondChoice | HostChoice | Value |
| 1 | Stay | Smaller | 0.3 |
| 2 | Stay | Smaller | 0.125 |
| 3 | Stay | Smaller | 0.075 |
| 1 | Switch | Smaller | 0.15 |
| 2 | Switch | Smaller | 0.15 |
| 3 | Switch | Smaller | 0.25 |
| 1 | Stay | Bigger | 0.3 |
| 2 | Stay | Bigger | 0.125 |
| 3 | Stay | Bigger | 0.075 |
| 1 | Switch | Bigger | 0.25 |
| 2 | Switch | Bigger | 0.6 |
| 3 | Switch | Bigger | 0.6 |

3. Sum out SecondChoice by restricting it in f5 to produce f6. Optimal policy for SecondChoice is Switch

|  |  |  |
| --- | --- | --- |
| FirstChoice | HostChoice | Value |
| 1 | Smaller | 0.3 |
| 1 | Bigger | 0.3 |
| 2 | Smaller | 0.15 |
| 2 | Bigger | 0.6 |
| 3 | Smaller | 0.25 |
| 3 | Bigger | 0.6 |

4. Sum out HostChoice from f6 to produce f7

|  |  |
| --- | --- |
| FirstChoice | Value |
| 1 | 0.6 |
| 2 | 0.75 |
| 3 | 0.85 |

5. Restrict f7 to get optimal policy for FirstChoice

|  |
| --- |
| Value |
| 0.85 |

6. Sum out the result to get expected utility of 0.85. Optimal policy for FirstChoice is to pick door 3. Optimal policy for SecondChoice to stay when FirstChoice is 1 and to switch otherwise. Expected utility of the optimal policy is 0.85.