

CSC263: Problem Set 3

October 8, 2019

1 Problem 1

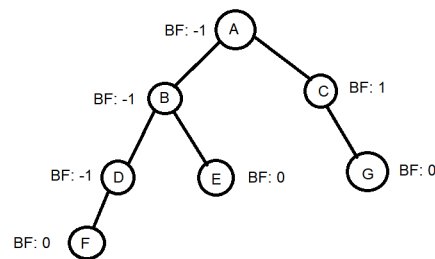


Figure 1: Height balanced tree that is not ideally height balanced.

- (a) This is an example of a height balanced tree that is not ideally height balanced. By definition an ideally height balanced tree has every leaf at depth h or $h - 1$, and every node of depth less than $h - 1$, has 2 children, and clearly Node C has only 1 child, Node G. Note that in this case, the tree in figure 1 has a height of 4 (assuming the root has a height of 1), and Node F has a depth of 4. Node C is at a depth of 2 which is less than $h - 1 = 4 - 1 = 3$.

However, this tree is height balanced. This is because for every node the height of the left subtree is within ± 1 of the height of the right subtree. To see that this is true for each node, we use balance factors. Balance Factor = $h(R) - h(L)$, where $h(R)$ and $h(L)$ stands for height of right subtree and height of left subtree respectively. Thus, if the balance factor is one of $-1, 0, 1$, then the tree is height balanced. As all balance factors in figure 1 are either $-1, 0$, or 1 , the tree is height balanced.