

## Appendix I - Bubble generation system and procedure

Figure A1a illustrates a schematic diagram of the experimental system. The operational process of the bubble generation system is as follows: First, a water pump transfers water from the reservoir through a liquid flow meter into the microbubble generator, while an air pump introduces air into the generator to achieve gas-liquid mixing. The water flow then passes through a pressure gauge and a valve before returning to the reservoir, completing the water circulation loop. The liquid flow rate is adjusted by regulating the water pump, while the air inflow is controlled using a gas flow meter. The pressure at the outlet section of the generator is fine-tuned via a valve.

The Venturi-tube microbubble generator, shown in Figure A1b, is made of polymethyl methacrylate. To minimize the influence of wall-reflected light on bubble measurements, the generator's flow channel and walls are designed in a rectangular shape and polished. The generator consists of five sections: an inlet section, contraction section, throat section, expansion section, and outlet section, with an overall width of 5 mm. The inlet section has a height of 12.7 mm and a length of 5 mm. The contraction section features a contraction angle of  $30^\circ$  and a length of 6.7 mm. The throat section, with a height of 5 mm and a length of 5 mm, includes a 1 mm diameter air pipe located at the center of its upper side. Following the throat section, the expansion section has an expansion angle of  $15^\circ$  and a length of 14.4 mm. The outlet section measures 12.7 mm in height and 45 mm in length. During the experiment, a light source and a camera are positioned on opposite sides of the generator's outlet section to capture the dynamic behavior of the microbubbles. The high-speed camera used in this study is the PCO DIMAX S1, featuring a sensor resolution of  $1008 \times 1008$  pixels with a pixel size of  $11 \times 11 \mu\text{m}$ . During image acquisition, the camera exposure time was set to  $20 \mu\text{s}$ , with a frame rate of 4467, a lens magnification of 1, and an aperture value of 4. A 4W LED green collimated light source was employed to ensure imaging quality and uniform illumination. In Figure A1b, the blue dashed line indicates the field of view (FOV) of the high-speed camera ( $11.1 \text{ mm} \times 11.1 \text{ mm}$ ), which largely covers the entire flow channel.

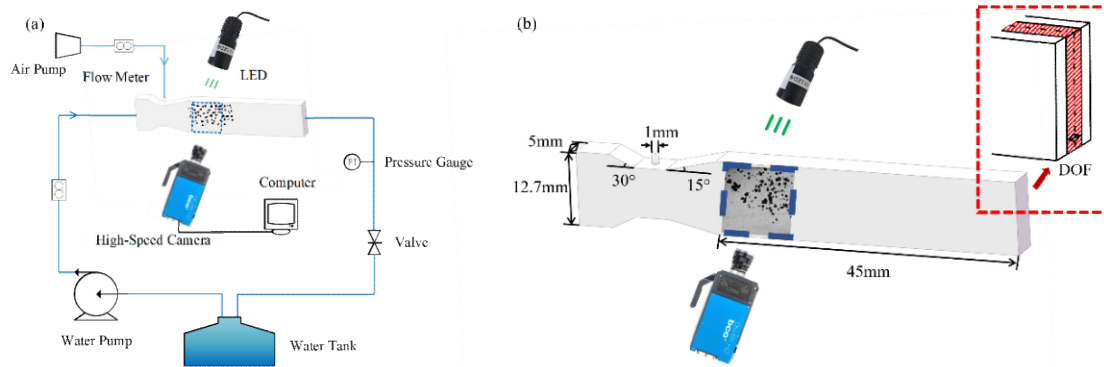


Figure A1. The schematic diagram of the experimental system (a) Bubble generation system (b) Venturi bubble generator

A total of seven experimental conditions were conducted under different water flow rates (6 L/min–9 L/min) and outlet back pressures (0–0.05 MPa), as detailed in Table A1. Once the flow conditions stabilized, continuous long-duration imaging was performed for each condition, and 4,000 images were selected for statistical analysis of bubble characteristics.

Table A1. Experimental conditions

Case	Water flow rate(L/min)	Gas flow rate (mL/min)	Outlet pressure (Mpa)
1	6	30	0
2	7	30	0
3	8	30	0
4	9	30	0
5	8	30	0.03
6	8	30	0.04
7	8	30	0.05

## Appendix II - Simulated bubble images generation strategy

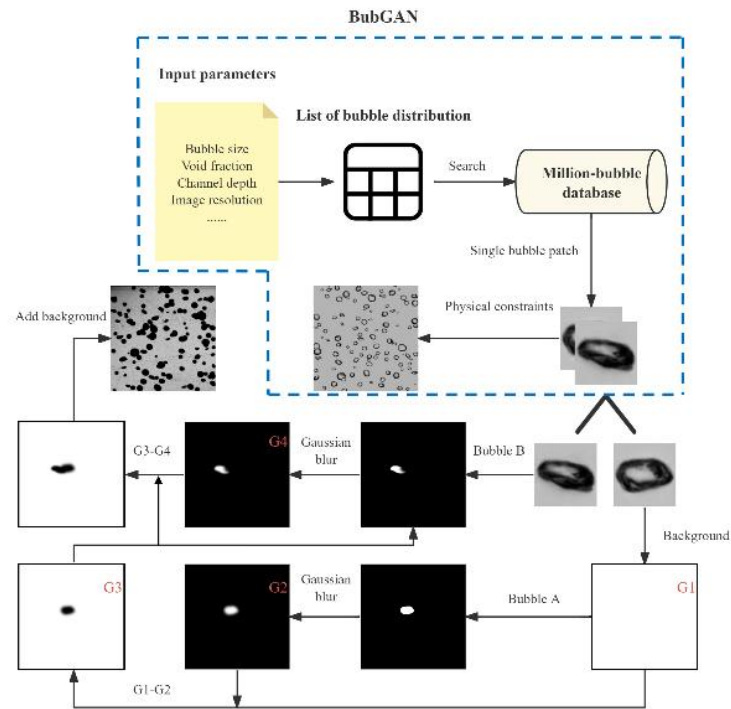


Figure A2. Flowchart for simulation images of overlapping bubbles

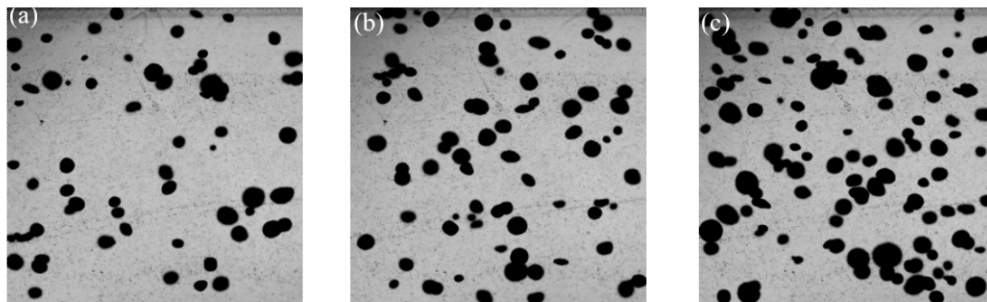


Figure A3. Simulated bubble images at different void fraction (a)  $\epsilon = 0.1$  (b)  $\epsilon = 0.15$  (c)  $\epsilon = 0.2$

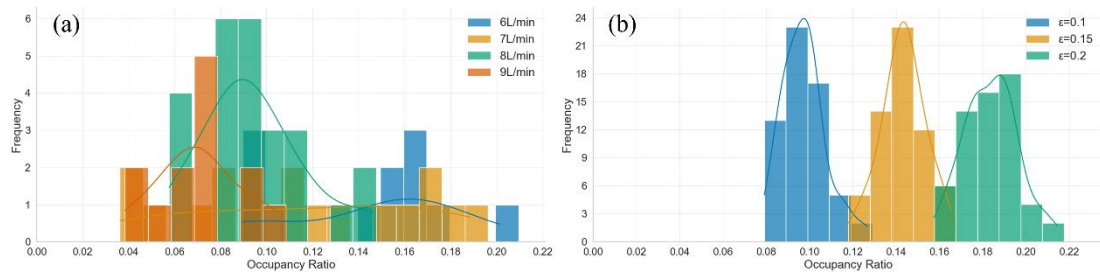


Figure A4. Distribution of bubble occupancy ratio in experimental and simulated images (a) Experimental images (b) Simulated images