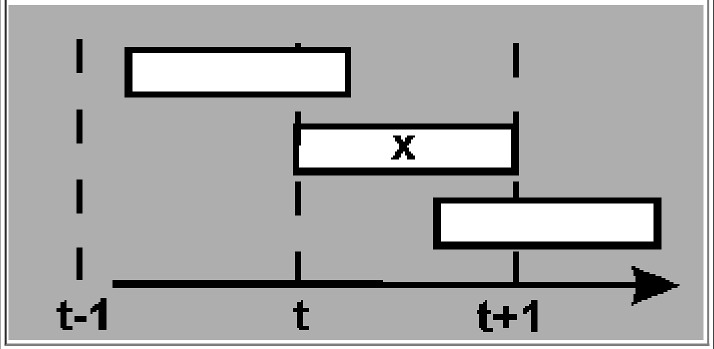
Summary: Analysis of the Carrier Sense Multiple Access (CSMA) scheme

- **Unslotted Aloha**
  - **When is a transmission *successful*:**
    - The **transmission *x*** is **successful**:



    - if and only if:
      - There are **no transaction attempts** that **begins (= arrives)** during the **time interval** **(t-1, t+1]**

  - **Therefore:**

```
     Probab[ a transmission attempt is successful ]
                 = Probab[ 0 arrivals in the period (t-1,t+1] ]

                 = Prabab[ 0 arrivals in 2 time units ]

                   (G × 2)^0 e^(-G × 2)
                 =  ----------------
                         0!
```

```
= e^-2G
```

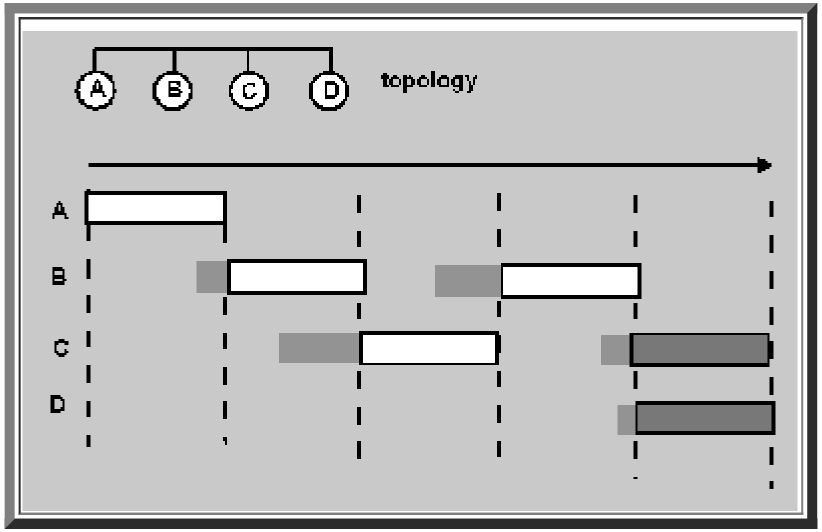
G = **offered load**

- **Throughput:**

```
Throughput = #transmission attempts × Probab[ attempt is successful ]

           = G × e^-2G
```

- **Slotted Aloha**
  - **When is a transmission *successful***:
    - The **transmission *x*** is **successful**:



    - if and only if:

- There are **no transaction attempts** that **begins (= arrives)** during the **time interval (t-1, t]**

- **Therefore:**

```
    Probab[ a transmission attempt is successful ]
                 = Probab[ 0 arrivals in the period (t-1,t] ]

                 = Prabab[ 0 arrivals in 1 time units ]

                    (G × 1)^0 e^(-G × 1)
                 =  ----------------
                          0!

                 = e^-G
```

$$\text{Probab[a transmission attempt is successful]} = \frac{(G \times 1)^0 e^{(-G \times 1)}}{0!} = e^{-G}$$

G = **offered load**

- **Throughput:**

```
Throughput = #transmission attempts × Probab[ attempt is successful ]

           = G × e^-G
```

- **Non-persistent CSMA**

  - **Non-persistent CSMA protocol:**

    - Station **listens** before **transmitting packet**
    - If station **detects *idle* channel**, it **transmits immediately**
    - **Otherwise** (station **detects *busy* channel**), the station **waits a random amount of time** and **tries again**

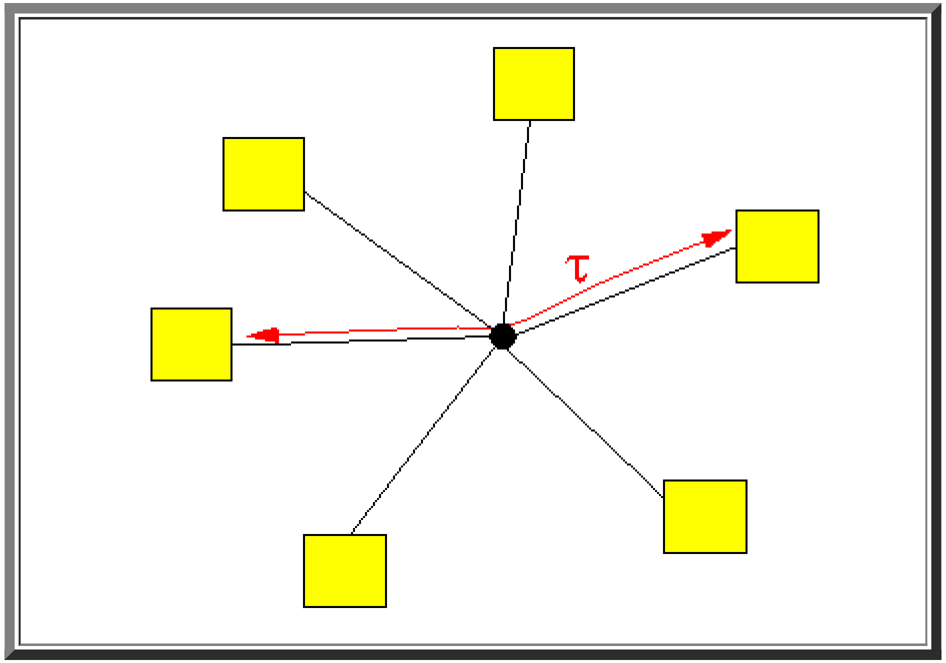
- **Simplifying assumptions:**

  - **All packets** have the **same length T**

    (So the **transmission time** for **any packet** is the **same duration**)

  - Assume that the **propagation delay between *any two nodes*** is equal to **τ seconds**

    Like this:



  - **Define:**

$$a = \frac{\tau}{T}$$

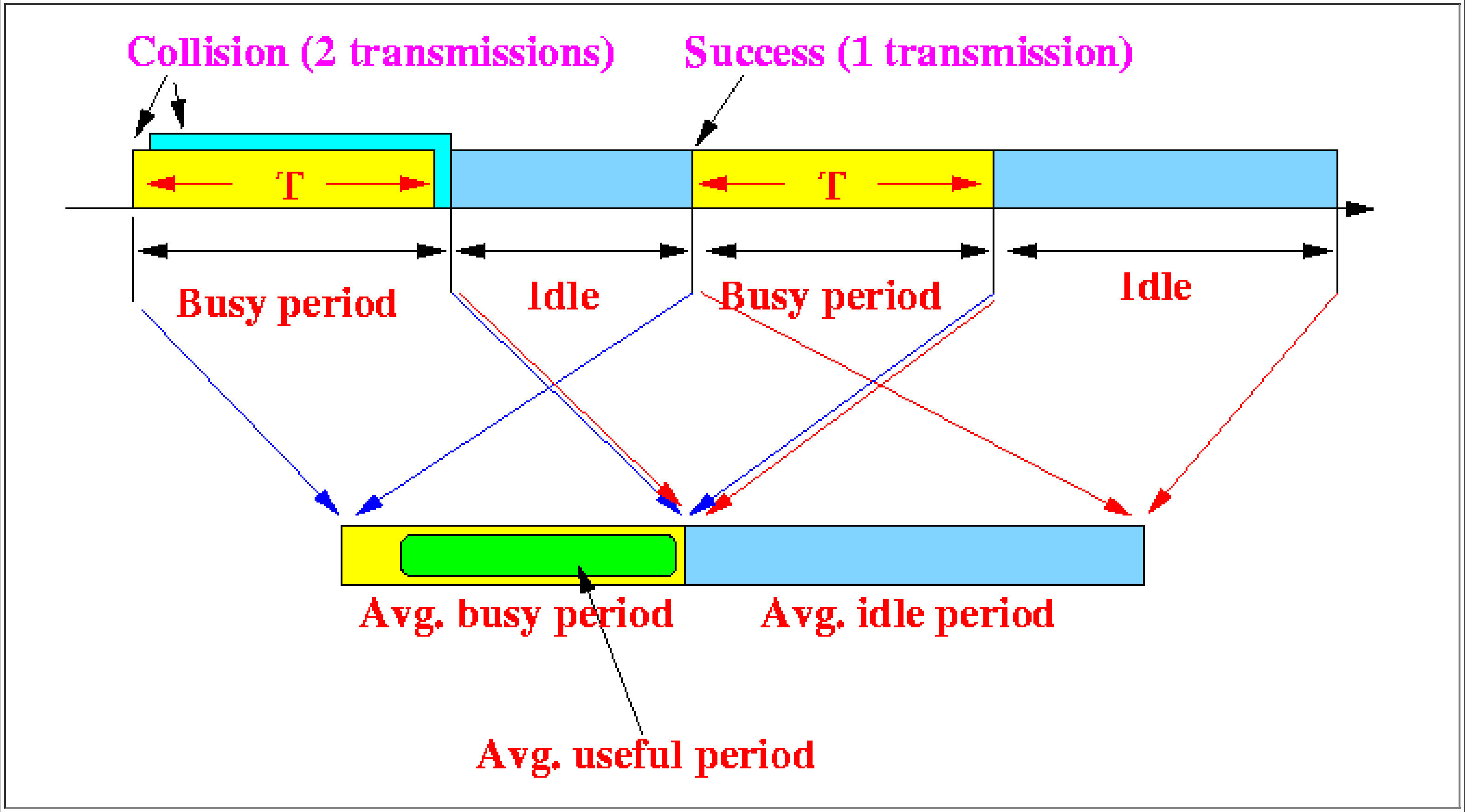
    which is the **normalized propagation delay**

- The **packet arrival process** is a **Poisson process** with rate λ

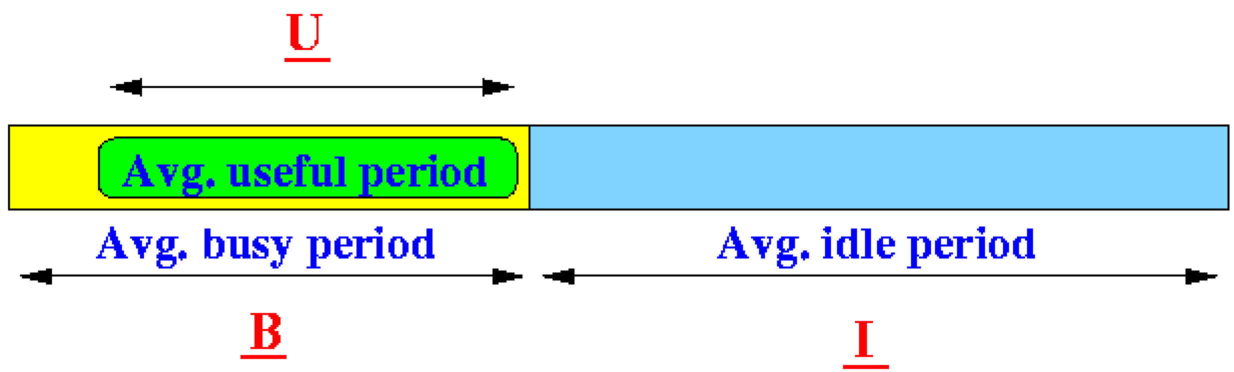
  The **offered load** (new arrivals and retransmission attempts combined) is **also** **Poisson distributed** with rate **g**

  (We will see that the **offered load g** is **dependent on** the **arrival rate λ**)

- **Throughput computation** of the **non-persistent CSMA** protocol:



- **Throughput of the *unslotted non-persistent* CSMA protocol:**



$$S_{\text{non-persistent CSMA}} = \frac{U}{B + I} \quad \ldots\ldots \ (1)$$

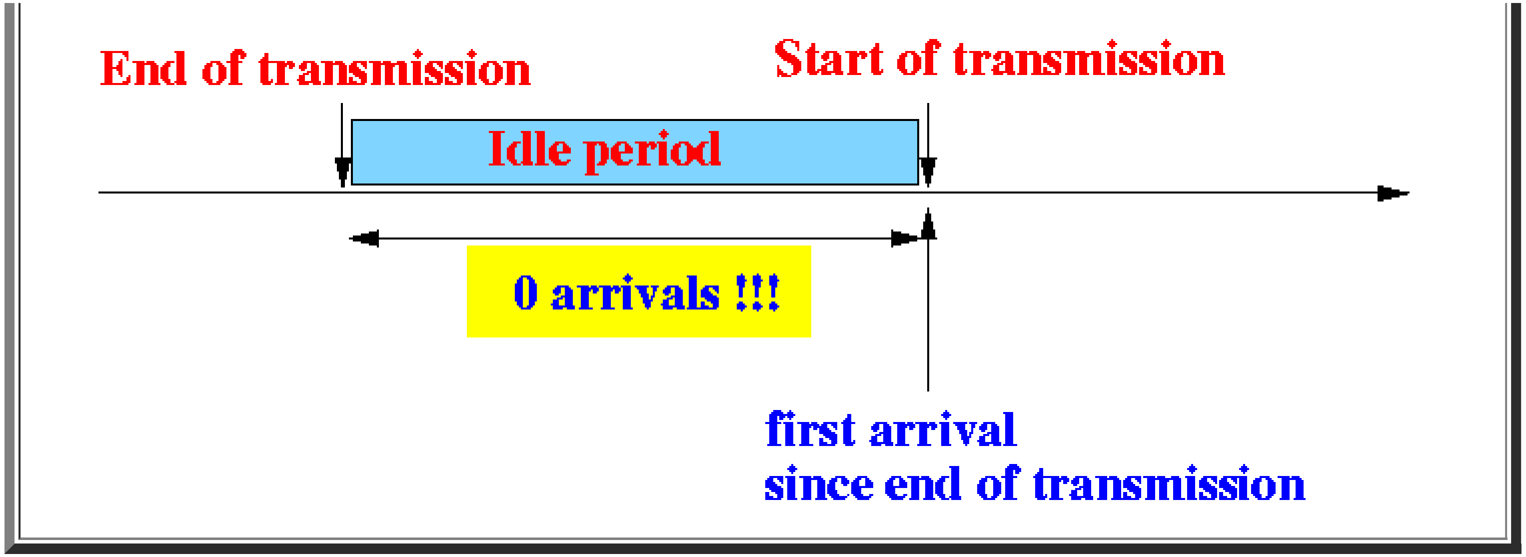
- where:
  - **B** = the **average duration** of a **busy period** in the **non-persistent CSMA cycle**

  - A **busy period** can be **useful (i.e., successful transmission)** or **not useful (i.e., collision)**

    U = the **average duration** of a **busy period** that is **useful** in the **non-persistent CSMA cycle**

  - **I** = the **average duration** of a **idle period** in the **non-persistent CSMA cycle**

- **Average length of an idle period (I)**
  - **Idle period *ends* with an *arrival*:**



- ***Cumulative*** **density function** of an **idle period I** is:

$$\Pr[\ I \le x\ ] = 1 - \Pr[\ I > x\ ]$$

$$= 1 - \Pr[\ \text{no packet arrives in } x \text{ sec}\ ]$$

$$\boxed{\Pr[\ k \text{ arrivals in } t \text{ sec}\ ] = \frac{(gt)^k}{k!}\, e^{-gt}}$$

$$\Pr[\ I \le x\ ] = 1 - \frac{(gx)^0}{0!}\, e^{-gx}$$

$$\Longleftrightarrow \quad \Pr[\ I \le x\ ] = 1 - e^{-gx}$$

- **Average duration of an idle period I:**

$$\underline{I} = \int_0^\infty x\ f_I(x)\ dx$$

$$\Longrightarrow \quad \underline{I} = \frac{1}{g}$$

- **Average length of a useful period U**

  - **Average length of the useful period:**

$$\underline{U} = T \times P[\text{ transmission successful }] \quad + \quad 0 \times (1 - P[\text{ transmission successful }])$$

  - **P[transmission successful]:**

$$P[\text{ transmission successful }] = P[\text{ 0 arrivals within "vulnerable period" }]$$

$$= P[\text{ 0 arrivals within } \tau \text{ sec }]$$

$$P[\text{ k arrivals in t sec }] = \frac{(gt)^k}{k!} e^{-gt}$$

$(g\tau)^0$

```
    P[ transmission successful ] = ----- e^(-gτ)
                                     0!

<=> P[ transmission successful ] = e^(-gτ)
```

- **Therefore:**

$$\underline{U} = T \times P[\text{ transmission successful }] + 0 \times (1 - P[\text{ transmission successful }])$$

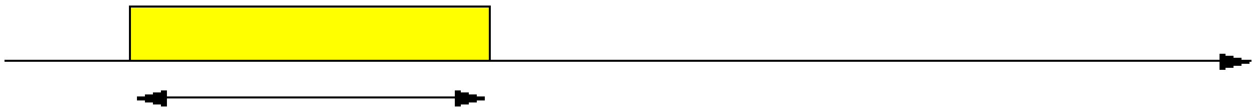
$$= T \times e^{-g\tau} + 0 \times (1 - e^{-g\tau})$$

$$\boxed{\underline{U} = T \times e^{-g\tau}}$$

- **Average length of busy period B**

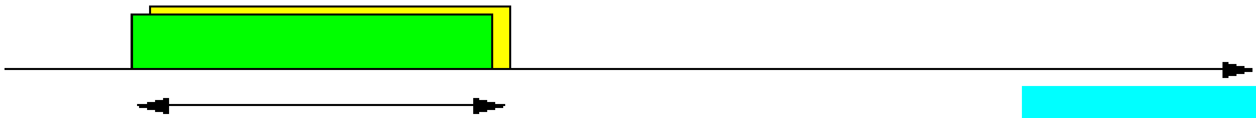
  - **Average length of a busy period B**

One arrival:



Length of busy period = T + τ sec

Multiple (nearly simultaneous) arrivals:



Length of busy period = T + y + τ sec

Note: y < τ

Notes:

```
      B = T + τ + y

 where:

      T = length of a packet transmission (constant)
      τ = (max) end-to-end delay (constant)
      y = time lag of the last transmission (random)

From Theory of probability:

      E[B] = E[ T + τ + y ]

<=>   E[B] = T + τ + E[y]

<=>   B = T + τ + y
```

- The **random variable** y is an ***hybrid* (mixed discrete/continuous) random variable**:

$$f_y(t) = e^{-g\tau} \times \&delta(t) + g\ e^{-g(\tau-t)} \qquad (\text{with } t \in [0, \tau))$$

- The **expected value** E[y]:

$$E[y] = {}_{-\infty}\int^{\infty} t \times f_y(t)\ dt$$

$$= {}_{0}\int^{\tau} t \times f_y(t)\ dt$$

- Split off the discontinuous point t = 0

$$\Rightarrow \quad E[y] = 0 \times P[\ y = 0\ ] + {}_{0+}\int^{\tau} t \times f_y(t)\ dt$$

$$= 0 \times e^{-g\tau} \quad + \quad {}_{0+}\int^{\tau} t \times g\ e^{-g(\tau-t)}\ dt$$

$$= \tau - \frac{1 - e^{-g\tau}}{g}$$

- **Therefore:**

$$\underline{B} = T + \tau + \left(\tau - \frac{1 - e^{-g\tau}}{g} \right)$$

$$= T + 2\tau - \frac{1 - e^{-g\tau}}{g}$$

- **Summary:**

$$\underline{I} = \frac{1}{g}$$

$$\underline{U} = T \times e^{-g\tau}$$

$$\underline{B} = T + 2\tau - \frac{1 - e^{-g\tau}}{g}$$

- **Therefore:**

$$S = \frac{\underline{U}}{\underline{B} + \underline{I}}$$

$$\Longrightarrow \quad S = \frac{gT \times e^{-agT}}{gT(1 + 2a) + e^{-agT}}$$

- **Performance graphs:**

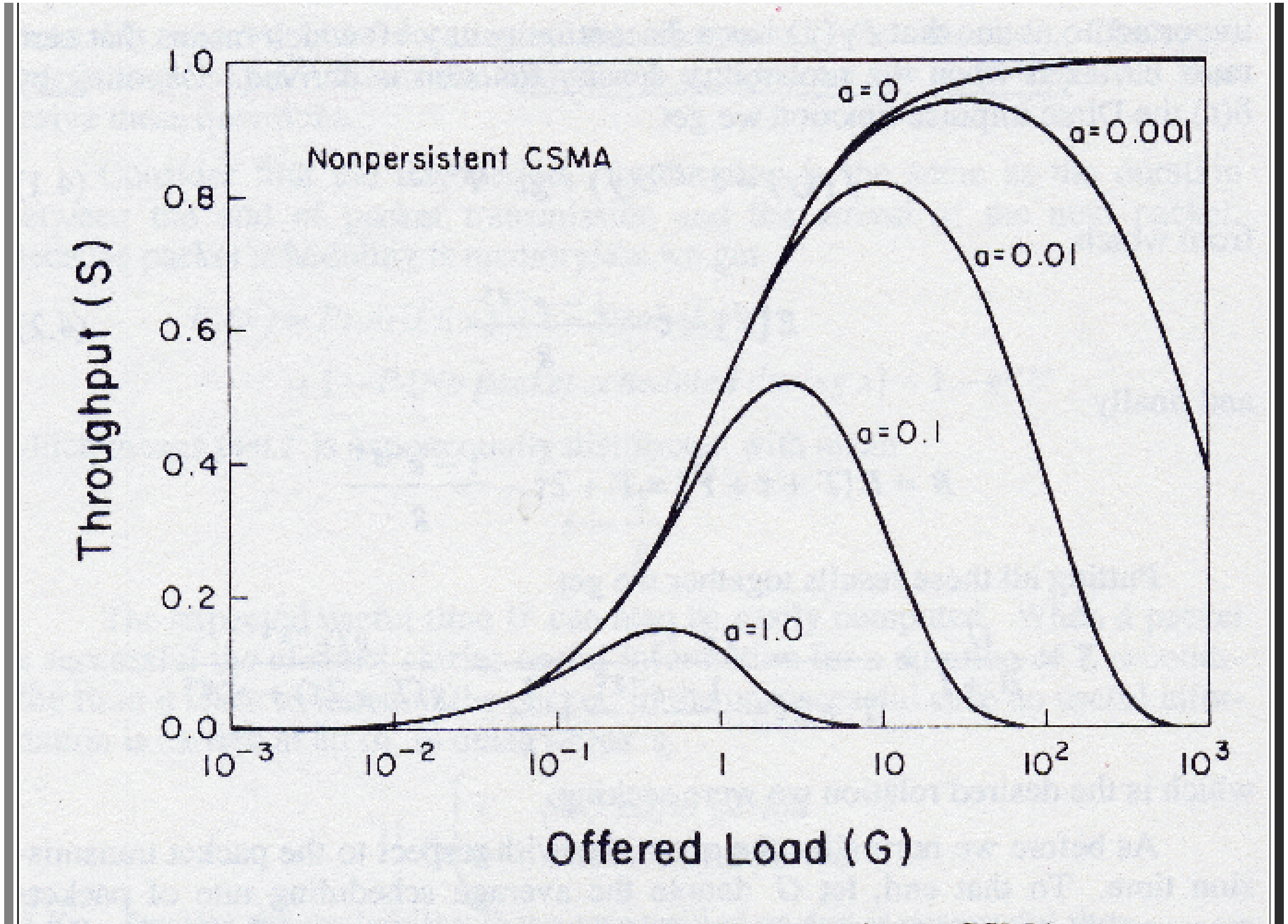


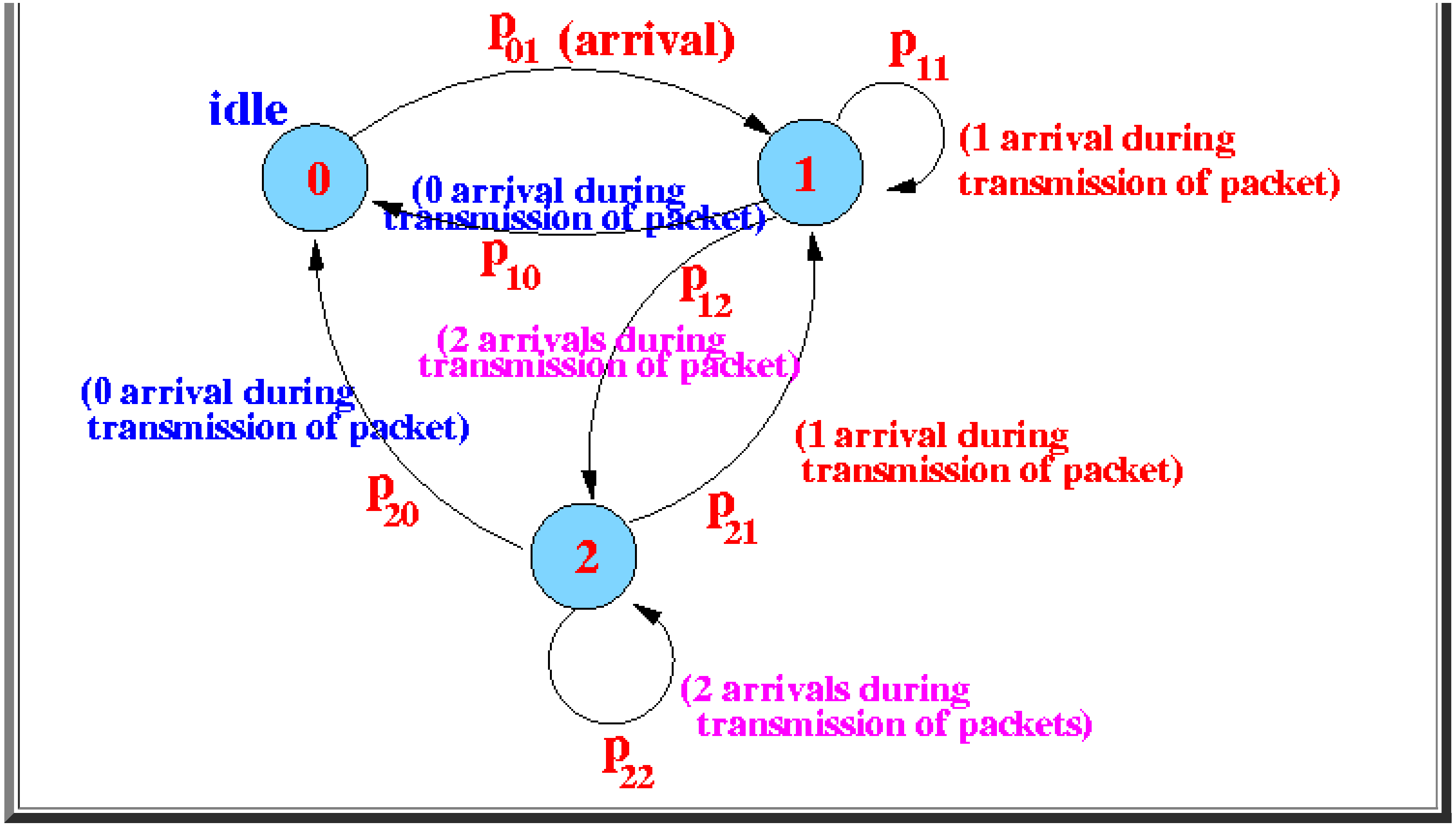
Figure 4.2: Throughput-Load of Nonpersistent CSMA

- **1-persistent CSMA**
  - **1-persistent CSMA protocol:**
    - Station **listens** before **transmitting packet**

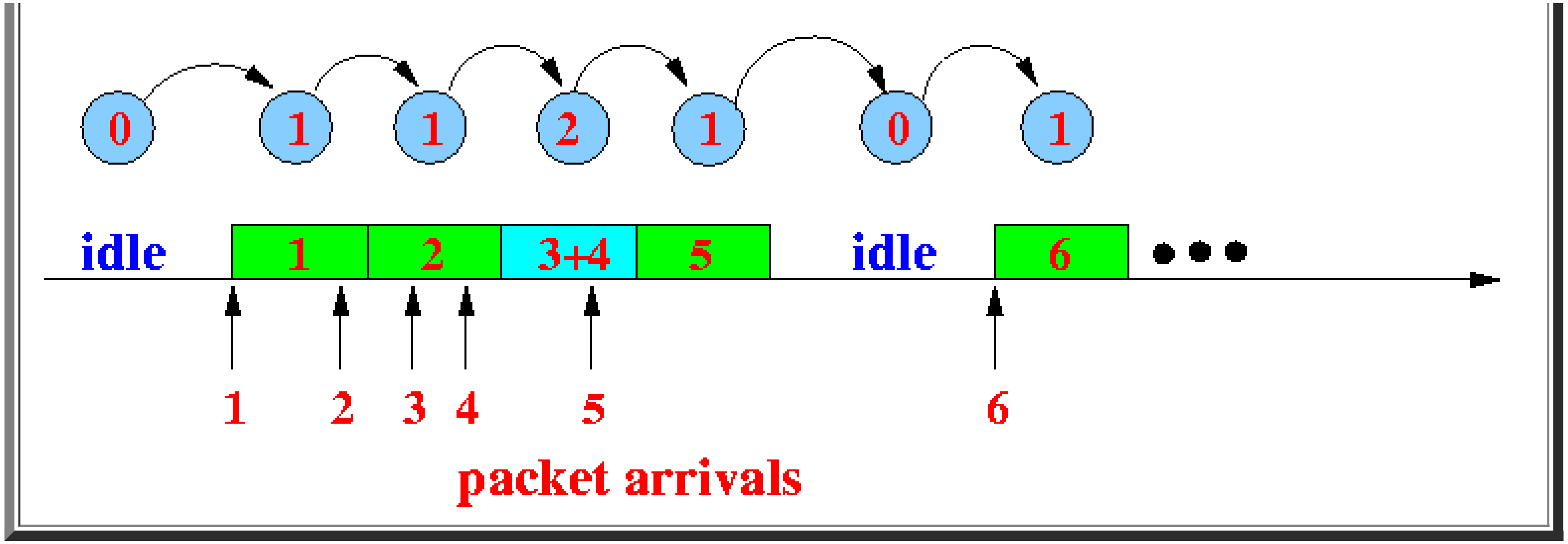
- If station **detects *idle* channel**, it **transmits immediately**
- **Otherwise** (station **detects *busy* channel**), the station **waits until transmission ends** and **transmits packet**

- **Definitions of the different states:**
  - **State 0** = the **number of packet arrivals at the *start*** of the **transmission period** is equal to **0**
  - **State 1** = the **number of arrivals at the *start*** of the **transmission period** is equal to **1**
  - **State 2** = the **number of arrivals at the *start*** of the **transmission period** is equal to **2**

- The **state transition diagram** of the **1-persistent (unslotted) CSMA** protocol:



- **Example state transitions:**



- **Preliminary Throughput expression of the unslotted 1-persistent CSMA protocol:**

$$S = \frac{\pi_0 T \times ℙ[\text{ success in } T_0 \text{ }] + \pi_1 T \times ℙ[\text{ success in } T_1 \text{ }] + \pi_2 T \times ℙ[\text{ success in } T_2 \text{ }]}{\pi_0 \underline{T}_0 + \pi_1 \underline{T}_1 + \pi_2 \underline{T}_2}$$

$$\Longleftrightarrow \quad S = \frac{\pi_0 T \times 0 + \pi_1 T \times ℙ[\text{ 0 arrival in } \tau \text{ sec }] + \pi_2 T \times 0}{\pi_0 \underline{T}_0 + \pi_1 \underline{T}_1 + \pi_2 \underline{T}_2}$$

$$\Longleftrightarrow \quad S = \frac{\pi_1 T \times e^{-g\tau}}{\pi_0 \underline{T}_0 + \pi_1 \underline{T}_1 + \pi_2 \underline{T}_2} \qquad \text{(T = packet length)}$$

- **T_0 = idle period:**

$$T_0 = \frac{1}{g} \qquad \text{(Idle period)} \quad \text{(See: click here)}$$

- The **average duration** of a **type 1 period and type 2 period** is:

$$E[T_1] = T + \tau + E[y]$$

$$E[T_2] = E[T_1]$$

Result:

$$T_1 = T + 2\tau - \frac{1 - e^{-g\tau}}{g}$$

$$T_2 = T + 2\tau - \frac{1 - e^{-g\tau}}{g}$$

- **Solving &pi$_0$, &pi$_1$ and &pi$_2$:**

$$\pi_0 = \frac{p_{10}}{1 + p_{10}}$$

$$\pi_1 = \frac{p_{10} + p_{11}}{1 + p_{10}}$$

$$\pi_2 = \frac{1 - p_{10} - p_{11}}{1 + p_{10}}$$

where:

$$p_{10} = P[\ 0 \text{ arrivals in } (T + y) \text{ sec }]$$

$$= (1 + g\tau)\, e^{-g(T+\tau)}$$

$$p_{11} = P[\text{ 1 arrivals in } (T + y) \text{ sec }]$$

$$= g\, e^{-g(T+\tau)}\, (T + g\tau(T + \tau/2))$$

- **Throughput of 1-persistent CSMA:**

$$S = \frac{gT\, e^{-g(T+2\tau)}\, [\, 1 + gT + g\tau(1 + gT + g\tau/2)\,]}{g(T + 2\tau) - (1 - e^{-g\tau}) + (1 + g\tau)e^{-g(T+\tau)}}$$