To find inverse of A:

Exists when determinant of $A \neq 0$

2 dimension $\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

3 dimension $\Rightarrow \text{rref}\left[A \mid I\right] = \left[I \mid A^{-1}\right]$

Subspace of $\mathbb{R}^n$: ① $A \subset \mathbb{R}^n$, ② $\vec{0} \in A$,

③ $\vec{x}, \vec{y} \in A \to \vec{x}+\vec{y} \in A$, ④ $\vec{x} \in A, \lambda \in \mathbb{R} \to \lambda\vec{x} \in A$

Solution space of homogeneous equation $\Leftrightarrow$ Subspace $\Leftrightarrow$ span of vectors

To find intersection of 2 subspaces:

Let $\vec{x_0} \in U \cap V$, $U = \text{span}\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}\right\}$, $V = \text{span}\left\{\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}-1\\1\\1\end{bmatrix}\right\}$,

$\vec{x_0} \in U \to \vec{x_0} = \alpha_1\begin{bmatrix}1\\0\\0\end{bmatrix} + \alpha_2\begin{bmatrix}0\\1\\0\end{bmatrix}$

$\vec{x_0} \in V \to \vec{x_0} = \beta_1\begin{bmatrix}1\\1\\1\end{bmatrix} + \beta_2\begin{bmatrix}-1\\1\\1\end{bmatrix}$

$$\alpha_1\begin{bmatrix}1\\0\\0\end{bmatrix} + \alpha_2\begin{bmatrix}0\\1\\0\end{bmatrix} - \beta_1\begin{bmatrix}1\\1\\1\end{bmatrix} - \beta_2\begin{bmatrix}-1\\1\\1\end{bmatrix} = \vec{0}$$

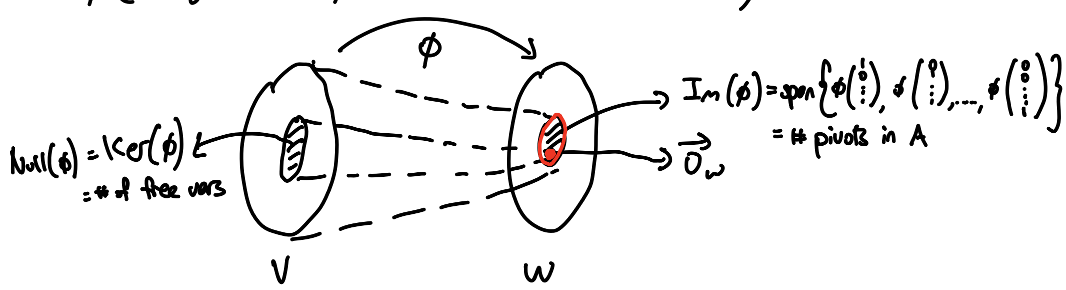
$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

Then sub in α_1 and α_2 in $\vec{x_0} \in U$.

Similarly for affine subspaces

Linear Map: $\phi(x_1 + x_2) = \phi(x_1) + \phi(x_2)$ (incl integration, differentiation, matrices)

$\phi(\lambda x) = \lambda\phi(x)$



$$\text{Rank}(A_{m\times n}) = \begin{array}{l}\text{\# non-zero rows in REF, or} \\ \text{\# linearly independent column vectors}\end{array} = \text{dimension of } \text{Im}(A_{m\times n})$$

$$\dim(\text{Im}(\phi)) + \dim(\ker(\phi)) = \text{\# of columns of } A$$

$\Phi_{TS} \Rightarrow$ transformation matrix from S to $T \Longrightarrow [w]_T = \Phi_{TS}[v]_S$

To find Φ_{TS} : $\text{rref}(T \mid S) = (I \mid \Phi_{TS})$ $\quad\Rightarrow I_{EV} = [v_1 \ldots v_n]$

$\Phi_{TS} = (\Phi_{ST})^{-1}$

Determinant : Cofactor expansion / Triangular Matrix

GE method: Multiply row by constant $\Rightarrow$ Det multiplied by same constant

Add two rows $\Rightarrow$ Det don't change

Swap two rows $\Rightarrow$ Det multiply negative 1

Eigenvectors: $\quad A\vec{x} = \lambda I\vec{x} \Rightarrow (A - \lambda I)\vec{x} = 0$

For eigenvectors to exists, $\det(A - \lambda I) = 0$, where $\det(A - \lambda I)$ is the char-poly of A

Sub back eigenvalue into $(A - \lambda I)$ to find eigenvectors.

Any point moves towards eigenvector with largest eigenvalue (PCA, Pagerank)

Diagonalization: $\quad I_{EV} D I_{VE} = A \Rightarrow D = I_{VE} A I_{EV}$

$\Rightarrow P^{-1}AP = D$ where columns of P are eigenvectors and D are eigenvalues

Cayley Hamilton Theorem : Let $\text{char-poly}(A) = a_0 + a_1\lambda + a_2\lambda^2 + \ldots + a_n\lambda^n$

then, $a_0 + a_1M + a_2M^2 + \ldots + a_nM^n = \vec{0}$

$$M(a_1I + a_2M + \ldots + a_nM^{n-1}) = a_0 I$$

$$M^{-1} = \frac{a_1I + a_2M + \ldots + a_nM^{n-1}}{a_0}$$

Dot Product: $\vec{a}\cdot\vec{b} = \sum_{i=1}^{n} a_i \times b_i = (\vec{a})^T \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Projection Matrix of $\vec{a}$ onto $\overset{\text{(span of)}}{B}$: $B(B^TB)^{-1}B$ B must be a basis (linearly independent)

$$B^T(\vec{a} - B\vec{x}) = \vec{0} \quad (B\vec{x} = \pi_B \vec{a})$$

$$B^T\vec{a} = B^TB\vec{x}$$

$$(B^TB)^{-1}B^T\vec{a} = \vec{x}$$

$$B(B^TB)^{-1}B^T\vec{a} = B\vec{x}$$

Rotation of Vectors:  $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$$M_{(\theta_1+\theta_2)} = M_{\theta_2}M_{\theta_1} = M_{\theta_1}M_{\theta_2}$$

$$\begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix}$$

$$\underset{y-z}{M_x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \qquad \underset{x-z}{M_y} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$z-x \quad \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Cauchy-Schwarz inequality: $\vec{a}\cdot\vec{b} \leq |\vec{a}||\vec{b}|$