

Time Complexity	Searching																												
$1 < \log n < \sqrt{n} < n < n \log n = \log n! < n^2 < 2^n$ <table> <tr> <th>Recurrence</th><th>Runtime</th></tr> <tr> <td>$T(n) = 2T(n-1) + O(1)$</td><td>2^n</td></tr> <tr> <td>$T(n) = 2T(n/2) + O(n)$</td><td>$n \log n$</td></tr> <tr> <td>$T(n) = T(n/2) + O(n)$</td><td>n</td></tr> <tr> <td>$T(n) = 2T(n/2) + O(1)$</td><td>n</td></tr> <tr> <td>$T(n) = S(0, \sqrt{n})T(i) + \sqrt{n}$</td><td>$n$</td></tr> <tr> <td>$T(n) = 2T(n/4) + O(1)$</td><td>$\sqrt{n}$</td></tr> <tr> <td>$T(n) = T(n/2) + O(1)$</td><td>$\log n$</td></tr> </table>	Recurrence	Runtime	$T(n) = 2T(n-1) + O(1)$	2^n	$T(n) = 2T(n/2) + O(n)$	$n \log n$	$T(n) = T(n/2) + O(n)$	n	$T(n) = 2T(n/2) + O(1)$	n	$T(n) = S(0, \sqrt{n})T(i) + \sqrt{n}$	n	$T(n) = 2T(n/4) + O(1)$	\sqrt{n}	$T(n) = T(n/2) + O(1)$	$\log n$	<table> <tr> <th>Search</th><th>Runtime</th></tr> <tr> <td>Linear</td><td>n</td></tr> <tr> <td>Binary</td><td>$\log n$</td></tr> <tr> <td>Quickselect</td><td>n</td></tr> <tr> <td>Knuth Shuffle</td><td>n</td></tr> <tr> <td>Merkel Tree</td><td>$\log n$</td></tr> </table> <p>Knuth Shuffle: for (1,n-1): swap(i,rand(0,i))</p>	Search	Runtime	Linear	n	Binary	$\log n$	Quickselect	n	Knuth Shuffle	n	Merkel Tree	$\log n$
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Sorting

Algorithm	Best	Average	Worst	Stable	Invariant
Bubble	n	n^2	n^2	Yes	Largest k items are in final k positions
Selection	n^2	n^2	n^2	No	Smallest k items are in smallest k positions
Insertion	n	n^2	n^2	Yes	First k items are sorted
Merge	$n \log n$	$n \log n$	$n \log n$	Yes	Groups of 2^x are sorted
Quick	$n \log n$ $n \log k$	$n \log n$ $n \log k$	n^2 nk	No	Array is partitioned around pivot $T(n) = kT(n/k) + O(n \log k) \rightarrow n \log n$
Reversal		$n(\log n)^2$			Quicksort with Mergesort around pivot

Trees

Structure	Search	Insert	Delete	Remark
Binary Search	h	h	h	Delete if x has 2 child: replace x with successor(x) Successor right.min() or recurse to (left of parent or root)

AVL	$\log n$	$\log n + 2R$	$\log n + \log n R$	$h < 2\log n$ or $n > 2^{(h/2)}$ v.left Left Heavy or Balanced: $\text{right}(v)$ v.left Right Heavy: $\text{left}(\text{v.left}), \text{right}(v)$
Trie	L	L	L	More space due to more overhead
(a,b)	$\log n$	$\log n$	$\log n$	split for insert, merge+share for delete
kd	h	h	h	Alternate splitting horizontally and vertically

Augmented Structure	Remark
Dynamic Order Statistic	Stores weight of subtree. During functions, $\text{rank} = \text{left.weight} + 1$ Select $\text{left.weight} < \text{rank}$: $\text{left.select}(k)$. Else: $\text{right.select}(k - \text{rank})$ Rank recurse to root, if node is right child: $\text{rank} += \text{parent.left.weight} + 1$
Interval Tree	Sort by left endpoint. Stores max endpoint in node's subtree Search ($\log n$) If $x > \text{max}$ or left is null, $\text{search}(\text{right})$. Else: $\text{search}(\text{left})$ All Overlap ($k \log n$) search node, add to list, delete node, repeat until null
Orthogonal Range Search	Store all points as leaves of a BST. Internal nodes stores max of left. Range Query ($k + \log n$) find split node. do left & right traversals. 2D Range Query ($k + (\log n)^2$) for node in x-tree, build y-tree using nodes in subtree.

Hashing

Must redefine **hashCode** default returns address and **equals** for **get** to work

Collision	Insert	Search	Space	Remarks
Linked List	$h+1 = 1$	$h+n/m = 1$	$m+n$	Simple Uniform Hashing Assumption Worst case for search = n
Open Addressing	$1/(1-a)$,	where $a = n/m$ $a < 1$	n	Uniform Hashing Assumption Linear Probing - Clusters Double Hashing - $h(k,i) = f(k) + ig(k) \bmod m$, ($m, g(k)$) Delete sets node to special value for search