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Time Complexity

logn=log(n^2) < n^1/2 < n < nlogn=logn! < n^2 < 2^n < 2^2n

Recurrence	Runtime
T(n) = S(0,n-1)T(i) = 2T(n-1)	2^n
T(n) = 2T(n/2) + O(n)	nlogn
T(n) = T(n/2) + O(n)	n
T(n) = 2T(n/2) + O(1)	n
T(n) = S(0,sqrt(n))T(i) + sqrt(n)	n
T(n) = 2T(n/4) + O(1)	sqrt(n)
T(n) = T(n/2) + O(1)	logn

Searching

Search	Runtime
Linear	n
Binary	logn
Quickselect	n

Knuth Shuffle	n
Merkel Tree	logn

Knuth Shuffle:
for (1,n-1): swap(i,rand(0,i))

Sorting

Algorithm	Best	Average	Worst	Stable	Space	Remarks
Bubble	n	n^2	n^2	Yes	1	**Largest k items are in final k positions**
Selection	n^2	n^2	n^2	No	1	**Smallest k items are in smallest k positions**
Insertion	n	n^2	n^2	Yes	1	**First k items are sorted**
Merge	nlogn	nlogn	nlogn	Yes	n	**Groups of 2^x are sorted**
Quick	nlogk	Median/ Random/ Check nlogk	Specific nk	No	logn	**Array is partitioned around pivot** T(n) = pT(n/p) + O(nlogp) = nlogn Duplicate: n^2 if no 3-way partition
Heap	nlogn	nlogn	nlogn	No	1	Build Heap: n, Get Sort: nlogn
Reversal		n(logn)^2				Quicksort with Mergesort around pivot

Data Trees

Structure	Operation	Remark
Binary Search	h	Full Tree: n = 2^(h+1) - 1 **Delete** if x has 2 child: replace x with successor(x) **Successor** right.min() or recurse to (left of parent or root)
Scapegoat	logn	Rebuild subtree rooted at scapegoat when triggered
AVL	logn	h < 1.44logn or n > 2^(h/1.44) v.left Left Heavy or Balanced: right(v) v.left Right Heavy: left(v.left), right(v) **Insert** 2 x R **Delete** 2logn x R
Trie	L	More space due to more overhead
(a,b)	logn	**Split** for insert, **Merge+Share** for delete
kd	h	Alternate splitting horizontally and vertically
Heap	logn	1. Heap Ordering: Pr(parent) >= Pr(child) 2. Complete Binary Tree **Delete** Swap(last), Bubble Down Array: Left(x)=2x+1, Right(x)=2x+2
Leftist Heap	logn	max rightRank = logn, max height = n **Merge** (logn) merge smaller root with right child of other swap if left>right, update rightRank **Insert** merge with single vertex, swap <= 1 **GetMax** remove root, merge child **Delete** update rightRank (stop if left child), swap <= 1

Augmented Structure	Remark
Dynamic Order Statistic	Stores weight of subtree. During functions, rank = left.weight + 1 **Select** left.weight < rank: left.select(k). Else: right.select(k-rank) **Rank** recurse to root, if node is right child: rank += parent.left.weight + 1
Interval Tree	Sort by left endpoint. Stores max endpoint in node's subtree **Search** (logn) If x > max or left is null, search(right). Else: search(left) **All Overlap** (klogn) search node, add to list, delete node, repeat until null
Orthogonal Range Search	Store all points as leaves of a BST. Internal nodes stores max of left. **Range Query** (k+logn) find split node. do left & right traversals. **2D Range Query** (k+(logn)^2) for node in x-tree, build y-tree using nodes in subtree. **Build2D** T(nlogn) S(nlogn)

Hashing

Must redefine **hashCode** *default returns address* and **equals** *for* ***get*** *to work*

Hash Table	Insert	Search	Space	Remarks
Linked List	h+1 = 1	h+n/m = 1	m+n	Simple Uniform Hashing Assumption: Equally mapped to every bucket Worst case for search = n
Linked List w/ Resize	amor(1)	exp(1)	m+n	Ideal: n==m -> Double Table . n<m/4 -> Half Table Increment O(n^2) . Double O(n) . Square O(n^2)
Open Addressing	h + 1/(1-a),	where a = n/m a< 1	n	Uniform Hashing Assumption: Equally mapped to every permutation Linear Probing - Clusters Double Hashing - h(k,i) = f(k) + ig(k) mod *m*, (m,g(k)) -> n^2 permutations !UHA **Delete** sets node to tombstone value for **search**
Fingerprint Bloom	k	k		False positive. No false negative. **Delete**(Counter/Tombstone) -> False negative *Let p = P(False positive)* FHT: p = 1-e^(-n/m) . n/m <= ln(1/1-p) BT: p = (1-e^(-kn/m)^k) . opt(k) = (m*ln2)/n

Graphs & Trees

Adjacency List O(V+E) Adjacency Matrix O(V^2) Edge List O(E)

SSSP	Runtime	Remarks
Bellman-Ford	VE	No negative cycles
Dijkstra	ElogV(AVL) E+VlogV(Fibo Heap)	No negative edge Vx(insert+deleteMin) + Ex(decreseKey)
Toposort Relax	V+E E	No directed cycles (Directed Acyclic Graph) Post-order DFS or Kahn's(get nodes w/o in-edges)
BFS	V+E(Queue)	Same Weight
DFS	V+E(Stack)	No cycle (Tree)
LCA	(logV)^2	Store depth and skip pointers to ancestors 2^n up Binary search isAncestor(*getAncestor(u,hops)*,v)
MED	nm	Toposort + Relax

MST	Runtime	Idea
Prim's	Dijkstra's E(known)	Add min edge on cut
Kruskal's	ElogV aE(known)	Add min edge not in same tree Sort + ExUF = ElogE + Ea(n)
Boruvska's	ElogV	Every step: Add min edge for every node Search min out-edge = V+E using B/DFS Update component ID = V

MST Property
1.Max edge in **cycles** NOT in MST
2.Min edge in **cut** IS in MST

Steiner Tree (<2xOPT)
1.Run APSP & build new graph
2.Run MST and remove duplicate edges

Rooted Directed Graph:
Add min incoming edge O(E)

Faster MST (EloglogV):
1.loglogV Boruvska's
2.Prim's using Fibo Heap

Union-Find	Find	Union
Quick Find	1	n
Quick Union	n	n

Union-Find	Find	Union
Weighted Union	logn	logn
Weighted Union + Path Compression	a(m,n)	a(m,n)

Dynamic Programming

DP	Runtime	Subproblem
LIS	n^2	**S[i]** = max(**S[j]**)+1 for all j>1 and S[j]>S[i]. Base: S[n]=0
Prize	kE	**P[v,k]** = max(**P[w,k-1]**)+W(w,v) for all w points to v. Base: P[v,0]=0
Vertex Cover	V	**S[v,0]** = sum **S[w,1]** and **S[v,1]** = 1 + sum min(**S[w,0]**, **S[w,1]**) for all w neighbour of v. Base: S[leaf,0]=0 S[leaf,1]=1
Road Trip	nL^2	**Cost[n,f]** = min(**Cost[n+1,f-d+i]**+c*i), start n with f fuel, d<=f<=L
APSP	V^3	S[v,w,n] = min(**S[v,w,n-1]**, **S[v,n,n-1]**+**S[n,w,n-1]**) Base: S[v,w,0]=W(v,w) P[v,w,n] = P[v,w,n-1] OR P[v,n,n-1] AND P[n,w,n-1]

APSP	Runtime	Method
Sparse +ve	(V^2)logV	Dijkstra's
Unweighted	VE	BFS
All	V^3	Floyd-Warshall

Solution for MiniMaxWidth (MST using Prim + Binary Heap)	Time Complexity		Space Complexity	
	Preprocess	Query	Preprocess	Query
Binary search on query-time trimmed graph	0	$O((V+E)\log k)$	0	$O(V)$
Binary search on preprocessed trimmed graphs	$O(k(V+E))$	$O(\log k)$	$O(kV)$	$O(1)$
Modified relax SSSP with Dijkstra's	0	$O((V+E)\log V)$	0	$O(V)$
Modified relax APSP with Dijkstra's	$O(V(V+E)\log V)$	$O(1)$	$O(V^2)$	$O(1)$
Modified relax APSP with Floyd-Warshall's	$O(V^3)$	$O(1)$	$O(V^2)$	$O(1)$
Query-time MST	0	$O(E\log V)$	0	$O(V)$
Preprocessed MST	$O(E\log V)$	$O(V)$	$O(V)$	$O(V)$
Preprocessed MST + Binary search + LCA	$O(E\log V+V^2)$	$O(\log^2 V)$	$O(V\log V)$	$O(1)$